Applications of Fuzzy Soft Sets over Some Semigroups based on Extended Averages

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Abstract—This paper introduces extended averages of fuzzy soft sets over some semigroups. The construction of a new algorithm for solving some decision-making problems is based on the extended averages of fuzzy soft sets, over some semigroups. In addition to that, the examples presented in this paper demonstrate that the new algorithm is practical for solving decision-making problems.

Index Terms—extended averages, fuzzy soft sets over some semigroups, decision-making problems.

I. Introduction

THE solution to real-world problems in many fields such as engineering, environment, computer science, medical science, economics involves data that contain uncertainties. There is a wide range of theories that can be used when dealing with uncertainties in data, such as the theory of probability, fuzzy sets [1], rough sets [2], intuitionistic fuzzy sets [3], interval mathematics [4], vague sets [5], as well as other mathematical tools. In 1999, Molodtsov [6] defined soft set theory as a new mathematical tool for dealing with uncertainties that are free from the difficulties. He pointed out several directions for the applications of soft sets, such as smoothness of functions, operations research, game theory, and probability. Since then the soft set theory has been worked on and developed by many researches [7], [8], [9], [10], which this paper outlines below.

The soft sets are extended to fuzzy soft sets by Maji et al. [11] (2001). They introduced fuzzy soft sets, fuzzy soft subsets, the intersection, union and investigated their properties. In 2007, Roy and Maji [12] discussed the application of the algorithm of fuzzy soft sets in decision-making problems. The score value of fuzzy soft sets in decision-making problems was computed using the Comparison table in the algorithm. Later in 2009, the algorithm of Roy and Maji was revised by Kong et al [13], this time by computing the score value of fuzzy soft sets in decision-making problems without the Comparison table in the algorithm. Next, the concept of mapping on classes of fuzzy soft sets was defined by Kharal and Ahmad [14] while studying properties of fuzzy soft images and inverse fuzzy soft images. In 2011, Cagman et al. [15] redefined fuzzy soft sets and defined fuzzy soft aggregation operator and applied an algorithm with fuzzy soft aggregation operator approach to decision-making problems. Neog and Sut [16] studied the union and intersection of fuzzy soft sets while presenting some properties as an idempotent property, a commutative property, an associative property, an absorption property, a distributive property of a union and intersection of fuzzy soft sets. Afterwards, a new algorithm of fuzzy soft sets in decision-making problems using grey theory was discussed by Kong et al. [17] by taking the choice value and score value evaluations. Gogoi et al. [18] also discussed an algorithm of fuzzy soft sets in decision-making problems, by computing an average of fuzzy soft sets and using the Comparison table in the algorithm. Next, an algorithm of fuzzy soft sets in decision-making problems based on grey relational analysis and Dempster-Shafer theory of evidence was presented by Tang [19]. Later, Alcantud and Mathew [20] defined separable fuzzy soft sets and discussed an algorithm for decision-making under the separable fuzzy soft sets.

The theory of fuzzy soft set is a good mathematical tool when it comes to dealing with uncertainty. However, it is also a new notion when it comes to applying it to abstract algebraic structures. In 2011, Yang [21] established fuzzy soft sets to fuzzy soft semigroups. He defined a fuzzy soft [left, right] ideal and a fuzzy soft semigroup over a semigroup. He provided sufficient and necessary conditions for α-level set, intersection and union of fuzzy soft [left, right] ideals. Afterwards in 2013, Naz et al. [22] defined a product of two fuzzy soft semigroups and presented some properties of fuzzy soft interior ideals [quasi-ideals, bi-ideals, generalized bi-ideals] over semigroups under some conditions. Later on, in 2015, Siripitukdet and Suebsan [23] defined semiprime, prime and strongly prime fuzzy soft bi-ideals over semigroups and presented their properties. Fuzzy soft bi-ideals over semigroups were also studied by Suebsan and Sriripitukdet [24] in 2018, followed by the presentation of their properties which proved that the image of fuzzy soft bi-ideals over semigroups are the fuzzy soft bi-ideals over semigroups. Julath and Siripitukdet [25] examined some characterizations of fuzzy bi-ideals and fuzzy quasi-ideals of semigroups.

When it comes to existing researches of fuzzy soft sets in decision-making problems, they are limited to some extent. The algorithm of Gogoi et al. [18] was designed for evaluating all parameters by all assessors. However, if assessors are evaluating for some specific parameters then the algorithm cannot be used to calculate the average. In a case where the maximum score equal to fuzzy soft sets the algorithm is unable to make a decision in decision-making problems.

The fuzzy soft set over a semigroup based on extended

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averages plays a key role in dealing with problems specified above. In this paper, we introduce extended averages of fuzzy soft sets over some semigroups. Here the extended averages are devised to be suitable for some or all parameters. The extended averages of fuzzy soft sets over some semigroups were used in the new algorithm. The results presented in this paper show that the model based on extended averages is practical for solving decision problems.

II. Preliminaries

This section provides some basic definitions of fuzzy soft sets and briefly reviews the algorithms used in decision-making problems.

Let S be a set and let $\alpha : S \to \mathbf{R}$ be a 1-1 function.

Define operations $\triangle$ and $\bigtriangledown$ on S as follows: For any $x, y \in S$, define

$$x \triangle y = \begin{cases} x & \text{if} \quad \alpha(\text{x}) \geq \alpha(\text{y}), \\ y & \text{if} \quad \alpha(\text{x}) < \alpha(\text{y}) \end{cases}$$

and

$$x \bigtriangledown y = \begin{cases} x & \text{if} \quad \alpha(\text{x}) < \alpha(\text{y}), \\ y & \text{if} \quad \alpha(\text{x}) \geq \alpha(\text{y}). \end{cases}$$

Then $(S, \triangle)$ and $(S, \bigtriangledown)$ are semigroups. These semigroups are called semigroups induced by a function α and denote S_α. We write $S_\alpha = \{x(\alpha(x)) | x \in S\}$.

In real-world problems, we can construct semigroups induced by 1-1 functions.

Example 1. A family is looking to purchase a water purifier. Let $S_\alpha = \{o_1(3),\ o_2(2),\ o_3(4),\ o_4(1),\ o_5(5)\}$ be a set of five water purifiers with a limited time warranty (year) under consideration. Then $(S_\alpha, \triangle)$ is a semigroup.

Example 2. A corporation is evaluating the decision of an investment opportunity. Let $S_\alpha = \{o_1(2),\ o_2(4),\ o_3(3),\ o_4(5),\ o_5(1)\}$ be a set of five investment avenues with risk assessments under consideration, where o_1:Bank Deposit, o_2:Shares, o_3:Mutual Fund, o_4:Stocks, o_5:Government Bonds and the levels of risk are 1:very low, 2:low, 3:moderate, 4:high, 5:very high. Then $(S_\alpha, \bigtriangledown)$ is a semigroup.

Now, we give some definition of soft sets over a common universe set.

Definition 1. [6] Let U be a common universe set and A be a non-empty subset of a set of parameters E and let $P(U)$ denotes a power set of U. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

Next, we give some definition of fuzzy soft sets over semigroups.

Definition 2. [11] Let A be a non-empty subset of a set of parameters E. A pair (F, A) is called a fuzzy soft set over a semigroup S, where $F : A \to \text{Fuz}(S)$ and $\text{Fuz}(S)$ is a set of all fuzzy sets on S.

Let (F, A) be a fuzzy soft set over S. For $p \in A, F(p) \in \text{Fuz}(S)$. Set $F_p := F(p)$. Thus $F_p \in \text{Fuz}(S)$.

The following example is an example of a fuzzy soft set over a semigroup.

Example 3. Let $S_\alpha = \{o_1(3),\ o_2(2),\ o_3(4),\ o_4(1),\ o_5(5)\}$ be a set of five water purifiers with a limited time warranty (year)under consideration. Let S_α be a semigroup with a binary operation $\triangle$ defined by Table I.

TABLE I
The Multiplication Table of a Semigroup S_α

$\triangle$	o_1	o_2	o_3	o_4	o_5
o_1	o_1	o_1	o_3	o_1	o_5
o_2	o_1	o_2	o_3	o_2	o_5
o_3	o_3	o_3	o_3	o_3	o_5
o_4	o_1	o_2	o_3	o_4	o_5
o_5	o_5	o_5	o_5	o_5	o_5

Let $E = \{e_1\{\text{strongly}\}, e_2\{\text{clean water}\}, e_3\{\text{easy using}\}, e_4\{\text{UV + RO available}\}, e_5\{\text{ultra filter available}\}, e_6\{\text{expensive}\}\}$ be a set of parameters, where UV:Ultraviolet, RO:Reverse osmosis, and let $A = \{e_2, e_4, e_6\}$. Let (F, A) be a fuzzy soft set over S_α such that

$F_{e_2} = \{o_1/0.3, o_2/0.6, o_3/0.1, o_4/0.7, o_5/0.6\}$,
$F_{e_4} = \{o_1/0.4, o_2/0.7, o_3/0.2, o_4/0.6, o_5/0.5\}$,
$F_{e_6} = \{o_1/0.4, o_2/0.5, o_3/0.6, o_4/0.5, o_5/0.6\}$.

Then (F, A) is a fuzzy soft set representing the "attractiveness of the water purifier" which Mr.X is going to buy.

In 2007, Roy and Maji [12] used the Comparison table approach in decision-making problems. Let $U = \{o_1, o_2, ..., o_n\}$ be an object set and let $E = \{e_1, e_2, ..., e_k\}$ be a set of parameters.

The Comparison table is a square table in which the number of rows and columns are equal, rows and columns both are labeled by the object names $o_1, o_2, ..., o_n$ of U, and the entries are $c_{ij}, i, j \in \{1, 2, ..., n\}$ given by c_{ij} = the number of parameters for which the membership value of o_i exceeds or equal to the membership value of o_j.

Obviously, $0 \leq c_{ij} \leq k$, and $c_{ii} = k$, for all i, j where, k is the number of all parameters in a fuzzy soft set. Thus, c_{ij} indicates a numerical measure, which is an integer number and o_i dominates o_j in c_{ij} number of parameters out of k parameters.

Roy and Maji [12] used the Comparison table in the algorithm as follows: Algorithm in [12],

1. Input the fuzzy soft sets $(F, A), (G, B)$ and (H, C).
2. Input the parameter set P as observed by the observer.
3. Compute (S, P) from the fuzzy soft sets $(F, A), (G, B)$ and (H, C).
4. Compute the Comparison table of the fuzzy soft sets (S, P) and compute t_i and r_i for o_i for all i.
5. Compute the score value of o_i for all i.
6. The decision is S_k if $S_k = \max_i S_i$.
7. If k has more than one value then any one of o_k maybe chosen.

In 2014, Gogoi et al. [18] computed an average of fuzzy soft sets and used the Comparison table in an algorithm with decision making problems. Algorithm in [18],

1. Input the fuzzy soft sets (F_i, A_i).
2. Find an average of the fuzzy soft sets.
3. Multiply the weight of the parameters such that $\Sigma w_j = 1$.
4. Compute the Comparison table of the fuzzy soft sets and compute t_i and r_i for o_i for all i.
5. Compute the score value of o_i for all i.
6. The decision is S_k if $S_k = \max_i S_i$.

TABLE II
The Average with Multiply the Weight of the Parameters of Fuzzy Soft Sets

	e_1	e_2	e_3	e_4	e_5	e_6
o_1	0.54	0.84	0.16	0.29	0.87	0.13
o_2	0.62	0.80	0.25	0.36	0.85	0.23
o_3	0.55	0.66	0.39	0.46	0.74	0.33
o_4	0.82	0.45	0.64	0.47	0.63	0.53
o_5	0.72	0.33	0.87	0.78	0.24	0.82

TABLE III
The Score Value Table

	Row sum	Column sum	Score value
o_1	14	22	−8
o_2	17	19	−2
o_3	17	19	−2
o_4	21	15	6
o_5	21	15	6

In the following example, we combined the data from [18] with an algorithm of Gogoi et al. [18].

Example 4. We calculate Steps 1-3, which results in Table II. We calculate Steps 4-5, which results in Table III. We observe that Gogoi et al's method is designed for evaluating all parameters by all assessors. Which presents an issue when some assessors are evaluating some specific parameters, then the average cannot be calculated using this method. In order to deal with the proposed problems, we define extended averages and present an algorithm of fuzzy soft sets over some semigroups in decision-making problems based on extended averages in the next section.

III. Fuzzy soft sets over semigroups in decision-making problems

The following section defines extended averages, by proposing it as a new idea of fuzzy soft sets over a semigroup, while also presenting an algorithm for identification of objects based on extended averages.

Definition 3. An extended average $\uplus_e$ of two fuzzy soft sets (F, A) and (G, B) over a semigroup S_α, is defined by $(F, A) \uplus_e (G, B) := (F \uplus_e G, A \cup B)$, where for each $p \in A \cup B$,

$$(F \uplus_e G)_p = \begin{cases} \frac{F_p+G_p}{2} & \text{if} \quad \mathrm{p} \in \mathrm{A} \cap \mathrm{B}, \\ F_p & \text{if} \quad \mathrm{p} \in \mathrm{A} - \mathrm{B}, \\ G_p & \text{if} \quad \mathrm{p} \in \mathrm{B} - \mathrm{A}. \end{cases}$$

Thus $(F, A) \uplus_e (G, B)$ is a fuzzy soft set over S_α.

Why does the extended average $\uplus_e$ of fuzzy soft sets over a semigroup S_α plays an important role for the decision?

The formula is designed for extending the average. In each situation of a corporation, we can select the best assessor for each item. If each parameter is evaluated by all assessors then see Example 5. In another case, we will consider as in Example 6 by selecting proper assessors.

Consider the following two examples of fuzzy soft sets over a semigroup S_α.

Example 5. A family is looking to purchase a water purifier. Let $S_\alpha = \{o_1(3), o_2(2), o_3(4), o_4(1), o_5(5)\}$ be a set of five water purifiers with a limited time warranty (year) under consideration. Let S_α be a semigroup with a binary operation $\triangle$ defined by the same Table I in the Example 3.

Let $E = \{e_1\{\text{strongly}\}, e_2\{\text{clean water}\}, e_3\{\text{easy using}\}, e_4\{\text{expensive}\}\}$ be the set of parameters, and let
$A = \{e_1, e_2, e_3, e_4\}$ and $B = \{e_1, e_2, e_3, e_4\}$.

Let (F, A) denotes a fuzzy soft set over S_α representing the “attractiveness of the water purifier” which a father is going to buy and let (G, B) denotes a fuzzy soft set over S_α representing the “attractiveness of the water purifier” which a mother is going to buy.

In this case, we see that both a father and a mother can evaluate the same parameters. We can use the Definition 3 for calculating the average. For the general case, if assessors evaluate the same parameters then we still use the Definition 3 for calculations. However, if we add a parameter $e_5\{\text{UV + RO available}\}$, $e_6\{\text{ultra filter available}\}$, where UV:Ultraviolet, RO:Reverse osmosis, then a mother may not have knowledge about the UV + RO available and the ultra filter available but she can evaluate another parameters.

If each parameter is evaluated by some expert assessors then we can also use the extended average of fuzzy soft sets for evaluating. It is shown by the following example.

Example 6. A corporation is evaluating the decision of an investment opportunity. Let $S_\alpha = \{o_1(2), o_2(4), o_3(3), o_4(5), o_5(1)\}$ be a set of five investment avenues with risk assessments under consideration, where o_1:Bank Deposit, o_2:Shares, o_3:Mutual Fund, o_4:Stocks, o_5:Government Bonds and the levels of risk are 1:very low, 2:low, 3:moderate, 4:high, 5:very high. Let S_α be a semigroup with a binary operation $\bigtriangledown$ defined by Table IV.

TABLE IV
The Multiplication Table of a Semigroup S_α

$\bigtriangledown$	o_1	o_2	o_3	o_4	o_5
o_1	o_1	o_1	o_1	o_1	o_5
o_2	o_1	o_2	o_3	o_4	o_5
o_3	o_1	o_3	o_3	o_3	o_5
o_4	o_1	o_4	o_3	o_4	o_5
o_5	o_5	o_5	o_5	o_5	o_5

The committee consider a set of parameters,
$E = \{e_1\{\text{maximum profit in minimum period}\}$, $e_2\{\text{high returns}\}, e_3\{\text{safety of funds}\}, e_4\{\text{stable return}\}$, $e_5\{\text{easy accessibility}\}\}$, and consider
$A = \{e_1, e_3, e_5\}, B = \{e_2, e_3, e_4, e_5\}$ and $C = \{e_3, e_5\}$.

Under the limited condition of the corporation, suppose that it has no an expert who evaluate both maximum profit in minimum period, high returns and stable return.

In this case, the fuzzy soft sets can be expressed as follows:

Let (F, A) denotes a fuzzy soft set over S_α representing the “attractiveness of the investment avenues” which the 1^{st} assessor is going to select. In this case, the 1^{st} assessor has no high returns and stable return knowledge. However, the 1^{st} assessor can evaluate the set of parameters $\{e_1, e_3, e_5\}$.

Let (G, B) denotes a fuzzy soft set over S_α representing the "attractiveness of the investment avenues" which the 2^{nd} assessor is going to select. In this case, the 2^{nd} assessor has no maximum profit in minimum period knowledge. However, the 2^{nd} assessor can evaluate the set of parameters $\{e_2, e_3, e_4, e_5\}$.

Let (H, C) denotes a fuzzy soft sets over S_α representing the "attractiveness of the investment avenues" which the 3^{rd} assessor is going to select. The 3^{rd} assessor has no maximum profit in minimum period, high returns and stable return knowledge but the 3^{rd} assessor can evaluate the set of parameters $\{e_3, e_5\}$.

In this case, we can select the best assessors for each parameters. Some assessors may not evaluate all parameters. In similar situations, the extended average $\uplus_e$ plays an important role in the evaluation of the investment avenues under the best choices of corporations.

We note that the associative law of fuzzy soft sets $(F, A), (G, B)$ and (H, C) over a semigroup S_α may not be true. We define an extended average of three fuzzy soft sets $(F, A), (G, B)$ and (H, C) over S_α.

Definition 4. An extended average $\uplus_e$ of three fuzzy soft sets $(F, A), (G, B)$ and (H, C) over a semigroup S_α,is defined by $(F, A) \uplus_e (G, B) \uplus_e (H, C) := (F \uplus_e G \uplus_e H, A \cup B \cup C)$, where for each $p \in A \cup B \cup C$,

$$(F \uplus_e G \uplus_e H)_p = \begin{cases} \frac{F_p+G_p+H_p}{3} & \text{if} \quad \mathrm{p \in A \cap B \cap C,} \\ \frac{F_p+G_p}{2} & \text{if} \quad \mathrm{p \in (A \cap B) - C,} \\ \frac{F_p+H_p}{2} & \text{if} \quad \mathrm{p \in (A \cap C) - B,} \\ \frac{G_p+H_p}{2} & \text{if} \quad \mathrm{p \in (B \cap C) - A,} \\ F_p & \text{if} \quad \mathrm{p \in A - (B \cup C),} \\ G_p & \text{if} \quad \mathrm{p \in B - (A \cup C),} \\ H_p & \text{if} \quad \mathrm{p \in C - (A \cup B).} \end{cases}$$

Thus $(F, A) \uplus_e (G, B) \uplus_e (H, C)$ is a fuzzy soft set over S_α.

The following example satisfying the Definition 4. The concept of the following example is similar to the Example 6.

Example 7. According to Example 6, let
$S_\alpha = \{o_1(2),\ o_2(4),\ o_3(3),\ o_4(5),\ o_5(1)\}$ be a semigroup.
Let $E = \{e_1\{\text{maximum profit in minimum period}\},$
$e_2\{\text{high returns}\}, e_3\{\text{safety of funds}\}, e_4\{\text{stable return}\},$
$e_5\{\text{easy accessibility}\}\}$, be a set of parameters and
$A = \{e_1, e_3, e_5\}$, $B = \{e_2, e_3, e_4, e_5\}$ and $C = \{e_3, e_5\}$.
Then $A \cup B \cup C = \{e_1, e_2, e_3, e_3, e_5\}$. Let $(F, A), (G, B)$ and (H, C) be three fuzzy soft sets over S_α, where
$F_{e_1} = \{o_1/0.3, o_2/0.4, o_3/0.3, o_4/0.6, o_5/0.8\}$,
$F_{e_3} = \{o_1/0.8, o_2/0.8, o_3/0.3, o_4/0.4, o_5/0.3\}$,
$F_{e_5} = \{o_1/0.4, o_2/0.3, o_3/0.8, o_4/0.6, o_5/0.7\}$,
$G_{e_2} = \{o_1/0.4, o_2/0.6, o_3/0.7, o_4/0.4, o_5/0.4\}$,
$G_{e_3} = \{o_1/0.3, o_2/0.6, o_3/0.7, o_4/0.4, o_5/0.5\}$,
$G_{e_4} = \{o_1/0.8, o_2/0.5, o_3/0.4, o_4/0.6, o_5/0.5\}$,
$G_{e_5} = \{o_1/0.2, o_2/0.4, o_3/0.4, o_4/0.5, o_5/0.6\}$,
$H_{e_3} = \{o_1/0.4, o_2/0.5, o_3/0.7, o_4/0.6, o_5/0.5\}$,
$H_{e_5} = \{o_1/0.7, o_2/0.4, o_3/0.7, o_4/0.6, o_5/0.8\}$.
Thus $(F, A) \uplus_e (G, B) \uplus_e (H, C)$ is a fuzzy soft sets over S_α, where
$(F \uplus_e G \uplus_e H)_{e_1} = \{o_1/0.3, o_2/0.4, o_3/0.3, o_4/0.6, o_5/0.8\}$,
$(F \uplus_e G \uplus_e H)_{e_2} = \{o_1/0.4, o_2/0.6, o_3/0.7, o_4/0.4, o_5/0.4\}$,
$(F \uplus_e G \uplus_e H)_{e_3} = \{o_1/0.5, o_2/0.633, o_3/0.6, o_4/0.467, o_5/0.433\}$,
$(F \uplus_e G \uplus_e H)_{e_4} = \{o_1/0.8, o_2/0.5, o_3/0.4, o_4/0.6, o_5/0.5\}$,
$(F \uplus_e G \uplus_e H)_{e_5} = \{o_1/0.433, o_2/0.367, o_3/0.733, o_4/0.567, o_5/0.7\}$.

We define an extended average of n fuzzy soft sets over semigroups.

Definition 5. Let $\{(F_i, A_i) | i \in I\}$ be a family of fuzzy soft sets over a semigroup S_α. Define

$$\{\uplus_e\}_{i \in I}(F_i, A_i) = (\{\uplus_e\}_{i \in I} F_i, \cup_{i \in I} A_i).$$

Definition 6. Let $\{(F_i, A_i) | i = 1, 2, ..., n\}$ be a family of fuzzy soft sets over a semigroup S_α. For all $p \in \cup^n{}_{i=1} A_i$, define by

$$(\{\uplus_e\}_{i=1}^n F_i)_p = \frac{(F_{i_1})_p + \cdots + (F_{i_s})_p}{s} \text{ if}$$
$$p \in \cap^s{}_{k=1, 1 \le i_k \le n} A_{i_k} - (\cup_{i \in \{1,\ldots,n\} - \{i_1,\ldots,i_s\}} A_i),$$

$s \in \{1, 2, ..., n\}$.
Thus $\{\uplus_e\}_{i \in I}(F_i, A_i) = (\{\uplus_e\}_{i \in I} F_i, \cup_{i \in I} A_i)$ is a fuzzy soft set over S_α.

In the following algorithm, we use the extended average of fuzzy soft sets over a semigroup in decision-making problems.

From the algorithm of Gogoi et al. [18], we revised by computing extended averages in a new algorithm.

Algorithm

Step 1. Input the fuzzy soft sets (F_i, A_i) over some semigroups.

Step 2. Compute the extended average of the fuzzy soft sets over some semigroups and compute the choice value.

Step 3. Multiply the weight of the parameters such that $\Sigma w_j = 1$.

Step 4. Construct the Comparison table of the extended average of the fuzzy soft sets over some semigroups.

Step 4. Compute row sum, column sum and the score value of o_i for i.

Step 6. The decision is S_k.

6.1. If $S_k = \max_i s_i$ then we choose S_k.

6.2. If S_k has more than one object then we choose o_k corresponding to operation of the semigroup.

IV. Applications

In this section, we use the new algorithm above in decision-making problems.

The following example using the new algorithm for the two fuzzy soft sets over some semigroups. The concept is similar to the Example 5.

Example 8. A family is looking to purchase a water purifier. Let $S_\alpha = \{o_1(3),\ o_2(2),\ o_3(4),\ o_4(1),\ o_5(5)\}$ be a set of five water purifiers with a limited time warranty (year) under consideration. Let S_α be a semigroup with a binary operation $\triangle$ defined by Table V.

Let $E = \{e_1\{\text{strongly}\}, e_2\{\text{clean water}\}, e_3\{\text{easy using}\},$
$e_4\{\text{UV + RO available}\}, e_5\{\text{ultra filter available}\},$
$e_6\{\text{expensive}\}\}$ be a set of parameters, where UV:Ultraviolet, RO:Reverse osmosis, and let
$A = \{e_1, e_2, e_4, e_5\}$ and
$B = \{e_1, e_2, e_3, e_6\}$.

Step 1. Let (F, A) be a fuzzy soft set over S_α representing the "attractiveness of the water purifier" which a father is going to buy, where
$F_{e_1} = \{o_1/0.4, o_2/0.5, o_3/0.6, o_4/0.6, o_5/0.7\}$,

TABLE V
The Multiplication Table of a Semigroup S_α

$\triangle$	o_1	o_2	o_3	o_4	o_5
o_1	o_1	o_1	o_1	o_1	o_5
o_2	o_1	o_2	o_3	o_4	o_5
o_3	o_1	o_3	o_3	o_3	o_5
o_4	o_1	o_4	o_3	o_4	o_5
o_5	o_5	o_5	o_5	o_5	o_5

$F_{e_2} = \{o_1/0.3, o_2/0.4, o_3/0.5, o_4/0.6, o_5/0.6\}$,
$F_{e_4} = \{o_1/0.7, o_2/0.5, o_3/0.3, o_4/0.4, o_5/0.4\}$.
$F_{e_5} = \{o_1/0.4, o_2/0.5, o_3/0.5, o_4/0.7, o_5/0.6\}$.
Let (G, B) be a fuzzy soft set over S_α representing the "attractiveness of the water purifier" which a mother is going to buy, where
$G_{e_1} = \{o_1/0.5, o_2/0.3, o_3/0.7, o_4/0.4, o_5/0.5\}$,
$G_{e_2} = \{o_1/0.4, o_2/0.4, o_3/0.3, o_4/0.5, o_5/0.8\}$,
$G_{e_3} = \{o_1/0.6, o_2/0.4, o_3/0.5, o_4/0.6, o_5/0.35\}$.
$G_{e_6} = \{o_1/0.7, o_2/0.5, o_3/0.4, o_4/0.6, o_5/0.3\}$.

Step 2. The extended average $(F, A) \uplus_e (G, B)$, where $A \cup B \cup C = \{e_1, e_2, e_3, e_4, e_5, e_6, \}$ and the choice values are shown in Table VI.

TABLE VI
The Extended average $(F, A) \uplus_e (G, B)$ Table

	e_1	e_2	e_3	e_4	e_5	e_6	Choice value
o_1	0.45	0.35	0.7	0.6	0.4	0.7	3.20
o_2	0.4	0.4	0.5	0.4	0.5	0.5	2.70
o_3	0.65	0.4	0.3	0.5	0.5	0.4	2.75
o_4	0.5	0.55	0.4	0.6	0.7	0.35	3.10
o_5	0.6	0.7	0.4	0.6	0.6	0.3	3.20

Step 3. Suppose that the family sets the preference weight of parameters as follows:
$e_1 : w_1 = 0.2, e_2 : w_2 = 0.1, e_3 : w_3 = 0.2,$
$e_4 : w_4 = 0.2, e_5 : w_5 = 0.1, e_6 : w_6 = 0.2$
such that $\Sigma w_j = 1$. The extended average $(F, A) \uplus_e (G, B)$ table with multiply the weight is shown in Table VII.

TABLE VII
The Extended Average $(F, A) \uplus_e (G, B)$ Table with Multiply the Weight

	e_1	e_2	e_3	e_4	e_5	e_6
o_1	0.09	0.035	0.14	0.12	0.04	0.14
o_2	0.08	0.04	0.1	0.08	0.05	0.1
o_3	0.13	0.04	0.06	0.1	0.05	0.08
o_4	0.1	0.055	0.08	0.12	0.07	0.07
o_5	0.12	0.07	0.08	0.12	0.06	0.06

Step 4. The Comparison table of the extended average $(F, A) \uplus_e (G, B)$ is shown in Table VIII.

Step 5. The score value is shown in Table IX.

Step 6. The decision is o_4 and o_5 which have the maximum score value.

A family buy o_5 for the water purifier corresponding to operation of the semigroup.

The following example using the new algorithm for the three fuzzy soft sets over some semigroups. The concept is similar to the Example 6.

TABLE VIII
The Comparison Table

c_{ij}	o_1	o_2	o_3	o_4	o_5
o_1	6	4	3	3	3
o_2	2	6	4	2	2
o_3	3	4	6	2	2
o_4	4	4	4	6	4
o_5	4	4	4	4	6

TABLE IX
The Score Value Table.

	Row sum	Column sum	Score value
o_1	19	19	0
o_2	16	22	-6
o_3	17	22	-5
o_4	22	17	5
o_5	22	17	5

Example 9. A corporation is evaluating the decision of an investment opportunity. Let $S_\alpha = \{o_1(2),\ o_2(4),\ o_3(3),\ o_4(5),\ o_5(1)\}$ be a set of five investment avenues with different risk assessments under consideration, where o_1:Bank Deposit, o_2:Shares, o_3:Mutual Fund, o_4:Stocks, o_5:Government Bonds and the levels of risk are 1:very low, 2: low, 3:moderate, 4:high, 5:very high. Let S_α be a semigroup with a binary operation $\bigtriangledown$ defined by Table V in the Example 8. The committee consider a set of parameters,
$E = \{e_1\{\text{safety of funds}\},\ e_2\{\text{high returns}\},\ e_3\{\text{maximum profit in minimum period}\},\ e_4\{\text{stable return}\},\ e_5\{\text{easy accessibility}\}\}$ and $A = \{e_1, e_3, e_4, e_5\}, B = \{e_1, e_2, e_3, e_5\}$ and $C = \{e_2, e_3, e_4, e_5\}$.

Step 1. Let (F, A) be a fuzzy soft set over S_α representing the "attractiveness of the investment avenues" which the 1^{st}committee are going to select, where
$F_{e_1} = \{o_1/0.4, o_2/0.5, o_3/0.6, o_4/0.7, o_5/0.6\}$,
$F_{e_3} = \{o_1/0.3, o_2/0.4, o_3/0.6, o_4/0.6, o_5/0.6\}$,
$F_{e_4} = \{o_1/0.7, o_2/0.5, o_3/0.6, o_4/0.4, o_5/0.4\}$.
$F_{e_5} = \{o_1/0.4, o_2/0.5, o_3/0.5, o_4/0.4, o_5/0.7\}$.
Let (G, B) be a fuzzy soft set over S_α representing the "attractiveness of the investment avenues" which the 2^{st}committee are going to select, where
$G_{e_1} = \{o_1/0.5, o_2/0.3, o_3/0.7, o_4/0.5, o_5/0.4\}$,
$G_{e_2} = \{o_1/0.4, o_2/0.4, o_3/0.6, o_4/0.8, o_5/0.5\}$,
$G_{e_3} = \{o_1/0.6, o_2/0.4, o_3/0.6, o_4/0.6, o_5/0.6\}$.
$G_{e_5} = \{o_1/0.7, o_2/0.5, o_3/0.4, o_4/0.5, o_5/0.4\}$.
Let (H, C) be a fuzzy soft set over S_α representing the "attractiveness of the investment avenues" which the 3^{st}committee are going to select, where
$H_{e_2} = \{o_1/0.5, o_2/0.3, o_3/0.5, o_4/0.5, o_5/0.4\}$,
$H_{e_3} = \{o_1/0.4, o_2/0.4, o_3/0.651, o_4/0.8, o_5/0.5\}$,
$H_{e_4} = \{o_1/0.6, o_2/0.3, o_3/0.5, o_4/0.5, o_5/0.4\}$.
$H_{e_5} = \{o_1/0.7, o_2/0.5, o_3/0.4, o_4/0.4, o_5/0.4\}$.

Step 2. The extended average $(F, A) \uplus_e (G, B) \uplus_e (H, C)$, where $A \cup B \cup C = \{e_1, e_2, e_3, e_4, e_5, \}$ and the choice values are shown in Table X.

Step 3. Suppose that the corporation sets the preference weight of parameters as

TABLE X
The Extended Average $(F, A) \uplus_e (G, B) \uplus_e (H, C)$ Table

	e_1	e_2	e_3	e_4	e_5	Choice value
o_1	0.45	0.45	0.433	0.65	0.6	2.583
o_2	0.4	0.35	0.4	0.4	0.5	2.05
o_3	0.65	0.55	0.617	0.55	0.433	2.8
o_4	0.6	0.65	0.667	0.45	0.433	2.8
o_5	0.5	0.45	0.567	0.4	0.5	2.417

$e_1 : w_1 = 0.2, e_2 : w_2 = 0.2, e_3 : w_3 = 0.2,$
$e_4 : w_4 = 0.2, e_5 : w_5 = 0.2$ such that $\Sigma w_j = 1$.
The extended average $(F, A) \uplus_e (G, B) \uplus_e (H, C)$ table with multiply the weight is shown in Table XI.

TABLE XI
The Extended Average $(F, A) \uplus_e (G, B) \uplus_e (H, C)$ Table with Multiply the Weight

	e_1	e_2	e_3	e_4	e_5
o_1	0.09	0.09	0.087	0.13	0.12
o_2	0.08	0.07	0.08	0.08	0.1
o_3	0.13	0.11	0.123	0.11	0.087
o_4	0.12	0.13	0.133	0.09	0.087
o_5	0.1	0.09	0.113	0.08	0.1

Step 4. The Comparison table of the extended average $(F, A) \uplus_e (G, B) \uplus_e (H, C)$ is shown in Table XII.

TABLE XII
The Comparison Table

c_{ij}	o_1	o_2	o_3	o_4	o_5
o_1	5	5	2	2	3
o_2	0	5	1	1	2
o_3	3	4	5	3	4
o_4	3	4	3	5	4
o_5	3	5	1	1	5

Step 5. The score value is shown in Table XIII.

TABLE XIII
The score value table

	Row sum	Column sum	Score value
o_1	17	14	3
o_2	9	23	-14
o_3	19	12	7
o_4	19	12	7
o_5	15	18	-3

Step 6. The decision is o_3 and o_4 which have the maximum score value.

The corporation chooses o_3:Mutual Fund for the investment avenues corresponding to operation of the semigroup.

Remark 1. 1. The extended averages are designed in such a way, that allows for all parameters to be evaluated by some or all assessors (if necessary), as shown in Example 6. That is, we can select the best assessors for evaluating each specific parameter.

2. The advantage of the new algorithm can be decided in case which has the maximum score equal for fuzzy soft sets but Gogoi et al's method is designed for evaluating all parameters by all assessors. Some assessors may not be the best assessor for some parameters.

V. Conclusion

In this paper, extended averages of fuzzy soft sets over some semigroups are introduced. The new algorithm for multiple evaluation in decision-making problems based on extended averages are presented. The proposed algorithm is an alternative for solving decision making problems under the best assessors in their corporations.

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