A Game-theoretic Cooperation Stimulus Routing Protocol in MANETs

Azadeh Omrani and Mehran S. Fallah

Abstract— **The existence of selfish nodes, who do not cooperate in routing and forwarding, menaces the applicability of mobile ad hoc networks (MANETs). In this paper, we propose a novel price-based method for stimulating cooperation among the nodes of a MANET. This method utilizes the game-theoretic notion of core to distribute the earnings of a cooperation coalition among its members in an optimal manner. In this way, a stable cost-efficient route is established between communicating parties.**

Index Terms— **Cooperation stimulation, cost-efficient routing, game theory, mobile ad hoc networks**

I. Introduction

The use of mobile devices has been proliferated in recent years. The advent of mobile ad hoc networks (MANETs) is a direct result of such proliferation in which a set of mobile nodes are connected without any pre-established infrastructure. In such networks, there is no specific-purpose router, and any node may undertake the responsibilities of a router. Therefore, cooperation among the nodes of a MANET is vital for network functionality. Such cooperation is an underlying assumption of a large number of routing protocols proposed so far. This assumption is only valid for the applications in which all the nodes belong to a single authority, e.g., military or emergency applications where the nodes share a common mission.

If the network nodes belong to different authorities, a node may not cooperate in network functions. This is because an intermediate node has to consume his limited resources, such as the battery power and bandwidth, for making a connection between two other nodes. Instead, the node prefers to save those resources for his own needs and consume them in the situations where he is beneficiary, like the ones in which he is either the source or the destination of the connection. Such a misbehaving node is said to be a *selfish node*. The existence of selfish nodes causes a MANET to be broken into pieces so that the network cannot provide services, such as route establishment and packet forwarding, for legitimate users. In this sense, the behavior of selfish nodes causes a passive denial-of-service in MANETs.

It has been shown that the presence of selfish nodes decreases the throughput of an ad hoc network [1]. It has also been proven that the probability of cooperation among nodes is very low where there is no specific mechanism for cooperation stimulation [2]. Hence, enforcing cooperation among nodes requires some encouraging mechanism in which cooperation is more beneficial than egocentric behavior.

There are two principal classes of encouraging mechanisms, reputation-based mechanisms and price-based mechanisms. In a reputation-based mechanism, the neighbors of a node continually log the actions taken by that node [2], [3], [4]. In this way, every node would have a record of his neighbors' reputations. The more cooperative a node is, the better reputation he gains and consequently, he will be better served. Notorious nodes, on the other hand, will be isolated from the network. Although the reputation-based mechanisms can be easily implemented, they suffer from the false detection of misbehaving nodes, that is very likely to happen in mobile environments [5].

In price-based mechanisms, what a cooperative node loses due to his cooperative behavior is compensated by some kind of virtual money [6], [7], [8], [9]. In [6] and [7], if a node relays a packet, he receives one unit of credit. Generally, the correctness of credit exchange is preserved by putting some kind of a tamper-proof hardware in any node, or deploying a specific-purpose secure exchange protocol. In the latter, as applied in [8], a Credit Clearance Service (CCS) makes payments in forwarding stage, when intermediate nodes have actually relayed the packets.

The price of relaying a packet may be different for each node. Therefore, an effective price-based mechanism should be supplemented by a technique which determines these prices accurately. In [9], a node calculates the price of a service according to the cost he incurs in providing that service. Then, he announces this price. In this approach, it is implicitly assumed that the nodes are truthful and announce their prices honestly. The source node calculates the flow rate on the basis of these prices as well as another factor of his called willingness-to-pay.

In ad hoc VCG, a routing protocol proposed in [10], the destination sets prices in such a way that dishonesty has no profit for the nodes, and therefore, they declare their costs truthfully. In doing so, it employs the idea stated in VCG (Vickrey-Clarke-Groves) that is an auction mechanism known

This work was supported in part by the Iran Telecommunication Research Center.

A. Omrani is an M.S. student in the department of IT and computer engineering, Amirkabir University of Technology, Tehran, Iran. Email: omrani@ce.aut.ac.ir.

M. S. Fallah is with the department of IT and Computer Engineering, Amirkabir University of Technology, Tehran, Iran. Email: msfallah@aut.ac.ir

(Advance online publication: 19 February 2008)

well for its efficient outcomes and incentives it provides for bidders to reveal their true valuation of the presented service. Nevertheless, the collusion problem in which selfish nodes cooperate with each other to gain better payoffs is not considered in ad hoc VCG protocol. The presence of such collusions threatens the stability of an established route.

In this paper, we propose a price-based routing mechanism, ASTR (Ad hoc STable Routing), which is very similar to ad hoc VCG but without the shortcoming stated above. In doing do, we employ game theory.

Several non-cooperative games have already been proposed to model the cooperation problem in ad hoc networks [2], [8], [10], [11], [12], [13]. In a non-cooperative game, players pick their actions according to their own interests. However, selfish players may achieve better payoffs if they cooperate with each other, i.e., if they make a collusion. This issue can be modeled more perfectly using cooperative (or better to say coalitional) games.

The routing protocol proposed in [14] uses coalitional game theory to analyze how a coalition of redirecting nodes is formed. It then distributes the earnings of the coalition, which is the frugality of energy consumption, among cooperating nodes in a fair manner. But, such a fair distribution does not necessarily preserve the stability of an established route.

We propose a payment scheme which pursues cost-efficiency, individual profitability, and stability of an established route as its main objectives. To do so, we define a cost function, and then deploy the notion of core in coalitional games to find a stable payment scheme. It is shown that the overpayment of our mechanism is less than the one in ad hoc VCG in the same circumstances.

This paper is organized as follows. Section 2 provides some basic definitions of coalitional game theory. Section 3 states the problem of cooperation stimulation in details. Section 4 explains our payment scheme. Section 5 shows simulation results. Section 6 outlines future works and concludes the paper.

II. Coalitional Game Theory

In this section we introduce those concepts of coalitional game theory used in our payment scheme.

The coalitional game theory [15] is a framework for finding the players' optimum strategies when they make coalitions for a common goal. The players in a coalition can do joint actions leading to possibly different payoffs for those players. Thus, the set of outcomes for a coalition is the set of all feasible payoffs for the coalition members. In *transferable payoff* games, an outcome is a distribution of total gain, obtained from cooperation, among the members with respect to the relative power or value of each member in the coalition.

Let N be a finite set of players. A transferable payoff coalitional game is shown by the pair (N,v), where v is the characteristic function $v: 2^N \to \mathbb{R}$ that maps any non-empty subset of N to a real number. The value of $v(S), S \subseteq N$, shows the maximum worth the members of S can make together.

A characteristic function is said to be *super-additive* if for all coalitions A and B with $A \cap B = \varnothing$, $v(A \cup B) \geq v(A) + v(B)$. In supper-additive coalitional games, the grand coalition, N, is necessarily formed.

Any payoff vector $x(S) = (x_i)_{i \in S}$ is called an S-feasible outcome if $\sum_{i \in S} x_i = v(S)$. An N-feasible outcome is called a feasible outcome. In super-additive games, a feasible outcome $x(N)$ is in the *core* if and only if every coalition receives a total payoff greater than or equal to what it is worth, i.e., $v(S) \leq \sum_{i \in S} x_i$ for any $S \subseteq N$.

The core is informally defined as the set of allocations for cooperative players in the grand coalition (N) which cannot be blocked by any other coalition. A coalition may block an allocation if it provides more payoffs for all of its members than the grand coalition can offer them. If the core is non-empty, there exists at least one stable outcome that makes it uninterested for any set of players to deviate from the grand coalition.

According to Bondareva-Shapley theorem, a transferable payoff coalitional game has a non-empty core if and only if it is *balanced*. A game (N,v) is balanced if $\sum_{S \subseteq N} \lambda_S v(S) \leq v(N)$ for every collection of weights $\sum_{S \subseteq N} \lambda_S = 1$.

III. Problem Statement

An ad hoc network is modeled as a directed graph $G = (V,E)$, where V and E are the set of nodes and the set of directed links respectively. When n_j is in the transmission range of n_i, the edge $e_{ij} \in E$ represents the directed link from n_i to n_j ($n_i, n_j \in V$). A route in the network is represented by a path in this graph, where its source and destination are denoted by n_s and n_d. If $e_{sd} \notin E$, n_d is not in the transmission range of n_s and the communication may be possible through a number of intermediate nodes realizing a path from n_s to n_d in the corresponding graph. In such a case, the cooperation among intermediate nodes, the nodes on the path except for the source and destination, is required for relaying data. A node $n_i \in V$ incurs a cost to send/forward a packet n_j that is in his transmission range (n_j is called a neighbor of n_i). Therefore, we assign a label c_{ij} to the link from n_i to n_j as the cost n_i incurs to communicate with n_j. Fig. 1 shows the model of an example ad hoc network.

The cost c_{ij} is calculated as the multiplication of the value of n_i's battery energy unit, which depends on the current battery level, and the amount of energy needed for transmission [16]. We assume that every node can adjust the level of his transmission power. In such a case, the minimum power required for transmission from n_i to n_j, $P_{\min}^{ij}$, is proportional to d_{ij}^{α}, where $2 \leq \alpha \leq 6$, and d_{ij} is the distance between n_i

and n_j. The parameter α depends on the environment circumstances [17].

The method proposed in this paper finds the cost-efficient route from the source to the destination. According to [16], [17], and [18], the selection of the cost-efficient route leads to an increase in both node and network survivability. We define a *stable time slot* as the duration of time in which the network topology does not change. The routing protocol proposed in this paper (ASTR) is an on-demand protocol which finds the cost-efficient route in a stable time slot. In such a protocol, the destination collects all the suggested routes and selects the least costly one.

In order to encourage the selfish intermediate nodes to cooperate in data transmission on a route R, the costs they incur as well as an extra benefit should be returned to them. As with the earlier price-based schemes for cooperation stimulation, our method considers a virtual account for each node. The source n_s has to pay the nodes on the cost-efficient route. This payment is withdrawn from n_s's account. As proposed in [8], such a money transfer is done in data transmission phase, i.e., the phase after establishing a route between sender and receiver. We stipulate that the remained money in an account should be nonnegative. Naturally, if a node has a zero or negative account he cannot gain any service from the network. Moreover, it is assumed that every node has a positive initial amount of money in his account enabling him to take his first action.

The source node n_s is willing to pay a specific amount $W(s)$ of money to set up a route to the destination node n_d. Clearly, this amount of money has to be less than what n_s owns in his account. The payment p_i to an intermediate node n_i should be greater than his cost in order to encourage him to cooperate with the nodes on the route R from n_s to n_d. Therefore, the payoff function for a node i with respect to a possible route R, on which i lies, is defined by

$$u(i,R)=\begin{cases} W(i)-\sum_{j\in R}p_j-c_i\ , & \text{if } i=s \\ p_i-c_i\ , & \text{if } i\neq s \text{ and } i \text{ is on } R \\ 0, & \text{otherwise}\end{cases} \quad (1)$$

It is more preferable for the source to pay less money compared to the value he considers for the route. Each intermediate node, on the other hand, is interested in the additional money he gains from cooperation. If a node does not cooperate, he gains and loses nothing.

We assume that the nodes can negotiate with each other on the price of services they provide (p_i). In the presence of such negotiation, an established route may not be stable because the sender may receive better suggestions from the nodes which are not on the current route. A new suggestion may lead to a cost-inefficient route. A cost-inefficient route results in a poor energy consumption pattern which threatens network survivability.

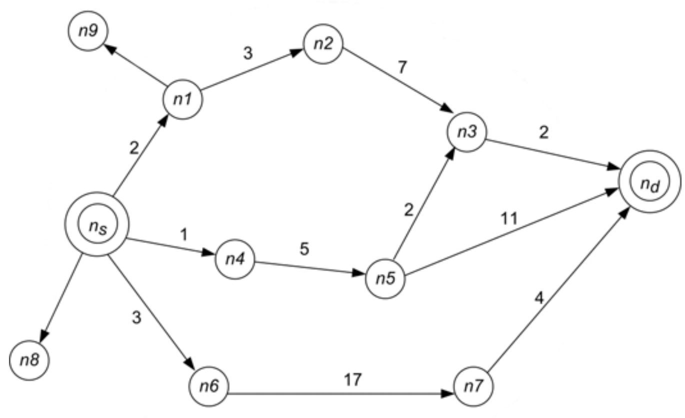

Fig. 1. The graph of an example ad hoc network. A label on the edge leaving a node is the cost that node incurs in data forwarding.

By the following example, we show that the negotiation among nodes may lead to an inefficient route in ad hoc VCG scheme [10]. As shown in Fig. 1, the set of all possible routes from n_s to n_d is $\Re=\{\langle n_1,n_2,n_3\rangle,\langle n_4,n_5,n_3\rangle,\langle n_4,n_5\rangle,\langle n_6,n_7\rangle\}$ where the cost-efficient route is $R_{best}=\langle n_4,n_5,n_3\rangle$ with the total cost of 10. If we assume $W(s)=35$, according to the ad hoc VCG scheme, every node on R_{best} will receive a payoff equal to his marginal contribution $u_i=cost(R_{best}^{-i})-cost(R_{best})$, where R_{best}^{-i} is the cost-efficient route from n_s to n_d when n_i does not contribute in routing. Note that the above equality is a result of truthfulness under which any node in the route announces the actual cost he incurs. For the network represented by Fig.1, we have

$$\begin{aligned} &u_1=u_2=u_6=u_7=0\,, \\ &u_4=u_5=14-10=4\,, \\ &u_3=17-10=7\,, \text{ and} \\ &u_s=W(s)-\textstyle\sum_{i\in R_{best}}p_i-c_s \\ &\quad=W(s)-\textstyle\sum_{i\in R_{best}\cup\{s\}}c_i-\sum_{i\in R_{best}}u_i \\ &\quad=35-10-15=10. \end{aligned}$$

On the other hand, another possible route from n_s to n_d is $R=\langle n_1,n_2,n_3\rangle$ with the total cost of 14. Assume that negotiation among the nodes in R results in the payoffs $u_1'=1$, $u_2'=1$, $u_3'=8$ and $u_s'=35-14-10=11$. Thus, the nodes n_s and n_3 will secede from R_{best} and form R together with n_1 and n_2, though R is not a cost-efficient route.

Another problem with the ad hoc VCG is its high ratio of overpayment. As stated in [10], the overpayment ratio is defined by

$$\frac{c_s+\sum_{i\in R_{best}}p_i}{cost(R_{best})},$$

in which $p_i=c_i+u_i$. It is proven that this ratio has an upper bound that is a constant multiple of $2^{\alpha+1}$ and thus can be very large in actual implementations. A payment scheme with a less overpayment ratio is more desirable.

In the next section, we propose the Ad hoc STable Routing protocol, ASTR, including a novel payment scheme. This protocol finds a cost-efficient route and provides incentives for cooperating entities in such a way that the established route remains stable. We examine the overpayment problem as well.

IV. Payment Scheme in ASTR

A typical on-demand routing protocol contains two phases; route discovery and data transmission. Our proposed protocol, ASTR, computes the value of payment to a node in route discovery phase and pays it in data transmission phase.

During the process of route discovery a coalition of nodes is formed which will transfer the packets from the source to the destination in data transmission phase. The stability of such a coalition is preserved through using the core prescription in coalitional game theory. It is also shown that the overpayment ratio of our scheme is less than the one in ad hoc VCG.

A. Route discovery and coalition formation

An on-demand routing protocol is proposed which deploys flooding of route request packets in its route discovery phase. The interaction between the source and any intermediate node in this phase is modeled as a non-cooperative game.

The source node n_s and all the nodes on a possible route from n_s to n_d form the set of players N. The action space for n_s is $\{send, not-send\}$. These actions represent source's decision on sending route discovery packets. For an intermediate node, the action space is $\{forward, drop\}$.

We assume that the cost of sending control packets is negligible because they are small in size and the routing operation is not very frequent.

The motivation for dispatching rout request packets is that the intermediate nodes hope to gain non-zero payoffs in data transmission phase. If the probability of being located on the selected route is ε for an intermediate node n_i, he gains the payoff $\varepsilon(p_i - c_i)$ according to (1). The source gains the corresponding payoff in (1) multiplied by γ that is the probability of finding at least one route from source to destination. The bimatrix of the single-shot game between the source and an intermediate node is shown in Fig. 2 in which $t = \sum_{i \in R} p_i + c_s$.

If $W(s) \geq t$ and $p_i \geq c_i$, the strategy profile (*send*, *forward*) will be a Nash equilibrium in the sense that deviation from this strategy has no profit for the deviator. The payment scheme leading to the Nash equilibrium above is called *individually profitable*. The players that adhere to the Nash prescription form a *cooperating coalition*.

		Intermediate node n_i	
		forward	drop
Source node	send	$\varepsilon(p_i - c_i)$, $\gamma(W(s) - t)$	0,0
	not-send	0,0	0,0

Fig 2. The bimatrix of the game between the source and an intermediate nod n_i, where $t = \sum_{i \in R} p_i + c_s$.

Now we describe the route discovery protocol. The source n_s broadcasts a route request packet by a predetermined level of transmission power that is known by any node in the network. This level of transmission power is the highest one a node can access. The reason for using the highest possible level of transmission power in the route discovery process is that all possible routes are discovered in this way. As will be seen, this is an important issue in solving the collusion problem.

The route request packet includes a route ID, ID_{s-d}, as well as the source and destination ID's, n_s and n_d. It also contains a payment offer $W(s)$ signed by the source, and the cost of one unit of energy in the source e_s. Each node n_j, receiving this request from his previous node n_i, senses the received signal strength. He then calculates the minimum transmission power $P_{\min}^{ij}$ needed for n_i to reach n_j (as formulated in [10]). The node n_j calculates the cost c_i by multiplying $P_{\min}^{ij}$ by e_i as the declared cost of one unit of energy in n_i. He then proceeds by substituting c_i for e_i, and appending $P_{\min}^{ij}$, n_j's ID, and e_j as the cost of one unit of energy in n_j. Finally, n_j broadcasts the route request packet.

During a stable time slot, for every route that could be found from n_s to n_d, the destination node receives a route request packet offering that route. For each offered route R, the destination node calculates $C_R = \sum_{i \in R} c_i$ and chooses the route with minimum C_R as the cost-efficient route. He also determines payments with respect to $W(s)$ and nodes' costs. Then, a route reply packet is unicasted from n_d to n_s along with the reverse of the selected route. This route reply packet contains those minimum transmission power levels the nodes on the selected route should know for data transmission. He also contains the payments, p_i, that should be made by the source in data transmission phase. The messages exchanged in the route discovery phase are as follows:

$n_s \to n_i : \langle RREQ, ID_{s-d}, n_s, n_d, \mathrm{W}(s), e_s \rangle$,
for any n_i in n_s's neighborhood,

$n_j \to n_k : \langle RREQ, ID_{s-d}, n_s, n_d, \mathrm{W}(s), \ldots, n_i, c_i, P_{\min}^{ij}, n_j, e_j \rangle$,
for any pair of adjacent nodes n_j and n_k, and

$n_d \to n_s : \langle RREP, ID_{s-d}, n_s, n_d, \ldots, n_i, p_i, P_{\min}^{ij}, n_j, p_j, P_{\min}^{jd} \rangle$.

Note that, we assume the nodes declare the values $W(s)$ and e_i truthfully and adhere to the integrity of c_i. In what follows, the values p_i are calculated using the concept of core in a coalitional game.

B. Coalitional Stability

A coalition of cooperating nodes is formed as a result of the ASTR route discovery process. More specifically, a subset of the grand coalition is determined as the cost-efficient route between the source and destination. In this section, a payment scheme is proposed that guarantees the stability of the cost-efficient route. In doing so, we use the notion of core in coalitional game theory, where the characteristic function reflects the benefit of different subsets of the grand coalition when they cooperate with each other.

The payment scheme is to satisfy the following properties.

1. It compensates the cost a node incurs in data transmission (individual profitability).
2. The payment distribution prevents any deviation from the solution (stability).
3. It makes as less payments as possible.

The first property is satisfied in our proposed scheme in which all the intermediate nodes are paid greater than the costs they incur in data transmission phase. Similarly, the source node pays less than his willingness to pay. For the second property we utilize the solution concept of coalitional game theory. The third property is investigated in the next section practically.

Consider a transferable payoff coalitional game (N, v^G). As shown in Fig. 3, the set of all players N includes the source node n_s and the intermediate nodes on the routes from n_s to n_d. For a subset S of N, we define S_G as the set of all possible paths from n_s to n_d in G whose nodes (except for n_d) are in S. The characteristic function v^G designates the total benefit the members of a coalition can make when they cooperate. This function is defined by

$$v^G(S) = \begin{cases} W(s) - \min_{R \in S_G} \{\sum_{i \in R} c_i + c_s\}, & n_s \in S \text{ and } S_G \neq \varnothing \\ 0, & \text{otherwise} \end{cases}. \tag{2}$$

The payment to the members of a coalition is the responsibility of the source, and is only possible for the coalitions the node n_s belongs to. Moreover, if no route exists in S, the coalition S can gain nothing. A coalition S including n_s and some intermediate nodes which can build some route to n_d gains the difference between the total money n_s is willing to pay and the total costs expended on the cheapest route in S. This is because every coalition tends to maximize $v^G(S)$. In this way, the cost-efficient route is selected among all possible routes. Note that $W(s)$ is determined at the beginning of route discovery process, and does not change later during that process.

As seen in Section 3, the distribution of the value produced by a cost-efficient route has an important role in the stability of that route. By using the core allocation rule, the stability of an outcome is guaranteed. This requires the core to be non-empty.

Theorem 1. *The core of the game (N, v^G) is non-empty.*

Proof. For any disjoint subsets A and B of N, if n_s is not in A or B, $v^G(A) = v^G(B) = v^G(A \cup B) = 0$. If n_s is in A but not in B, $v^G(B) = 0$ and $v^G(A) \le v^G(A \cup B)$. This is because the set $A \cup B$ contains the routes in A together with some new routes through which attaining a route of less cost is possible. A similar argument holds for remaining cases. Therefore, the game is super-additive, and the grand coalition is formed. From (2), it is evident that $v^G(S) \le v^G(N)$ for any $S \subseteq N$. Now, consider a balanced collection of weights $\sum_{S \subseteq N} \lambda_S = 1$. This is immediate that $\sum_{S \subseteq N} \lambda_S v^G(S) \le v^G(N)$. Hence, the game (N, v^G) is balanced with a non-empty core.□

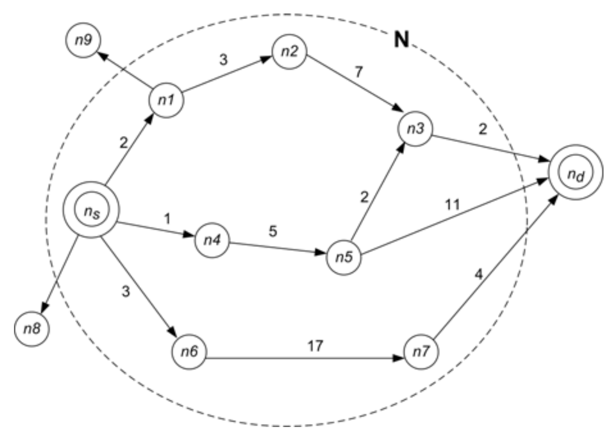

Fig. 3. The example ad hoc network and the set *N* of game players.

Consider the example network in Fig. 3. We have

$$\begin{aligned} &v^G(\{n_s, n_1, n_2, n_3\}) = 35 - 14 = 21, \\ &v^G(\{n_s, n_4, n_5, n_3\}) = 35 - 10 = 25, \\ &v^G(\{n_s, n_6, n_7\}) = 35 - 24 = 11, \\ &v^G(N) = 25. \end{aligned}$$

A stable payoff allocation (x) by the use of the core solution is

$$\begin{aligned} &x_1 = x_2 = x_6 = x_7 = 0, \\ &x_s + x_3 \ge 21, \\ &11 \le x_s \le 25, \\ &x_s + x_4 + x_5 + x_3 = 25. \end{aligned}$$

As seen in Section 3, the payoff allocation of ad hoc VCG lies outside the core, and therefore, the cost-efficient route is instable. This is because the selfish nodes are in collusion for more benefits. The allocations made by the core of (N, v^G) in ASTR would prevent such collusion if all possible routes were known before solving the system of core. In other words, despite using the payments prescribed by the core, an unconsidered route may still compromise the stability of the cost-efficient route. In order to have all possible routes before solving the core inequalities, it is sufficient to use the highest level of transmission power in broadcasting route request packets. Moreover, no route request of the same route ID has to be discarded unless it contains the data seen before.

In the route discovery process presented in the previous section, the destination node acquires several route offers in a stable time slot. We assume the duration of stable time slot to be more than the time the destination needs to discover the complete graph. This holds if the length of a route is bounded above by L. Under such an assumption, none of the source or destination nodes are interested in routes longer than L, since it leads to unacceptable delays. Thus, all possible routes are discovered during a limited time.

After collecting all possible routes, the destination node solves the system of linear inequalities to determine the nodes' payoffs. If there are n nodes in the network, the system has 2^{n-2} inequalities of $n-2$ unknowns in the worst case. Thus, solving this problem is of exponential order.

Nevertheless, this system can be reduced to (3) where the number of inequalities is noticeably decreased. In the system of (3), $\Re$ is the set of discovered routes and $R_{best} \in \Re$ is the cost-efficient one.

1. $\sum_{i \in N} x_i = v^G(N)$,
2. $x_s + \sum_{i \in R_{best}} x_i \geq v^G(N)$,
3. $x_s \geq \max_{S \in \Re'} \{v^G(S)\}$ with
   $\Re' = \{R \mid R \in \Re, R \cap R_{best} = \varnothing\}$,
4. $x_s + \sum_{i \in S} x_i \geq v^G(S)$ for any $S \in \Re''$, where
   $\Re'' = \{R \mid R \in \Re, R \cap R_{best} \neq \varnothing, R_{best} \not\subseteq R\}$. (3)

In (3), the number of inequalities is proportional to the number of discovered routes. The maximum number of discovered routes is n^L. Moreover, we should make at most n comparisons to find the constant coefficients of each of these inequalities. Therefore, the problem of finding core inequalities is of $O(n^{L+1})$. Then, we have a linear programming problem to solve that is of $O(an^y)$ [20]. Hence, the overall complexity of the core problem in ASTR is $O(n^{L+1}) + O(an^y)$. By using the interior point method and considering ε-exact solutions, we have $y = 3$ and $a = log(1/\varepsilon)$ [20].

On the other hand, it has been shown that in a directed graph of n vertices and m edges, solving the VCG problem requires $\Omega(n(nlogn + m))$ computations [21]. However, in practical settings of ad hoc VCG routing protocol, the problem is of $O(n^{L+1})$. This is because for each of the nodes (there are n nodes in the worst case) on the best route, the minimum-cost route excluding that node has to be determined. A search for such a route takes n^L computations in the worst case.

Therefore, the problem of payment assignment is of the same complexity, $O(n^{L+1})$, in both methods. However, as will be shown in simulations results, ASTR takes more time than ad hoc VCG in actual examples. This is owing to the hidden multiplicative constants in the asymptotic notations stated above.

C. Overpayment ratio

The overpayment ratio is defined as the ratio of total payment to total cost in a payment scheme. Evidently, a scheme with less overpayment ratio is more desirable. In this section, we compare the overpayment ratio in ASTR with that in ad hoc VCG.

Let C be the cost of the cost-efficient route R_{best}. Further, assume there exists a route having the following properties: it shares no node with R_{best}, its cost is greater than C, say $C(1+\tau)$, and it is the least costly route in the absence of R_{best}. We compare ASTR with ad hoc VCG when such an assumption holds.

In ad hoc VCG, each node on R_{best} earns, $C(1+\tau) - C = C\tau$. Thus, if there are k nodes on R_{best}, the overpayment ratio is $(kC\tau + C)/C = k\tau + 1$. In ASTR, we have $x_s \geq W(s) - C(1+\tau)$. This means that the maximum payment occurs when $x_s = W(s) - C(1+\tau)$. According to (3), in the case of maximum payment, we have $W(s) - C(1+\tau) + \sum_{i \in R_{best}} x_i = W(s) - C$. This implies that $\sum_{i \in R_{best}} x_i = C\tau$, and therefore, the maximum overpayment ratio is $(C\tau + C)/C = \tau + 1$.

Hence, under the assumption stated above, the overpayment ratio in ASTR is less than the one in ad hoc VCG. In this sense, it enjoys a better performance. The simulation results confirm this, even in the cases the above assumption does not hold.

V. Simulation Results

This section presents MATLAB simulations which help compare ad hoc VCG and ASTR payment schemes. In these simulations, a routing protocol based on DSR [19] is adopted which is very similar to the cost-aware routing protocol used in [10]. The number of nodes is 40, and they are randomly located on a 50m×50m terrain. This is repeated 50 times (each results in a random topology), and in each topology, 30 pairs of source-destination nodes are selected randomly. The routing protocols are applied to these pairs and the results are then averaged. In addition, it is assumed that $\alpha = 4$, and the maximum length of a route equals the square root of the number of nodes, i.e., $L = \sqrt{40}$.

In order to solve the system of core, the function linprog.m with the large-scale model is employed. This function uses the LIPSOL algorithm [22] as a polynomial time algorithm based on the primal-dual interior point method.

The resultant overpayment ratios in ad hoc VCG and ASTR are shown in Fig. 4. As seen, the average overpayment ratios in ASTR lie below the ones in ad hoc VCG.

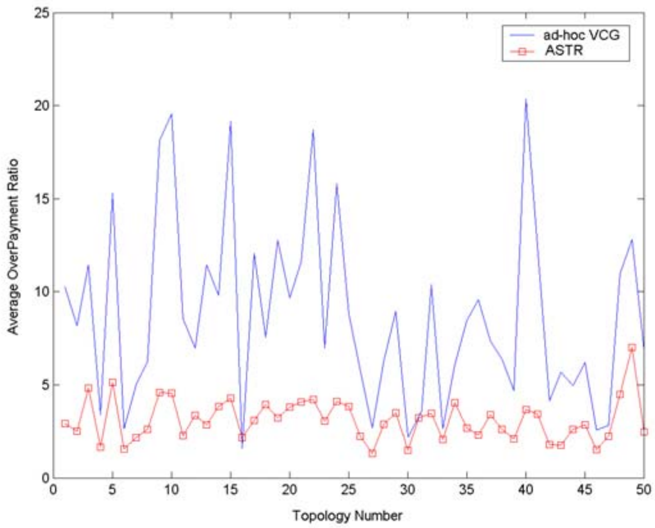

Fig 4.The average overpayment ratios in ASTR and in ad hoc VCG.

As we mentioned in Subsection B of Section 4, both VCG and ASTR payment computations are done in polynomial time. Fig. 5 compares the time needed for computations in the two methods. As seen, the time taken by ASTR is more than the one taken by ad hoc VCG. This is because ASTR performs extra computations by solving the system of core to find stable payments. This introduces a tradeoff between the execution time and the overpayment. The less overpayment one intends, the more computations he should perform. A question arises here: Are these extra computations economic? In order to answer this question, we should have an effective method to compare the cost of extra computations with the benefits produced by low overpayments.

We denote the difference between total payments in ASTR and ad hoc VCG by F_s. This is the amount of money a source node can save when he uses ASTR in lieu of ad hoc VCG. Similarly, D_t is the extra time a destination node spends when he uses ASTR instead of ad hoc VCG. Assume that in a unit of time the destination node consumes one unit of energy. As the cost of one unit of energy is e_d in the destination, the amount of money he should expend for the extra computations equals $M_d = D_t e_d$. Therefore, the actual benefit produced by ASTR, in comparison to ad hoc VCG, is $Q = F_s - M_d$. Fig. 6 shows the average of $Q = F_s - M_d$. As seen, it is mostly positive, i.e., $F_s > M_d$ in most topologies. In other words, the money saved is often more that the money spent for extra computations. Hence, ASTR is more beneficial than ad hoc VCG.

VI. Conclusion

In this paper, we focused on providing incentives for selfish nodes in a MANET to participate in routing and forwarding. In doing so, we deployed coalitional game theory to propose an optimal price-based scheme aimed at energy-efficiency, individual profitability, and stability. As its distinctive feature, it was shown that the proposed scheme called ASTR preserves the stability of the cost-efficient route. In addition, it has a better performance than ad hoc VCG.

In our scheme, it is assumed that the nodes declare their costs truthfully. This is not necessarily valid and deserves further studies.

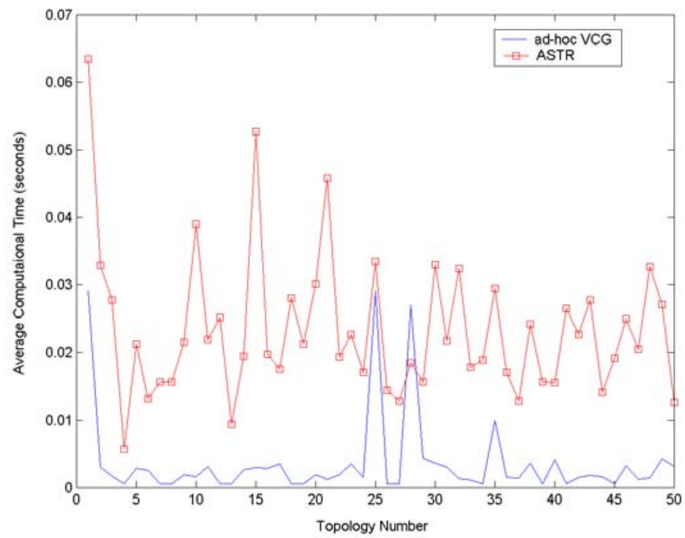

Fig 5. The time taken by ASTR in comparison to the one taken by ad hoc VCG.

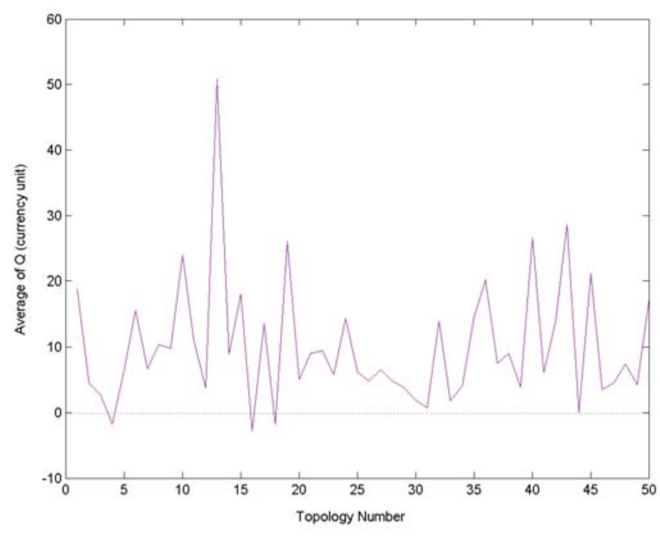

Fig 6. The average of $Q = F_s - M_d$ as the actual benefit produced by ASTR.

References

[1] S. Marti, T.J. Giuli, K. Lai, and M. Baker, "Mitigating routing misbehavior in mobile ad hoc networks," in *Proc. 6th Annu. Int. Conf. Mobile Computing and Networking*, Boston, MA, 2000, pp. 255-265.

[2] M. Félegyházi, J.-P. Hubaux, L. Buttyán, "Nash equilibria of packet forwarding strategies in wireless ad hoc networks," *IEEE Trans. Mobile Computing*, vol. 5, no. 5, May 2006, pp. 463-476.

[3] S. Buchegger and J.-Y. Le Boudec. "Performance analysis of the CONFIDANT protocol: cooperation of nodes fairness in dynamic ad-hoc networks," in *Proc. IEEE/ACM Symp. Mobile Ad Hoc Networking and Computing*, Lausanne, CH, 2002, pp. 226-236.

[4] P. Michiardi, R. Molva, “Core: a collaborative reputation mechanism to enforce node cooperation in mobile ad hoc networks,” in *Proc. 6th IFIP-Communication and Multimedia Security Conf.*, Portoroz, Slovenia, 2002 ,pp. 107-121.

[5] R. Carruthers, I. Nikolaidis, “Certain limitations of reputation-based schemes in mobile environments,” in *Proc. 8th Int. Symp. Modeling Analysis and Simulation of Wireless and Mobile Systems*, Montreal, Canada, 2005, pp. 2-11.

[6] L. Buttyán and J.-P. Hubaux, “Stimulating cooperation in self-organizing mobile ad hoc networks,” *ACM J. Mobile Networks and Applications*, vol. 8, no. 5, Oct. 2003, pp. 579-592.

[7] L. Blazevic, L. Buttyán, S. Capkun, S. Giordano, J.-P. Hubaux, and J.-Y. Le Boudec, “Self-organization in mobile ad hoc networks: the approach of Terminodes,” *IEEE Communications Magazine*, vol. 39, no. 6, Jun. 2001, pp. 166-174.

[8] S. Zhong, J. Chen, and Y. R. Yang, “Sprite: a simple, cheat-proof, credit-based system for mobile ad hoc networks,” in *Proc. 22nd Annu. Joint Conf. IEEE Computer and Communications Societies*, San Franciso, CA, 2003, pp. 1987-1997.

[9] J. Crowcroft, R. Gibbens, F. Kelly, and S. Östring, “Modelling incentives for collaboration in mobile ad hoc networks,” *Performance Evaluation*, vol. 57, no. 4, Aug. 2004, pp. 427-439.

[10] L. Anderegg and S. Eidenbenz. "Ad hoc-VCG: a truthful and cost-effcient routing protocol for mobile ad hoc networks with selfsh agents," in *Proc. 9th Int. Conf. Mobile Computing and Networking*, San Diego, CA, 2003, pp. 245-259.

[11] S. Zhong, L. Liy, Y. G. Liu, Y. R. Yang, “On designing incentive-compatible routing and forwarding protocols in wireless ad-hoc networks: an integrated approach using game theoretical and cryptographic techniques,” in *Proc. 11th Annu. Int. Conf. Mobile Computing and Networking*, Cologne, Germany, 2005, pp. 117-131.

[12] A. Urpi, M. Bonuccelli and S. Giordano, “Modeling cooperation in mobile ad hoc networks: a formal description of selfishness,” *presented at the 1st Int. Symp. Modeling and Optimization in Mobile, Ad-Hoc and Wireless Networks*, Sophia-Antipolis, France, 2003.

[13] V. Srinivasan, P. Nuggehalli, C.-F. Chiasserini, and R. R. Rao, “Cooperation in wireless ad hoc networks,” in *Proc. 22nd Annu. Joint Conf. IEEE Computer and Communications Societies*, San Franciso, CA, 2003, pp. 808-817.

[14] J. Cai, U. Pooch, “Allocate fair payoff for cooperation in wireless ad hoc networks using Shapley value,” *Int. Workshop on Algorithms for Wireless, Mobile, Ad Hoc and Sensor Networks*, Santa Fe, NM, 2004.

[15] M. J. Osborne and A. Rubinstein, *A course in game theory*, MIT press, 1994, pp. 255-263.

[16] S. Singh, M. Woo, and C.S. Raghavendra, “Power-aware routing in mobile ad hoc networks,” in *Proc. 4th Annual ACM/IEEE Int. Conf. Mobile computing and Networking*, Dallas, TX, 1998, pp. 181-190.

[17] S. Basagni, M. Conti, S. Giordano, I. Stojmenovic, Mobile Ad hoc networking. New York: IEEE Press and John Wiley and Sons Inc. , 2004, pp. 301-327.

[18] J. Gomez, A. T. Campbell, M. Naghshineh and C. Bisdikian, “Conserving transmission power in wireless ad hoc networks”, in *Proc. 9th Int. Conf. Network Protocols*, Riverside, CA, 2001, pp. 24-34.

[19] D. Johnson, Y.-C. Hu, and D. Maltz, “The dynamic source routing protocol (DSR) for mobile ad hoc networks for IPv4,” RFC 4728, Feb. 2007.

[20] K. Ross, T. Terlaky and J.-P. Vial, *Theory and Algorithms for Linear Optimization An Interior Point Approach*, John Wiley & Sons Wiley – Interscience Series in Discrete Mathematics and Optimization Chichester, 1997.

[21] J.Hershberger, S. Suri and A. Bhosle, “On the difficulty of some shortest path problems,” *ACM Trans. Algorithms,* vol. 3, no.1, Feb 2007.

[22] Y. Zhang, “Solving large-scale linear programs by interior-point methods under the MATLAB environment,” Department of Mathematics and Statistics, University of Maryland, Baltimore, MD, Tech. Rep. TR96-01, Jul.1995.