Power Spectra Estimation Using the Combination of Goodman and Belsher Method with a Constant on Gamma Camera Images

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Abstract—**In this paper, we introduce an algorithm that combines between Goodman and Belsher method with a constant to estimate the value of the power spectra of images produced by a gamma camera. The information about power spectra of an images will allow one to investigate other characteristics of the imaging system and to increase the quality of the images. In this work, we demonstrate the use of the power spectra values to restore the images of gamma camera using Wiener filtering technique. The imaging system is modeled by the Monte Carlo N Particle code version X (MCNPX). The object that was used in this paper is a square source which was located in air and a 1570cm^3 water cylinder. The results show that, in terms of the signal to noise ratio, the best estimation was given by the proposed method.**

Keywords: medical image processing, gamma camera, power spectra, Goodman and Belsher, MCNP

1 Introduction

For gamma camera images, one of the important characteristic is the power spectra or spectral density. The power spectra of the images carry the information of the power at each frequency components. However, since the images are obtained from analogue photons distribution, hence the power spectra of those images are hidden. In this paper, we will also look at the leakage of the power spectrum. The leakage of the power spectrum refers to the power that is lost or exists at the wrong frequency components. In the literature, there are many methods have been proposed to estimate the power spectra of images obtained from the gamma camera.

The most common and easiest way is by treating it as a constant. In this paper, we also investigate the estimation techniques proposed by Press et al. [6] and Goodman and Belsher [2], and compare all of these approaches. Based on the observation as well as those reported in previous works of the natural behavior of the power spectrum of the imaged object and the noise, we combined the constant method with Goodman and Belsher.

One of the applications to the estimation of the power spectra is to extract the original signal and remove the noise component. Based on Penney et al. [5] and Vastola and Poor [7], the error produces in Wiener filtering method could be reduced with a good estimation of power spectra. In this paper, we used the estimation method as in the literature, as well as the proposed combination method with the Wiener filter to find the best estimation among the presented methods. Our results showed that our proposed combination method supersedes other technique in terms of the signal-to-noise ratio and mean-square-error of the images.

In the next section, we present the image restoration to improve the quality of the captured images using Wiener filtering technique. Section 3 discusses about the current and proposed estimation of power spectra for images obtained from the gamma camera. In Section 4, the MCNPX model of the gamma camera is presented. The results and discussion is given in Section 5, before we conclude our findings in Section 6.

2 Restoration using Wiener Filtering

There are many techniques available to the area of image restoration, e.g. inverse filtering, maximum entropy deconvolution and etc. In this paper, we choose to restore the images using Wiener filtering method. The image restoration method is used in many areas, like in astronomy as well as in medical imaging. In image restoration, we have the knowledge of the point spread function beforehand. Using this characteristic, we are able to traceback the propagation of the point source to its original location.

The basic image formation process of a camera (and valid for almost all image formation system) with an additive noise may be modeled as follows:

$$G(\mu,\nu) = H(\mu,\nu)F(\mu,\nu) + N(\mu,\nu) \quad (1)$$

where $G(\mu,\nu)$ is the Fourier transform of the recorded image, $H(\mu,\nu)$ is the Fourier transform of the the point spread function of the imaging device, $F(\mu,\nu)$ is the

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Fourier transform of the the imaged object and $N(\mu,\nu)$ is the Fourier transform of the noise field. This is a linear system of equations in terms of the unknown values of $F(\mu,\nu)$. Its solution is also expected to be linear, i.e. we expect the unknown values of $F(\mu,\nu)$ to be recoverable as linear combinations of the known values $G(\mu,\nu)$:

$$F(\mu,\nu) = W(\mu,\nu)G(\mu,\nu) \tag{2}$$

In Wiener filtering, the weights $W(\mu,\nu)$ is given by:

$$W(\mu,\nu) = \frac{H^*(\mu,\nu)}{|H(\mu,\nu)|^2 + \frac{S_{vv}(\mu,\nu)}{S_{ff}(\mu,\nu)}} \tag{3}$$

where $\frac{S_{vv}(\mu,\nu)}{S_{ff}(\mu,\nu)}$ is the ratio of the power spectrum of the noise to the power spectrum of the imaged object and * means taking the complex conjugate.

To obtain the best result, the value of the power spectrum of the imaged object, $S_{ff}(\mu,\nu)$ and the power spectrum of the noise, $S_{vv}(\mu,\nu)$ must close to their original value. This is the department that we try to improve. The better the estimation technique for the power spectra, the better the result of the Wiener filtering.

For this approach to be applicable, there are a few assumptions on the imaging process [3]. First, the patient' body must remain stationary for a few minutes while the image is being captured. Since the patient is asked to rest and lay on a bed, we may assume that this condition is fulfilled. Second, the transfer function $H(\mu,\nu)$ which corresponds to a point spread function is shift invariant. This assumption may be slightly violated without major consequences in the case of a planar gamma camera such as ours, but for a three-dimensional imaging device, such as a 3D SPECT machine, this assumption is very important, and if it is violated an incorrect solution may result which may lead to an incorrect diagnostic of the depth location of a tumor.

The third assumption is that the point spread function of the imaging system is known. In this work, we used a water cylinder to approximate the liquids in the human body, because 60% of our body liquid is water. By simulating a point source in it, we worked out the point spread function (PSF) of our camera.

3 Estimation of Power Spectra

In this work, we try to estimate the value of the power spectra. Figure 1 shows an example of a power spectrum of an output image, S_{gg} extracted from an image produced by a gamma camera.

There are many ways to estimate the power spectrum of the noise and the power spectrum of the object. The most common and easiest way is by treating their ratio as a constant (Γ), and use a trial and error method to find

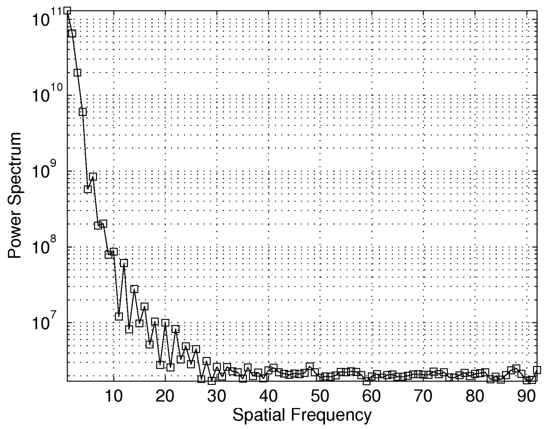

Figure 1: An example of a power spectrum of an image obtained from a gamma camera.

the best value of the constant. Using this method, we assume that the changes of both power spectra is similar. The advantage of this method is there is no analysis of S_{gg} needed beforehand.

Alternatively, we may use the estimation technique proposed by Press *et. al* [6], where they estimated the power spectrum of the noise (S_{vv}) as the mean of the spectrum of the recorded values, after a cut-off frequency, f_c. According to King *et al.* [4], f_c is defined as the frequency beyond which the power spectrum of the recorded image is mainly dominated by noise. This can be expressed as follows:

$$S_{ff}(\mu,\nu) = S_{gg}(\mu,\nu) - S_{vv}(\mu,\nu) \tag{4}$$

where S_{gg} is the power spectrum of the recorded image.

Another estimation technique has been proposed by Goodman and Belsher [2]:

$$S_{ff}(\mu,\nu) = \frac{S_{gg}(\mu,\nu) - S_{vv}(\mu,\nu)}{|H^2(\mu,\nu)|} \tag{5}$$

Also, according to them, for an image degraded by Poisson noise, the power spectrum of the noise, S_{vv} is a constant and it can be approximated by the total count of photons at the detector.

The algorithm proposed in this paper is by combining the first method, i.e. taking the ratio as a constant, and the method proposed by Goodman and Belsher. Based on the pattern of the power spectrum in 1, the power spectrum value is settling down at one frequency, where after that frequency, value of S_{gg} is stable. That particular frequency is known as the cut-off frequency, f_c. The proposed algorithm takes Goodman and Belsher method until f_c, and a constant, Γ after the f_c. By implementing the method, value of Γ will be higher than if we had used it for the entire frequency range. This is because $\Gamma = \frac{S_{vv}(\mu,\nu)}{S_{ff}(\mu,\nu)}$, and since according to Press *et. al*, the frequency above f_c is mainly dominated by noise, thus $S_{vv}(\mu,\nu)$ is dominating Γ.

4 Monte Carlo Simulation

The methodology of simulating the gamma camera is largely following the work by Vries *et al.* [1]. In this work, the whole model is simulated using the combination of MCNPX code and an in-house software. A schematic representation of the gamma camera simulated by the MCNPX is shown in figure 2. As shown in the figure, the gamma camera is bounded by a box boundary used to define the limits of the simulated environment and this is referred to as the MCNPX calculation boundary. This is one of the requirement by MCNPX code that makes the code runs faster because the software does not need to measure any photon that escapes from this boundary.

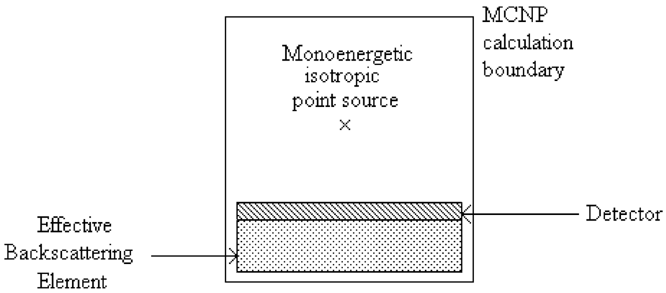

Figure 2: The gamma camera simulated by the MCNP Code. The box outside the detector and the effective backscattering compartment is the MCNP calculation boundary. It has no material and does not offer any interaction with photons.

The photons detected by this camera is then been processed by the in-house software. The characteristics of the modeled gamma camera largely follows the specifications of Toshiba GCA-7100A at the Royal Surrey County Hospital, Guildford, Surrey, United Kingdom.

5 Preliminary Phantom Study

The original square source images were obtained by the simulated gamma camera. Then, we restored the images using the presented techniques and compared the images using the signal-to-noise ratio (SNR). The SNR was defined using the contrast of the image, as the ratio between the gray value of the known content of the source to the average gray value outside the source. In image restoration, we try to find the best approximate value of $F(\mu,\nu)$ with the information of $G(\mu,\nu)$ and $H(\mu,\nu)$. $H(\mu,\nu)$ or the point spread function (PSF) is obtained by an ideal point source inside air and inside the water cylinder. Table 1 presents the comparison of all the techniques.

From this table, we can see that the best estimation technique is the proposed estimation method, indicated by the highest value of the SNR. In this experiment, the value of constants and f_c are obtained by using trial and error method. In table 1, we only show the best SNR value taken from the optimized value of the constants and f_c.

The second best method in this experiment is given by the Goodman and Belsher method. We also see that the Press's method is not very bad either, and its results is between that of the constant technique and that of the Goodman and Belsher's technique. As we have predicted, the estimation technique using a constant only is worst of all. Figure 3 shows the original square image in air and inside the water cylinder, and the restored image using the original Wiener filtering with the Goodman and Belsher's method of estimation. Based on the image profile in figure 4, the restored images have sharper edges and less noise than the original images.

Table 1: SNR of restored images by the various estimation techniques in Wiener filtering.

Estimation Types	SNR
A square source in air:	
Original image	952.89
Constant	1.18×10^3
Press *et al.* method	1.20×10^3
Goodman and Belsher method	1.31×10^3
Goodman and Belsher + constant	1.32×10^3
A square source in a 1570cm^3 water cylinder:	
Original image	193.2052
Constant	320.0093
Press *et al.* method	320.9275
Goodman and Belsher method	325.8001
Goodman and Belsher + constant	330.7927

6 Conclusions

The study presented in this paper demonstrated that the proposed method supersedes other estimation techniques in a simple experimental set up of sources in air and water cylinder. The results show the potential of the technique to be extended for a more complicated scenario of gamma camera imaging.

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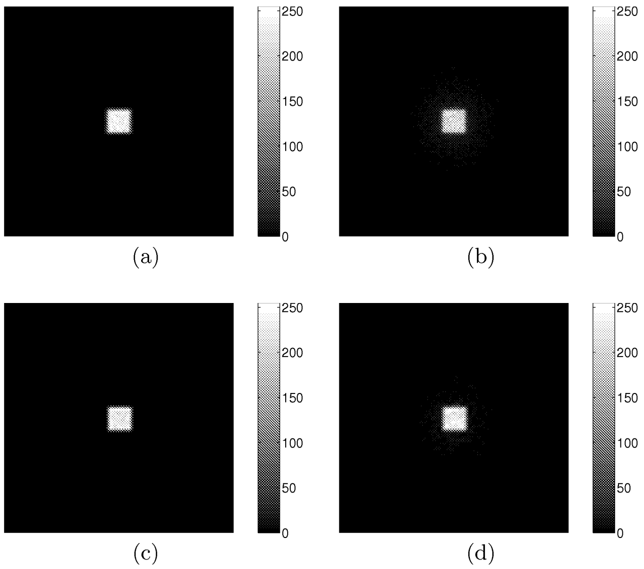

Figure 3: The recorded image of a square source inside air (a) and inside water (b). (c) and (d) images are the restored (a) and (b) images using Wiener filtering technique with the proposed estimation method, respectively.

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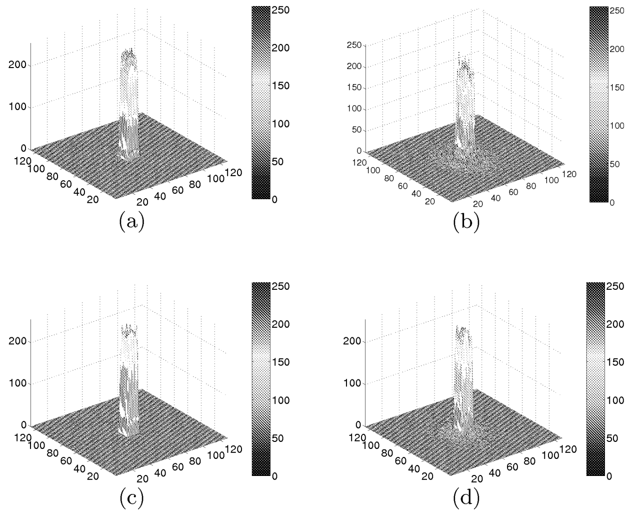

Figure 4: The respective image profile of figure 3.