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Analytical Model for Loss and Delay Behavior of the Switch under Self-Similar Variable Length Packet Input Traffic

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Abstract— **It has been reported that Internet Protocol (IP) packet traffic exhibits self-similarity or long range dependence (LRD) and causes the degradation of switch performance. Therefore, it is crucial for an appropriate buffer design of a switch. In this paper, we investigate the packet loss and delay behavior of the asynchronous switch under self-similar variable length packet traffic by modeling it as $MMPP/M/1/K$ queueing system wherein MMPP (Markov-Modulated Poisson Process) is fitted by equating the variance of MMPP and that of self-similar traffic. MMPP model is already validated one to emulate the self-similar characteristics. We investigate the packet loss probability, mean waiting time against system parameters, traffic parameters and fitting parameters. Numerical results show that analysis presented in this paper is useful in dimensioning the switch.**

Index Terms — MMPP, queueing system, self-similar traffic, packet loss probabilities, Quality of Service (QoS), computation complexities.

I. INTRODUCTION

Seminal studies reveal that IP packet traffic in both Ethernet and Wide Area Network (WAN) tends to be bursty in many time-scales [1-3]. This bursty traffic can be characterized mathematically as self-similar or long range dependent (LRD). Earlier results have shown that LRD traffic has an ill affect on the performance of network nodes such as routers or switches. Since the switch buffer will overflow easily under bursty traffic, it is worthy to consider the influence of self-similar traffic while designing the network. Most of the proposed parsimonious self-similar models are asymptotic in nature; hence they are less effective in analyzing the queueing based performance measures when the buffer size is small. Self-similar traffic modeled by Markovian Arrival Process (MAP) has been proposed to emulate self-similarity over the desired time-scales [4, 5]. These works involve Markov Modulated Poisson Process (2^d -MMPP), which is a superposition of d 2-state Interrupted Poisson Processes (IPPs) and a Poisson process. MMPP is fitted by equating the second order statistics of resultant MMPP and that of self-similar traffic over desired time-scales. MMPP is a particular case of MAP and IPP is a particular case of MMPP. These models are computationally tractable; therefore they are more suitable models especially at shorter buffer depth.

Deeper understanding of the switch behavior is necessary, because switch is the critical component in providing Quality of Service (QoS) guarantee in internet. The QoS parameters such as packet loss probability and packet delay are significant in dimensioning the switch. In [5], first, switch is modeled as $MMPP/D/1/K$ queueing system and then the impacts of time-scale and Hurst parameter (H) on the packet loss probability are investigated by means of both analytical and simulation results, wherein the MMPP is generated so as to match the variance of self-similar traffic over specified time-scales. From [5], it is found that generalized variance based fitting is robust on queuing behavior in terms of time-scale and number of components (d) in superposition. In general, switches could be divided into two categories based on their operation modes: slotted synchronous and un-slotted asynchronous. When the switches are operated under slotted synchronous mode, packets need to be aligned before entering the switch and involves the cost of packet synchronous. On the other hand, asynchronous network could let the packets enter without alignment. Since IP packets are, in general, variable in length, switch is required to possess the ability to switch the variable length packets. Hence the concern about asynchronous variable length packet traffic is increasing. Therefore, performance analysis of switch by means of $MMPP/D/1/K$ queueing system wherein service time is deterministic may not be appropriate. In the papers [6, 7], switch with the variable length packet traffic is modeled as $MMPP/M/1/K$ system wherein service time is exponential. That is, packet length is assumed to follow exponential distribution to make the performance analysis of switch handling self-similar traffic with variable length packets. In the said paper, MMPP emulating self-similar traffic is modeled as superposition of four IPPs and a Poisson process. It is illustrated in the paper [8] that superposition of four two-state MMPPs suffices to model the second order self-similar behavior over several time-scales. In the present paper, first, two MMPPs emulating self-similar traffic are fitted by taking the superposition of three IPPs and four IPPs (that is d =3 and 4), respectively, and then queueing analysis is made for the comparison of these cases by means of simulation results.

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The rest of the paper is organized as follows. In section II, we first, overview the definitions of self-similar processes and MMPP. We then present the computational complexity and the analytical results of $MMPP/M/1/K$ system in section III. In section IV, we present some numerical results pertaining to packet loss probability and mean waiting time of $MMPP/M/1/K$ queueing system and illustrate the effects of parameters such as Hurst parameter, traffic intensity, number of components in superposition, and time scales. Finally some conclusions are given in section V.

II. Self-similar traffic and Markov modulated Poisson process (MMPP)

The definition of exact second-order self-similar processes is given as follows. If we consider X as a second -order stationary process with variance σ^2, and divide time axis into disjoint intervals of unit length, we could define $X = \{X_t / t = 1,2,3........\}$ to be the number of points (packet arrivals) in the t^{th} interval. A new sequence $X^{(m)} = \{X_t^{(m)}\}$, where

$X_t^{(m)} = \frac{1}{m}\sum_{i=1}^{m} X_{(t-1)m+i}$, $t = 1,2,3.....$, is the average of the original sequence in m non-overlapping blocks. Then the process X is defined as an exact second order self-similar process with the Hurst parameter, $H = 1-\beta/2$,

$$\text{if} \quad Var(X^{(m)}) = \sigma^2 m^{-\beta}, \forall m \geq 1. \tag{1}$$

Now we present the fundamentals of MMPP. MMPP is a doubly stochastic process in which arrival rate is given by $\lambda[J_t]$, where $J_t, t \geq 0$ is an m-state Markov process. The arrival rate can therefore take on only m values, namely $\lambda_1, \lambda_2, ..., \lambda_m$. It is equal to λ_j whenever the Markov process is in the state j, $1 \leq j \leq m.$ The MMPP is fully parameterized by the infinitesimal generator Q of the Markov process and the vector $\lambda = (\lambda_1, \lambda_2, ..., \lambda_m)$ of the arrival rates. Let Λ be the diagonal matrix with $\Lambda_{jj} = \lambda_j$, $1 \leq j \leq m.$ In the case of two states, Q and Λ are given as follows:

$$Q = \begin{bmatrix} -c_1 & c_1 \\ c_2 & -c_2 \end{bmatrix}, \qquad \Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}. \tag{2}$$

The mean arrival rate λ of MMPP is given by $\lambda = \vec{\pi}\Lambda e$, where $\vec{\pi}$ is the stationary probability vector of Q, i.e. $\vec{\pi}Q = 0, \vec{\pi}e = 1$ and e= [1, 1, 1…1] ^{T} is an all -1 column vector with designated dimension. If we let N_t, t ≥0, be the number of arrivals in (0, t], for the stationary MMPP, the Mean of N_t is

$$E[N_t] = \frac{c_2\lambda_1 + c_1\lambda_2}{c_1 + c_2} t. \tag{3}$$

The Variance of N_t is

$$Var[N_t] = E[N_t] + \frac{2c_1c_2(\lambda_1-\lambda_2)^2}{(c_1+c_2)^3} t - \frac{2c_1c_2(\lambda_1-\lambda_2)^2}{(c_1+c_2)^4} [1-e^{-(c_1+c_2)t}]. \tag{4}$$

The interesting feature of MMPP is that a superposition of MMPPs is still MMPP.

III. MMPP/M/1/K Queueing System

Asynchronous switch with self-similar variable length packet input traffic is modeled as $MMPP/M/1/K$ queuing system. In $MMPP/M/1/K$ system, the packets arrive according to the MMPP of states m and is characterized by the matrices Q, Λ, where Q, Λ are $m \times m$ matrices. The service time is exponential with service rate μ. Let D_k, $k \geq 0$ denotes the matrix of order $m \times m$ whose (i, j) element is the probability that given departure at time 0, which left at least one packet in the system and the process is in state i, the next departure occurs when the arrival process in j, and during that service time there were k arrivals. Then D_k satisfies the following equation:

$$\sum_{k=0}^{\infty} D_k z^k = \mu \int_0^{\infty} e^{[Q-\Lambda+\Lambda z]x} e^{-\mu x} dx \tag{5}$$

$$i.e. \sum_{k=0}^{\infty} D_k z^k = \mu\left(\mu I - \left(Q - \Lambda + \Lambda z\right)\right)^{-1}$$

$$i.e. \sum_{k=0}^{\infty} D_k z^k = \mu^2 \sum_{k=0}^{\infty} \left(\frac{Q - \Lambda + \Lambda z}{\mu}\right)^k, \tag{6}$$

where I is the unit matrix of designated dimension. Now we compute the D_k s by extending the methodology [9, 10] from the deterministic service time distribution to the exponential service time distribution. For $k = 0$ in (6), we have

$$D_0 = \mu^2 \left(I + \sum_{r=0}^{\infty} \left(\frac{Q-\Lambda}{\mu}\right)^r \right). \tag{7}$$

For $l, n \geq 1$, let $T(n,l)$ be the coefficient of z^{l-1} in $\mu^2 \left(\frac{Q-\Lambda+\Lambda z}{\mu}\right)^n$, n^{th} term of the series on right hand side of (6). From (6), we have

$$T(1,1) = \mu^2 \left(\frac{Q-\Lambda}{\mu}\right), \qquad T(1,2) = \mu^2 \frac{\Lambda}{\mu}$$

$$T(n,1) = \mu^2 \left(\frac{Q-\Lambda}{\mu}\right)^n, \; T(n,l) = 0, \text{ if } l > n+1, \text{ and}$$

$$T(n,1)+T(n,2)z+T(n,3)z^2+...+T(n,l)z^{l-1}+........+$$
$$T(n,n+1)z^n=\mu^2\left(\frac{Q-\Lambda+\Lambda z}{\mu}\right)^n.$$

Multiplying both sides by $\mu^2\left(\frac{Q-\Lambda+\Lambda z}{\mu}\right)$, we obtain

$$[T(n,1)+T(n,2)z+T(n,3)z^2+.....+T(n,l)z^{l-1}$$
$$+.....+T(n,n+1)z^n]\mu^2\left(\frac{Q-\Lambda+\Lambda z}{\mu}\right)=$$
$$T(n+1,1)+T(n+1,2)z+\ldots\ldots$$

Equating the coefficients of like powers of z, we obtain,

$$T(n+1,1)=T(n,1)\mu^2\left(\frac{Q-\Lambda}{\mu}\right),$$
$$T(n+1,2)=T(n,2)\mu^2\left(\frac{Q-\Lambda}{\mu}\right)+T(n,1)\mu^2\frac{\Lambda}{\mu},$$
$$\ldots \quad \ldots \quad \ldots \quad \ldots$$
$$T(n+1,q)=T(n,q)\mu^2\left(\frac{Q-\Lambda}{\mu}\right)+T(n,q-1)\mu^2\frac{\Lambda}{\mu}$$

$q \in \mathrm{N}$.

(8)

From (6), we have

$$D_s=\sum_{k=s}^{\infty}T(k,s+1),\ s=1,2,.. \tag{9}$$

We compute the matrices D_s s using the recurrence formulae (7) - (9). Now we consider the embedded Markov chain $\{L(n),J(n)/n\geq 0\}$ at the departure epochs of the queueing system $MMPP/M/1/K$ on the state space $S=\{(b,i)/0\leq b\leq K-1, 1\leq i\leq m\}$, where $L(n)$ denotes buffer occupancy and $J(n)$ denotes the state of MMPP. Then the pertaining embedded Markov chain has transition probability matrix:

$$P=\begin{bmatrix} GD_0 & GD_1 & ... & GD_{K-2} & GE_{K-1} \\ D_0 & D_1 & ... & D_{K-2} & E_{K-1} \\ 0 & D_0 & ... & D_{K-3} & E_{K-2} \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & D_1 & E_2 \\ 0 & 0 & ... & D_0 & E_1 \end{bmatrix}, \tag{10}$$

where $G=(\Lambda-Q)^{-1}\Lambda$, consisting of conditional probabilities that system is not busy and $E_i=\sum_{k=i}^{\infty}D_k$.

Let $\overrightarrow{y_k}$, $(0\leq k\leq K-1)$ be an $1\times m$ vector whose i^{th} element is the stationary conditional probability that the number of packets in the system is k and the state of underlying arrival process is in i at an arbitrary time. The packet loss probability (PLP) as given in the paper [11] is

$$PLP=1-\frac{\left(1-\overrightarrow{y_0}e\right)}{\rho}, \tag{11}$$

where $\rho=\frac{\lambda}{\mu}$, traffic intensity, and λ is the mean arrival rate of MMPP and is given by $\lambda=\vec{\pi}\Lambda e$.

Now we shall discuss the computational complexity of the stationary probability vector $\vec{y}=(\overrightarrow{y_0},\overrightarrow{y_1}.....\overrightarrow{y_{K-1}})$ of the transition probability matrix P following the method [12]. The matrix P is in canonical form of $M/G/1$ system. The stationary probability vector is given by $\vec{y}=[0,0,0,.....0,1][I-P_1]^{-1}$, where I is the unit matrix of appropriate dimension and P_1 is the matrix P in which the last column is replaced by $[-1,-1,-1,........0]^T$. Multiplying the permutation matrix S by $(I-P_1)$, where

$$S=\begin{bmatrix} 0 & I & 0 & ... & 0 & 0 \\ 0 & 0 & I & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 0 & I \\ I & 0 & 0 & ... & 0 & 0 \end{bmatrix}$$

and the product can be put in the following form

$$S(I-P_1)=\begin{bmatrix} A & B \\ C & D \end{bmatrix}.$$

Dimensions of the blocks of matrices A,B,C and D are $(K-1)m\times(K-1)m, (K-1)m\times m, m\times(K-1)m$ and $m\times m$, respectively. Using Schur-Banachiewicz formula for the inverse of block matrices leads to

$$(S(I-P_1))^{-1}=\begin{bmatrix} A^{-1}+E\Delta^{-1}F & -E\Delta^{-1} \\ -\Delta^{-1}F & \Delta^{-1} \end{bmatrix},$$

where $\Delta=D-CA^{-1}B$, the Schur complement of A, $E=A^{-1}B$, and $F=CA^{-1}$. The stationary probability vector is the last row of the matrix $(\Delta^{-1}\quad -\Delta^{-1}F)$ since $(I-P_1)^{-1}=(S(I-P_1))^{-1}S$. The matrix Δ is non-singular, if A is non-singular. The matrix A is upper triangular Toeplitz matrix whose inverse is easy to compute. The computation complexity to compute its inverse is of the order $O\left((K-2)^2m^3\right)$. The computation complexity to compute F is of the order $O\left((K-2)^2m^3\right)$. The computation complexities to compute FB and $\Delta^{-1}F$ are of the same order and equals to $O\left((K-2)\ m^3\right)$. Therefore, overall complexity to compute the stationary probability vector is $O\left((K-2)^2m^3\right)$.

IV. Numerical results

In this section, we present some numerical results of packet loss probability and investigate its behavior in terms of traffic intensity, time-scale, Hurst parameter H, and d. Also, mean waiting time against traffic intensity is presented. First, transition rate matrix Q and arrival rate matrix Λ of MMPP are fitted according to the generalized variance based method [5] for the self-similar traffic pertaining to the values $H = 0.7, 0.8, 0.9$, variance $\sigma^2 = 0.6$, arrival rate $\lambda = 1$ over the time-scales $[10^2, 10^5]$, $[10^2, 10^6]$, and $[10^2, 10^7]$ as in the paper [5]. Next, the stationary probability vector $\overrightarrow{y}$ is computed by using the method discussed in earlier section, and then packet loss probability is computed against traffic intensity using (11). Numerical calculations are performed using MATLAB and are compared with that of published tool SIRIUS ++ in [13], for the validation of MATLAB program. The difference between these two analytical results is found to be very small. These validated analytical results are compared with the simulation results of self-similar traffic. Self- similar traffic is generated by using ad-hoc discrete event simulator (DES) using raw C++ code. We use aggregated multiple sub-streams, each consisting of alternative Pareto-distribution on/off periods. The Pareto distribution is heavy-tailed distribution with the probability density function $f(x) = \frac{\alpha b^{\alpha}}{x^{\alpha+1}}$, $(x \geq b)$ has the degree of self-similarity (Hurst parameter) $H = \frac{(3-\alpha)}{2}$, where $\alpha (= 1.6)$ is a shape parameter, and $b (= 4000)$ is a location parameter. Results are depicted in Figs.1-4. For all the cases, variance σ^2, arrival rate λ of the traffic and buffer depth K are assumed to be 0.6, 1 and 10, respectively [5]. Fig.1 depicts packet loss probability as a function of traffic intensity over three time-scales $[10^2, 10^5]$, $[10^2, 10^6]$, and $[10^2, 10^7]$. Parameter settings for Fig.1 are $H = 0.7$ and $d = 4$. From the Fig.1, it is clear that as time-scale increases PLP decreases as in the papers [5, 9]. Fig.2. depicts the variation of packet loss probability as function of traffic intensity over the time-scale $[10^2, 10^7]$ for various H. In

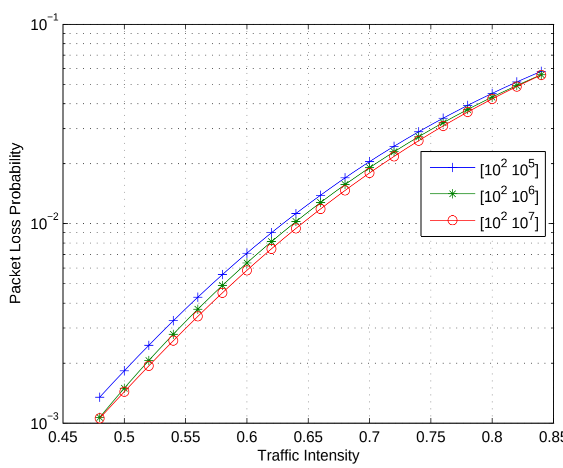

Fig.1. Variation of Packet Loss Probability with Traffic Intensity when H=0.7 and d=4.

the Fig.2, solid line represents analytical results whereas dotted line represents simulation results. It is clear from the figure that the difference between analytical results and for $H = 0.7, 0.8, 0.9$, over the time–scales $[10^2, 10^5]$ and $[10^2, 10^7]$, when $d = 4$. Fig. 3 infers that as self-similarity (H) and traffic intensity increase PLP increases. When (H) is greater, the values pertaining to $[10^2, 10^7]$ are greater than that of $[10^2, 10^5]$. However, when $H = 0.7$, the discrepancy is not seen. This indicates that time-scale matters when H is greater. Fig.4. depicts the packet loss probability as a function of traffic intensity over the time-scale $[10^2, 10^7]$ for the cases $d = 3$ and 4. From Fig.4, we could conclude that it suffices to superpose four IPPs as in [5, 7]. Mean waiting time (W) can be computed by means of queue length distribution as follows:

$$W = \frac{\rho}{\lambda} \sum_{k} k(\vec{y}_k e)$$

Mean waiting time against traffic intensity is computed for $H = 0.7$ and $d = 4$, results are compared with that of software Q -SQUARED, and are presented in Fig-5. It is clear that both the results are in agreement, and mean waiting time increases as traffic intensity increases as expected. From the above results, we can conclude that four IPP components over larger time-scale are enough to model the self-similar traffic. In all the above cases, we observe that packet loss probability and mean waiting time become large as ρ is getting larger. This tendency agrees with our intuition.

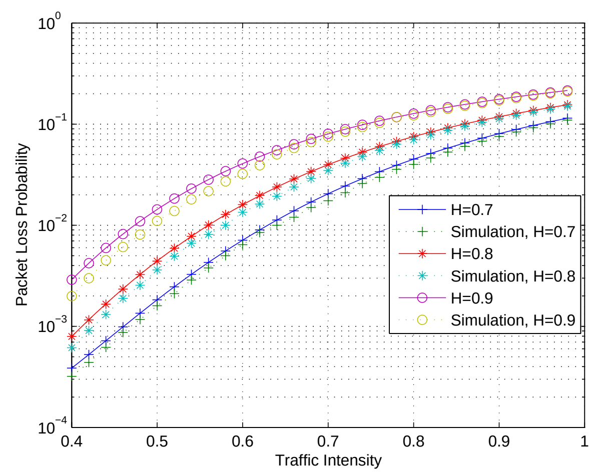

Fig.2. Variation of Packet Loss Probability with Traffic Intensity over the Time-Scale $[10^2, 10^7]$ when d=4.

V. Conclusions

In this paper, the asynchronous switch with the self-similar variable length packet traffic is modeled as $MMPP/M/1/K$ system, and then the loss behavior of said queuing system is investigated, where MMPP is generated so as to match the variance of self-similar traffic over a time-scale. In this model, service time distribution is considered as exponential rather than deterministic, since IP packets are, in general, variable in length. Our numerical results reveal that time-scale and number of components in fitting (d) do have impact on packet loss probability. Packet loss probability increases as H and ρ increase. Based on the analysis presented in this paper, one could select the appropriate time-scale in the generalized variance based fitting method [5] to meet the QoS requirement. This kind of analysis is useful in dimensioning the switch under self-similar variable length packet traffic.

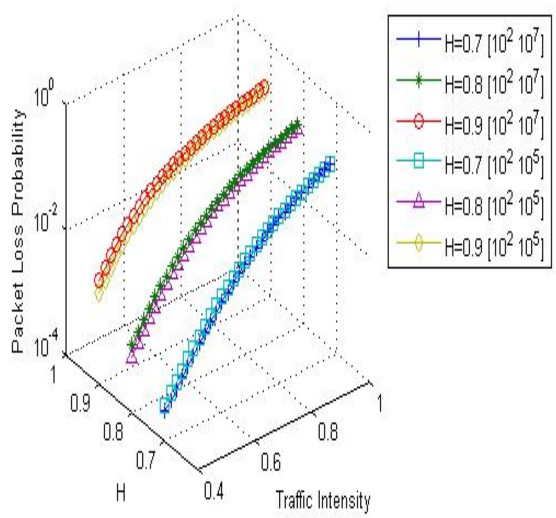

Fig.3. Variation of Packet Loss Probability with Traffic Intensity and H when $d = 4$.

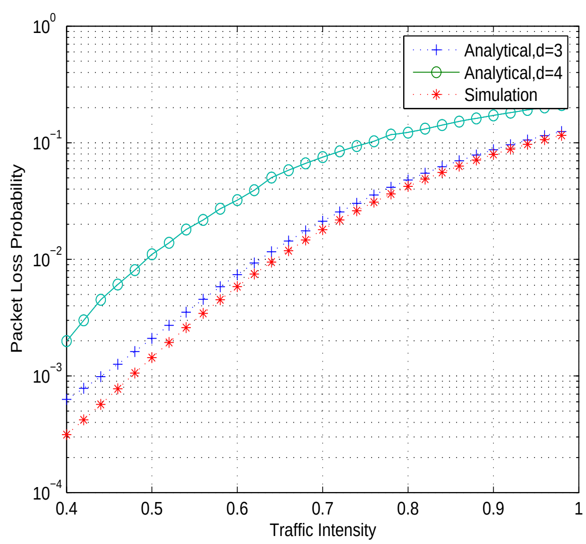

Fig.4. Variation of Packet Loss Probability with Traffic Intensity over the Time-Scale [10^2,10^7] when d=3, 4 for analytical results and when H= 0.7

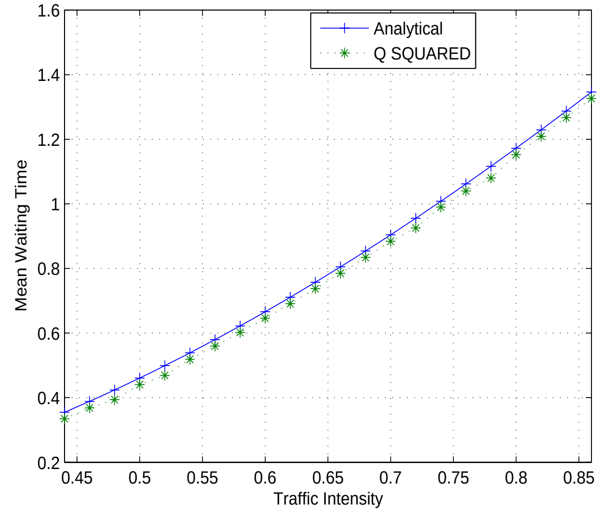

Fig. 5.Variation of Mean Waiting Time with Traffic Intensity over the Time-Scale [10^2,10^7], when H= 0.7, d=4.

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