Adaptive Synchronization for Fractional-order Biomathematical Model of Muscular Blood Vessel with Input Nonlinearity

Xiaomin Tian, Zhong Yang

Abstract—**In this paper, the adaptive synchronization of fractional-order muscular blood vessel (MBV) model subject to input nonlinearity is investigated. The parameters of controlled systems are assumed to be unknown in advance, moreover, the effects of model uncertainties and external disturbances are fully taken into account. On the basis of frequency distributed fractional integrator model and Lyapunov stability theory, a robust control law and fractional-order type parametric update laws are designed to ensure the synchronization. Simulation results demonstrate that the proposed control scheme can steer the abnormal muscular vessel into normal orbit with good robustness.**

Index Terms—**Adaptive synchronization, Muscular blood vessel model, Input nonlinearity, Fractional-order nonlinear system.**

I. Introduction

Biological systems are often complicated and highly nonlinear. Due to many biological systems can behave chaotically, the research for biological systems based on chaotic theory has grown significantly over past decades and became a popular topic.

Muscular blood vessel model is one of intriguing nonlinear biological systems. It has been reported by WHO that cardiovascular disease is the main cause of death in developed countries. Particularly, coronary artery lesions are deemed to be the major reason of heart attacks. Coronary artery refers to the vessel which supplies the oxygen and nutriment to the vessel of cardiac muscle and known as muscle type vessel. Obstruction of coronary artery can leads to many disease, such as stenocardia, angina, myocardial infarction, etc. According to the chaotic theory of coronary artery, the key problem is how to make the biomedical model of MBV synchronize with a prescribed chaotic system of a normal vessel. That is, from the medical perspective, the chaos synchronization means that the state trajectories of the vessel with pathological changes can be synchronized with those of the normal vessel, and then the treatment can be achieved. At present, some researches about the coronary artery have been reported [1-4].

However, all of above mentioned literatures are only focus on the integer-order model of MBV. It is well known that with the help of fractional-order calculus, systems can be described more accurately. Lately, Aghababa et al. [5] first developed the fractional-order model of MBV, and studied the finite-time chaos control. But, it is worth noting that the system's parameters in [5] are assumed to be known. As a matter of fact, many systems' parameters cannot be exactly determined in advance. The chaos control/synchronization will be not achieved under the effect of unknown uncertainties.

On the other hand, since the presence of input nonlinearities can lead to unpredictable and undesirable behaviors, thus the effect of nonlinear inputs should be considered in the synchronization of chaotic systems. However, to the best of our knowledge, up until now, there is no information available about the synchronization of fractional-order MBV model with unknown parameters and nonlinear inputs.

Motivated by the above discussions, in this paper, we design a robust controller to realize the synchronization of two fractional-order MBV systems with input nonlinearity. The systems' parameters are assumed to be unknown in advance. Moreover, the effects of model uncertainties and external disturbances are fully taken into account. To prove the robustness and stability of the proposed scheme, the frequency distributed fractional integrator model and Lyapunov stability theory are applied. Finally, a simulation example is provided to verify the effectiveness and applicability of the proposed control scheme.

II. Preliminaries

The Caputo definition is the most commonly used definition of fractional calculus.

Definition 1 The Caputo fractional derivative of order α is defined as

$$ {}_{t_0}D_t^\alpha f(t)=\begin{cases} \frac{1}{\Gamma(m-\alpha)}\int_{t_0}^{t}\frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}}d\tau, & m-1<\alpha<m \\ \frac{d^m}{dt^m}f(t), & \alpha=m \end{cases} \quad (1)$$

where m is the smallest integer number, larger than α.

In the rest of this paper, we will use D^α instead of ${}_0D_t^\alpha$.

Lemma 1 (see [6]) Consider a nonlinear fractional-order system

$$D^\alpha x(t) = f(x(t)) \quad (2)$$

where $\alpha \in (0,1)$. Then the system can be equivalently converted to the following continuous frequency distributed model

$$\begin{aligned} \frac{\partial z(\omega,t)}{\partial t} &= -\omega z(\omega,t) + f(x(t)) \\ x(t) &= \int_0^\infty \mu(\omega) z(\omega,t) d\omega \end{aligned} \quad (3)$$

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where $\mu(\omega) = ((\sin(\alpha\pi))/\pi)\omega^{-\alpha}$. In above equations, $z(\omega, t)$ is the true state variable, and $x(t)$ is the pseudo state variable.

III. Main Results

According to the contribution from Ref.[5], fractional-order MBV can be mathematically model as follows

$$\begin{aligned} D^{\alpha}x_1 &= -bx_1 - cx_2 \\ D^{\alpha}x_2 &= -(\lambda + b\lambda)x_1 - (\lambda + c\lambda)x_2 + \lambda x_1^3 + E\cos t \quad (4) \end{aligned}$$

where $\alpha \in (0, 1)$, x_1 is the change of internal diameter of vessel, x_2 is the pressure change of vessel, $E\cos t$ is the periodical stimulating disturbance term, b, c and λ are system parameters. It has been proved by Aghababa, when $b = 0.15$, $c = -1.7$, $\lambda = -0.65$, $E = 0.3$, $\omega = 1$ and $0.01 \le \alpha \le 0.99$, system (4) can behave chaotically. Selecting $x_1(0) = x_2(0) = 0.1$, the strange attractor of system (4) for $\alpha = 0.99$ is shown in Fig. 1.

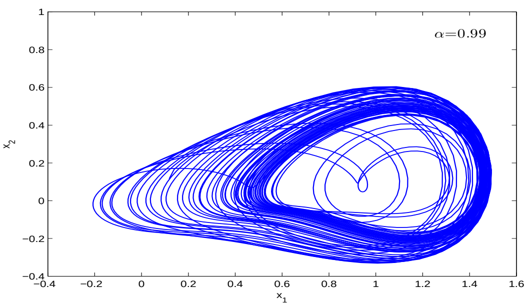


Fig. 1. Strange Attractors of System (4) with Different Fractional Orders

Take system (4) as master system, the slave system with model uncertainties and external disturbances is described by

$$\begin{aligned} D^{\alpha}y_1 &= -by_1 - cy_2 + \Delta f_1(y) + d_1(t) + \phi_1(u_1(t)) \\ D^{\alpha}y_2 &= -(\lambda + b\lambda)y_1 - (\lambda + c\lambda)y_2 + \lambda y_1^3 + E\cos t \\ &\quad + \Delta f_2(y) + d_2(t) + \phi_2(u_2(t)) \quad (5) \end{aligned}$$

where $y = (y_1, y_2)^T$, $\Delta f_i(y)$ and $d_i(t)$, $i = 1, 2$ are model uncertainties and external disturbances, respectively. $u(t) = (u_1(t), u_2(t))^T$ is a vector of controller to be designed later. $\phi_i(u_i(t))$, $i = 1, 2$ are continuous nonlinear functions inside the sector $[\rho_{i1}, \rho_{i2}]$, $\rho_{i1} > 0$, and satisfying

$$\rho_{i1}u_i^2(t) \le u_i(t)\phi_i(u_i(t)) \le \rho_{i2}u_i^2(t) \quad (6)$$

A typical nonlinear input is shown in Fig.2.

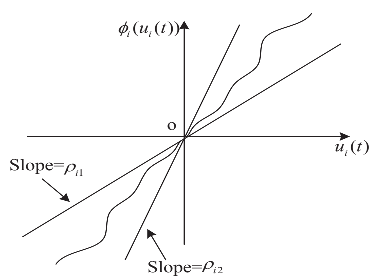


Fig. 2. A Typical Nonlinear Input

Letting $B = \lambda + b\lambda$, $C = \lambda + c\lambda$, and subtracting (4) from (5), it yields

$$\begin{aligned} D^{\alpha}e_1 &= -be_1 - ce_2 + \Delta f_1(y) + d_1(t) + \phi_1(u_1(t)) \\ D^{\alpha}e_2 &= -Be_1 - Ce_2 + \lambda e_1(e_1^2 + 3e_1x_1 + 3x_1^2) + \Delta f_2(y) \\ &\quad + d_2(t) + \phi_2(u_2(t)) \quad (7) \end{aligned}$$

Obviously, the synchronization problem between systems (4) and (5) is equivalently transformed to the stabilization problem of error system (7).

Our goal in this paper is to design an appropriate controller to ensure the stabilization of system (7) with unknown parameters. To make the proposed control scheme more reasonable and practical, an assumption is given.

Assumption 1 In general, the model uncertainties, external disturbances are bounded by

$$|\Delta f_i(y) + d_i(t)| \le \delta_i, \quad i = 1, 2. \quad (8)$$

where δ_i are known positive constants.

Theorem 1 Consider the error system (7), if the system is controlled by the controller

$$u_i(t) = -\xi_i\eta_i sgn(e_i) \quad (9)$$

where $i = 1, 2$, $\xi_i = \frac{1}{\rho_{i1}}$, $\eta_1 = |e_1||\hat{b}| + |e_2||\hat{c}| + \delta_1 + k_1|e_1| > 0$, $\eta_2 = |e_1||\hat{B}| + |e_2||\hat{C}| + |(e_1^2 + 3e_1x_1 + 3x_1^2)e_1||\hat{\lambda}| + \delta_2 + k_2|e_2| > 0$, k_1 and k_2 are two positive constants.

The parametric update laws are selected as

$$\begin{aligned} D^{\alpha}\widetilde{b} &= D^{\alpha}\hat{b} = -e_1^2 \\ D^{\alpha}\widetilde{c} &= D^{\alpha}\hat{c} = -e_1e_2 \\ D^{\alpha}\widetilde{B} &= D^{\alpha}\hat{B} = -e_1e_2 \\ D^{\alpha}\widetilde{C} &= D^{\alpha}\hat{C} = -e_2^2 \\ D^{\alpha}\widetilde{\lambda} &= D^{\alpha}\hat{\lambda} = (e_1^2 + 3e_1x_1 + 3x_1^2)e_1e_2 \quad (10) \end{aligned}$$

where $\widetilde{b} = \hat{b} - b$, $\widetilde{c} = \hat{c} - c$, $\widetilde{B} = \hat{B} - B$, $\widetilde{C} = \hat{C} - C$, $\widetilde{\lambda} = \hat{\lambda} - \lambda$ are estimate parameter errors, $\hat{b}$, $\hat{c}$, $\hat{B}$, $\hat{C}$, $\hat{\lambda}$ are estimate values of b, c, B, C, λ, respectively.

Then the synchronization between systems (4) and (5) can be achieved.

Proof. According to Lemma 1, system (7) and adaptation laws (10) constitute the following closed-loop system

$$\begin{aligned} \frac{\partial z_{e_1}(\omega,t)}{\partial t} &= -\omega z_{e_1}(\omega,t) - be_1 - ce_2 + \Delta f_1(y) \\ &\quad + d_1(t) + \phi_1(u_1(t)) \\ e_1(t) &= \int_0^{\infty} \mu(\omega)z_{e_1}(\omega,t)d\omega \\ \frac{\partial z_{e_2}(\omega,t)}{\partial t} &= -\omega z_{e_2}(\omega,t) - Be_1 - Ce_2 + \lambda e_1(e_1^2 + 3e_1x_1 \\ &\quad + 3x_1^2) + \Delta f_2(y) + d_2(t) + \phi_2(u_2(t)) \\ e_2(t) &= \int_0^{\infty} \mu(\omega)z_{e_2}(\omega,t)d\omega \\ \frac{\partial z_{\widetilde{b}}(\omega,t)}{\partial t} &= -\omega z_{\widetilde{b}}(\omega,t) - e_1^2 \\ \widetilde{b} &= \int_0^{\infty} \mu(\omega)z_{\widetilde{b}}(\omega,t)d\omega \\ \frac{\partial z_{\widetilde{c}}(\omega,t)}{\partial t} &= -\omega z_{\widetilde{c}}(\omega,t) - e_1e_2 \\ \widetilde{c} &= \int_0^{\infty} \mu(\omega)z_{\widetilde{c}}(\omega,t)d\omega \end{aligned}$$

$$\begin{aligned}
\frac{\partial z_{\widetilde{B}}(\omega,t)}{\partial t} &= -\omega z_{\widetilde{B}}(\omega,t) - e_1e_2\\
\widetilde{B} &= \int_0^\infty \mu(\omega) z_{\widetilde{B}}(\omega,t)d\omega\\
\frac{\partial z_{\widetilde{C}}(\omega,t)}{\partial t} &= -\omega z_{\widetilde{C}}(\omega,t) - e_2^2\\
\widetilde{C} &= \int_0^\infty \mu(\omega) z_{\widetilde{C}}(\omega,t)d\omega\\
\frac{\partial z_{\widetilde{\lambda}}(\omega,t)}{\partial t} &= -\omega z_{\widetilde{\lambda}}(\omega,t) + (e_1^2+3e_1x_1+3x_1^2)e_1e_2\\
\widetilde{\lambda} &= \int_0^\infty \mu(\omega) z_{\widetilde{\lambda}}(\omega,t)d\omega \qquad (11)
\end{aligned}$$

Selecting a Lyapunov function in the form of

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t)+V_5(t)+V_6(t)+V_7(t) \quad (12)$$

where

$$\begin{aligned}
V_1(t) &= \int_0^\infty \mu(\omega)v_1(\omega,t)d\omega,\ v_1(\omega,t)=\frac{1}{2}z_{e_1}^2(\omega,t)\\
V_2(t) &= \int_0^\infty \mu(\omega)v_2(\omega,t)d\omega,\ v_2(\omega,t)=\frac{1}{2}z_{e_2}^2(\omega,t)\\
V_3(t) &= \int_0^\infty \mu(\omega)v_3(\omega,t)d\omega,\ v_3(\omega,t)=\frac{1}{2}z_{\widetilde{b}}^2(\omega,t)\\
V_4(t) &= \int_0^\infty \mu(\omega)v_4(\omega,t)d\omega,\ v_4(\omega,t)=\frac{1}{2}z_{\widetilde{c}}^2(\omega,t)\\
V_5(t) &= \int_0^\infty \mu(\omega)v_5(\omega,t)d\omega,\ v_5(\omega,t)=\frac{1}{2}z_{\widetilde{B}}^2(\omega,t)\\
V_6(t) &= \int_0^\infty \mu(\omega)v_6(\omega,t)d\omega,\ v_6(\omega,t)=\frac{1}{2}z_{\widetilde{C}}^2(\omega,t)\\
V_7(t) &= \int_0^\infty \mu(\omega)v_7(\omega,t)d\omega,\ v_7(\omega,t)=\frac{1}{2}z_{\widetilde{\lambda}}^2(\omega,t) \quad (13)
\end{aligned}$$

Taking time derivative of $V(t)$, it yields

$$\begin{aligned}
\dot{V}(t) &= \int_0^\infty \mu(\omega)z_{e_1}\frac{\partial z_{e_1}}{\partial t}d\omega + \int_0^\infty \mu(\omega)z_{e_2}\frac{\partial z_{e_2}}{\partial t}d\omega\\
&+\int_0^\infty \mu(\omega)z_{\widetilde{b}}\frac{\partial z_{\widetilde{b}}}{\partial t}d\omega + \int_0^\infty \mu(\omega)z_{\widetilde{c}}\frac{\partial z_{\widetilde{c}}}{\partial t}d\omega\\
&+\int_0^\infty \mu(\omega)z_{\widetilde{B}}\frac{\partial z_{\widetilde{B}}}{\partial t}d\omega + \int_0^\infty \mu(\omega)z_{\widetilde{C}}\frac{\partial z_{\widetilde{C}}}{\partial t}d\omega\\
&+\int_0^\infty \mu(\omega)z_{\widetilde{\lambda}}\frac{\partial z_{\widetilde{\lambda}}}{\partial t}d\omega \qquad (14)
\end{aligned}$$

Inserting (11) into (14), we obtain

$$\begin{aligned}
\dot{V}(t) &= \int_0^\infty \mu(\omega)z_{e_1}[-\omega z_{e_1}-be_1-ce_2+\Delta f_1(y)\\
&+d_1(t)+\phi_1(u_1(t))]d\omega+\int_0^\infty \mu(\omega)z_{e_2}[-\omega z_{e_2}\\
&-Be_1-Ce_2+\lambda e_1(e_1^2+3e_1x_1+3x_1^2)+\Delta f_2(y)\\
&+d_2(t)+\phi_2(u_2(t))]d\omega+\int_0^\infty \mu(\omega)z_{\widetilde{b}}[-\omega z_{\widetilde{b}}\\
&-e_1^2]d\omega+\int_0^\infty \mu(\omega)z_{\widetilde{c}}[-\omega z_{\widetilde{c}}-e_1e_2]d\omega\\
&+\int_0^\infty \mu(\omega)z_{\widetilde{B}}[-\omega z_{\widetilde{B}}-e_1e_2]d\omega+\int_0^\infty \mu(\omega)\times\\
&z_{\widetilde{C}}[-\omega z_{\widetilde{C}}-e_2^2]d\omega+\int_0^\infty \mu(\omega)z_{\widetilde{\lambda}}[-\omega z_{\widetilde{\lambda}}+(e_1^2\\
&+3e_1x_1+3x_1^2)e_1e_2]d\omega\\
&= -J+\big[-be_1-ce_2+\Delta f_1(y)+d_1(t)\\
&+\phi_1(u_1(t))\big]\int_0^\infty \mu(\omega)z_{e_1}d\omega+\big[-Be_1-Ce_2\\
&+\lambda e_1(e_1^2+3e_1x_1+3x_1^2)+\Delta f_2(y)+d_2(t)\\
&+\phi_2(u_2(t))\big]\int_0^\infty \mu(\omega)z_{e_2}d\omega-e_1^2\int_0^\infty \mu(\omega)z_{\widetilde{b}}d\omega\\
&-e_1e_2\int_0^\infty \mu(\omega)z_{\widetilde{c}}d\omega-e_1e_2\int_0^\infty \mu(\omega)z_{\widetilde{B}}d\omega\\
&-e_2^2\int_0^\infty \mu(\omega)z_{\widetilde{C}}d\omega+(e_1^2+3e_1x_1+3x_1^2)e_1e_2\times\\
&\int_0^\infty \mu(\omega)z_{\widetilde{\lambda}}d\omega\\
&= -J+[-be_1-ce_2+\Delta f_1(y)+d_1(t)\\
&+\phi_1(u_1(t))]e_1+[-Be_1-Ce_2+\lambda e_1(e_1^2+3e_1x_1\\
&+3x_1^2)+\Delta f_2(y)+d_2(t)+\phi_2(u_2(t))]e_2-e_1^2\widetilde{b}\\
&-e_1e_2\widetilde{c}-e_1e_2\widetilde{B}-e_2^2\widetilde{C}+(e_1^2+3e_1x_1+3x_1^2)\times\\
&e_1e_2\widetilde{\lambda}\\
&= -J-e_1^2\hat{b}-e_1e_2\hat{c}+(\Delta f_1(y)+d_1(t))e_1\\
&+e_1\phi_1(u_1(t))-e_1e_2\hat{B}-e_2^2\hat{C}\\
&+(e_1^2+3e_1x_1+3x_1^2)e_1e_2\hat{\lambda}+(\Delta f_2(y)\\
&+d_2(t))e_2+e_2\phi_2(u_2(t)) \qquad (15)
\end{aligned}$$

where $J = \int_0^\infty \mu(\omega)\omega z_{e_1}^2 d\omega + \int_0^\infty \mu(\omega)\omega z_{e_2}^2 d\omega + \int_0^\infty \mu(\omega)\omega z_{\widetilde{b}}^2 d\omega + \int_0^\infty \mu(\omega)\omega z_{\widetilde{c}}^2 d\omega + \int_0^\infty \mu(\omega)\omega z_{\widetilde{B}}^2 d\omega + \int_0^\infty \mu(\omega)\omega z_{\widetilde{C}}^2 d\omega + \int_0^\infty \mu(\omega)\omega z_{\widetilde{\lambda}}^2 d\omega > 0$.

Through surveying (6) and (9), one has

$$\begin{aligned}
u_i(t)\phi_i(u_i(t)) &= -\xi_i\eta_i sgn(e_i)\phi_i(u_i(t))\\
&\geq \rho_{i1}u_i^2(t)=\rho_{i1}\xi_i^2\eta_i^2 sgn^2(e_i) \quad (16)
\end{aligned}$$

since $\xi_i=\frac{1}{\rho_{i1}}$, $\eta_i>0$, then

$$-sgn(e_i)\phi_i(u_i(t)) \geq \eta_i sgn^2(e_i) \qquad (17)$$

Multiplying both sides of (17) by $|e_i|$, according to $|e_i|sgn(e_i)=e_i$, then we obtain

$$e_i\phi_i(u_i(t)) \leq -\eta_i|e_i| \qquad (18)$$

Substituting (18) into (15), using Assumption 1, we have

$$\begin{aligned}
\dot{V}(t) \leq\ & -J+|e_1^2||\hat{b}|+|e_1e_2||\hat{c}|+\delta_1|e_1|-\eta_1|e_1|+|e_1e_2||\hat{B}|\\
&+|e_2^2||\hat{C}|+|(e_1^2+3e_1x_1+2x_1^2)e_1e_2||\hat{\lambda}|\\
&+\delta_2|e_2|-\eta_2|e_2|\\
=\ & -J-k_1e_1^2-k_2e_2^2<0 \qquad (19)
\end{aligned}$$

According to Ref.[6], we known that system (7) is asymptotically stable, thus, the proof is completed.

IV. Simulation example

In this section, some simulation results are provided to demonstrate our theoretical results. Take the system (7) as the controlled system. Let $\alpha=0.99$, the unknown parameters $b=0.15$, $c=-0.17$, $\lambda=-0.65$, the initial conditions are randomly chosen as $x(0)=0.1$, $y(0)=0.2$, $\hat{b}(0)=0.1$, $\hat{c}(0)=0.1$, $\hat{B}(0)=0.11$, $\hat{C}(0)=0.11$, $\hat{\lambda}(0)=0.1$. The model uncertainties and external disturbances as follows

$$\begin{aligned}
\Delta f_1(y)+d_1(t) &= -0.01\cos(y_1t)\\
\Delta f_2(y)+d_2(t) &= -0.01\cos(y_2t) \qquad (20)
\end{aligned}$$

the nonlinear inputs are

$$\phi_i(u_i(t)) = [1 - 0.5\cos(u_i(t))]u_i(t), \qquad i = 1, 2. \quad (21)$$

clearly, $\rho_{i1} = 0.5$, $\xi_i = 2$. According to Theorem 1, select the positive control constants $k_1 = k_2 = 2$, the controller and parametric update laws can be designed. When the controller is activated, we can obtain the desired state trajectories of system (7), meanwhile, for observe the control effect of the proposed control strategy, the state trajectories of master (4) and slave system (5) are also shown, the simulation results are presented in Figs. 3, 4 and 5.

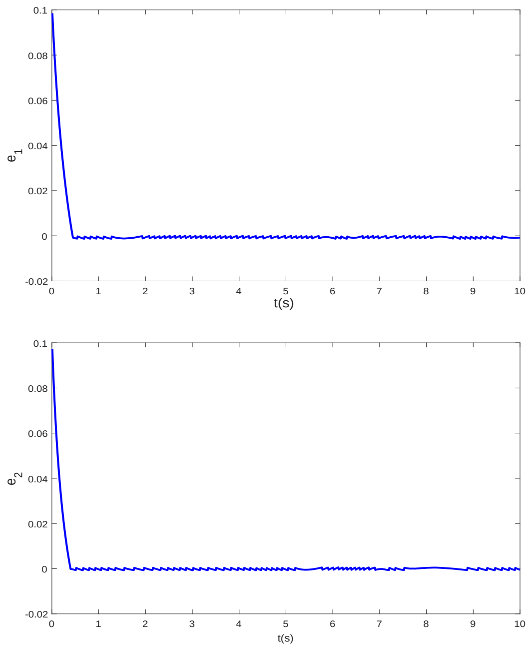
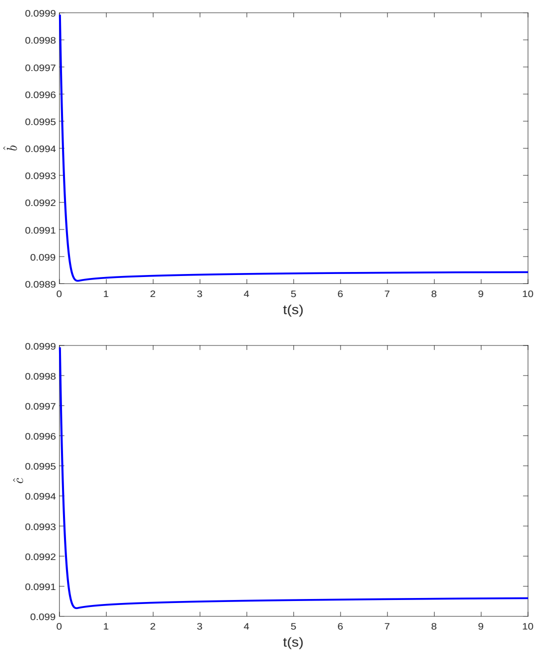


Fig. 3. State Trajectories of Error System (7) with Controller Activated



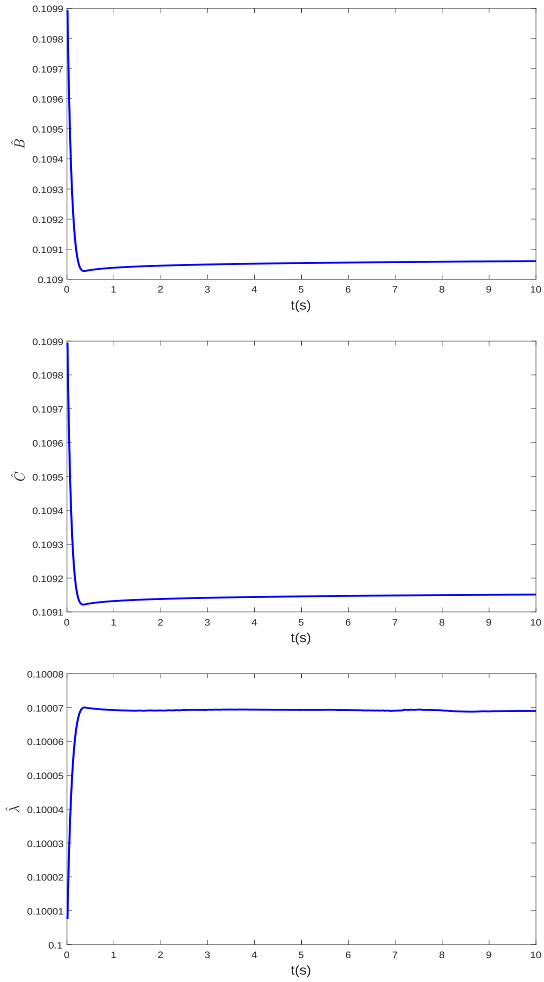


Fig. 4. Time Response of Estimate Parameters in Error System (7)

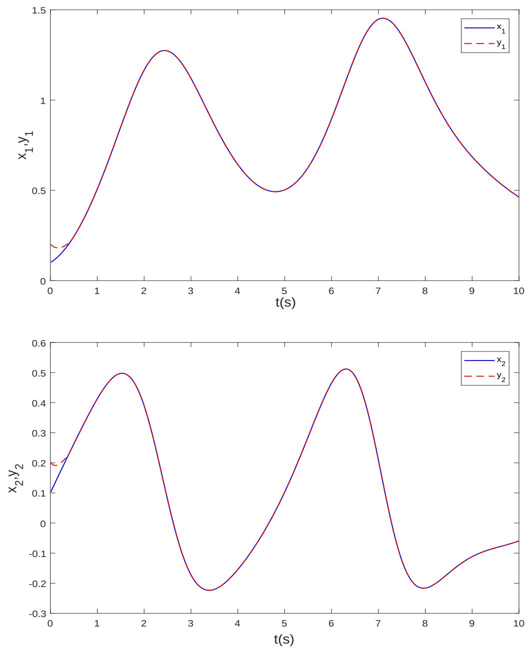


Fig. 5. State Trajectories of Master (4) and Slave System (5) with Controller Activated

From Fig. 2, we can see that all error states asymptotically converge to zero, which implies that the designed controller is applicable. In Fig. 3, it is obvious that all unknown parameters gradually converge to their actual values, which implies that the proposed parametric update laws are correct.

In Fig. 4, which can be seen that the master and slave system can reach synchronization asymptotically. All simulation results demonstrate the effectiveness and feasibility of our control scheme.

V. Conclusion

This paper researched the synchronization of fractional-order MBV systems with unknown parameters. The effects of model uncertainties, external disturbances and nonlinear inputs are fully taken into consideration. The frequency distributed model of fractional integrator and Lyapunov stability theory are applied to prove the asymptotic stability of closed-loop system. Simulation results have verified the correctness and applicability of the proposed synchronization scheme. Since our results are very useful to both bioengineering and medical science, we believe that there is high potential in the proposed approach.

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