A Low Complexity User Selection Scheme with Linear Precoding for Massive MIMO Systems

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***Abstract*—Massive MIMO is the future of wireless communication systems as it promises 1000 times increase in data rates per cell. With large number of base station (BS) antennas, simple linear precoding schemes like zero forcing precoding would become computationally expensive due to the inversion of large channel matrices. User selection is an approach which extracts the effective part of the channel matrix. Additionally in conjunction with linear precoding schemes, user selection is an efficient way to maximize the sum rate per cell at the reduced computational cost. Computational complexity of existing multi-user selection approaches grows cubically with the total number of users (K) or double with the number of base station antennas (M). Very little focus has been drawn for user selection with massive MIMO assumptions. In this paper, we propose a low complexity greedy user selection scheme with linear zero forcing precoding at the base station. Complexity of our proposed algorithm is remarkably low i.e. $O(MK)$ while the performance is same as that of best greedy algorithm known so far.**

***Index Terms*—Massive MIMO, channel hardening, user selection, linear precoding, subset selection.**

I. INTRODUCTION

CURRENTLY research efforts in defining the 5G communication systems architectures are at their peak. Recent 4G mobile communication systems are unable to meet future multi-giga bit data rate demands. Multiple disruptive research directions have already been defined that promise a multitude (around 1000 times) increase in data rates [1]. Massive MIMO is one among many proposals that are considered as major enabling technologies for 5G. Massive MIMO is an alternative name of conventional multiuser MIMO (MU-MIMO) systems, but with very large number of BS antennas. Massive MIMO could be realized in two distinct radio frequency (RF) architectures depending upon the carrier frequencies of the MIMO channel. At centimeter waves massive MIMO, BS antenna elements are connected with low cost and fully digitized RF modules [2]. These RF modules are controlled by very simple signal processing algorithms such as linear precoding [3] and matched filtering techniques. At millimeter waves (mmwave), BS antenna arrays are considered to be connected with very few RF chains with either analog RF components only or hybrid architectures with both analog and digital components [4].

In MU-MIMO downlink system, different precoding techniques in conjunction with user selection are used to maximize the aggregate rate of the systems [5]. User selection is a combinatorial NP-Hard problem [6] and apart from expensive exhaustive search, optimal solutions are very less understood. Authors in [5], have proposed user selection techniques based on exhaustive combinatorial search. These algorithms become computationally prohibitive particularly when the number of users becomes large. Proposed methods like zero-forcing precoding based selection (ZFS) in [7], semi-orthogonal user selection (SUS) in [8] and greedy user selection with swap (GUSS) [9] operate at the computational cost of $O\left(MK^3\right)$. Cubic increase in the complexity with the number of users per cell makes these algorithms impractical for dense multiuser MIMO scenarios such as stadiums and airports. On the other hand complexity of decremental and incremental algorithms in [10], [11] is approximately around $O\left(M^2KL_s\right)$ where $L_s \leq K$ corresponds to the number of selected users. Two step selection approach in [12] operates at computational cost $O\left(M^2L_sr\right)$ where r corresponds to the number of required iterations to reach near optimal results. One may note that complexity of selection algorithms in [10]–[12] increases quadratically with M. Hence, for massive MIMO systems with large BS antennas such algorithms are computationally prohibitive. Delete minimum lambda (DML) algorithm proposed in [13] initializes by selecting all users. Under a certain channel propagation and SNR setup, it greedily deletes users with minimal channel gain till the aggregate rate reaches its maximum value. Computational cost of DML is $O\left(MK^2\right)$ that makes it suitable for massive MU-MIMO scenarios where large number of users per cell need to be selected.

Apart from complexity issues, arguments in [14] have pointed out that large number of BS antennas lead towards *channel hardening* in massive MU-MIMO system. In other words, it means that all users can be spatially separated and hence all time and frequency resources can be allocated equally to each user. Additionally, if channel hardening assumption is taken into account, MU-MIMO sum rate formulas become independent of number of BS antennas.

With the best of our knowledge, channel hardening assumption is yet quite optimistic as multiple channel propagation properties play a vital role in defining inter user correlation setup. In Section II, multiple propagation aspects that make channel hardening far from reality have been briefly discussed. This survey study is based on some recent results obtained from realistic and model based channel evaluations. Considering the studies in Section II as the primary motivation, this paper proposes a greedy user selection algorithm that performs nearly the same as that of best greedy algorithm known from literature at a remarkably low computational cost i.e. $O(MK)$. A detailed step by step complexity analysis of the algorithm is also shown.

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II. CHANNEL HARDENING: SOME CONTRADICTING ASPECTS

If $\boldsymbol{H} \in \mathbb{C}^{M \times K}$ is a massive MIMO downlink channel matrix then mathematically *channel hardening* implies that the off-diagonal entries of the instantanous covariance matrix $\boldsymbol{H}^H \boldsymbol{H}$ become significantly lower than diagonal entries of it. This implies that singular value spread of the covariance matrix converges to zero. One may note that singular value spread of a channel matrix is highly sensitive to correlation setup of the channel. Below some important spatial correlation defining aspects have been discussed.

A. Number of multipath clusters

During the channel modeling process, all the multipaths are normally grouped into clusters for simplification purpose. Studies in [15] show that the number of multipath clusters have significant effect on defining the correlation setup of MIMO channels. Results show that channel spatial correlation increases with reduction of number of multipath clusters whereby it decreases with an increase in number of clusters.

B. Angular spreads of clusters

In [15], it has also been reported that lower angular spread of the cluster tend to increase the spatial correlation whereby it reduces with higher angular spreads.

C. Polarimetric structures of antenna arrays

Channel measurement results in [16] are quite encouraging and have shown that channel hardening indeed works. Authors have conducted a measurement campaign with dual polarized (DP) cylindrical array with 128 antenna ports at the BS. Results show that 8 closely located LOS users can be spatially separated with 128 BS antenna ports. However, this comes at the cost of 128 expensive RF chains at the BS. Therefore, same authors have proposed suboptimal BS (transmit) antenna subset selection technique in [17] which is again a NP-Hard problem.

One may also expect that due to cylindrical shape of the BS antenna array, multiple distinct clusters may arrive at the MS. Though encouraging, but authors of [16] have not described the number of multipath clusters and their corresponding angular spreads. If it is a rich multi-clustered channel, then offcourse users can be spatially separated, but in an unlucky situation it may also be possible that the multipath clusters are significantly reduced (hence increased correlation) which can make spatial separation of users more difficult as channel hardening no longer exist.

In contrast to DP base station antenna array in [16], authors of [18], [19] have reported correlation results of single and dual polarized antenna arrays in an urban macro scenario. Results show that in LOS scenario spatial correlation is more than 0.9 whereby it reduces with dual polarized antenna array.

From the discussion above, it can concluded that channel hardening depends upon the polarization and structure of antenna arrays. If a circular or cylindrical massive antenna array is chosen at the base station, other geometrical properties of the channel such as number of clusters and their angular spreads can make channel hardening hard to realize.

D. Sum rate maximization

Assuming that number of BS antenna elements are fixed, this paper shows that channel hardening work up a particular number of users i.e. all users can be selected. However, this *select all* strategy does not ensure maximum cell throughput. Therefore, depending upon the channel spatial correlation setup, link adaption techniques like user selection are important to maximize the aggregate rates per cell.

III. SYSTEM MODEL

Let M be the number of antennas at the BS and K be the number of users in the cell. The downlink MU-MIMO channel is defined as

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{h}_1 & \boldsymbol{h}_2 & \cdots & \boldsymbol{h}_K \end{bmatrix} \quad \in \mathbb{C}^{M \times K} \tag{1}$$

where, $\boldsymbol{h}_i \in \mathbb{C}^{M \times 1}$ is the channel impulse response between the BS and the i^{th} user. It is assumed that the BS is equipped with $K \leq M$ RF chains and has perfect knowledge of MU-MIMO downlink channel $\boldsymbol{H}$. Ideally, if the channel is fully uncorrelated, a full complexity MIMO system should serve $\min(M, K)$ mobile users. For simplicity, it is assumed that the total transmit power P is distributed equally to each transmitted data symbol s_i to the i^{th} user. The receive signal y_i is

$$y_i = \begin{bmatrix} \boldsymbol{w}_1 s_1 & \boldsymbol{w}_2 s_2 & \cdots & \boldsymbol{w}_i s_i & \cdots & \boldsymbol{w}_K s_K \end{bmatrix}^T \boldsymbol{h}_i + \boldsymbol{n}_i. \tag{2}$$

where, $\boldsymbol{w}_i \in \mathbb{C}^{M \times 1}$ is the weight vector used to transmit data symbol s_i to the i^{th} user and $\boldsymbol{n}$ is the uncorrelated Additive White Gaussian Noise (AWGN).

A. User selection in Massive MIMO systems at micorwave frequency band

At microwave frequency bands, the number of BS antennas and the low cost fully digitized RF chains are realized to be equal. This implies that ideally $\min(M, K)$ data streams can be transmitted under the assumption that channel is fully uncorrelated. If the channel is correlated, the number of transmitted data streams depend upon the rank of the channel. With increasing spatial correlation, rank of MIMO channel will reduce accordingly and transmission of lower number of data streams can maximize the data rate [20]. If the channel has very low rank, decremental selection scheme proposed in [13] would be computationally prohibitive. One may note that high spatial correlation assumption is quite reasonable, particularly when the physical size of MIMO antenna array becomes small.

B. User selection in Massive MIMO systems at millimeter wave frequency bands

Due to design and implementation cost constraints, the number of RF chains are realized to be far much lower than BS antennas at mmwave frequency bands. Hence, only few users may be served at a particular time snapshot. This contradicts to the large number of users selection assumption in [13] and hence DML algorithm of [13] would become computationally expensive as large number users need to be discarded.

IV. USER SELECTION WITH LINEAR ZERO FORCING PRECODING

Assuming that the zero-forcing precoding is used at the BS with equal power allocation per user, the post processing SNR of the i^{th} user is defined as

$$\text{SNR}_i^{\text{ZF}} = \frac{P}{N_0 K \left[\left(\boldsymbol{H}^H \boldsymbol{H} \right)^{-1} \right]_{i,i}} \tag{3}$$

Taking the QR decomposition of $\boldsymbol{H}$, Eq. (3) becomes

$$\begin{aligned} \text{SNR}_i^{\text{ZF}} &= \frac{P}{N_0 K \left[\left(\boldsymbol{R}^H \boldsymbol{Q}^H \boldsymbol{Q} \boldsymbol{R} \right)^{-1} \right]_{i,i}} \\ &= \frac{P}{N_0 K \left[\left(\boldsymbol{R}^H \boldsymbol{R} \right)^{-1} \right]_{i,i}} \end{aligned} . \tag{4}$$

In Eq. (4), $\boldsymbol{R}$ is an upper triangular matrix and can be expressed as

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{0} & \boldsymbol{C} \end{bmatrix} . \tag{5}$$

where, $\boldsymbol{A} \in \mathbb{C}^{L_s \times L_s}$, $\boldsymbol{B} \in \mathbb{C}^{L_s \times (K-L_s)}$ and $\boldsymbol{C} \in \mathbb{C}^{(M-L_s) \times (K-L_s)}$. If the effective rank of matrix $\boldsymbol{H}$ is low, it is quite likely that selecting of all K users in a cell may not result in maximum sum rate. Therefore, it is often required to distribute power only among $L_s \leq K$ selected users under the condition that the selected channel submatrix is as far away from singularity as possible. Let $\boldsymbol{H}_s$ and $\boldsymbol{H}_d$ are the channel impulse responses matrices of selected and discarded users respectively

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_s & \boldsymbol{H}_d \end{bmatrix} . \tag{6}$$

The QR decompositions of $\boldsymbol{H}_s$ and $\boldsymbol{H}_d$ in Eq. (6) are coupled with the QR decomposition of full multiuser MIMO channel $\boldsymbol{H}$ such that

$$\boldsymbol{H}_s = \boldsymbol{Q} \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{0} \end{bmatrix} \qquad \text{and} \qquad \boldsymbol{H}_d = \boldsymbol{Q} \begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{C} \end{bmatrix}$$

This implies that if two columns i and j are permuted between matrices $\boldsymbol{H}_s$ and $\boldsymbol{H}_d$ inside the matrix $\boldsymbol{H}$, it corresponds to the same permutation of columns i and j inside $\boldsymbol{R}$. The post processing SNR of the i^{th} user is defined as

$$\text{SNR}_i^{\text{ZF}} = \frac{P}{N_0 L_s \left[\left(\boldsymbol{H}_s^H \boldsymbol{H}_s \right)^{-1} \right]_{i,i}} \tag{7}$$

The total post equalization $\text{SNR}_{\text{Tot}}^{\text{ZF}}$ can now be written as

$$\begin{aligned} \text{SNR}_{\text{Tot}}^{\text{ZF}} &= \frac{P}{N_0 L_s \left(\text{Tr} (\boldsymbol{H}_s^H \boldsymbol{H}_s)^{-1} \right)} \\ &= \frac{P}{N_0 L_s \left(\text{Tr} (\boldsymbol{A}^H \boldsymbol{A})^{-1} \right)} \end{aligned} \quad \because \boldsymbol{H}_s = \boldsymbol{Q} \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{0} \end{bmatrix} \tag{8}$$

Note that the total post equalization $\text{SNR}_{\text{Tot}}^{\text{ZF}}$ in Eq. (8) can be maximized by minimizing the trace $\text{Tr} \left(\left(\boldsymbol{A}^H \boldsymbol{A} \right)^{-1} \right)$. i.e.

$$\arg\min \left(\text{Tr} \left(\left(\boldsymbol{A}^H \boldsymbol{A} \right)^{-1} \right) \right) \tag{9}$$

Taking the singular value decomposition (SVD) of $\boldsymbol{A}$, and using the property $\text{Tr}\left(\boldsymbol{X}\boldsymbol{Y}\right) = \text{Tr}\left(\boldsymbol{Y}\boldsymbol{X}\right)$, we can write

$$\begin{aligned} \text{Tr} \left(\left(\boldsymbol{A}^H \boldsymbol{A} \right)^{-1} \right) &= \text{Tr} \left(\left(\boldsymbol{V}_s^H \boldsymbol{\Sigma}_s^H \boldsymbol{U}_s^H \boldsymbol{U}_s \boldsymbol{\Sigma}_s \boldsymbol{V}_s \right)^{-1} \right) \\ &= \text{Tr} \left(\boldsymbol{\Sigma}_s^{-1} \left(\boldsymbol{\Sigma}_s^H \right)^{-1} \right) \\ &= \sum_{i=1}^{L_s} \frac{1}{\mu_i^2}, \text{where} \quad \mu_i \left(\boldsymbol{A} \right) = \mu_i \left(\boldsymbol{H}_s \right) \\ &= \left\| \boldsymbol{H}_s^{+} \right\|_F^2 \\ &= \left\| \boldsymbol{A}^{-1} \right\|_F^2 \end{aligned} \tag{10}$$

where, μ_i corresponds to the i^{th} singular value of $\boldsymbol{H}_s$ or $\boldsymbol{A}$. Eqs. (8) and (10) show that post equalization SNR of L_s selected users is maximized by selecting a subchannel matrix with minimum $\left\| \boldsymbol{H}_s^{+} \right\|_F^2$. An optimal selection of subchannel matrix $\boldsymbol{H}_s$ requires exhaustive search over all $\binom{K}{L_s}$ possible matrix inversions which have exponential computational complexity.

A. *Sum rate with zero forcing beamformer*

The total sum rate $R_{\text{Tot}}^{\text{ZF}}$ of selected users with zero-forcing beamforming at the BS is defined in [5] and can be written as

$$\begin{aligned} R_{\text{Tot}}^{\text{ZF}} &= \sum_{i=1}^{L_s} \log_2 \left(\text{SNR}_i^{\text{ZF}} \right) \\ &= L_s \log_2 \left(\frac{P}{N_0 L_s} \right) + \sum_{i=1}^{L_s} \log_2 \left(\frac{1}{\left[\left(\boldsymbol{A}^H \boldsymbol{A} \right)^{-1} \right]_{i,i}} \right) \end{aligned} \tag{11}$$

V. PROPOSED MULTIUSER SELECTION SCHEME

A computationally efficient greedy algorithm is proposed. The algorithm selects columns (users) in $\boldsymbol{H}_s \in \boldsymbol{H}$ such that all selected column are *as non singular as possible*. Basic methodology is based on the identities discussed in Section IV. Proposed algorithm starts with an empty set $\boldsymbol{S} = \Phi$. It then greedily selects columns indices J_s such that frobenious norm $\left\| \boldsymbol{H}_l^{+} \right\|_F^2 = \left\| \boldsymbol{A}^{+} \right\|_F^2 = \sum_{i=1}^{L_s} \frac{1}{\mu_i^2}$ is minimized. After the selection of each column, its projections are removed from rest of the elements of $\boldsymbol{H}$.

1) Compute the Euclidean norms of each column of full channel matrix $\boldsymbol{H}$.
2) Select a column J_s with largest norm $\boldsymbol{w}_j, \forall j \notin \boldsymbol{S}$.
3) Compute an orthogonal matrix $\boldsymbol{Q}_n$ to re-triangularize $\boldsymbol{R}_n$ matrix.
4) Compute $\boldsymbol{A}_n^{-1}$ using backward substitution and calculate change in the sum rate as $\Delta R^{\text{ZF}} = R_n^{\text{ZF}} - R_{n-1}^{\text{ZF}}$.
5) If $\Delta R^{\text{ZF}} > 0$, then update $\boldsymbol{w}_j \leftarrow \left\| \boldsymbol{h}_j \right\|^2 - \left(\boldsymbol{h}_j^H \boldsymbol{h}_{J_s} \right)^2 \left\| \boldsymbol{h}_{J_s} \right\|^{-2}$ for all $j \notin \boldsymbol{S}$ and go back to step 2, otherwise terminate.

The steps of the proposed Algorithm 1 are inspired from Golub-Businger algorithm [21] for computing QR decomposition using column pivoting strategy.

A. *Computational complexity*

Let a floating point operation (flop) corresponds to the any $\times, \div, +, -$ operations. The computational cost of each step of Algorithm 1 is computed. Initialization at Line 2 requires the computations of K Euclidean norms which costs a total

Algorithm 1 Greedy user selection algorithm

1: **procedure** MAXVOL($\boldsymbol{H}$)
2: $\boldsymbol{S} \leftarrow \Phi, n = 0$ and compute Euclidean norms $\boldsymbol{w}_j$ of each column of $\boldsymbol{H}$.
3: **while** $\Delta R^{\text{ZF}} > 0$ **do**
4: Find $J_s = \arg\max \{\boldsymbol{w}_j : j \notin \boldsymbol{S}\}$.
5: $\boldsymbol{S} \leftarrow \boldsymbol{S} \cup J_s$
6: Compute an orthogonal matrix $\boldsymbol{Q}_n$ for upper triangularization of $\boldsymbol{A}_n$.
7: Compute $\boldsymbol{A}_n^{-1}$.
8: Compute $\Delta R^{\text{ZF}} = R_n^{\text{ZF}} - R_{n-1}^{\text{ZF}}$.
9: Update $\boldsymbol{w}_j$ as $\boldsymbol{w}_j \leftarrow \|\boldsymbol{h}_j\|^2 - \left(\boldsymbol{h}_j^H \boldsymbol{h}_{J_s}\right)^2 \|\boldsymbol{h}_{J_s}\|^{-2}$ for all $j \notin \boldsymbol{S}$
10: **end while**
11: **return** R_n^{ZF}
12: **end procedure**

of $K(2M-1)$ flops. Consider the case when $L_s > 1$ and a new column is inserted into $\boldsymbol{A}$. As a result $\boldsymbol{A}$, becomes an upper Hessenberg matrix. The fast Givens rotations [22] is used to zero out the elements below the main diagonal in $\boldsymbol{R}_n$. Let $\boldsymbol{Q}_n$ be an orthogonal matrix formed as the product of the Givens rotation matrices. Let

$$\boldsymbol{Q}_n = \begin{pmatrix} \boldsymbol{Q}_A & \boldsymbol{Q}_C \end{pmatrix} \tag{12}$$

where, $\boldsymbol{Q}_A$ and $\boldsymbol{Q}_C$ are used to zero out the elements below the diagonals in the matrices $\boldsymbol{A}$ and $\boldsymbol{C}$ respectively.

$$\boldsymbol{R}_{n+1} = \boldsymbol{Q}_n \boldsymbol{R}_n = \begin{pmatrix} \boldsymbol{Q}_A \boldsymbol{A}_n & \boldsymbol{Q}_A \boldsymbol{B}_n + \boldsymbol{Q}_C \boldsymbol{C}_n \end{pmatrix} \tag{13}$$

As $\boldsymbol{A}$ is an upper Hessenberg matrix, therefore the cost of computing both $\boldsymbol{Q}_A \boldsymbol{A}_n$ and $\boldsymbol{Q}_A \boldsymbol{B}_n$ is less than $3L_s(2K - L_s)$ flops. The cost of computing $\boldsymbol{Q}_C \boldsymbol{C}_n$ is around $4(M - L_s)(K - L_s)$ flops. After re-triangularization, $\boldsymbol{A}$ becomes an upper triangular matrix and hence, the computation of $\boldsymbol{A}^{-1}$ using backward substitution requires L_s^2 flops. Note that in Line 9, norms $\|\boldsymbol{h}_j\|$ and $\|\boldsymbol{h}_{J_s}\|$ are known from earlier iteration. Therefore, update of $\boldsymbol{w}_j$ at Line 9 only requires computation of vector-vector product $\boldsymbol{h}_j^H \boldsymbol{h}_{J_s}$ which requires $2M(K - l)$ flops. Let C_p corresponds to the computational complexity of the proposed algorithm then the total cost of our algorithm is summation of all flops defined for each step of Algorithm 1.

$$\begin{aligned} C_p &= K(2M-1) + \ldots \\ &\quad \sum_{l=1}^{L_s} 3l(2K-l) + 2(K-l)(3M-2l) + l^2 \\ &= K(2M-1) + \sum_{l=1}^{L_s} 2Kl + 6(MK - Ml) + 2l^2 \\ &= K(8M-1) + \sum_{l=1}^{L_s} 2Kl - 6Ml + 2l^2 \end{aligned} \tag{14}$$

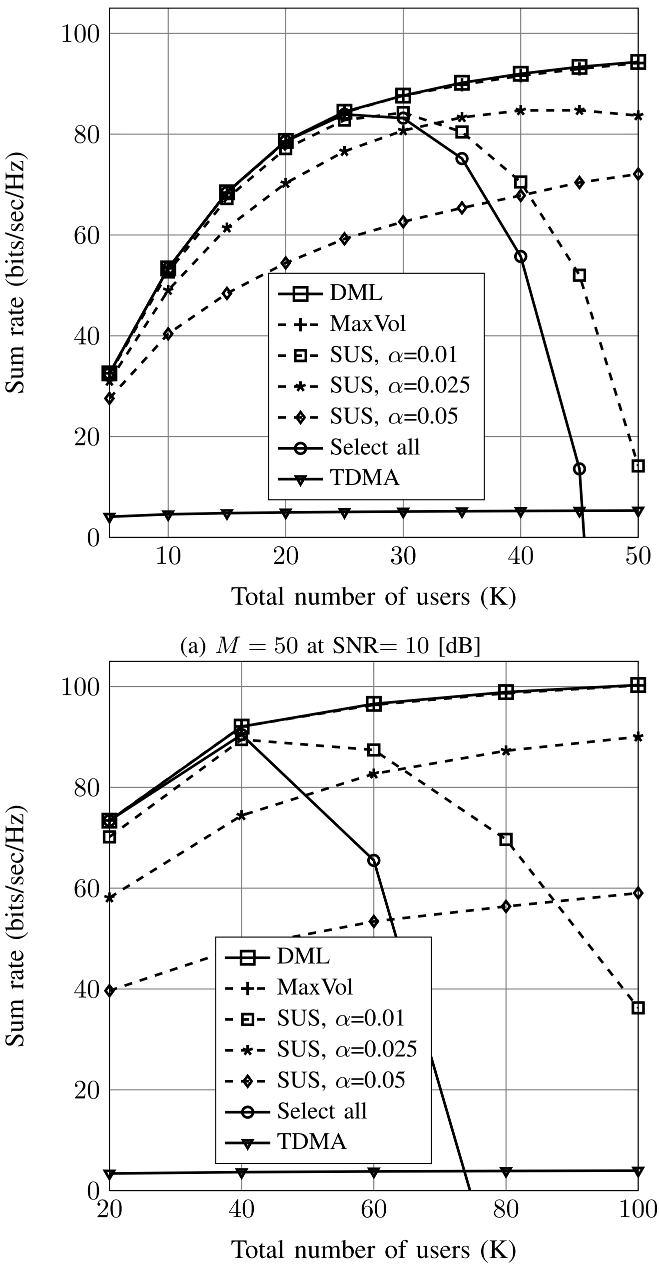
Considering a massive MIMO scenario [2] with a very large number of BS antennas i.e. $M \gg 1$, the summation term in Eq. (14) $\sum_{l=1}^{L_s}(\bullet)$ asymptotically becomes less significant due to subtraction of factor $6Ml$. Hence, the overall complexity can be approximated as

$$C_p \approx 8MK \tag{15}$$

whereby the effective computational order is $O(MK)$.

VI. SIMULATION RESULTS

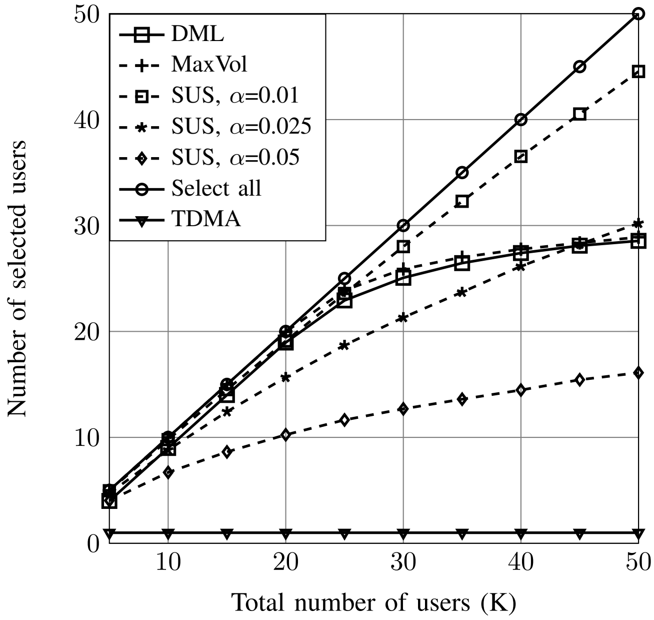
In this section, the proposed algorithm is compared with existing suboptimal user selection algorithms. The literature study revealed that greedy ZFS algorithm [7] is considered to be the performance benchmark. DML algorithm in [13] has shown the same performance as that of ZFS with far much lower computational cost. GUSS algorithm in [9] is slight extension of same DML algorithm with very little performance gain. Complexity of SUS algorithm [8] is also comparable to ZFS and GUSS. Table I shows that complexity of ZFS, GUSS, SUS and DML depends little on the BS antennas as compared to the total number of users. Therefore, for massive MIMO systems with very large M, these algorithms are quite scalable. On the other hand, complexity of algorithms [10]–[12] doubles with M. For massive MIMO applications these algorithms become computationally expensive. Therefore,



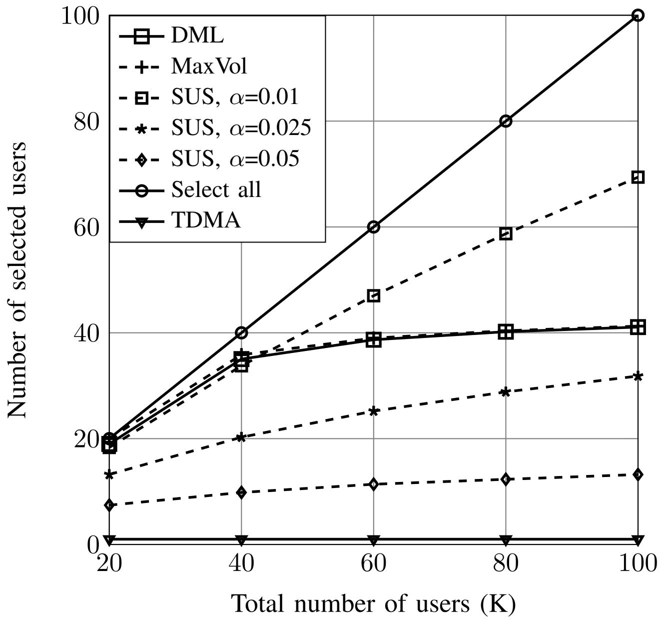
(a) $M = 50$ at SNR= 10 [dB]

(b) $M = 100$ at SNR= 5 [dB]

Fig. 1: Average sum rate vs. total number of users (K)



(a) $M = 50$ at SNR= 10 [dB]



(b) $M = 100$ at SNR= 5 [dB]

Fig. 2: Average sum rate vs. total number of users (K)

for simplicity only DML and SUS algorithm are used as performance benchmarks.

Standardized antenna independent channel models like SCM and WINNER [23], [24] does not support very large antenna arrays beyond the stationarity interval and spherical waves [25]. Generally algorithms for massive MIMO systems are evaluated at flat faded, independent identically distributed (i.i.d.) Rayleigh channel. Such channel model assumption is quite reasonable as superposition of large number of faded channels from each BS antenna will nearly result in i.i.d like channel. Therefore, due to the unavailability of proper channel models, monte carlo simulations of Rayleigh flat fading channel have been used for performance evaluation in this paper.

Results in Figs. 1a and 1b show that the proposed MaxVol algorithm shows the same performance behavior as that of DML algorithm. Optimality and complexity of classical SUS algorithm depends upon the parameter α. Results show SUS algorithm is efficient with smaller values of α for the cases when $K \leq M$. As $K \to M$, SUS algorithm losses its efficiency drastically with small α. Note that when $K \to M$ larger values of α result in better performance. Therefore, for SUS algorithm, optimal values of α needs comprehensive investigation. Sum rates provided by time division multiple access schemes are not convincing and hence may not be the use case in large scale MIMO systems. Results in Fig. 1 show that *select all* is clearly not an optimal strategy for massive MIMO systems. It means that when the total number of users in a cell grow large (i.e. $K \to M$), user selection becomes more and more important to maximize the cell throughput.

Design of DML is inspired by the *select all* strategy. DML assumes that, as only few users need to be discarded, therefore a greedy decremental selection approach would be more appropriate. Results in Fig. 2 show a comparison of number of selected users for the plots shown in Fig. 1. In Fig. 2a, one may note that as $K \to M$, selection of only 28 users maximize the sum rate. Hence, DML selection approach must have to delete 22 users in order to ensure maximum sum rate. Similarly in Fig. 2b, one can observe that as $K \to M$, DML deletes 60 out 100 users to get final ≈ 40 users to ensure maximum sum rate. These observations show that DML becomes computationally inefficient as K grows large. Also note that DML algorithm initializes with highly expensive pseudo inverse of full channel $\boldsymbol{H}$. The proposed MaxVol algorithm starts with the inversion of a single column of channel matrix and greedy updates the selected channel till there is no further increase in sum rate. Results in Figs. 2a and 2b show that for small $\alpha = 0.01$, SUS algorithm selects large number of users with high multiplexing gain but the over all sum rate reduces as $L_s \to K$. For considerably larger values of $\alpha = 0.025, 0.05$, SUS selects fewer number of users but the subset selection efficiency of SUS algorithm is not good enough. Reason is that SUS algorithm is based on Gram-Schmidt orthogonalization which has poor numerical properties [22].

VII. CONCLUSION

With large number of BS antennas, it is largely assumed that multiple users can be spatially separated and hence no multiuser interference exist between users in the cell. In contrast, this paper presents some aspects which prevent spatial separation of users. User selection techniques are effective tools to maximize the overall sum rate of the system. Therefore, performance and complexity of existing user selection approaches with linear zero forcing precoding have been analyzed. Complexity of existing approaches grows significantly with an increase in the number of BS antennas or the number of cell users. Therefore, the paper propose a low complexity algorithm with same performance as that of the best greedy algorithm known from literature. Main advantage of our algorithm is in computational cost that is $O(MK)$ which is remarkably lower than existing algorithms.

TABLE I: Summary of computational complexity of different selection algorithms

Algorithm	Complexity
Optimal [5]	# of searched sets $\times O\left(MK^2\right)$
ZFS [7]	$O\left(MK^3\right)$
SUS [8]	$O\left(\frac{7}{6}MK^3\right)$
GUSS [9]	$O\left(\frac{1}{2}MK^3\right)$
Decremental [10]	$O\left(M^2KL\right)$
Incremental [11]	$O\left(M^2KL\right)$
Two-step [12]	$O\left(M^2Lr\right)$, r=# of iterations
DML [13]	$O\left(MK^2\right)$
MaxVol (Proposed)	$O\left(MK\right)$

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