Tracking Control of Higher Order Reference Signal Using Integrators and State Feedback

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Abstract—**The purpose of the research is to stabilize a system with a different kind of reference signals such as a unit-ramp, unit-parabolic or higher order signals (Higher Order Reference). The proposed controller consists of state feedback and integrators. State feedback controller makes the system have good performance. Meanwhile, integrators have natural characteristic for eliminating the steady-state error. By using new design of integrators, the system is able to be stable and reach the reference value (tracking control). To design the proposed controller design, the Coefficient Diagram Method (CDM) will be used and the pole location will be examined first to ensure the stability of the system. The design then is validated in simulation by implementing it in some system models, such as triple integrator system, DC motor, and inverted pendulum. The proposed controller design then is compared to PID Ziegler Nichols to evaluate system performances. The result proves that the proposed controller can eliminate steady-state error and stabilize the system with different kind of reference signals within 3 second.**

Index Terms—**Tracking Control, Higher Order Reference, Integrators, State Feedback, Coefficient Diagram Method.**

I. Introduction

There are many controllers that have been implemented in the industrial world. One of them is the Proportional-Integral-Derivative (PID) controller which is very famous and popular [1]. PID is easy to understand and to be implemented. PID has fast response characteristic, thus it can give good system performance. However, PID is not suitable for higher order than the second order system [2]. For higher order systems with multiple complex conjugate pole pairs (oscillations at multiple frequencies), the methods are not suitable [3]. Also, PID needs some method to tune the gains of proportional, integral, and derivative.

The other controller is Fuzzy Logic Controller (FLC). Same with PID, it is also very famous and popular [4]. FLC is built based on human knowledge or human logic so it is easy to understand. FLC also has some disadvantages. To design FLC, it needs data from the system specifications and limitations. It also will be difficult to be designed to the system that has many inputs and will perform slow response too.

Neural network (NN) control is also famous in industrial technology [5]. But, it needs a lot of data and a fast computer to train the controller. If it does not, it will waste time to train. However, NN has good performance because it can adapt to the new variable if there is any change within the system. This can occur because it had a training with a lot of data. This is the reason why it needs time to train and a fast computer.

There are six important types of input changes used in industrial practice for the purposes of modelling and control. Those signals are an unit-impulse, unit-step, rectangular pulse, unit-ramp, sinusoidal, and random signal [6]. Most commonly used in the control system signals are unit-step, unit-ramp and unit-parabolic. Those signals are categorized into several orders. The sequence starts from unit-step as the lowest order reference signal. The next sequences are a ramp, then parabolic signal consequently from low to high order.

In the control system design, a system which has unit-step or unit-ramp signal as reference input is easy to be stabilized, particularly in eliminating the steady-state error [7]. It's controller also will be easy to be designed, as well as tuning the controller gain. The problem arises on how to stabilize system with parabolic signals or higher order reference signal [8]. It will be even harder to get the controller gain as higher the order of the reference signal.

A controller that can eliminate the steady-state error of the system is important in the industrial process [9]. Despite the type of the reference input signal, the controller must be able to eliminate the steady-state error or tracking error. PID and Fuzzy Controller can eliminate steady-state error quite easily on the system that has low order reference signal. However, controlling a system with high order reference input signal using PID and Fuzzy Controller will be a hard task [10] [11]. Also, it will be a difficult task to tune the controller parameter.

State feedback controller is one of the simplest controllers in modern control system that can make the system having good performance. State feedback controller consists of gain controllers which respond to every state in the system. Hence, it can be modified following the dynamics of the system [12]. The application of state feedback such as inverted pendulum on cart [13], crane system [14], magnetic levitation system [15], quadrotor [16], balancing robot [17] [18].

The idea of the research is coming from eliminating the steady-state error of the system when the reference signal is given using an or some integrators with the state feedback controller. It is quite simple and easy to be implemented. The natural characteristic of an integrator is to eliminate the steady-state error. During the experiment, the number of integrators needed depends on the order of the reference input.

State space representation will be used in the research. The reason to use state space model is that the state feedback controller needs a model in state space representation. State

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space model also can be used to represent the Multi-Input Multi-Output (MIMO) system. Besides, by using state space representation, the dynamics of the system can be known.

The difficulty of the controller's parameter design will be solved using an optimization method. Some proposed methods to design the controller parameter gains are Pole Placement (Pole Assignment) [19] [20], Linear Quadratic Regulator (LQR) [21], Genetic Algorithm [22], and firefly algorithm [23].

The weakness of Pole Placement [24] and LQR [25] are there are no standard parameters to determine the parameter gains of the controller, thus it is still half trial and error method. While Genetic Algorithm needs a time of iteration to find the best parameter.

Another method is Coefficient Diagram Method (CDM) which can be defined as improved LQG [26]. With this method, trial and error can be avoided and determining a gain controller will be effortless and timeless [15]. Hence, this research will also implement the CDM in tuning the proposed controller gains.

The paper will be divided into some parts. The first part is the introduction. The second part presents system models which will be used in this research. The third part discusses the proposed controller that consists of state feedback with integrators. Next part explains the theory of CDM. The fifth part is the controller design. The sixth part is about numerical simulation and result that consists of Triple Integrator system, DC Motor system and Inverted Pendulum system. Then the last part is a conclusion and future work.

II. Proposed Controller

Consider, the model of the system in the state space representation is

$$\dot{x} = \mathbf{A}x + \mathbf{B}u \tag{1}$$

$$y = \mathbf{C}x \tag{2}$$

where x is state vector of the system (m-vector), $\dot{x}$ is the first derivative, u is control signal, y is the response of the system (output signal), $\mathbf{A}$ is $m \times m$ constant matrix, $\mathbf{B}$ is $m \times 1$ constant matrix and $\mathbf{C}$ is $1 \times m$ constant matrix.

The proposed controller is consists of Integrators and State Feedback. The block diagram of the system with the proposed controller is shown in Figure 1. The block n−Int is the Integrators and the structure is shown in Figure 2. Also, the block $-\mathbf{K}$ is the state feedback controller that is shown in Figure 3.

The basic principle of the proposed method is to add some integrators between error comparator and plant to remove the steady-state error. The number of Integrators depends on the reference input signal. If the reference is a unit-step ($i = 1$), the system uses one Integrator. For the unit-ramp ($i = 2$), the system uses two Integrators and so on.

A. State Feedback plus Feed Forward

Assume that the reference is unit-step so the system only need one integrator. The equation from Figure 1 is

$$\dot{z} = \hat{\mathbf{A}}z + \hat{\mathbf{B}}u + \mathbf{F}r \tag{3}$$

$$y = \hat{\mathbf{C}}z \tag{4}$$

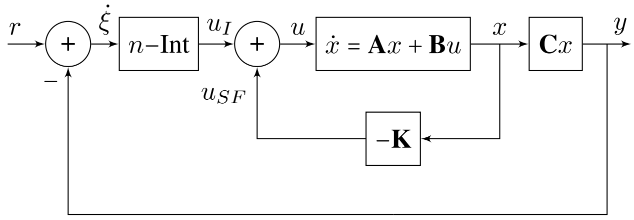

Fig. 1. The Block Diagram of Augmented System

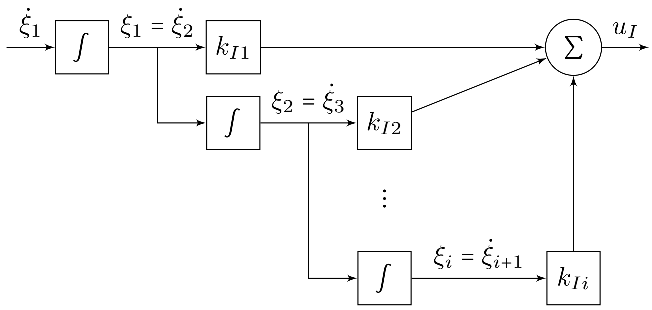

Fig. 2. The Block Diagram of n-Integrators

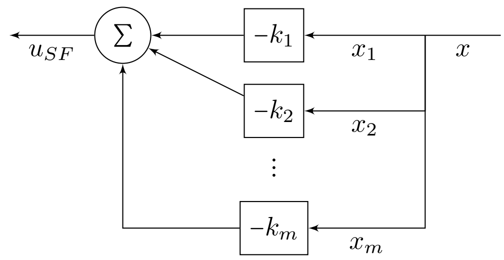

Fig. 3. The Block Diagram of State Feedback

where

$$z = \begin{bmatrix} x \\ \xi \end{bmatrix}$$

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & 0 \\ -\mathbf{C} & 0 \end{bmatrix} \quad \hat{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} \quad \hat{\mathbf{C}} = \begin{bmatrix} \mathbf{C} \\ 0 \end{bmatrix}^T \quad \mathbf{F} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

then, z is the new state vector of the plant (n-vector, $n = m+i$), ξ is the state variable of the system, r is the reference input signal, $\hat{\mathbf{A}}$ is $n \times n$ constant matrix, $\hat{\mathbf{B}}$ is $n \times 1$ constant matrix, $\hat{\mathbf{C}}$ is $1 \times n$ constant matrix, T is transpose matrix and $\mathbf{F}$ is reference matrix.

The u is the number of the control signal that consist of Integrators control signal, u_I, and state feedback control signal, u_{SF}. The number of the control signal, u, consist of Integrators control signal and State feedback control signal. It can be written as

$$u = u_I + u_{SF}. \tag{5}$$

In state feedback scheme, the control signal, u_{SF}, that determined by instantaneous state [9] is written as

$$u_{SF} = -\begin{bmatrix} k_1 & k_2 & \dots & k_m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_m \end{bmatrix} \tag{6}$$

$$= -\mathbf{K}x, \tag{7}$$

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TABLE I
LIST NUMBER OF ORDERS REFERENCE

Orders Reference (i)	Reference Signal	Integrator(s)	Time Function
1	unit-step	1	t^0
2	unit-ramp	2	t^1
3	unit-parabolic	3	t^2
⋮			
i	...	i	t^{i-1}

where $\mathbf{K}$ is the state feedback gain matrix.

B. Integrators Control

The natural characteristic of Integral control can eliminate steady-state error. To eliminate steady-state error because of the reference signal (ramp, parabolic and above), it can be done by adding some integrators.

However, there is no guidelines yet on designing the structure of those integrators. This includes where the integrators and gains are placed in a system.

The structure of Integrators is shown in figure 2. The output of the first integrator becomes the input of the second integrator, and the output of the second integrator becomes the input of the third integrator, and so on continuously. Those output of Integrators will be multiplied by a gain to fasten the response on eliminating steady-state error.

Based on Figure 2, the Integrators control signal is

$$\begin{aligned} u_I &= \begin{bmatrix} k_{I1} & k_{I2} & \dots & k_{Ii+1} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ . \\ . \\ . \\ \xi_{i+1} \end{bmatrix} \\ &= \mathbf{K_I}\xi \end{aligned} \tag{8}$$

where i is the order of the reference as shown in Table I.

The Integrators state is

$$\begin{aligned} \dot{\xi}_1 &= r - y = r - \mathbf{C}x \\ \dot{\xi}_2 &= \xi_1 \\ \dot{\xi}_3 &= \xi_2 \\ \vdots \; & \; \vdots \\ \dot{\xi}_{i+1} &= \xi_i \end{aligned} \tag{9}$$

therefore, the matrix of Integrators is

$$\dot{\xi} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & & \vdots \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xi + \begin{bmatrix} r - \mathbf{C}x \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{10}$$

Next, let assume that the reference signal is a unit-ramp or unit-parabolic signal, the matrix of the integrator is,

$$\mathbf{I_{ramp}} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \tag{11}$$

$$\mathbf{I_{parabolic}} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{12}$$

for the High Order Reference (HOR), the matrix of the integrator is,

$$\mathbf{I_{HOR}} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{13}$$

Based on (10) until (13), the first row which is $r - \mathbf{C}x$ part can be eliminated from the integrators matrix. Those equations also have an identity matrix as part of the integrators matrix. Hence, the equation can be rewritten it as a general equation as

$$\mathbf{I_{HOR}} = I_{i \times i} \tag{14}$$

where I is identity matrix and i is the order of reference.

C. The Augmented System

The new system design can be written as,

$$\dot{z} = \hat{\mathbf{A}}z + \hat{\mathbf{B}}u + \mathbf{F}r, \tag{15}$$

$$y = \hat{\mathbf{C}}z, \tag{16}$$

where

$$z = \begin{bmatrix} x \\ \xi \end{bmatrix}$$

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & 0 \\ -\mathbf{C} & 0 \\ 0 & \mathbf{I_{HOR}} \end{bmatrix} \; \hat{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ 0 \\ 0 \end{bmatrix} \; \hat{\mathbf{C}} = \begin{bmatrix} \mathbf{C} \\ 0 \\ 0 \end{bmatrix}^T \; \mathbf{F} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{17}$$

Because unit-ramp has one order of reference, $i = 1$, then the system design is written as

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & 0 \\ -\mathbf{C} & 0 \\ 0 & 1 \end{bmatrix} \; \hat{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ 0 \\ 0 \end{bmatrix} \; \hat{\mathbf{C}} = \begin{bmatrix} \mathbf{C} \\ 0 \\ 0 \end{bmatrix}^T,$$

While the system design for unit-parabolic that has two order of reference, $i = 2$, is stated as

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & 0 & 0 \\ -\mathbf{C} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \; \hat{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ 0 \\ 0 \\ 0 \end{bmatrix} \; \hat{\mathbf{C}} = \begin{bmatrix} \mathbf{C} \\ 0 \\ 0 \\ 0 \end{bmatrix}^T.$$

The overall control signal is

$$\begin{aligned} u &= u_{SF} + u_I \\ &= -\mathbf{K}x + \mathbf{K_I}\xi \\ &= -\hat{\mathbf{K}}z \end{aligned} \tag{18}$$

where $\hat{\mathbf{K}}$ is gain of control signal and consists of state feedback gains and integrators gains. It can be written as

$$\hat{\mathbf{K}} = \begin{bmatrix} k_1 & k_2 & \cdots & k_m & | & -k_{I1} & -k_{I2} & \cdots & -k_{Ii} \end{bmatrix} \tag{19}$$

Later, $\mathbf{K}$ and $\mathbf{K_{I_n}}$ will be designed using the CDM and matrix transformation.

Then, the state error equation can be obtained by substituting (18) into (15) as

$$\dot{z} = (\hat{\mathbf{A}} - \hat{\mathbf{B}}\hat{\mathbf{K}})z + \mathbf{F}r \tag{20}$$

$$y = \hat{\mathbf{C}}z. \tag{21}$$

To apply the proposed controller, the system must fulfil controllability condition, $\mathbf{M}$, and state controllable condition, $\mathbf{P}$. Respectively, the condition is

$$\mathbf{M} = \begin{bmatrix}\mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \dots & \mathbf{A}^{m-1}\mathbf{B}\end{bmatrix} \tag{22}$$

$$\mathbf{P} = \begin{bmatrix}\mathbf{A} & \mathbf{B} \\ -\mathbf{C} & 0\end{bmatrix}. \tag{23}$$

III. Coefficient Diagram Method

Coefficient Diagram Method or CDM is one of the algebraic design approach or polynomial method in control system design that firstly introduced by Manabe [27]. It will be used here to design the parameter gains of the proposed controller (the n-integrators gain and state feedback gain). The advantage of CDM is to have the standard parameter to design the polynomial characteristic so it can avoid the trial and error method [28]. Also, it will save effort and time to obtain the parameter gains. The polynomial characteristic of the closed loop system is defined as

$$\begin{aligned}\mathbf{P(s)} &= |s\mathbf{I} - \hat{\mathbf{A}}| \\ &= \alpha_n s^n + \alpha_{n-1}s^{n-1} + \dots + \alpha_1 s + \alpha_0,\end{aligned} \tag{24}$$

where α_n is the coefficient of closed loop system.

The fundamental parameters of CDM are the equivalent time constant, τ and the stability index, γ. The equivalent time constant is corresponding to the response of the system and the stability index is corresponding to the stability of the system. The standard form of the equivalent time constant is

$$\tau = \frac{1}{3}t_s, \tag{25}$$

where t_s is desired sampling time. While, the standard form of the stability index is

$$\gamma_{n-1} = \dots = \gamma_3 = \gamma_2 = 2, \quad \gamma_1 = 2.5. \tag{26}$$

The characteristic polynomial, $\mathbf{P_T}$, is known as the desired characteristic equation which is expressed by a coefficient, a_0, the equivalent time constant, τ, and stability index, γ. It can be written as

$$\begin{aligned}\mathbf{P_T}(s) &= \alpha_0\left[\left\{\sum_{i=2}^{n}\left(\prod_{j=1}^{i-1}\frac{1}{\gamma_{i-j}^{j}}\right)(\tau s)^i\right\} + \tau s + 1\right] \\ &= \alpha_n s^n + \alpha_{n-1}s^{n-1} + \dots + \alpha_1 s + \alpha_0\end{aligned} \tag{27}$$

where α_n is the coefficient of the desired characteristic equation, n is the $p+1$ of $\mathbf{A}$ matrix of the system, and α_0 [14] can be written as

$$\alpha_0 = \frac{\prod_{j=1}^{n-1}\gamma_{n-j}^{j}}{\tau^n}. \tag{28}$$

Based on (27), the stable pole location as $\mu_1, \mu_2, ...\mu_n$, in the LHP (Left Half Plane) of s-plane are

$$(s-\mu_1)(s-\mu_2)(s-\mu_3)\cdots(s-\mu_n) = 0 \tag{29}$$

IV. Controller Design

The Matrix Transformation, $\mathbf{T}$, will be used to obtain gains of the Integrators and state feedback from Coefficient Diagram Method. The Matrix Transformation is,

$$\mathbf{T} = \mathbf{MW}, \tag{30}$$

where $\mathbf{M}$ is the controllability matrix and it can be defined as,

$$\mathbf{M} = [\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A}^2\mathbf{B} \quad \dots \quad \mathbf{A}^{n-1}\mathbf{B}], \tag{31}$$

while

$$\mathbf{W} = \begin{bmatrix}\beta_1 & \beta_2 & \cdots & \beta_{n-1} & 1 \\ \beta_2 & \beta_3 & \cdots & 1 & 0 \\ \vdots & \vdots & \iddots & \vdots & \vdots \\ \beta_{n-1} & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0\end{bmatrix}, \tag{32}$$

where β_n is the open loop polynomial of the system (15) given by

$$\begin{aligned}\mathbf{P_{OL}}(s) &= |s\mathbf{I} - \hat{\mathbf{A}}| \\ &= \beta_n s^n + \beta_{n-1}s^{n-1} + \dots + \beta_1 s + \beta_0.\end{aligned} \tag{33}$$

Let, define a new state vector by $\hat{z}$ as

$$z = \mathbf{T}\hat{z}, \tag{34}$$

then, if the rank of controllability matrix is full, it means that the matrix $\mathbf{T}$ can be inverse. The system then can be modified in the controllable canonical form as

$$\dot{\hat{z}} = \mathbf{T}^{-1}\mathbf{AT}\hat{z} + \mathbf{T^{-1}B}u. \tag{35}$$

Choose the desired eigenvalues as in (29), then the desired characteristic is (27). Now, define $\hat{\mathbf{K}}\mathbf{T}$ as

$$\hat{\mathbf{K}}\mathbf{T} = \begin{bmatrix}k_1 & k_2 & \dots & k_m & | & k_{I1} & k_{I2} & \dots & k_{Ii}\end{bmatrix}. \tag{36}$$

When control signal $u = -\hat{\mathbf{K}}\mathbf{T}\hat{z}$ is used to control the system (35), the system equation becomes,

$$\begin{aligned}\mathbf{P_{CL}}(s) &= |s\mathbf{I} - \mathbf{T}^{-1}\mathbf{AT} + \mathbf{T^{-1}B}\hat{\mathbf{K}}\mathbf{T}| \\ &= s^n + (\alpha_{n-1}+k_1)s^{n-1} + (\alpha_{n-2}+k_2)s^{n-2} + \dots \\ &+ (\alpha_1 + k_{I(n-1)})s + (\alpha_0 + k_{In}).\end{aligned} \tag{37}$$

According to CDM, the desired polynomial characteristic based on the stability index and the equivalent time constant is needed. Therefore, the coefficients of the desired polynomials a_0 to a_{n-1} can be obtained from (27). Finally by equating (33) and (37), the gain matrix K_a for the augmented system can be found as

$$\hat{\mathbf{K}} = \begin{bmatrix}k_1 & k_2 & \dots & k_m & | & -k_{I1} & -k_{I2} & \dots & -k_{Ii}\end{bmatrix}\mathbf{T}^{-1} \tag{38}$$

where,

$$\begin{aligned}k_1 &= \alpha_{n-1} - \beta_{n-1} \\ k_2 &= \alpha_{n-2} - \beta_{n-2} \\ &\vdots \\ -k_{I(n-1)} &= \alpha_1 - \beta_1 \\ -k_{In} &= \alpha_0 - \beta_0.\end{aligned}$$

V. Algorithm

The proposed controller gains can be derived using the CDM. It is effortless than using trial and error method which takes a lot of time. The procedure to determine controller gains is explained briefly as

1) Determine the matrix size (m), based on the $\mathbf{A}$ Matrix's of the system, the reference order (for ramp

$i = 1$), and the $\hat{\mathbf{A}}$ matrix size of the augmented system, $n = m + i$. The matrix system is,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & 0 \\ -\mathbf{C} & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

2) Determine the polynomial target (desired polynomial) that can be obtained using (27) which has parameter as in (26) and (28). The polynomial target is,

$$\alpha_0 = \frac{\gamma_4\gamma_3^2\gamma_2^3\gamma_1^4}{\tau^5},$$

$$\mathbf{P_T} = \alpha_0\Big(\frac{\tau^5}{\gamma_4\gamma_3^2\gamma_2^3\gamma_1^4}s^5 + \frac{\tau^4}{\gamma_3\gamma_2^2\gamma_1^3}s^4 + \frac{\tau^3}{\gamma_2\gamma_1^2}s^3 + \frac{\tau^2}{\gamma_1}s^2 + \tau s + 1\Big)$$

$$\mathbf{P_T} = \alpha_5 s^5 + \alpha_4 s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0.$$

The polynomial target code from the CDM can be seen in the Appendix.

3) Determine the open loop polynomial of augmented system using (33), $\mathbf{W}$ matrix using (32) and controllability matrix using (31). The open loop polynomial is,

$$\mathbf{P_{OL}} = \beta_5 s^5 + \beta_4 s^4 + \beta_3 s^3 + \beta_2 s^2 + \beta_1 s + \beta_0$$

4) Calculate the transformation matrix $\mathbf{T}$ using (30). The inverse transformation matrix is,

$$\mathbf{T^{-1}} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

5) Calculate the gains of proposed controller $\hat{\mathbf{K}}$ using (38). The gains is,

$$\hat{\mathbf{K}} = [-2500 \quad -2500 \quad 1000 \quad 200 \quad 20]\mathbf{T^{-1}}. \quad (39)$$

Steps 3-5 can be replaced with easier ways. The code **acker** (from Ackermann's formula) in MATLAB also can be used. After getting the polynomial target, the poles can be obtained. Then, by using 'acker' command, the gains can be determined.

VI. Numerical Simulation and Discussion

The simulation will be generated using Matlab Simulink. The equivalent time constant (τ) in simulation is 1 second and the stability index (γ_i) are in (26). There is three experiment in the section. The first is poles location experiment that is used to ensure the stable system. The second is response comparison between a system with proposed controller and conventional controller. The third is comparison response between the proposed controller and PID Ziegler-Nichols.

A. Pole Location Experiment

First of all, pole locations of desired polynomials that were designed using CDM will be examined to ensure its stability which can be shown in Table II and Figure 4. Based on Table II, all poles have a negative value. Negative poles are located on a left-half plane (LHP). Since all poles position is on LHP, the system will be stable. So CDM can give the stable system based on the location of the poles.

While, based on Figure 4, the dominant poles are located on real axis so that the system performs no overshoot in response. Besides, they are located close enough to the imaginary axis hence the generated control signal is not too big.

TABLE II
Pole Location in s-Plane

m	Desired Polynomial	Pole Position
4	$s^4 + 10s^3 + 50s^2 + 125s + 125$	$u_{1,2} = -2.5 \pm 3.44i$ $u_{3,4} = -2.5 \pm 0.81i$
5	$s^5 + 20s^4 + 200s^3 + 1000s^2$ $+2500^s + 2500$	$u_{1,2} = -5.56 \pm 6.3983i$ $u_{3,4} = -3.02 \pm 1.7642i$ $u_5 = -2.84$
6	$s^6 + 40s^5 + 800s^4 + 8000^3$ $+4 \times 10^4 s^2 + 10^5 s + 10^5$	$u_{1,2} = -11.14 \pm 13.04i$ $u_{3,4} = -3.22 \pm 1.86i$ $u_5 = -8.3253$ $u_6 = -2.9567$
7	$s^7 + 80s^6 + 3200s^5 + 64000s^4$ $+6.4 \times 10^5 s^3 + 3.2 \times 10^6 s^2$ $+8 \times 10^6 s + 8 \times 10^6$	$u_{1,2} = -22.25 \pm 26.07i$ $u_{3,4} = -3.21 \pm 1.85i$ $u_5 = -14.64$ $u_6 = -11.49$ $u_7 = -2.95$
	$\vdots$	
10	$s^1 + 640s^6 + 204800s^5$ $+3.2768 \times 10^7 + 2.6214 \times 10^9$ $+1.0486 \times 10^{11} + 20972 \times 10^{12}$ $+2.0972 \times 10^{13} + 1.0486 \times 10^{14}$ $+2.6214 \times 10^{14} + 2.6214 \times 10^{14}$	$u_{1,2} - 178.02 \pm 208.53i$ $u_{3,4} - 3.21 \pm 1.85i$ $u_5 - 119.97; u_6 - 84.04$ $u_7 = -39.96; u_8 = -19.86$ $u_9 = -10.76; u_{10} = -2.95$

By using CDM, the desired pole location can be designed so that the system is guaranteed in its stability and well performance. It will eliminate the trial and error method and lessen the effort in determining the pole. Table II shows that even in a high order system (for example is a system with 10 poles), CDM provides the desired polynomials with 'best' designated pole location in a very easy way.

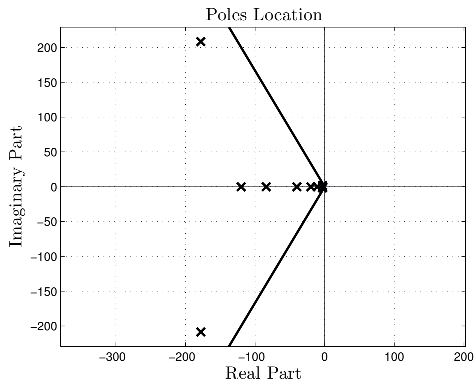

Fig. 4. Poles Location

B. Proposed Controller Experiment

The simulation will use a unit-ramp and unit-parabolic as a reference signal. It will compare the result between the system with proposed controller and an conventional integrator control. The augmented system in Simulink can be seen in Figure 13 at Appendix.

Some system models will be used to examine the proposed controller in this research through simulation. The system models are a triple integrator, DC motor, and Inverted Pendulum in state space representation. The system model can be written generally as

$$\dot{x} = \mathbf{A}x + \mathbf{B}u,$$
$$y = \mathbf{C}x.$$

The matrix constants of triple integrator system are

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{B}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \mathbf{C}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T. \tag{40}$$

The matrix constants of DC motor system are

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -5 & 10 \\ 0 & -0.2 & -4 \end{bmatrix} \quad \mathbf{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \quad \mathbf{C}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T. \tag{41}$$

The matrix constants of inverted pendulum system are

$$\mathbf{A}_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 20.601 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.4905 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{B}_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0.5 \end{bmatrix} \quad \mathbf{C}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}^T. \tag{42}$$

Not every system can be controlled by this proposed controller (state feedback with integrators controller). The system must fulfil the criterion of the controllability matrix and must be a state controllable system. By using (7) and (8), the criterion of the systems can be obtained. Those systems have more than 1 rank of controllability matrix and state controllable. Hence, state feedback with integrators controller can be applied to those systems.

The result of the simulation using a unit-ramp and unit-parabolic reference is shown in Figure 5 - 10. Figure 5 and 6 are the response of the triple integrator system, Figure 7 and 8 are the response of the DC motor and Figure 9 and 10 are the response of the Inverted Pendulum.

Based on Figure 5, triple integrator system with the proposed controller gives a response that can achieve the reference value of the ramp signal. Meanwhile, the same system that is given only one integrator cannot reach reference signal value and makes a steady-state error. The same result with the different system can be seen in Figure 7 and Figure 9.

As seen in Figure 6, a similar result is shown on the same system response which the parabolic signal reference unit is given. By applying the proposed controller, the system is able to follow the reference signal. However, when only one integrator is applied to the system, it responds quite bad performance with a bigger steady-state error. The same result with the different system also can be seen in Figure 8 and Figure 10.

To stabilize the system, the proposed controller does not take a long time. The parameter gains of the proposed controller are taken from the CDM. In the research, the standard parameter of CDM is used and can give the good result to stabilize the system while reaching the reference signal. Even, it only takes under 4 seconds of time.

Based on Figure 5-10, the response of the system that using the proposed controller can follow the reference signal. By using a unit-ramp and unit-parabolic as the reference signal, the proposed controller (with Integrators) can make the system stable and can eliminate the steady-state error. While, for the system with conventional integrator control, it is not able to follow the reference signal.

Based the simulation, it is clearly defined that there is no steady-state error. To prove it mathematically, the analysis is done by doing input substitution [29],

$$e_{ss(ramp)}(\infty) = \lim_{t\to\infty} \langle [1 + \hat{\mathbf{C}}\hat{\mathbf{A}}^{-1}\hat{\mathbf{B}}]t + \hat{\mathbf{C}}(\hat{\mathbf{A}})^{-1})^2\hat{\mathbf{B}}\rangle \tag{43}$$

$$\begin{aligned} e_{ss(parabolic)}(\infty) = \lim_{t\to\infty} \langle [1 + \hat{\mathbf{C}}\hat{\mathbf{A}}^{-1}\hat{\mathbf{B}}]t^2 \\ + [1 + \hat{\mathbf{C}}\hat{\mathbf{A}}^{-1}\hat{\mathbf{B}}]t + \hat{\mathbf{C}}(\hat{\mathbf{A}})^{-1})^2\hat{\mathbf{B}}\rangle \end{aligned} \tag{44}$$

where,

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} - \mathbf{BK} & \mathbf{B}k_{I1} & \mathbf{B}k_{I2} \\ -\mathbf{C} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \hat{\mathbf{B}} = \begin{bmatrix} 0 \\ \mathbf{F} \\ 0 \end{bmatrix} \quad \hat{\mathbf{C}} = \begin{bmatrix} \mathbf{C} \\ 0 \\ 0 \end{bmatrix}^T.$$

For the triple integrators system with ramp-unit as reference, the steady-state error is,

$$e_{ss}(\infty) = \lim_{t\to\infty} \langle 0t + 0\rangle = 0, \tag{45}$$

while with parabolic-unit as reference,

$$e_{ss}(\infty) = \lim_{t\to\infty} \langle 0t^2 + 0t + 0\rangle = 0. \tag{46}$$

Based on (45) and (46), zero result means there is no steady state error by mathematical calculation. Thus, it is proven on simulation and mathematically that the steady-state error has been eliminated by proposed controller.

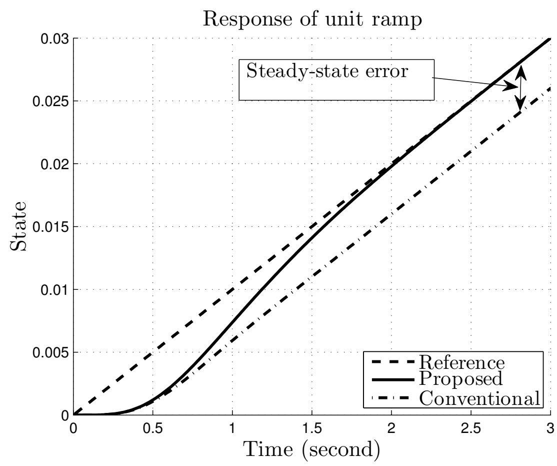

Fig. 5. Triple integrator response of unit-ramp

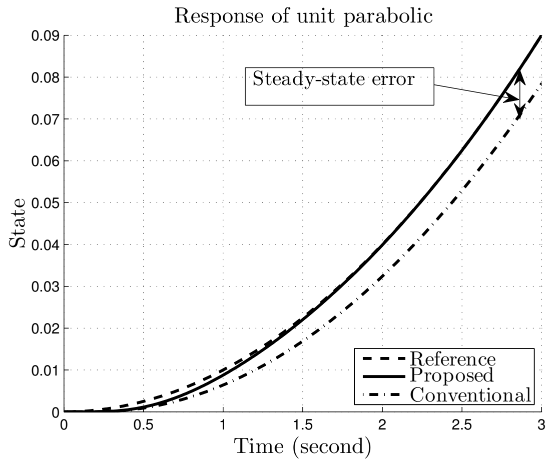

Fig. 6. Triple integrator response of unit-parabolic

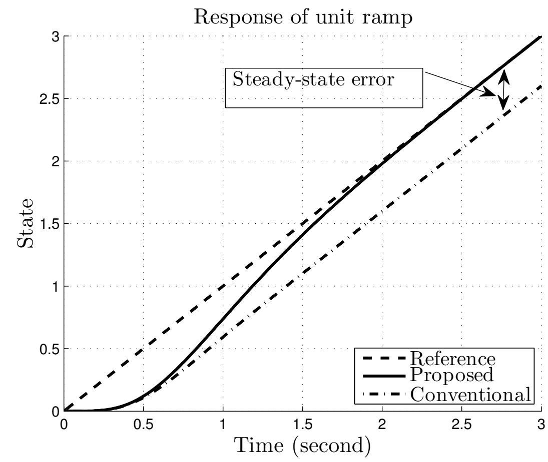

Fig. 7. DC motor response of unit-ramp

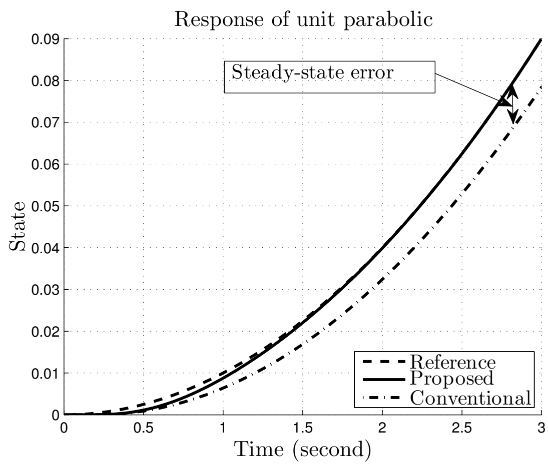

Fig. 8. DC motor response of unit-parabolic

C. Comparison with PID Ziegler-Nichols

The third simulation is to compare the proposed controller with PID Ziegler-Nichols. Both controllers are implemented in DC Motor system model. As in CDM, the PID Ziegler-Nichols also has a standard parameter to determine the controller parameter. The result is shown in Figure 11 (response of unit-ramp) and Figure 12 (response of unit-parabolic).

Overall, there is no steady-state error when PID Controller

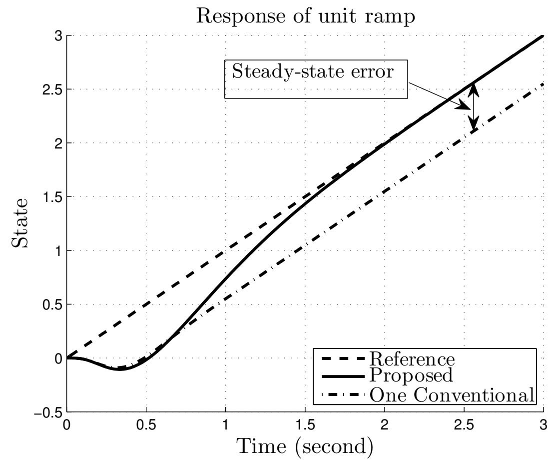

Fig. 9. Inverted pendulum response of unit-ramp

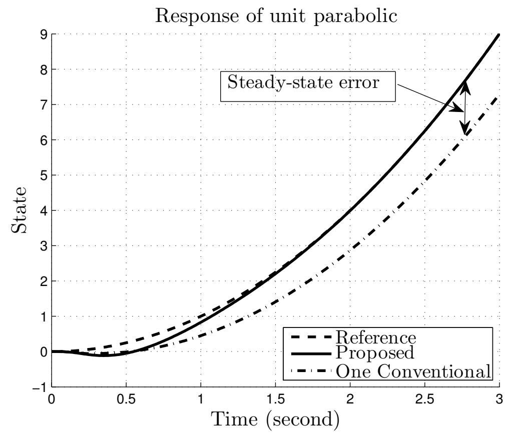

Fig. 10. Inverted pendulum response of unit-parabolic

is implemented to the system with a ramp reference is given, even though the system responds a bit overlapping in the beginning. Meanwhile, there is a difference value of 0.0021 between the response and the reference (steady-state error) when parabolic reference is given.

Since there is a steady-state error in the system response of unit-parabolic, modification of PID Controller is needed to stabilize the system. Thus, the design could be repeated until the augmented system performs no steady-state error.

The proposed controller is able to make the system reach the reference value both when the parabolic and the unit-ramp is given as reference. Although the system also responds a bit overlapping with the ramp reference in the beginning, it is still able to achieve reference value. Also, the augmented system is able to follow the reference value when the parabolic reference is given. Thus, no modification is needed when the proposed controller is implemented.

VII. Conclusion and Future Work

In the paper, the experiment of the proposed controller, the Integrators and state feedback has been done by simulation in Matlab Simulink. By using the proposed controller, the steady-state error of the system that caused by any reference signal can be eliminated and the system can be stable. The optimization method, Coefficient Diagram method, also can be used to design the parameter gain of the controller to accelerate the controller design process. As long as the order

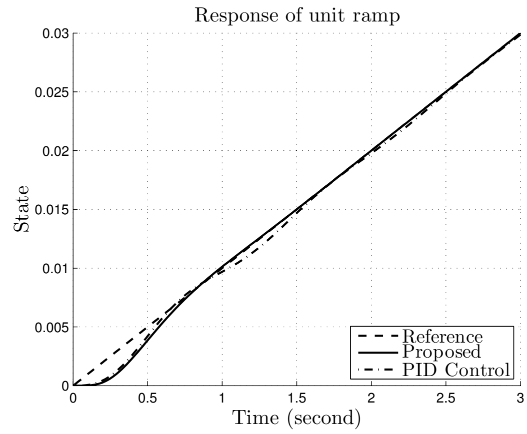

Fig. 11. The response of unit-ramp

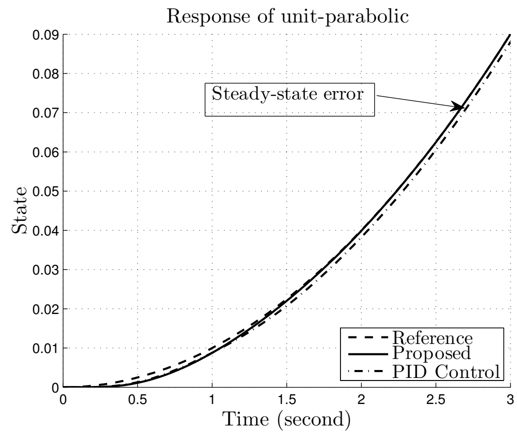

Fig. 12. The response of unit-parabolic

of the reference signal is known and the system is modelled in state space representation, the controller can be designed.

The flexibility of the proposed controller is still an issue which means not all system cannot be applied by it. There are some requirements that the system would meet before can be implemented with the proposed controller.

Another issue which this paper still cannot comply is that addition of Integrators still needs to be done through controller design. In other words, if the reference signal type is changed after the design process, the parameter must be redesigned. The parameter cannot be changed automatically when the reference signal is changed.

APPENDIX

PLANT SIMULATION IN SIMULINK

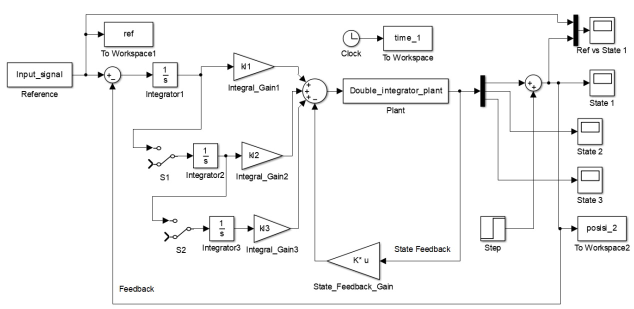

Fig. 13. Plant in simulink

COEFFICIENT DIAGRAM MATLAB CODE

```
clear; close; clc

tau = 1; %Equivalent time constant

G=[2.5 2 2 2] %Standard stability index

%Calculate alpha_0 - alpha_5
a0=(G(4)*G(3)^2*G(2)^3*G(1)^4)/(tau^5);
a1=a0*tau;
a2=a0*[(tau^2)/G(1)];
a3=a0*[(tau^3)/(G(2)*G(1)^2)];
a4=a0*[(tau^4)/(G(3)*G(2)^2*G(1)^3)];
a5=a0*[(tau^5)/(G(4)*G(3)^2*G(2)^3*...
G(1)^4)];

%Desired polynomial
data3=[a5 a4 a3 a2 a1 a0]

J=roots(data3)' %Poles

zplane(J') %Pole location
```

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