Development of a Semi-definite Programming Weighted Sum Based Approach for Solving Stochastic Multi-objective Economic Dispatch Problems Incorporating CHP Units

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***Abstract*—This paper has presented a weighted sum based semidefinite programming (SDP) optimization technique for solving stochastic multi-objective economic dispatch (MOED) model that incorporates Combined Heat and Power (CHP) units. The stochastic multi-objective model was transformed into its deterministic equivalent through their expectation, with the assumption that involved random variables are normally distributed. The multi-objective problem was recast in matrix form as a SDP relaxation problem and subsequently solved with a MATLAB programming suite. The system inequality and equality constraints uncertainty were entered into YALMIP, which is a linear matrix inequality parser. Simulations were performed on modified IEEE 6 and 20 units' networks with 2 CHP units. The efficiency of the proposed method is determined by investigating reformulated problems in stochastic and deterministic models on power dispatch. The standard weighted sum method is utilized in generating the Pareto-optimal solution between two objectives' functions. An optimal selection of control weight selection k_1 parameter that provides a better convergence property and moderately good extent of the Pareto distributions was empirically determined. The proposed SDP method performed well in accuracy of results and providing lower operational cost in the Pareto set produced.**

***Index Terms*—SDP, stochastic Multi-objectives problem, Pareto Distribution.**

I. Introduction

THE reduction of operational cost of power production in electrical power system analysis can be simply referred to as economic dispatch (ED) [1]. Thus, the problem in economic dispatch becomes multi-objective optimization when two or more objectives' functions are considered in the optimization model such as the total running fuel cost and the total emission are to be minimized simultaneously by adjusting the output power of every single generator while meeting the load demand and satisfying the system's constraints.

However, in recent years, cogeneration unit popularly known as combined heat and power (CHP) unit has become an essential energy production technology in many countries due to its advanced efficiency in the production of total energy which can produce sufficient different heat and power generation. Moreso, the heat generated by CHP units can be used for heating or industrial purposes. It is more important to know that the load demand is unstable in nature [2]. Therefore, problems in CHP are usually formulated as stochastic model and based on this, different power dispatch plans can be modeled. The main objectives in CHP units dispatch are to ensure sufficient production of power and heat, and fuel costs minimization.

Some of the works that are relevant to this study are hereby presented. Particle Swarm Optimization method for solving Stochastic Multi-Objective Dispatch Problems has been reported in [2], [3], [4], [5], [6], [7], [8], [9], [10], [11].

A bi-objective economic dispatch model incorporating wind power units has been formulated in [12], whereby operational cost and security effects are considered as conflicting objectives. Some evolutionary optimization methods based on stochastic searching techniques have been presented in [13], [14], [15], [16], [17], [18] to achieve optimal power flow problems resolutions. Problems such as smooth, non-smooth and piecewise fuel cost objectives were considered in the presented works.

Quasi-oppositional teaching learning based optimization (QOTLBO) has been proposed by [19] to solve non-linear multi-objective economic emission dispatch (EED) formulated problem of electric power generation with valve point loading. Also, recent studies on optimal power flow problems have been solved by the hybridization of stochastic searching-based optimization techniques proposed by [20], [21]. Similarly, a study on PID controller design for an automatic generation control of multi-area power units network has been addressed in [22] using Firefly algorithm. A three level decomposition technique has been presented in [23] for solving problems on ac-unit commitment and a robust commitment schedule to resist the stochastic wind power generation. More so, a mixed integer linear programming has been proposed for solving multi-carrier power systems problems presented in [24].

An optimization approach based on generalized bender decomposition has been presented in [25] for solving voltage problems in transmission and distribution networks of distributed generation units such as wind and solar power systems. Furthermore, a direct search optimization technique has been presented in [26] for the reduction of fuel cost taken into account the inter-area power flow and reserve capacity constraints.

Many techniques involved in handling the multi-objective

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CHP problem (both deterministic and stochastic) are all heuristic in nature [27]. These evolutionary methods are population based algorithms and can generate a number of solutions over several runs. However, because they are stochastic in nature, the attainment of the Pareto solutions are not guaranteed to converge to the ideal optimal solution set: they involve multiple runs and different solutions obtained in each run which result to keeping the statistical data by obtaining the best and worst optimal solutions. Another problem about these evolutionary methods is their less capacity of dealing with problem constraints, which produces non-feasible solutions. Examples of these algorithms are genetic algorithms (GAs), particle swarm optimization technique (PSO), they consumed much time in evaluating a large number of functions. On the other hand, semi-definite programming (SDP)-based weighted sum approach proposed in this paper is not a population based algorithm but convex optimization technique and have been shown to be useful in attaining the global optimal solution over several runs; therefore, the global optimality of its solution is assured, if the problem is convex. Likewise, for non-convex problem, semi-definite relaxation of the problem gives an estimated convex form that generates an approximation bound for the problem [27]. The applications of SDP to optimal power flow (OPF) and economic dispatch (ED) problems can be found in [27], [28], [29], [30], [31], [32], [33]. The paper is structured as follows: Section I presents the introduction and literature reviews, Section II describes the formulation of stochastic multi-objective problems, constraints and their SDP relaxation forms, Section III discusses the description of semi-definite programming approach. Section IV reports the simulations and results and Section V is the conclusion.

II. Problem Objectives

A. Total cost function

The objective function for the Total cost (J_1) is formulated as [7]

$$J_1 = \sum_{i=1}^{N_p} C_i(P_i) + \sum_{j=1}^{N_c} C_j(\Theta_j, H_j) + \sum_{k=1}^{N_h} C_k(T_k) \quad (1)$$

where N_p are the numbers of conventional power units, N_c are the numbers of electrical and thermal power outputs units and N_h are the numbers of heat units, respectively. The expected stochastic objective cost function J_1 is further expressed as follows [7]:

$$\begin{aligned} J_1 = &\sum_{i=1}^{N_p}\{\alpha_i + \beta_i\bar{P}_i + \gamma_i(\bar{P}_i^2 + Var(P_i))\} + \sum_{j=1}^{N_c}\{\alpha_j + \\ &\beta_j\bar{\Theta}_j + \gamma_j(\bar{\Theta}_j^{\,2} + Var(\Theta_j)) + \delta_j\bar{H}_j + \Theta_j(\bar{H}_j^{\,2} + \\ &Var(H_j)) + \xi_j(\bar{\Theta}_j\bar{H}_j + Cov(\Theta_j, H_j))\} + \sum_{k=1}^{N_h}\{\alpha_k + \\ &\delta_k\bar{T}_k + \theta_k(\bar{T}_k^2 + Var(T_k))\} \end{aligned} \quad (2)$$

whereby α_i, β_i, γ_i are the running cost coefficients of the *i*th thermal unit, P_i is the power output of the *i*th unit, Θ_j and H_j are the electrical and thermal power output coefficients of the *j*th chp unit respectively. Also, α_j, β_j, γ_j, δ_j, θ_j, ξ_j are the cost coefficients of the *j*th chp unit and α_i, δ_k, θ_k are the cost coefficients of the *k*th heat-only unit.

Where the terms $Var(P_i) = V^2(P_i)\bar{P}_i^{\,2}$, $Var(T_k) = V^2(T_k)\bar{T}_k^{\,2}$, $Cov(\Theta_j, H_j) = C^2(\Theta_j, H_j)\bar{\Theta}_j^{\,2}\bar{H}_j$, and $V_{()}, C_{()}$ are the variance coefficients and correlation coefficients of all the random variables respectively. The coefficient of variance of all the involved random variables is chosen as 0.2, and the correlation coefficient of each pair of random variables is set as 0.3 [7].

$$\begin{aligned} \bar{J}_1 = &\sum_{i=1}^{N_p}\{\alpha_i + \beta_i\bar{P}_i + \gamma_i(\bar{P}_i^2 + V^2(P_i))\bar{P}_i^{\,2}\} + \sum_{j=1}^{N_c}\{\alpha_j + \\ &\beta_j\bar{\Theta}_j + \gamma_j(\bar{\Theta}_j^{\,2} + V^2(\Theta_j)\bar{\Theta}_j^{\,2}) + \delta_j\bar{H}_j + \theta_j(\bar{H}_j^{\,2} + \\ &V^2(H_j)\bar{H}_j^{\,2}) + \xi_j(\bar{\Theta}_j\bar{H}_j + C^2(\Theta_j, H_j)\bar{\Theta}_j\bar{H}_j\} + \sum_{k=1}^{N_h} \\ &\{\alpha_k + \delta_k\bar{T}_k + \theta_k(\bar{T}_k^2 + V^2(T_k)\bar{T}_k^{\,2})\} \end{aligned} \quad (3)$$

$$\begin{aligned} \bar{J}_1 = &\sum_{i=1}^{N_p}\{\alpha_i + \beta_i\bar{P}_i + \gamma_i(1 + V^2(P_i))\bar{P}_i^{\,2}\} + \sum_{j=1}^{N_c}\{\alpha_j + \beta_j\bar{\Theta}_j \\ &+ \gamma_j(1 + V^2(\Theta_j))\bar{\Theta}_j^{\,2} + \delta_j\bar{H}_j + \theta_j(1 + V^2(H_j))\bar{H}_j^{\,2} \\ &+ \xi_j(1 + C^2(\Theta_j, H_j))\bar{\Theta}_j\bar{H}_j\} + \sum_{k=1}^{N_h}\{\alpha_k + \delta_k\bar{T}_k + \theta_k(1 + \\ &V^2(T_k))\bar{T}_k^{\,2}\} \end{aligned}$$

$$\begin{aligned} \bar{J}_1 = &\sum_{i=1}^{N_p}\{\alpha_i + \beta_i\bar{P}_i + \gamma_i(1 + 0.04)\bar{P}_i^{\,2}\} + \sum_{j=1}^{N_c}\{\alpha_j + \beta_j\bar{\Theta}_j + \\ &\gamma_j(1 + 0.04)\bar{\Theta}_j^{\,2} + \delta_j\bar{H}_j + \theta_j(1 + 0.04)\bar{H}_j^{\,2} + \\ &\xi_j(1 + 0.09)\bar{\Theta}_j\bar{H}_j\} + \sum_{k=1}^{N_h}\{\alpha_k + \delta_k\bar{T}_k + \theta_k(1 + 0.04)\bar{T}_k^{\,2}\} \end{aligned}$$

The conversion of Eq. (II-A) to its equivalent deterministic model becomes;

$$\begin{aligned} \bar{J}_1 = &\sum_{i=1}^{N_p}\{\alpha_i + \beta_i\bar{P}_i + \gamma_i(1.04)\bar{P}_i^2\} + \sum_{j=1}^{N_c}\{\alpha_j + \beta_j\bar{\Theta}_j + \\ &\gamma_j(1.04)\bar{\Theta}_j^{\,2} + \delta_j\bar{H}_j + \theta_j(1.04)(\bar{H}_j^{\,2}) + \xi_j(1.09)\bar{\Theta}_j\bar{H}_j\} \\ &+ \sum_{k=1}^{N_h}\{\alpha_k + \delta_k\bar{T}_k + \theta_k(1.04)\bar{T}_k^2\} \end{aligned}$$

$$\bar{J}_1 = trace(X^T\Gamma X) + \Delta^T X + \Omega + \delta_j^T\bar{H}_j + \bar{\Theta}_j^{\,T}\bar{H}_j \quad (4)$$

where X represents the vector variables in matrix form i.e $X = [\bar{P}_i, \bar{\Theta}_j, \bar{H}_j, \bar{T}_k]^T$,

$\Gamma = blkdiag[diag(\gamma_1, \cdots, \gamma_i); diag(\gamma_1, \cdots, \gamma_j); \cdots diag(\theta_1, \cdots, \theta_j); diag(\theta_1, \cdots, \theta_k)] * 1.04$,

$\Delta = [(\beta_1, \cdots, \beta_i); (\beta_1, \cdots, \beta_j); (\delta_1, \cdots, \delta_j)]^T$

$\Omega = \sum_{i=0}^{N_p}\alpha_i + \sum_{j=0}^{N_c}\alpha_j + \sum_{k=0}^{N_k}\alpha_k$,

B. Expected emissions SO_2/NO_x and CO_2

The total emissions in ton/h of SO_2 and NO_x are given as follows by a function of units power output with an exponential factor for the conventional units [7]:

$$\begin{aligned}\bar{J_2} &= \sum_{i=1}^{N_p} 10^2(\alpha_i + \beta_i P_i + \gamma_i P_i^2) + \zeta_i e^{(\lambda_i, P_i)} \\ &+ \sum_{j=1}^{N_c} (\theta_j + \eta_j)\bar{\Theta_j} + \sum_{k=1}^{N_h} (\pi_k + \rho_k) T_k\end{aligned} \tag{5}$$

where P_i is the power output generated by the conventional generators, power produced by the CHP units is denoted as Θ_j and the coefficients of the emission for the thermal units are α_i, β_i, γ_i, ζ_i, λ, the emissions coefficients for the CHP units are given as θ_j, η_j and the emissions coefficients for the heat-only units are given as π_k, ρ_k respectively. The expectation values of the random variables in Eq. (5) can be further expressed by taking the Taylor series expansion for the exponential factor:

$$\begin{aligned}\bar{J_2} &= \sum_{i=1}^{N_p} 10^2(\alpha_i + \beta_i \bar{P_i} + \gamma_i(\bar{P}_i^2 + Var(P_i))) \\ &+ \zeta_i + \zeta_i\lambda_i\bar{P_i} + \frac{\zeta_i\lambda_i}{2}(\bar{P}_i^2 + Var(P_i)) + \\ &\sum_{j=1}^{N_c} (\theta_j + \eta_j)\bar{\Theta_j} + \sum_{k=1}^{N_h} (\pi_k + \rho_k)\bar{T_k}\end{aligned} \tag{6}$$

Therefore, the sdp relaxation of (6) is as follows;

$$\begin{aligned}\bar{J_2} &= trace(\bar{P_i}\Gamma_i\bar{P_i}^T) + \Delta^T\bar{P_i} + \Omega_i) \\ &+ \zeta_i + \zeta_i\lambda_i\bar{P_i} + \frac{\zeta_i\lambda_i}{2}(\bar{P}_i^2 + Var(P_i)) \\ &+ \sum_{j=1}^{N_c} \{(\theta_j + \eta_j)\}\bar{\Theta_j} + \sum_{k=1}^{N_h} \{(\pi_k + \rho_k)\}\bar{T_k}\end{aligned} \tag{7}$$

$\Gamma = diag[(\gamma_1, \cdots, \gamma_i)](1.04)$;
$\Delta = [(\beta_1, \cdots, \beta_i)]^T$
$\Omega = \sum_{i=0}^{N_p} \alpha_i$,

Also, the stochastic approximation of CO_2 emissions can be expressed as a linear equation of units' power output as follows [7]:

$$\bar{J_{2c}} = \sum_{i=1}^{N_p} \tau_i\bar{P_i} + \sum_{j=1}^{N_c} k_j\bar{\Theta_j} + \sum_{k=1}^{N_h} \sigma_k\bar{T_k} \tag{8}$$

where τ_i, k_j, σ_k are the coefficients of CO_2 emissions.

C. Expected power deviation

The model for the expected deviation is obtained by finding the difference between the scheduled electric power generation and demand by taking the expectation of the square of unsatisfied demand, during the dispatch calculation, and is stated in [7] as:

$$\bar{J_3} = E\left\{\left(p_D + p_L - \sum_{i=1}^{N_p} P_i - \sum_{j=1}^{N_c} \Theta_j\right)^2\right\} \tag{9}$$

where the power demand is denoted as p_D, the power loss is p_L. Eq. (9) can be expressed further as [7]:

$$\begin{aligned}\bar{J_3} &= \sum_{i=1}^{N_p} Var(P_i) + \sum_{j=1}^{N_c} Var(\Theta_j) + 2\sum_{i=1}^{N_{p-1}}\sum_{m=i+1}^{N_p} Cov(P_i, \\ &P_m) + 2\sum_{j=1}^{N_{c-1}}\sum_{m=j+1}^{N_p} Cov(\Theta_i, \Theta_m) + 2\sum_{i=1}^{N_p}\sum_{j=1}^{N_c} Cov(P_i, \\ &\Theta_j)\end{aligned} \tag{10}$$

$$\begin{aligned}\bar{J_3} &= \sum_{i=1}^{N_p} V^2(P_i)\bar{P_i}^2 + \sum_{j=1}^{N_c} V^2(\Theta_j)\bar{\Theta_j}^2 + 2\sum_{i=1}^{N_{p-1}}\sum_{m=i+1}^{N_p} \\ &C^2(P_i, P_m)\bar{P_i}\bar{P_m} + 2\sum_{j=1}^{N_{c-1}}\sum_{m=j+1}^{N_p} C^2(\Theta_i, \Theta_m)\bar{\Theta_i}\bar{\Theta_m} \\ &+ 2\sum_{i=1}^{N_p}\sum_{j=1}^{N_c} C^2(P_i, \Theta_j)\bar{P_i}\bar{\Theta_j}\end{aligned}$$

Eq. (10) can be transformed into its equivalent deterministic matrix form as,

$$\begin{aligned}\bar{J_3} &= 0.04 \times P_i^2 + 0.04 \times \bar{\Theta_j}^2 + 2 \times 0.09 \times \bar{P_i}^T\bar{P_m} + 2\times \\ &0.09 \times \bar{\Theta_i}^T\bar{\Theta_m} + 2 \times 0.09 \times \bar{P_i}^T\bar{\Theta_j}\end{aligned} \tag{11}$$

Similarly, the expected heat generation deviation can be expressed as an objective function $\bar{J_3}$ formulated as follows:

$$\bar{J_4} = E\left\{\left(h_D - \sum_{j=1}^{N_c} H_j - \sum_{k=1}^{N_h} T_k\right)^2\right\} \tag{12}$$

where h_D is the heat deviation.

D. Problem constraints

The total electric power generation comprises both the total electric power demand and the real power losses, given as follows [7]:

$$\bar{P_D} + \bar{P_L} - \sum_{i=1}^{N_p} \bar{P_i} = 0 \tag{13}$$

The inner matrix representation of Eq. (13) is as follows:

$$\begin{bmatrix} (\bar{P_D} + \bar{P_L}) & -1 \\ -\bar{P_i} & 1 \end{bmatrix} \succeq 0 \tag{14}$$

where the expectation value of the power losses is denoted as $\bar{P_L}$. The power losses P_L otherwise known as J_4 can be expressed further by utilizing the Kron's B-loss coefficients as [7]:

$$\begin{aligned}\bar{P_L} &= \sum_{i=1}^{N_p}\sum_{m=1}^{N_p} P_i B_{im} P_m + \sum_{i=1}^{N_p}\sum_{j=1}^{N_c} P_i B_{ij}\Theta_j \\ &+ \sum_{j=1}^{N_c}\sum_{n=1}^{N_c} \Theta_j B_{jn}\Theta_n\end{aligned} \tag{15}$$

where the coefficients of the power loss for a line branch connecting units i and j is represented as B_{ij}. The sdp relaxation of Eq. (15) can be written as,

$$J_4 = P_i^T B_{im} P_m + P_i^T B_{ij}\Theta_j + \Theta_j^T B_{jn}\Theta_n \tag{16}$$

TABLE I
THERMAL UNITS COEFFICIENTS

Units	α_i	β_i	γ_i	P_{GI}^{min}	P_{GI}^{max}
G_1	100	200	10	0.05	0.5
G_2	120	150	10	0.05	0.6
G_3	40	180	10	0.05	1.00
G_4	60	100	10	0.05	1.20

TABLE II
CHP AND BOILER COEFFICIENTS

Units	α_j	β_j	γ_j	δ_j	θ_j	ξ_j
CHP_1	265	145	34.5	42	30	31
CHP_2	125	360	43.5	6	27	11
$HEAT_1$	110	41	23	-	-	-

The stochastic expression for Eq. (15) can be found in [1] which is further expressed as:

$$\bar{J}_4 = \sum_{i=1}^{N_p}\sum_{m=1}^{N_p} \bar{P}_i B_{im} \bar{P_m} + \sum_{i=1}^{N_p}\sum_{j=1}^{N_c} \bar{P}_i B_{ij} \bar{\Theta}_j + \sum_{j=1}^{N_c}\sum_{n=1}^{N_c} \bar{\Theta}_j B_{jn} \bar{\Theta}_n + \sum_{i=1}^{N_p} B_{ii} Var(P_i) + \sum_{j=1}^{N_c} B_{jj} Var(\Theta_j) + 2\sum_{i=1}^{N_{p-1}}\sum_{m=i+1}^{N_p} B_{im} Cov(P_i, P_m) + 2\sum_{j=1}^{N_{c-1}} \sum_{n=j+1}^{N_c} B_{jn} Cov(\Theta_j, \Theta_n) + \sum_{i=1}^{N_p}\sum_{j=1}^{N_c} B_{ij} Cov(P_i, \Theta_j) \quad (17)$$

$$\bar{J}_4 = \sum_{i=1}^{N_p}\sum_{m=1}^{N_p} \bar{P}_i B_{im} \bar{P_m} + \sum_{i=1}^{N_p}\sum_{j=1}^{N_c} \bar{P}_i B_{ij} \bar{\Theta}_j + \sum_{j=1}^{N_c}\sum_{n=1}^{N_c} \bar{\Theta}_j B_{jn} \bar{\Theta}_n + \sum_{i=1}^{N_p} B_{ii} V^2(P_i) \bar{P}_i^{\,2} + \sum_{j=1}^{N_c} B_{jj} V^2(\Theta_j) \bar{\Theta}_j^{\,2} + 2\sum_{i=1}^{N_{p-1}}\sum_{m=i+1}^{N_p} B_{im} C^2(P_i, P_m) \bar{P}_i \bar{P}_m + 2\sum_{j=1}^{N_{c-1}}\sum_{n=j+1}^{N_c} B_{jn} C^2(\Theta_j, \Theta_n) \bar{\Theta}_j \bar{\Theta}_n + \sum_{i=1}^{N_p}\sum_{j=1}^{N_c} B_{ij} C^2(P_i, \Theta_j) \bar{P}_i \bar{\Theta}_j$$

Eq. (17) can be relaxed as follows,

$$\begin{aligned} \bar{J}_4 &= \bar{P}_i^T B_{im} \bar{P_m} + \bar{P}_i^T B_{ij} \bar{\Theta}_j + \bar{\Theta}_j^T B_{jn} \bar{\Theta}_n \\ &+ 0.04 B_{ii}^T P_i^2 + 0.04 B_{jj}^T \Theta_j^2 + 2(0.09(B_{im} \bar{P}_i^T \bar{P_m})) \\ &+ 2(0.09(B_{jn} \bar{\Theta}_i^T \bar{\Theta}_n)) + 0.09(B_{ij} \bar{P}_i^T \bar{\Theta}_j) \end{aligned} \quad (18)$$

The expected values are limited within the ranges of the minimum and maximum limits given below,

$$I^T P_i^{min} \le \bar{P}_i \le I^T P_i^{max} \quad i = 1, \ldots, N_p \quad (19)$$

$$I^T \Theta_j^{min} \le \bar{\Theta}_j \le I^T \Theta_j^{max} \quad j = 1, \ldots, N_c \quad (20)$$

$$I^T H_j^{min} \le \bar{H}_j \le I^T H_j^{max} \quad j = 1, \ldots, N_c \quad (21)$$

$$I^T T_k^{min} \le \bar{T}_k \le I^T T_k^{max} \quad k = 1, \ldots, N_h \quad (22)$$

Tables I, II, III are obtained from [7] while Table IV and the B matrix of the transmission loss coefficient for 20 units network are available in [34].

TABLE III
CHP AND BOILER CAPACITY

Unit	Θ_j^{min}	Θ_j^{max}	H_j^{min}	H_j^{max}
CHP_1	0.05	1.0	0	0.6
CHP_2	0.05	0.6	0	0.6
$HEAT_1$	-	-	0	2

TABLE IV
DATA FOR THE TWENTY THERMAL UNITS OF GENERATING UNIT CAPACITY AND COEFFICIENTS

Unit	$P_i^{min}(pu)$	$P_i^{max}(pu)$	$\gamma(\$/pu^2)$	$\beta(\$/pu)$	$\alpha(\$)$
1	1.50	6.00	6.80	1819	1000
2	0.50	2.00	7.10	1926	970
3	0.50	2.00	65.00	1980	600
4	0.50	2.00	50.00	1910	700
5	0.50	1.60	73.80	1810	420
6	0.20	1.00	61.20	1926	360
7	0.25	1.25	79.00	1714	490
8	0.50	1.50	81.30	1892	660
9	0.50	2.00	52.20	1827	765
10	0.30	1.50	57.30	1892	770
11	1.00	3.00	48.00	1669	800
12	1.50	5.00	31.00	1676	970
13	0.40	1.60	85.00	1736	900
14	0.20	1.30	51.10	1870	700
15	0.25	1.85	39.80	1870	450
16	0.20	0.80	71.20	1426	370
17	0.30	0.85	89.00	1914	480
18	0.30	1.20	71.30	1892	680
19	0.40	1.20	62.20	1847	700
20	0.30	1.00	77.30	1979	850

III. SEMI-DEFINITE PROGRAMMING

Semi-definite programming is a solution method for convex optimization problems which simplifies the linear program (LP) by replacing the vector variables by matrix variables. Moreover, the component-wise non-negativity condition is replaced by positive semidefiniteness of the matrices. Therefore, the general SDP optimization problem is stated below as [35];

$$\begin{array}{ll} \textit{minimize} & \langle \mathbf{A_0}, \mathbf{X} \rangle \\ \textit{subject to:} & \langle \mathbf{A_i}, \mathbf{X} \rangle = b_i, \quad i = 1, \ldots, m \\ & \mathbf{X} \succeq 0 \end{array} \quad (23)$$

where $\mathbf{X} \in \mathcal{S}^n$ is the decision variable, $b \in \mathcal{R}^n$ and $\mathbf{A_0}, \mathbf{A_i} \in \mathcal{S}^n$ while $\mathcal{S}^n$ is refer to as a set of all symmetric matrices in $\mathcal{R}^{n \times n}$. The inner product between two matrices $\mathbf{M}, \mathbf{N} \in \mathcal{S}^n$ is defined as $\langle \mathbf{M}, \mathbf{N} \rangle = trace(\mathbf{MN})$

A. SDP Relaxation

Semidefinite programming (SDP) approach is a recent approach that is becoming widely used for solving various power system optimization problems. SDP involves the minimization of a linear problem subject to the constraints that are affine combination of symmetric matrices is semi-definite [36]. Semidefinite programming is considered as an extension of linear programming whereby the elements of the inequalities vectors are substituted by matrix inequalities, otherwise, the first orthant is substituted by the cone of positive semi-definiteness of the matrices [36]. Several normal problems such as linear and quadratic programming are combined using semi-definite programming and discovers a lot of uses in the field of engineering and combinatorial optimization [36]. More so, SDPs are gaining much recognition compared to linear programs, SDPs are not much harder to solve. Most interior-point methods for linear programming have been simplified to semidefinite programs. As in linear programming, these methods have polynomial worst-case complexity and perform very well in practice.

Most importantly, semi-definite programs can be effectively executed, both in theory and practice [37]. Semi-definite programs have been successfully applied to non-convex or combinatorial optimization. For instance, given an optimization problem in a quadratic form:

$$\begin{array}{ll} \textit{minimize} & f_0(x) \\ \textit{subject to:} & f_i(x) \leq 0, \ i = 1, \dots, l, \end{array} \tag{24}$$

where $f_0(x) = x^T A_0 x + 2{b_0}^T x + c_0$, $f_i(x) = x^T A_i x + 2{b_i}^T x + c_i, \quad i = 1, \cdots, l$

The matrices of A_i are indefinite, and thus, Eq. (24) is a difficult, non-convex optimization problem and involves polynomial objective problem and polynomial constraints.

With $A_0, A_i \in \mathcal{R}^{n\times n}$; $b_0, b_i \in \mathcal{R}^n$; and $c_0, c_i \in \mathcal{R}$; $i = 1, \cdots, l$. Each of the quadratic functions is convex if $A_i \geq 0$. A lifting variable $X = xx^T$ is introduced to convert the problem in Eq. (24) to its SDP relaxation form, by further reducing the constraint in equality form to an inequality constraint X $\geq xx^T$. Eq. (24) becomes

$$\begin{array}{ll} \textit{minimize} & Tr\langle XA_0\rangle + 2{b_0}^T x + c_0 \\ \textit{subject to:} & Tr\langle XA_i\rangle + 2{b_i}^T x + c_i \leq 0, \ i = 1, \dots, l, \\ & \begin{bmatrix} X & x \\ x^T & 1 \end{bmatrix} \geq 0, \end{array} \tag{25}$$

where $X = X^T \in \mathcal{R}^{k\times k}$ and $x \in \mathcal{R}^k$ are the variables. The constraint $\begin{bmatrix} X & x \\ x^T & 1 \end{bmatrix} \geq 0$ is similar to $X \geq xx^T$. A relaxation of the original problem (24) is the semi-definite program in (25) which is expressed as

$$\begin{array}{ll} \textit{minimize} & Tr\langle XA_0\rangle + 2{b_0}^T x + c_0 \\ \textit{subject to:} & Tr\langle XA_i\rangle + 2{b_i}^T x + c_i \leq 0, \ i = 1, \dots, l, \\ & X = xx^T. \end{array} \tag{26}$$

The only difference between (26) and (25) is the replacement of the non-convex constraint $X = xx^T$ with the convex relaxation $X \geq xx^T$. The relaxed problem (26) and the problem in Eq. (25) are equivalent to each other if $\begin{bmatrix} X & x \\ x^T & 1 \end{bmatrix}$ is of rank one. Furthermore, every of the quadratic representation in Eq. (26) can be relaxed in their SDP equivalent, in which the optimization problem can be deduced to the standard SDP form in Eq. (23) as

$$\begin{array}{ll} \textit{minimize} & \left\langle \begin{bmatrix} A_o & b_o \\ b_o^T & c_o \end{bmatrix}, \begin{bmatrix} X & x \\ x^T & 1 \end{bmatrix} \right\rangle \\ \textit{subject to:} & \left\langle \begin{bmatrix} A_i & b_i \\ b_i^T & c_i \end{bmatrix}, \begin{bmatrix} X & x \\ x^T & 1 \end{bmatrix} \right\rangle \leq 0, \quad i = 1, \cdots, p \\ & \begin{bmatrix} X & x \\ x^T & 1 \end{bmatrix} \geq 0, \end{array} \tag{27}$$

It is essential to have a good computation on the lower bounds for the ideal value of (23) using Shor's relaxation [35] by getting the dual SDP:

$$\begin{array}{ll} \textit{maximize} & \phi \\ \textit{subject to:} & \begin{bmatrix} A_0 & b_0 \\ b_0^T & c_0 - \phi \end{bmatrix} + \tau_1 \begin{bmatrix} A_1 & b_1 \\ b_1^T & c_1 \end{bmatrix} + \\ & \cdots + \tau_L \begin{bmatrix} A_L & b_L \\ b_L^T & c_L \end{bmatrix} \geq 0, \\ & \tau_i \geq 0, \quad i = 1, \dots, L. \end{array} \tag{28}$$

The constraint in the non-convex problem (24) can be relaxed as follows:

$$f_i(x) = \begin{bmatrix} x \\ 1 \end{bmatrix}^T \begin{bmatrix} A_i & b_i \\ b_i^T & c_i \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} \leq 0 \tag{29}$$

$$f_i(x) = \begin{bmatrix} x \\ 1 \end{bmatrix}^T \left(\begin{bmatrix} A_0 & b_0 \\ b_0^T & c_0 - \phi \end{bmatrix} + \tau_1 \begin{bmatrix} A_1 & b_1 \\ b_1^T & c_1 \end{bmatrix} + \cdots + \tau_L \begin{bmatrix} A_L & b_L \\ b_L^T & c_L \end{bmatrix} \right) \begin{bmatrix} x \\ 1 \end{bmatrix} \geq 0 \tag{30}$$

$$f_0(x) - \phi + \tau_1 f_1(x) + \cdots + \tau_L f_L(x) = 0$$

$$f_0(x) - \phi \geq 0$$

Simply, the derivation of the problem (28) is obtained by using Lagrangian duality.

IV. Lagrangian Relaxations

Lagrangian relaxation is another lesser way of achieving a more computable lower bound on an optimal value of the nonconvex quadratic optimization problem given as [38];

$$\begin{array}{ll} \textit{minimize} & x^T A_0 x + {b_0}^T x + c_0 \\ \textit{subject to:} & x^T A_i x + {b_i}^T x + c_i \leq 0, \quad i = 1, \dots, l, \end{array} \tag{31}$$

This method utilizes the dual of a problem which is always convex to achieve a solvable problem. The lagrangian form of the above Eq. (31) is given as

$$L(x,\lambda) = x^T \left(A_0 + \sum_{i=1}^{l} \lambda_i A_i \right) x + \left(b_0 + \sum_{i=1}^{l} \lambda_i b_i \right)^T x + c_0 + \sum_{i=1}^{l} \lambda_i c_i \tag{32}$$

To obtain the dual form of Eq. (31), given a function

$$\inf_{x\in\mathbf{R}} x^T Ax + b^T x + c = \begin{cases} \text{c-}\frac{1}{4}\ \text{b}^T Ab, & \text{if } A \succeq 0 \text{ and } b \in \mathbf{R}(A) \\ \text{-}\infty, & otherwise \end{cases} \tag{33}$$

Then, the dual function is

$$g(\lambda) = \inf_{x\in\mathbf{R}} L(x,\lambda) \tag{34}$$

$$= \frac{1}{4}\left(A_0 + \sum_{i=1}^{l} \lambda_i A_i \right)^T x + \left(b_0 + \sum_{i=1}^{l} \lambda_i b_i \right)^T \left(A_0 + \sum_{i=1}^{l} \lambda_i A_i \right) + \sum_{i=1}^{l} \lambda_i c_i + c_0 \tag{35}$$

The dual form of Eq. (31) using Schur complements becomes

$$\begin{array}{ll} \textit{maximize} & \gamma + \sum_{i=1}^{l} \lambda_i A_i + c_0 \\ \textit{subject to:} & \begin{bmatrix} (A_0 + \sum_{i=1}^{l} \lambda_i A_i) & (b_0 + \sum_{i=1}^{l} \lambda_i b_i)/2 \\ (b_0 + \sum_{i=1}^{l} \lambda_i b_i)^T/2 & -\gamma \end{bmatrix} \geq 0, \\ & \lambda_i \geq 0 \quad i = 1, \dots, l \end{array} \tag{36}$$

where the variable $\lambda \in R^m$. The dual of the nonconvex quadratically constrained quadratic programs (QCQP) in Eq. (31) is a convex program, otherwise known a Semidefinite program which is easier to solve, and gives a lower bound on the optimal value of the nonconvex QCQP.

A. Unpredictability of a Nonconvex Optimization Problem

The sdp relaxation in Eq. (25) is used to produce a positive semidefinite and covariance of the matrix with the constraint limit condition on the objective [38]. However, if x is taken as a normal distribution variable with $x \sim N(x, X - xx^T)$, the nonconvex quadratic problem in (25) can be solved by considering the mean distribution of x, i.e:

$$\begin{array}{ll} \textit{minimize} & E(Tr(XA_0) + 2{b_0}^T x + c_0) \\ \textit{subject to:} & E(Tr(XA_i) + 2{b_i}^T x + c_i) \leq 0, \quad i = 1, \ldots, l, \\ & \begin{bmatrix} X & x \\ x^T & 1 \end{bmatrix} \geq 0, \end{array} \tag{37}$$

A "good" feasible solution can be determined by sampling x over a large number of times, which results to keeping the best statistical solution.

B. Weighted sum method

Considering the weight vector $w = [w_1, \cdots, w_p]^T \in \mathcal{R}_p$, the vector objective function $f(x) = [f_1(x), \cdots, f_p(x)]^T \in \mathcal{R}_p$ and the map $\phi(f, w) : \mathcal{R}^p \times \mathcal{R}^p \longmapsto \mathcal{R}$. The weighted sum method includes a linear or convex combination of the objectives $f_i(x), i = 1, \cdots, p$, details can be obtained in [27]. Each of the objectives $f_i(x)$ is multiplied by a weight factor w_i and later added up to provide the scalar objective, $\phi(x, w)$, as

$$\phi(f, w) = \sum_{i=1}^{p} w_i f_i(x) = w^T f(x) \tag{38}$$

where p stands for the size of the objectives and

$$\sum_{i=1}^{p} w_i = 1, w_i \geq 0, \quad i =, \cdots, p. \tag{39}$$

This vector optimization problem in (38) is transformed to a scalar of the form:

$$\begin{array}{ll} \textit{minimize} & \phi(f, w) \\ \textit{s.t:} & x \in X \end{array} \tag{40}$$

The p-dimensional objective space are mapped onto the positive real line $\mathcal{R}$ and each of the optimal (non-dominated) points are mapped to the same point on the line. Let's consider when $p = 2$ for the bi-objective problem, then both Eqs. (38) and (39) ca be deduced to

$$\phi(f, w) = w_1 f_1(x) + w_2 f_2(x) \tag{41}$$

and

$$w_1 + w_2 = 1, \quad w_1, w_2 \geq 0 \tag{42}$$

If the weights in (41) is constrained by λ, i.e $w_1 = \lambda$ and $w_2 = 1 - \lambda$, therefore the gradient of w is defined as

$$\tan \theta_w = \frac{1 - \lambda}{\lambda} \tag{43}$$

and sensitivity of the gradient as

$$\frac{d}{d\lambda} \tan \theta_w = \frac{d}{d\lambda} \left(\frac{1 - \lambda}{\lambda} \right) = -\frac{1}{\lambda^2} \tag{44}$$

C. The adaptation of weight selection in improving weighted sum method

Let's assume that the weights in Eq. (41) are parameterized by λ, such that $w_1 = \lambda$ and $w_2 = 1 - \lambda$, a consistent set value of λ does not generate a consistent space distribution on the Pareto front (PF) [27]. Although, when the weight is parameterized such that k is parameterized on the surface of an ellipsoid, the improved spreading of the Pareto solutions are obtained on the Pareto front. In the parameterizations, setting

$$w_1 = \frac{\lambda_1^2}{k_1^2}, w_2 = \frac{\lambda_2^2}{k_2^2} \tag{45}$$

and substituting (45) in (42), the elliptical equation becomes

$$\frac{\lambda_1^2}{k_1^2} + \frac{\lambda_2^2}{k_2^2} = 1 \tag{46}$$

where the elliptical axes are denoted as k_1 and k_2. The normalization of the expression is obtained by fixing the value of $k_2 = 1$. Let $\lambda_1 = \lambda$ and $k_2 = 1$ in Eq. (46), the slope becomes

$$\tan \theta_w = \frac{k_1^2 - \lambda^2}{\lambda^2} \tag{47}$$

and the sensitivity of the slope becomes

$$\frac{d}{d\lambda} \tan \theta_w = \frac{-2k_1^2}{\lambda^3} \tag{48}$$

This indicates that the minor axis of the elliptical surface is set to unit value. Though, k_1 is selected from any value greater than 1. Variation in k_1 value allows for the curvature control of the ellipsoidal surface. Therefore, the non-linear weight selection provides a higher sensitivity and achieves further sensitivity improvement through the free parameter k_1. The value of k_1 can be used to control the solution points such that the gathered solutions can be distributed out, thus enabling an improving computational efficiency of the technique.

V. Simulation and Results

The standard modified IEEE 6 and 20 units' networks with 2 CHP units to each of the networks were considered to investigate the effectiveness of the SDP technique presented in this paper. The conversion of the SDP problem into the standard primal/dual form was achieved using YALMIP parser [27]. However, in the generation of the Pareto-front solution, a standard weighted sum method was used in generating the Pareto-optimal solution between two objectives functions. Different values of the control weight selection parameter were used in the generation of Pareto points.

Fifty one (51) runs were performed for each parameter value to explore the impact of changes in control weight selection k_1 and compare different cases. Figs. 1-9 show the Pareto curves at the values of control weight selection k_1= 1, 5, and 10 respectively. Only less distinct points were obtained from 51 runs with control weight selection k_1=1. This shows that different values of λ achieved very close values at different runs. This is regarded as a waste of computational effort. As the value of control selection k_1 is increased, the Pareto points were distributed uniformly out. As the value of k_1 further increases, a gradual progression

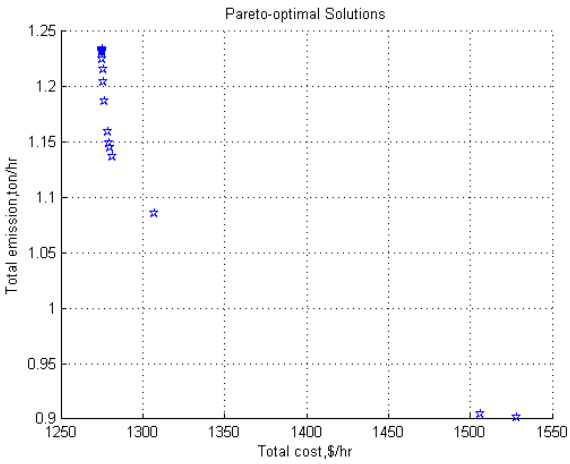

Fig. 1. Pareto front at weight selection k_1=1 for total cost and total emission functions using a modified IEEE 6 units' network.

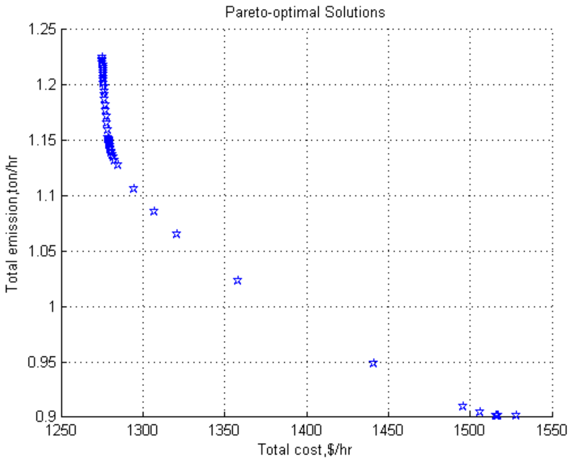

Fig. 2. Pareto front at weight selection k_1=5 for total cost and total emission functions using a modified IEEE 6 units' network.

in the spread up of the Pareto points is noticed, and the gathering of the Pareto points stop to exit. Conversely, it can be observed that the solutions points near to the lower extreme point are not captured. When control weight selection k_1=1, more points were missed from the middle part of the curve while more spread solution points were noticed at the middle part of the curve as the k_1 is further increased from 1. It is observed in the Pareto fronts (PFs) solutions that for every case of control selection parameter $k_1 = 10$, the optimal solutions are widely distributed on the tradeoff surface using the proposed SDP algorithm. Therefore, the decision maker can select an appropriate solution based on his/her choice from a generated group of Pareto optimal solutions in the multi-objective optimization. Also, one of the disadvantages of the weighted sum method is its unavailability to produce uniform spread of the solutions on the Pareto surface with uniform values of the weight factor w [27]. The Adaptation of the weight selection into the weighted sum method using non-linear weight selection however, controls and improves the distribution of Pareto points [27].

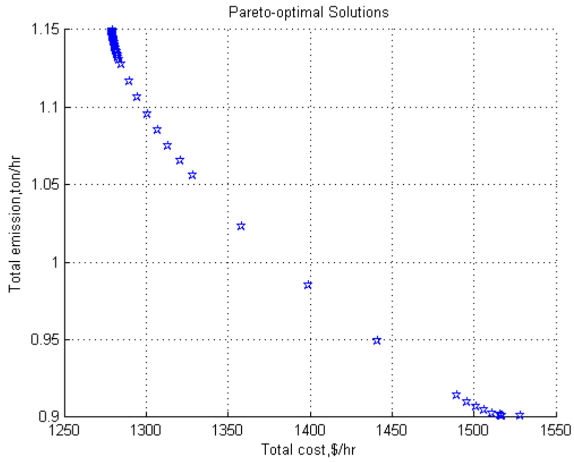

Fig. 3. Pareto front at weight selection k_1=10 for total cost and total emission functions using a modified IEEE 6 units' network.

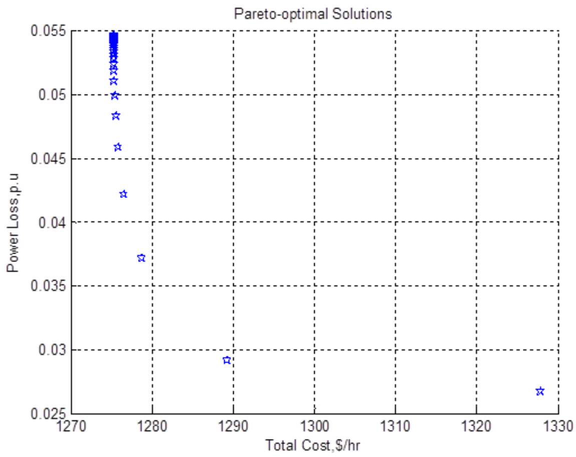

Fig. 4. Pareto front at weight selection k_1=1 for total cost and power loss functions using a modified IEEE 6 units' network.

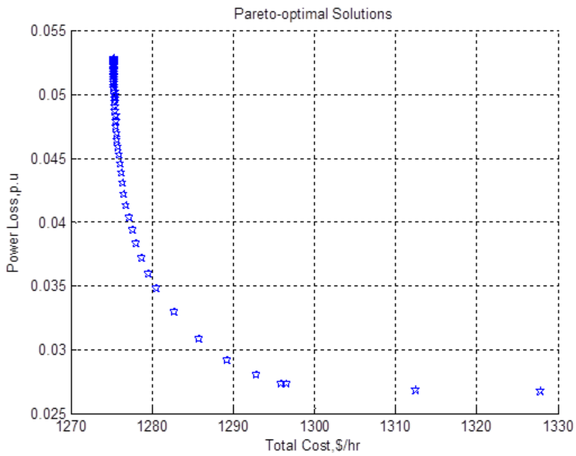

Fig. 5. Pareto front at weight selection k_1=5 for total cost and power loss functions using a modified IEEE 6 units' network.

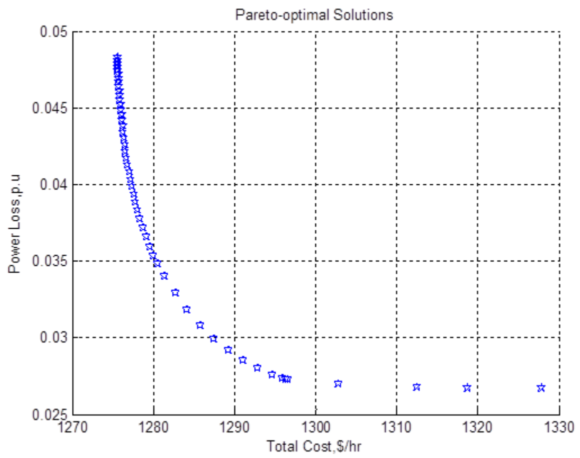

Fig. 6. Pareto front at weight selection k_1=10 for total cost and power loss functions using a modified IEEE 6 units' network.

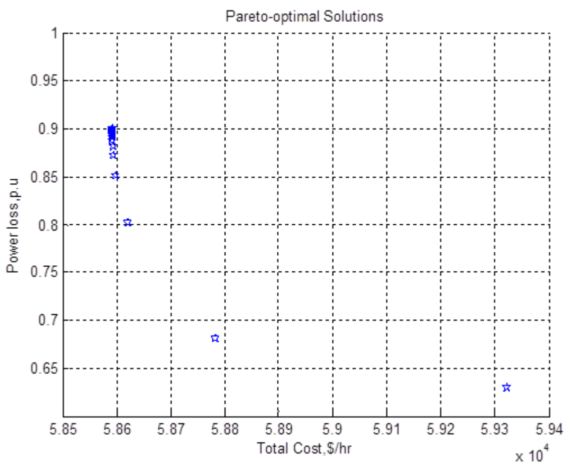

Fig. 7. Pareto front at weight selection k_1=1 for the total cost and power loss functions using a modified IEEE 20 units' network.

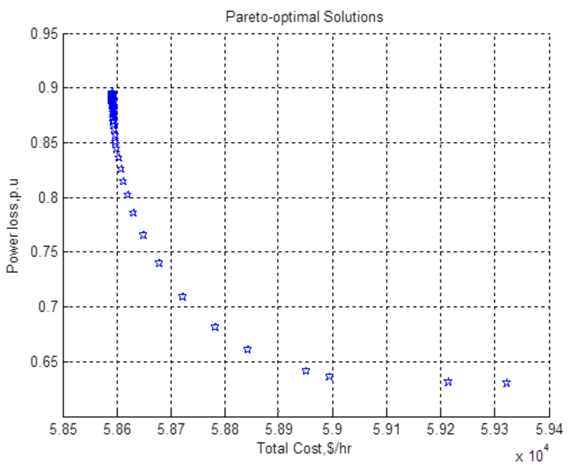

Fig. 8. Pareto front at weight selection k_1=5 for the total cost and power loss functions using a modified IEEE 20 units' network.

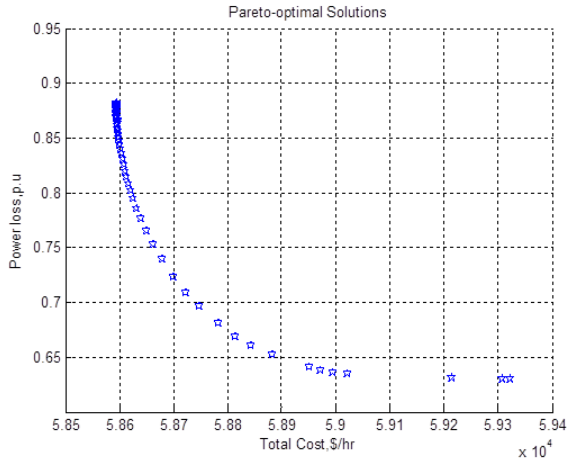

Fig. 9. Pareto front at weight selection k_1=10 for the total cost and power loss functions using a modified IEEE 20 units' network.

A. Case study I: Modified IEEE six units

In this study, a modified IEEE six units, 30 bus test network with 2 CHP units is considered based on the simulation analysis obtained from [7]. Total power demand is 2.834pu and heat demand is 0.8pu. The coefficient of variance of all the involved random variables is chosen as 0.2, and the correlation coefficient of each pair of random variables is set as 0.3 [7]. The cogeneration units emissions coefficients used are θ_j=0.00015, η_j=0.0015 and k_j=0.2 for SO_2, NO_x and CO_2, respectively, and for thermal units only, π_k=0.0008, ρ_k=0.001 and σ_k=0.4. All the B-coefficients are given in per unit (p.u.) on a 100 MVA base capacity.
The optimization results obtained from SDP technique compared to the results of a modified multi-objective particle swarm optimization (MOPSO), genetic algorithms (GA) and the weighted aggregation (WA) reported in the literature are shown in Table V. The cost reduction results obtained from other methods are close, while SDP approach achieved better computational results when compared with the results from the literature.
It can be observed in Table VI that there is diversity of results in the minimum values of the operational costs which differentiated the stochastic and deterministic power dispatch models as a result of uncertainties of the power and heat demands. A comparison between stochastic and deterministic models is presented in Tables VIII and IX using the results obtained from the Pareto set, for the lowest value of each problem objective.

B. Case study II: Modified IEEE Twenty-units system

This case study consists of eighteen thermal and two CHP units. This system supplies a total load demand of P_D = 25.00 pu. The data table for IEEE Twenty-units system and the B matrix of the transmission line loss coefficient are available in [34]. More so, the stochastic and deterministic models optimization results for modified IEEE twenty units network obtained from the Pareto set, for the lowest value of each problem objective are shown in Tables X and XI.

The B_{ij} matrix of the transmission loss coefficient for

TABLE V
COST REDUCTION ON A MODIFIED IEEE SIX UNITS' NETWORK INCORPORATING TWO CHP UNITS.

	SDP	MOPSO	GA	WA
$\overline{P_1}$	0.0500	0.2980	0.4597	0.0500
$\overline{P_2}$	0.6000	0.4576	0.5290	0.6000
$\overline{P_3}$	0.5260	0.6519	0.4721	0.7737
$\overline{P_4}$	1.2000	0.7826	0.8623	0.7205
$\overline{\Theta_1}$	0.4629	0.3468	0.4856	0.0600
$\overline{\Theta_2}$	0.0500	0.1523	0.0699	0.0600
$\overline{H_1}$	0.0000	0.2422	0.3684	0.0000
$\overline{H_2}$	0.6000	0.2330	0.1507	0.2000
$\overline{T_1}$	0.2000	0.2972	0.2309	0.5000
Cost	**1275.3**	**1305.0**	**1322.4**	**1303.6**
CO_2	1.0823	1.5960	1.7212	1.6001
NO_x,SO_2	0.1519	0.1272	0.1285	0.1361
Power dev.	0.5265	0.4784	0.5447	0.4882
Heat dev.	0.0268	0.0309	0.0292	0.0206

TABLE VI
COST REDUCTION INCORPORATING TWO CHP UNITS CONSIDERING A MODIFIED IEEE SIX UNITS' NETWORK USING SDP APPROACH

Gen/Obj	Sto. Model (SM)	Det. Model (DM)
$\overline{P_1}(P_1)$	0.0500	0.0828
$\overline{P_2}(P_2)$	0.6000	0.6000
$\overline{P_3}(P_3)$	0.5260	0.5086
$\overline{P_4}(P_4)$	1.2000	1.2000
$\overline{\Theta_1}(\Theta_1)$	0.4629	0.4493
$\overline{\Theta_2}(\Theta_2)$	0.0500	0.0500
$\overline{H_1}(H_1)$	0.0000	0.0000
$\overline{H_2}(H_2)$	0.6000	0.6000
$\overline{T_1}(T_2)$	0.2000	0.2000
Cost	**1275.3**	**1274.3**
CO_2	1.0821	1.0836
NO_x,SO_2	0.1519	0.1470
Total Emission	1.2340	1.2306
Power Loss	0.0531	0.0567
Power dev.	0.5265	0.5311
Heat dev.	0.0268	0.0268
Total Power Output	2.8871	2.8907
Total Heat Output	0.8000	0.8000

TABLE VII
COST REDUCTION ON A MODIFIED IEEE TWENTY UNITS' NETWORK INCORPORATING TWO CHP UNITS USING SDP APPROACH

Gen/Obj	Opt. val	Gen/Obj	Opt. val
$\overline{P_1}$	4.3094	$\overline{P_{16}}$	0.8000
$\overline{P_2}$	1.8332	$\overline{P_{17}}$	0.6618
$\overline{P_3}$	1.3017	$\overline{P_{18}}$	0.8610
$\overline{P_4}$	1.0920	$\overline{\Theta_1}$	1.0000
$\overline{P_5}$	0.9691	$\overline{\Theta_2}$	0.6000
$\overline{P_6}$	0.5924	$\overline{H_1}$	0.0000
$\overline{P_7}$	1.0950	$\overline{H_2}$	0.6000
$\overline{P_8}$	1.2927	$\overline{T_1}$	0.2000
$\overline{P_9}$	0.9415	Cost($/pu)	58593.0
$\overline{P_{10}}$	0.8183	Power Loss	0.9000
$\overline{P_{11}}$	1.4582	Power dev.	54.4161
$\overline{P_{12}}$	2.7944	Heat dev.	0.0268
$\overline{P_{13}}$	1.4293	Total Power Output	25.9000
$\overline{P_{14}}$	0.2000	Total Heat Output	0.8000
$\overline{P_{15}}$	1.8500		

TABLE VIII
MULTI-OBJECTIVE MINIMIZATION RESULTS FOR STOCHASTIC MODEL CONSIDERING A MODIFIED IEEE SIX UNITS TEST SYSTEM USING SDP APPROACH

min$\{J_1\}$	min$\{J_2\}$	min$\{J_3\}$	min$\{J_4\}$	min$\{J_5\}$
1275.3	1528.1	1422.1	1415.7	1327.9
1.2342	**0.9013**	1.2213	1.3585	1.3387
0.5265	0.5165	**0.4567**	0.5210	0.5482
0.0268	0.0416	0.0334	**0.0256**	0.0341
0.0546	0.1902	0.0873	0.0854	**0.0267**

TABLE IX
MULTI-OBJECTIVE MINIMIZATION RESULTS FOR DETERMINISTIC MODEL CONSIDERING A MODIFIED IEEE SIX UNITS TEST SYSTEM USING SDP APPROACH

min$\{J_1\}$	min$\{J_2\}$	min$\{J_3\}$	min$\{J_4\}$	min$\{J_5\}$
1274.3	1519.9	1420.4	1411.6	1345.9
1.2306	**0.8881**	1.2142	1.3516	1.3014
0.5311	0.5084	**0.4551**	0.5165	0.5287
0.0268	0.0416	0.0333	**0.0256**	0.0341
0.0567	0.1664	0.0907	0.0727	**0.0259**

IEEE six units is given by

$$\begin{bmatrix} 0.1382 & -0.0299 & 0.0044 & -0.0022 & -0.0010 & -0.0008 \\ -0.0299 & 0.0487 & -0.0025 & 0.0004 & 0.0016 & 0.0041 \\ 0.0044 & -0.0025 & 0.0182 & -0.0070 & -0.0066 & -0.0041 \\ -0.0022 & 0.0004 & -0.0070 & 0.0137 & 0.0050 & 0.0033 \\ -0.0010 & 0.0016 & -0.0066 & 0.0050 & 0.0109 & 0.0005 \\ -0.0008 & 0.0041 & -0.0066 & 0.0033 & 0.0005 & 0.0244 \end{bmatrix} \tag{49}$$

Furthermore, Figs. 10 and 11 illustrate the performance of SDP technique on stochastic and deterministic models respectively by performing fifty-one (51) iterations which were investigated on standard modified IEEE six and twenty units' networks with two CHP units to each of the networks. It is established that there is significant variation in the comparative convergence profiles for both stochastic and deterministic models presented in both Figs. 10 and 11 using a standard modified IEEE six units' network with two CHP units and also, Fig. 12 shows a comparative convergence profiles for both stochastic and deterministic models considering a standard modified IEEE twenty units' network with two CHP units. There is a deviation in the results of stochastic and deterministic models presented in Figs. 10, 11 and 12 as a result of random effect on the power generation systems. Therefore, in real life implementation,

TABLE X
MULTI-OBJECTIVE MINIMIZATION RESULTS FOR STOCHASTIC MODEL CONSIDERING A MODIFIED IEEE TWENTY UNITS USING SDP APPROACH

Obj	min$\{J_1\}$	min$\{J_3\}$	min$\{J_4\}$	min$\{J_5\}$
Cost	**58593.0**	59285.0	60129.0	59323.0
Power dev.	54.4161	**52.9812**	56.5431	53.5166
Heat dev.	0.0268	0.0332	**0.0256**	0.0332
Power Loss	0.9000	0.7419	1.0661	**0.6298**

TABLE XI
MULTI-OBJECTIVE MINIMIZATION RESULTS FOR DETERMINISTIC MODEL CONSIDERING A MODIFIED IEEE TWENTY UNITS USING SDP APPROACH

Obj	min$\{J_1\}$	min$\{J_3\}$	min$\{J_4\}$	min$\{J_5\}$
Cost	**29826**	29834	29835	29834
Power dev.	8.5139	**8.5139**	8.5139	8.5139
Heat dev.	0.5937	0.5357	**0.5297**	0.5358
Power Loss	0.1436	0.1436	0.1436	**0.1436**

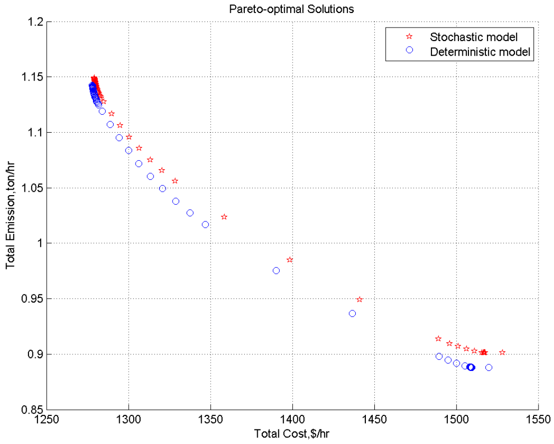

Fig. 10. Pareto fronts for the Stochastic and Deterministic models

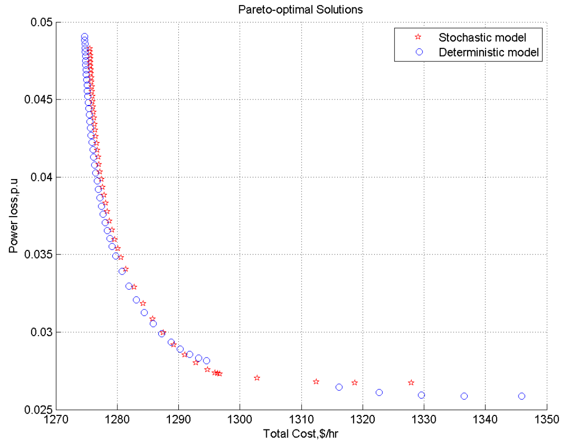

Fig. 11. Pareto fronts for the Stochastic and Deterministic models

it is advisable to express problems in CHP as stochastic model so as to cover the effect of the uncertain factors [7]. Figs. 13 and 14 illustrate the Pareto fronts solutions for the simultaneous minimizations of stochastic multi-objective problems respectively. It is shown in Fig. 13 that the lower extreme solution produces the lowest total cost, at maximum power loss and minimum total emission, while at the upper extreme point generates maximum total cost at minimum power loss and maximum total emission among all the solutions in the Pareto front. Fig. 14 shows that the lower extreme solution generates lowest value of total cost, at maximum power deviation and minimum heat deviation whereas as the total cost increases, heat deviation increases and power deviation decreases among all the solutions in the Pareto front. If randomness is to be considered in the power systems, there will be increase in total cost, power and heat deviations [1].
Also, the Pareto solutions for the simultaneous minimization of deterministic multi-objectives functions are presented in Figs. 15 and 16. It can be deduced from the Fig. 15 that the lower extreme solution indicates that at minimum total cost, maximum power loss and minimum total emission are generated while at the upper extreme solution gives maximum total cost, at minimum power loss and maximum total emission among the solutions in Pareto front. On the other hand, Fig. 16 illustrates that at minimum total cost, maximum power deviation and minimum heat deviation are generated at the lower extreme solution.

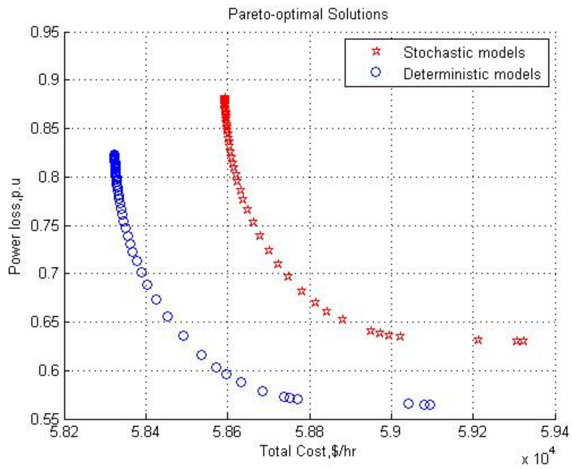

Fig. 12. Pareto fronts for the Stochastic and Deterministic models

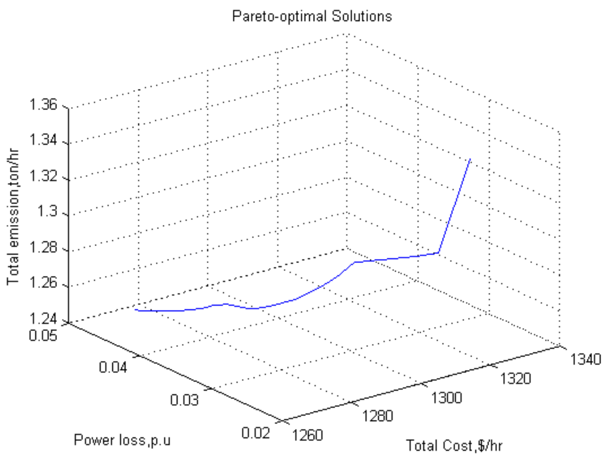

Fig. 13. Pareto front Solution for the Stochastic objective functions

VI. Conclusion

The proposed SDP method performed well in accuracy of results and provides lower operational cost in the Pareto set produced. The results for the multi-objectives formulation problems are presented using SDP approach, indicating that the decision maker can choose his/her preferred solution while satisfying multiple criteria. The SDP method solves a stochastic problem by minimizing the expectation of the multi-objective functions using the statistics of Gaussian distribution. Also, further investigations were performed on the comparison of the stochastic model for the multi-objective functions and the deterministic approach, resulting to diversity in total operational cost which covers the

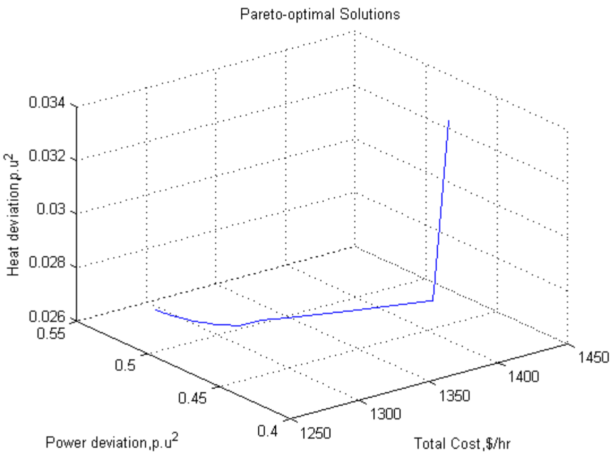

Fig. 14. Pareto front Solution for the Stochastic objective functions

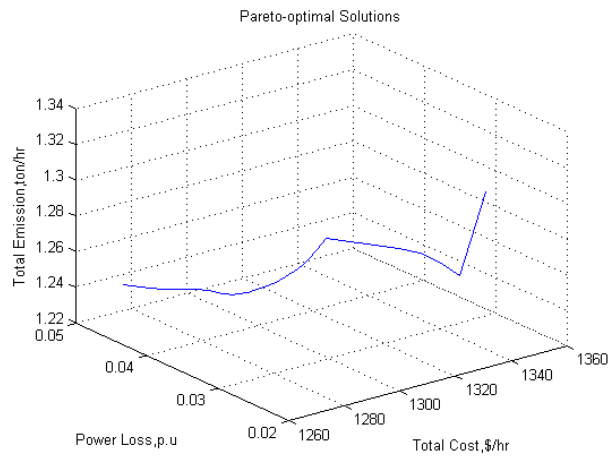

Fig. 15. Pareto front Solution for the Deterministic objective functions

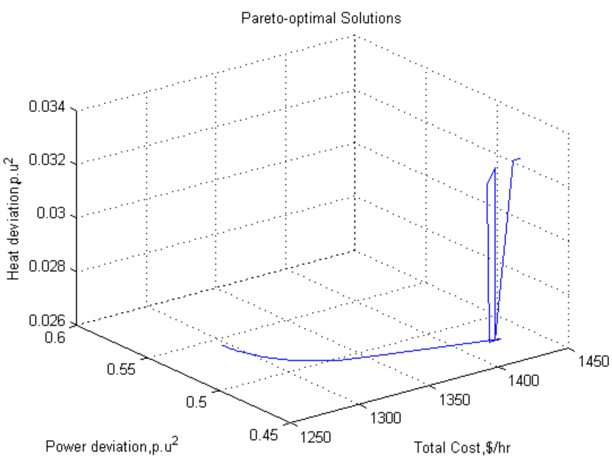

Fig. 16. Pareto front Solution for the Deterministic objective functions

uncertainties of power and head demand.
It is obvious that the adaptation of weight selection k_1 into the weight sum method achieves more uniform distribution of the solution points as the k_1 value increases. An optimal selection of k_1 parameter that generates a comparative uniform spread out of the algorithm is practically determined.
Future work can be conducted in the implementation of chance constraints to capture the stochastic characteristic of the power system generation and distribution which is more practical than the deterministic constraints.

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