A Framework of Importance-Performance Analysis Based on the Multiple Determination Coefficient

Jiunn-I Shieh and Hsin-Hung Wu

***Abstract*—This study uses derived importance based on the multiple determination coefficient to replace self-stated importance for importance-performance analysis. The traditional importance-performance analysis assumes that there are no interactions among the survey items. Without considering the interactions among the survey items, some items might be either underestimated or overestimated in terms of importance for quadrant classifications, which might result in misunderstanding the major strengths (weaknesses) to minor strength (weaknesses) and vice versa. Thus, the improvement efforts might be in vain. In this study, the proposed framework based on the multiple determination coefficient considers the items interactions to be under the other items influence. A case is illustrated to show how this framework differs from the traditional importance-performance analysis when interactions among the survey items are taken into consideration.**

***Index Terms*—multiple determination coefficient, importance performance analysis, fuzzy measure, Shapley value**

I. INTRODUCTION

Importance-performance analysis (IPA) is a useful marketing research technique that can be easily and effectively applied in a wide variety of areas to suggest successful marketing strategies [1]-[5]. When applying the IPA method, we need both performance and importance data from the survey results with the underlying assumption that the importance of a particular item is not influenced by the other items. In practice, the importance among items is interdependent [6].

Traditional importance-performance analysis uses self-stated importance data directly from the survey results [7], [8]. However, there are two main disadvantages of using self-stated evaluations. First, respondents often find it difficult to differentiate the degree of importance and respondents' answers may be influenced by social norms or political correctness [9]. Second, self-stated evaluations could be either overestimated or underestimated when there are significant interactions among self-stated items.

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Jiunn-I Shieh is with Department of M-Commerce and Multimedia Applications, Asia University, Taichung City, Taiwan (e-mail: jishieh@asia.edu.tw).

Hsin-Hung Wu is with Department of Business Administration, National Changhua University of Education, Changhua City, Taiwan and Faculty of Education, State University of Malang, Malang, East Java, Indonesia.

On other hand, the derived importance evaluation approach uses a less direct way of uncovering the evaluations that are most reliable to reflect the respondents' views from the survey. In the literature, the decision-making trial and evaluation laboratory (DEMATEL) method determine the importance by considering the interactions among the items [10]. This method, however, lacks of any solid theoretical background and the number of questionnaires is limited for an effective study [11]. To overcome this problem, we propose a new framework of using the Shapley value based on a fuzzy measure to evaluate the derived importance of a particular item influenced by the other items.

This paper is organized as follows: Section 2 briefly reviews multiple determination coefficient, fuzzy measure, Shapley value, and importance-performance analysis. The methodology is described in Section 3. The results are depicted in Section 4. Finally, conclusions are drawn in Section 5.

II. LITERATURE REVIEW

A. Multiple Determination Coefficient

In a simple linear regression, the determination coefficient tells us which percentage of the variance of the response variable is explained by the fitted linear mapping of the explanatory variable. It is a measure of the goodness of fit of the relationship between the dependent and independent random variables in a regression analysis. It is also known as *R*-square (R^2). The value of the determination coefficient must lie between 0 and 1. It is easy to generalize the multiple determination coefficient which is a measure of the goodness of fit of the relationship between the dependent and independent random variables (at least two independent random variables) in a multiple regression analysis. Let Y be a dependent random variable, X_1, X_2, …, and X_p be independent random variables, and $\hat{y}_i = f(x_1, x_2, \ldots, x_p)$ be the best multiple regression equation where $\hat{y}_i$ is predicted (by the multiple regression model) values for the sample survey data. Let n be the number of sample and $\bar{y} = (y_1+y_2+\ldots+y_n)/n$. Then, the multiple determination coefficient is defined as follows [12]:

$$R^2_{Y_{12\ldots p}} = \frac{SSR(X_1, X_2, \ldots, X_p)}{SST} = 1 - \frac{SSE(X_1, X_2, \ldots, X_p)}{SST}, \quad (1)$$

where $SSR(X_1, X_2,...,X_P) = \sum_{i=1}^{n}\left(\hat{y}_i - \bar{y}\right)^2$, $SSE(X_1, X_2,...,X_P) = \sum_{i=1}^{n}\left(\hat{y}_i - y_i\right)^2$, and $SST(X_1, X_2,...,X_P) = \sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2$.

B. Fuzzy Measure

In general, the interactions among the items should be considered while aggregating the items evaluations. For aggregating the items, the common methods which are based on the covariance structure principle and additive measure have been studied at length. Note that the linear assumption is needed for the above methods. However, the linear assumption happens to be often rather far from the reality, and models with low accuracy are produced. For this reason, non-additive measures or fuzzy measures are introduced to solve these problems.

The central concept of fuzzy measure theory was introduced by Choquet in 1953 and independently defined by Sugeno in 1974 in the context of fuzzy integrals [13], [14]. Let $X = \{X_1, X_2, \ldots, X_n\}$ be a finite set and $P(X)$ be class of subsets of universal set X. We have the following definition [15], [16]. A real-valued set function $\mu : P(X) \to R$, is a fuzzy measure if it satisfies

(i) $\mu(\phi) = 0$;

(ii) If $A, B \in P(X)$ and $A \subseteq B$, then $\mu(A) \leqq \mu(B)$.

It is easy to check that the multiple determination coefficient is fuzzy measure. Fuzzy measure can deal with multi-criteria decision problems under the group interactions without the other items influencing the decision-making process. To deal with the group interactions of other items influencing cases, the Shapley value is introduced.

C. Shapley Value

The Shapley value measures a variation on a characteristic function g when item i enters a set (or coalition) of items. In game theory, the Shapley value or Shapley index is used to indicate the weight of a game. Shapley values based on a fuzzy measure can be calculated in order to give some indication of the importance of each singleton among items interactions under the other items influence [17]. In the case of additive measures (that is, all items are mutually independent), the Shapley value will be the same as the derived importance of each singleton for which there is no influence from the other items. For a given fuzzy measure g and $|X| = n$, the Shapley value is defined for every $i \in X$ as follows [17]:

$$\phi(i) = \sum_{M \subseteq X \setminus \{i\}} \frac{(n - |M| - 1)!|M|!}{n!}\left[g(M \cup \{i\}) - g(M)\right]. \quad (2)$$

Shapley values based on a fuzzy measure are needed when there are significant interactions among items. In this study, we used the Shapley value based on a fuzzy measure to evaluate the derived importance of each item, specifically taking into consideration the interaction of items under the influence of other items.

D. Importance-Performance Analysis

A commonly seen importance-performance analysis is a two-dimensional grid and can be constructed by plotting mean ratings of performance and importance to form four quadrants to identify the major strengths and weaknesses as shown in Fig. 1 [2], [3], [18]-[20]. Importance is labeled as the x-axis, whereas performance is labeled as the y-axis. These four quadrants include “keep up the good work” in Quadrant I, “possible overkill” in Quadrant II, “low priority” in Quadrant III, and “concentrate here” in Quadrant IV [4], [5].

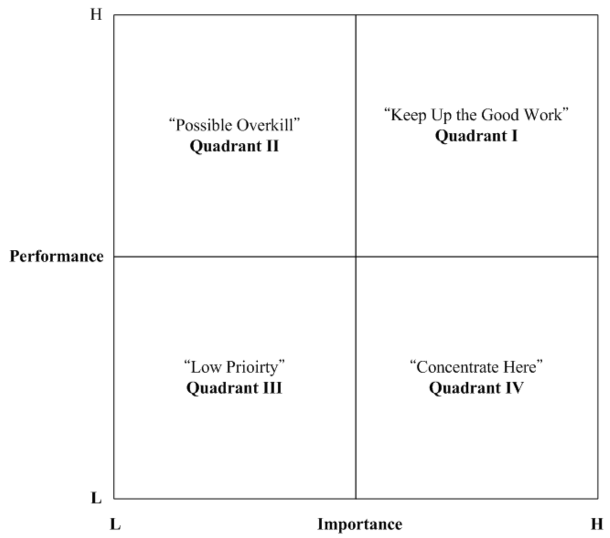

Fig. 1. Importance-performance analysis

The meanings of these four quadrants are described below [4], [5], [21]. Items in Quadrant I with both high performance and high importance are viewed as major strengths for organizations to achieve or maintain a competitive advantage. Items in Quadrant II with high performance but low importance might imply that the resources committed to these items are excessive and should be deployed elsewhere. Items in Quadrant III with both low performance and low importance might indicate that they do not require additional efforts. Finally, items in Quadrant IV with low performance but high importance can be viewed as major weaknesses for an organization. Immediate attention for improvement is required since the inability to identify the items in Quadrant IV might result in low customer satisfaction. In fact, immediate improvement efforts should be placed in highest priority when major weaknesses are identified, while items regarded as major strengths should be maintained, leveraged, and heavily promoted [2], [4], [5].

III. METHODOLOGY

In order to construct these four quadrants of importance-performance analysis, both performance and importance data are required from the survey results. In addition, the basic underlying assumption is that the importance of a particular item is not influenced by the others. However, if the assumption is violated, it results in either overestimating or underestimating the importance when there are significant interactions among items. In practice, the importance among items is interdependent. In our study, we use the Shapley value based on a fuzzy measure to evaluate the derived importance of a particular item which is influenced by the other items. To compute the Shapley value, we need to have a fuzzy measure *FM* based on the multiple determination coefficient. Specifically, compute the fuzzy measures of all subsets of all questionnaire items except the item of overall satisfaction by Equation (1). Then, compute the Shapley value for each item by Equation (2). Finally, the derived importance data can be calculated and be used for IPA method. The details are as follows.

Suppose that there are α items in the questionnaire and the number of valid questionnaire responses was β. Let x_{ij} be the performance value from 1 to 5 for *j*-th respondent in *i*-th item, where $i = 1, 2, \ldots, \alpha$ and $j = 1, 2, \ldots, \beta$. Let z_j be the overall performance value from 1 to 5 for *j*-th respondent. The procedures of the proposed framework are summarized as follows:

1. Calculate the fuzzy measure *FM* based on the multiple determination coefficient by Equation (1).
2. Calculate the Shapely values with respect to the fuzzy measure *FM* for each item by Equation (2).
3. Set the derived importance of item *i*, denoted as $impo_i$, to be the Shapely value $\phi(i)$, i.e., $impo_i = \phi(i)$.
4. Plot IPA diagram. The performance value for each item is computed by $perf_i = \sum_{j=1}^{\beta} x_{ij} / \beta$, while the importance value for each item is $impo_i = \phi(i)$. Draw the vertical reference line *L* with the formula of $y = \sum_{i=1}^{\alpha} perf_i / \alpha$ and the horizontal reference line *M* with the formula of $x = \sum_{i=1}^{\alpha} impo_i / \alpha$. Finally, plot *i* pairs of ($impo_i$, $perf_i$).

IV. RESULTS

The questionnaire in this study was designed based on the SERVQUAL model proposed by Parasuraman, Zeithaml and Berry [22] with 21 items as shown in Table I. Each respondent was asked to fill out both the importance and performance for each item. The respondent can express feelings of very satisfactory (importance), satisfactory (importance), neutral, dissatisfactory (unimportance), and very dissatisfactory (unimportance) by numerical values of 5, 4, 3, 2, and 1, respectively. The survey was taken among 113 patients or their families at a geriatric long-term care center which is the ancillary organization of Show Chwan Memorial Hospital (located in Changhua City, Taiwan) and its related organizations from August 15, 2012 to August 24, 2012. A total of 102 valid questionnaires were received, and the valid return rate was 95.5%.

TABLE I
TWENTY-ONE ITEMS IN THE SURVEY

Item	Description
1	Detailed description of hospitalization
2	Well-equipped medical equipment
3	Clear marked signs in the hospital
4	Easy to make a family visit
5	Quality of meals plan and dining
6	Detailed physical condition and demand for each patient by nursing staff
7	Cordial attitude of service staff
8	Comfort and safe environments
9	Recorded the patients' physiological conditions precisely
10	Reaction efficiency on patients' complaints
11	Responding ability for emergency
12	Sufficient staff to quickly respond the patients' needs
13	Service staff with good communication skills
14	Quality sanitary conditions and maintenance
15	Medical staff with professional abilities
16	Proper arrangement of rehabilitation and daily activities
17	Reasonable charge for service and care
18	Sense of security from the service provided by the institution
19	Friendly attitude on staff
20	Emphasis on patients' feelings and personal needs
21	Staff's concern and assist when patients encounter difficulties

The reliability of the survey measured by Cronbach's α was well above 0.959, indicating that the scales of the formal questionnaire have considerable reliability [23]. The structures for performance questions in factor analysis went well with the structure of the questionnaire with the Kaiser-Meyer-Olkin statistic of 0.931. The construct validity was also supported by factor loadings. Therefore, the construct validity of the questionnaire was good [24].

The numerical values of these twenty-one questions in terms of importance and performance are summarized in Table II. The commonly seen importance-performance analysis plot is depicted in Fig. 2. In contrast, based on the proposed framework, the first step is to compute the fuzzy measure *FM* based on multiple determination coefficient. The number of fuzzy measures for all subset in power set of $\{X_1, X_2, \ldots, X_{21}\}$ is 2^{21} = 2097152 including the empty set. Note that the percentage of the variance of the response variable is explained by the fitted linear mapping of the empty set is zero. Therefore, the fuzzy measure of the empty set is zero. By Equation (1), the largest value of the fuzzy measure of $\{X_1, X_2, \ldots, X_{21}\}$ is 0.7377, while the smallest value is 0.2701 except for the empty set. These twenty-one fuzzy measure values are 0.2701, 0.3951, 0.3028, 0.2786, 0.4015, 0.3483, 0.5130, 0.4643, 0.4482, 0.3745, 0.3966, 0.3878, 0.3311, 0.4403, 0.2786, 0.4430, 0.4431, 0.4221, 0.4998, 0.4713, and 0.5123, respectively.

TABLE II
VALUES OF IMPORTANCE AND PERFORMANCE OF TWENTY-ONE QUESTIONS

Question Item	Importance Value	Performance Value
1	4.1250	3.5490
2	4.1518	3.4804
3	4.0179	3.6373
4	4.1441	3.5588
5	4.3661	3.3725
6	4.2321	3.5098
7	4.2703	3.5000
8	4.2883	3.6078
9	4.2232	3.4804
10	4.1081	3.3922
11	4.3750	3.6471
12	4.2232	3.3529
13	4.2768	3.5098
14	4.4196	3.6078
15	4.3750	3.6961
16	4.2500	3.5686
17	4.1607	3.4706
18	4.1441	3.4608
19	4.1696	3.6078
20	4.2232	3.6863
21	4.2589	3.5882
Grand Average	4.2287	3.5373

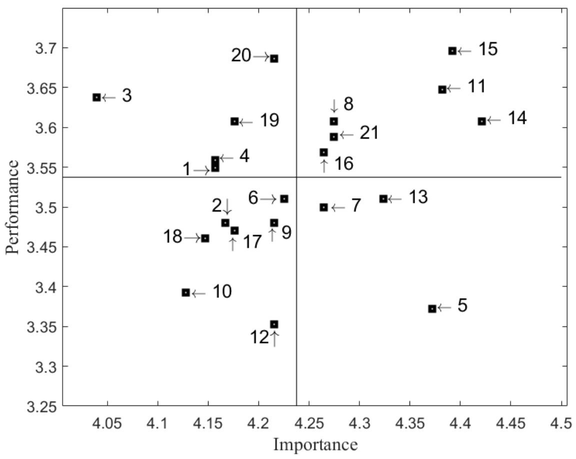

Fig. 2. Original importance-performance analysis plot

TABLE III
DERIVED IMPORTANCE AND PERFORMANCE OF TWENTY-ONE ITEMS

No.	Derived Importance	Performance
1	0.0182	3.5490
2	0.0293	3.4804
3	0.0217	3.6373
4	0.0266	3.5588
5	0.0372	3.3725
6	0.0276	3.5098
7	0.0482	3.5000
8	0.0385	3.6078
9	0.0650	3.4804
10	0.0256	3.3922
11	0.0303	3.6471
12	0.0333	3.3529
13	0.0232	3.5098
14	0.0329	3.6078
15	0.0179	3.6961
16	0.0485	3.5686
17	0.0384	3.4706
18	0.0302	3.4608
19	0.0494	3.6078
20	0.0438	3.6863
21	0.0519	3.5882

In the second step, the Shapley value for each item can be calculated by Equation (2), and these twenty-one values are 0.0182, 0.0293, 0.0217, 0.0266, 0.0372, 0.0276, 0.0482, 0.0385, 0.0650, 0.0256, 0.0303, 0.0333, 0.0232, 0.0329, 0.0179, 0.0485, 0.0384, 0.0302, 0.0494, 0.0438, and 0.0519, respectively. Please refer to Table III for the combinations of derived importance and performance. The larger the Shapley value is, the more likely the importance of the item. Moreover, since all Shapley values are greater than zero, it indicates that the importance of a particular item is influenced by the importance of the other items.

The third step is to set the importance of each item to be the correspondent Shapley value for each item. The fourth step, the performance value for each item can be computed by $perf_i = \sum_{j=1}^{102} x_{ij} / 102$ and these values are 3.5490, 3.4804, 3.6373, 3.5588, 3.3725, 3.5098, 3.5000, 3.6078, 3.4804, 3.3922, 3.6471, 3.3529, 3.5098, 3.6078, 3.6961, 3.5686, 3.4706, 3.4608, 3.6078, 3.6863, and 3.5882, respectively. The vertical reference line L, based on the formula of $y = \sum_{i=1}^{21} perf_i / 21$, is 3.5373, i.e., ((3.5490 + 3.4804 + 3.6373 + 3.5588 + 3.3725 + 3.5098 + 3.5000 + 3.6078 + 3.4804 + 3.3922 + 3.6471 + 3.3529 + 3.5098 + 3.6078 + 3.6961 + 3.5686 + 3.4706 + 3.4608 + 3.6078 + 3.6863 + 3.5882)/21). The horizontal reference line M with the formula of $x = \sum_{i=1}^{21} impo_i / 21$ is 0.0351, i.e., ((0.0182 + 0.0293 + 0.0217 + 0.0266 + 0.0372 + 0.0276 + 0.0482 + 0.0385 + 0.0650 + 0.0256 + 0.0303 + 0.0333 + 0.0232 + 0.0329 + 0.0179 + 0.0485 + 0.0384 + 0.0302 + 0.0494 + 0.0438 + 0.0519)/21). Finally, plot twenty-one pairs of ($impo_i$, $perf_i$) as shown in Fig. 3.

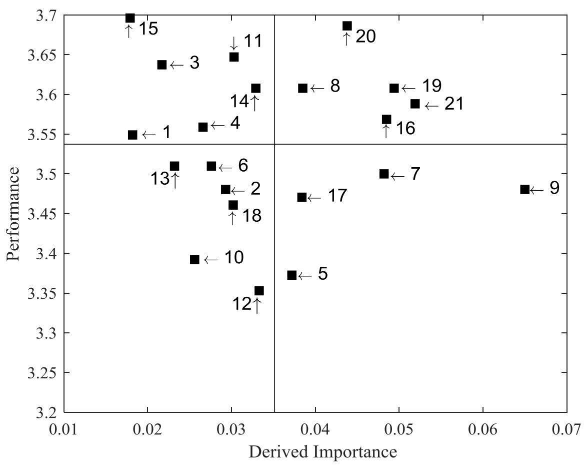

Fig. 3. Importance-performance analysis plot by the multiple determination coefficient

By comparing the original IPA plot and IPA plot with the Shapley value, the results are somewhat different. Please refer to Table IV for further information. Using different importance scales results in different located quadrants horizontally. That is, some items originally situated in

Quadrant I (III) will be shifted to Quadrant II (IV) and vice versa. This indicates that the importance for each item will be classified significantly different. For instance, a particular item originally viewed as the minor strength (Quadrant II) assuming the independence of importance will be viewed as the major strength when interdependence of importance is taken into consideration. This will lead to different marketing strategies and effectiveness of actions taken. From Table II and Table III, the importance of item 9 is ranked in the middle when the interactions are not taken into consideration but it becomes the most important item when the interactions among items are significant and taken into consideration. Therefore, the importance of item 9 is underestimated. By the same token, the importance of items 19 and 20 is underestimated when the interactions are not taken into consideration. On the contrary, items 11, 14, and 15 are overestimated.

TABLE IV
QUADRANTS DIFFERENCES BETWEEN PLOTS

Quadrant	I	II	III	IV
Original IPA	8, 11, 14, 15, 16, 21	1, 3, 4, 19, 20	2, 6, 9, 10, 12, 17, 18	5, 7, 13
IPA with Shapley value	8, 16, 19, 20, 21	1, 3, 4, 11, 14, 15	2, 6, 10, 12, 13, 18	5, 7, 9, 13

V. CONCLUSIONS

A framework of importance-performance analysis based on the multiple determination coefficient method is proposed without assuming that the importance of a particular item is independent of influence from the other items. Traditional importance-performance analysis needs the importance data, but the method we proposed do not need it and could deal with more general cases. The self-stated importance from the survey results is replaced by derived importance to form a two-dimensional grid with derived importance labeled as x-axis and performance labeled as y-axis. A case is illustrated to show how this proposed framework works and how this framework differs from the traditional importance-performance analysis. The results show that Items 9, 19, and 20 are underestimated when the interactions among items are not taken into consideration. By contrast, Items 11, 14, and 15 are overestimated when the interactions among items are not taken into consideration. Therefore, the proposed framework of IPA based on the multiple determination coefficient is very practical to use. In fact, the proposed framework can be viewed as a general framework no matter what the interactions among items are.

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