Underdetermined DOA Estimation Based on Target Space Diversity

Chen Miao, Hui Tang, Yue Ma ,and Peishuang Ni

***Abstract*—In the underdetermined DOA scenario, the performance of the conventional direction of arrival (DOA) estimation algorithms often degrades greatly, in the worst case, these algorithms even fail to estimate the direction of targets. To overcome this problem, an underdetermined DOA estimation based on target space partitioning is proposed. Firstly, the IF signal of the LFMCW radar is processed by two-dimensional FFT to get target distribution information. And then the range-velocity target space is divided into several subspaces according to the targets' distribution. In the process of partitioning, ensure that every subspace meets the overdetermined conditions. Now, various classic DOA estimation algorithms can be used. The multiple signal classification (MUSIC) algorithm is applied in this paper. Simulation results show that the method well solved the underdetermined DOA estimation problem. Under certain conditions, the distance, speed, and angle of the target can also be estimated simultaneously.**

***Index Terms*—LFMCW, Underdetermined DOA, the MUSIC algorithm**

I. Introduction

The direction of arrival (DOA) estimation is an important application of array signal processing [1]-[2]. It has been a research hotspot and difficulty since it was proposed in the 1970s. The traditional subspace-like DOA estimation algorithm can break through the constraint of the Rayleigh limit [3]. When the number of sources is less than the number of array elements, the DOA can be estimated by employing the subspace approach. In other words, these subspace approaches are only for an overdetermined DOA scenario. In the underdetermined situation where the number of targets greater than the number of array elements, the performance of these algorithm drops drastically, in the worst case, these algorithms even fail to estimate the angle of the targets. However, in practical applications, the underdetermined scenario often occurs due to the existence of ambient targets and the limited number of sensors. To further exploit the array's physical structure, underdetermined DOA estimation is studied, which is beneficial to save the resources and improving the overall performance of the array [4].

Currently, most of the proposed methods for solving underdetermined DOA estimation tried to solve the problem by modifying the array structure or improving the array manifold matrix [5]-[7]. In [8]-[10], scholars proposed a KR-subspace DOA estimation method based on the properties of the Kronecker product. This method can increase the number of estimated targets from *N* to *2N-1*. However, the above algorithm requires the received signal to be a quasi-stationary signal. In addition, it is mainly used in linear arrays, which limits the range of applications. In [11]-[12], scholars used the non-zero properties of the ellipse covariance matrix of non-circular signals to solve the underdetermined problem. By changing the array manifold of the received signal, the number of targets can be estimated reaches to *2N-1*, but the received signal is required to be a non-circular signal. In [13], the characteristics of non-circular signals are also used to solve the underdetermined DOA estimation problem, and the array is also required to be a coprime array. In [14], the author proposed an underdetermined DOA estimation method for wideband signals, which has low complexity but is only suitable for uniform linear array (ULA) with specially designed spacing and system settings. The advantage of these methods is that they can be applied to various radar systems. But they are not universal and have many prerequisites.

A novel idea is proposed in this paper to solve the underdetermined problem. Firstly, get the distribution information of the targets by performing two-dimensional FFT on the IF echo signal of the LFMCW radar, then the range-velocity space called target space is divided into several subspaces according to the distribution of targets. In the process of partitioning, ensure that each subspace can meet the overdetermined conditions. Finally, various classic DOA algorithms [15]-[16] can be executed. The innovation of this article lies in the clever fusion of two existing technologies to solve the difficult problem of underdetermined DOA estimation. The multiple signal classification (MUSIC [15]) algorithm is used in this paper for DOA estimation. This article also proposes a variety of methods for partitioning targets. Through simulation comparison, we can know that the method of dividing target space which only takes the data around the peak position in each subspace has the best performance. Besides this division method can also estimate the angle-distance-velocity of the target simultaneously.

Notations : $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^*$ are the operators of transpose, conjugate transpose, conjugate, respectively. I_N is the $N \times N$ identity matrix. $E[\cdot]$ denotes the mathematical

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expectation. $j=\sqrt{-1}$ is the imaginary unit. $\lfloor \cdot \rfloor$ denotes round down.

II. Algorithm Principle and Implementation

A. MUSIC Algorithm of LFMCW Radar

As shown in Fig. 1, an uniform linear array has N isotropic elements, and the element spacing is d.

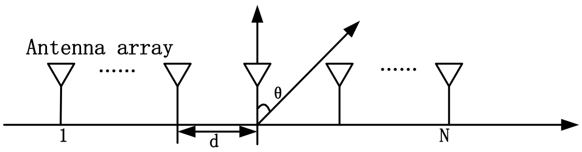

Fig. 1. N-element uniform linear array.

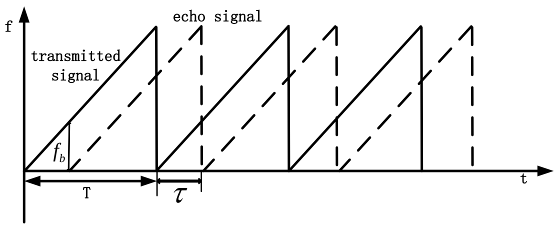

Fig. 2 Time-frequency diagram of chirped sawtooth wave

The radar transmits sawtooth chirp signals, as shown in Fig. 2. The signal sweep period is T and the bandwidth is B, so the slope is $k=B/T$.

Assume there are M targets, and the incident direction of each target is $\theta_{\mathrm{m}}(m\in[1,M])$. The relative distance between the reference element(first element) and the m-th target is r_m, and the relative velocity is v_m. Then the intermediate frequency (IF) signal of the m-th target received by reference array element can be expressed as:

$$s_m(t)=\exp(j*2\pi(f_0\tau_{0m}+f_{dm}t+f_{Rm}(t-nr*T))) \tag{1}$$

where $m=1,2,\ldots M$; c is the velocity of light; f_0 is the carrier frequency; $\tau_{0m}=2r_m/c$ is time delay of the m-th target relative to the reference array element ; $f_{dm}=2v_m f_0/c$ is the doppler frequency shift; $f_{Rm}=2kr_m/c$ is the frequency difference caused by the distance of the m-th target; $nr=\lfloor t/T \rfloor$ is the number of the cycles.

So the intermediate frequency (IF) signal of the n-th element can be expressed as [17]-[18]:

$$\begin{aligned} x_n(t)&=\sum_{m=1}^{M}\exp(j*(n-1)k_0 d\sin\theta_m)s_m(t)+n_n(t) \\ &=\sum_{m=1}^{M}\exp(j2\pi f_{nm})s_m(t)+n_n(t) \end{aligned} \tag{2}$$

where $n=1,2,\ldots N$, k_0 is the propagation constant and express as $k_0=2\pi/\lambda$; $f_{nm}=d(n-1)\sin\theta_m/\lambda$, $n_n(t)$ is noise of the n-th element at time t. This noise is modeled as Additive White Gaussian Noise (AWGN).

The received signal vector is therefore given by:

$$X(t)=AS(t)+N(t) \tag{3}$$

where $S(t)=[s_1(t),\cdots s_M(t)]^T$ is the complex envelope of the signal received by the reference array element. $N(t)=[n_1(t),n_2(t),\cdots n_N(t)]^T$ is Additive White Gaussian Noise vector. The array manifold matrix $A=[a(\theta_1),a(\theta_2),\cdots,a(\theta_M)]$ with each column vector $a(\theta_m)$ as the steering vector corresponding to the m-th source signal, expressed as:

$$a(\theta_m)=[1,\mathrm{e}^{jk_0 d\sin\theta_m},\cdots,\mathrm{e}^{jk_0(N-1)d\sin\theta_m}]^T \tag{4}$$

Next, The MUSIC algorithm [15] is used to DOA estimation. The array covariance matrix of the received data is:

$$\begin{aligned} R&=E[XX^H] \\ &=AE[SS^H]A^H+\sigma^2 I_N \\ &=AR_S A^H+\sigma^2 I_N \end{aligned} \tag{5}$$

Perform eigenvalue decomposition on matrix R and sort the eigenvalues in descending order, that is:

$$\lambda_1\ge\lambda_2\ge\cdots\lambda_N>0 \tag{6}$$

In (6), the largest M eigenvalues correspond to the target signal, and the rest *M+1 to N* eigenvalues correspond to the noise. Suppose that u_i is the eigenvector corresponding to the eigen value λ_i. Due to the independence of signal and noise, the covariance matrix can be decomposed into two parts related to signal and noise respectively:

$$R=\sum_{i=1}^{M}U_S\lambda_i U_S^H+\sum_{i=M+1}^{N}U_N\lambda_i U_N^H \tag{7}$$

where $U_s=[u_1,\cdots u_M]$ is the signal subspace; $U_N=[u_{M+1},\cdots u_N]$ is the noise subspace.

In the practical situation, the length of the received data is finite, then the covariance matrix of data can be replaced with maximum likelihood estimation:

$$\hat{R}=\frac{1}{Y}\sum_{y=1}^{Y}XX^H \tag{8}$$

where Y is the length of data.

Due to the fact that the signal subspace is orthogonal to the noise subspace, the steering vector corresponding to the source is also orthogonal to the noise subspace:

$$a^H(\theta)U_N=0 \tag{9}$$

where $a^H(\theta)$ is the conjugate transpose of $a(\theta)$. The DOA of the input signals can be estimated by the determined MUSIC spatial spectral peaks, which are given as follows:

$$P(\theta)_{\text{music}}=\frac{1}{a^H(\theta)\hat{U}_N\hat{U}_N^H a(\theta)} \tag{10}$$

where $a(\theta)$ is a scan vector scans over all possible angles of incidence. Whenever the MUSIC spectrum reaches peak value, the corresponding angle θ must be the signals' DOA. This shows that the MUSIC algorithm is only useful when $M<N$. Because when $M>N$, the noise subspace U_N can not be obtained.

B. Underdetermined DOA Estimation Algorithm Based on Dividing Target Space

The number of targets that can be estimated by the traditional subspace DOA estimation algorithm is limited by the number of array elements. In order to estimate the DOA of targets as much as possible, this paper proposes a DOA estimation method based on target space partitioning. Dividing the target space into multiple subspaces so that each subspace can satisfy the overdetermined condition.

Suppose that Nr repetition periods are sampled for the echo signal $x_n(t)$, and the number of sampling points per cycle is *Ns*. Sampling frequency is 1/*Ts*, and the time of one sampling period is *Tr*. Then the discrete form of the IF signal is:

$$x_n(nr,ns)=\sum_{m=1}^{M}\exp(j*2\pi f_{nm})*s_m(nr,ns)+n_n(nr,ns) \quad (11)$$

where *ns*= 0, 1, ..., *Ns*-1; *nr*= 0, 1, ..., *Nr*-1; $x_n(nr,ns)$ is the *ns*-th sampling point of the *nr*-th repetition period.

According to (2) and (11), the range and velocity information are implied in frequency and phase term of Exp function. So, in order to obtain the target's distance and doppler information, a two-dimensional FFT is carried out for each channel of the IF signal firstly. FFT along *ns* axis is called range transformation, and results in range spectrum of target, in which the peak value appears at the range frequency f_{Rm}. Similarly, FFT along *nr* axis is called velocity transformation, and leads to doppler spectrum of target, where the peak value appears at the doppler frequency f_{dm} [19].

So the two-dimensional FFT of the IF signal received by the *n*-th element can be expressed as the following formula:

$$\begin{aligned} w_n(k,l) &= \sum_{nr=0}^{N_r-1}\sum_{ns=0}^{N_s-1} x_n(nr,ns)\exp\left(\frac{-j2\pi k*ns}{N_S}\right)*\exp\left(\frac{-j2\pi l*nr}{N_r}\right) \\ &= \sum_{m=1}^{M}\exp(j*2\pi f_{nm})*s_m(l,k)+n_n(l,k) \end{aligned} \quad (12)$$

where *l*= 0, 1, ..., *Nr* -1; k= 0, 1, ..., *Ns* -1; *ns*= 0, 1, ..., *Ns*-1; *nr*= 0, 1, ..., *Nr*-1.

It is easy to know that $w_n(k,l)$ is $Ns\times Nr$ matrix. Due to the discrete character of FFT, the interval between spectrum lines of range transformation is $1/(Ts*Ns)$, and each spectrum line corresponds to a range bin. Assume that the peak value appears at *k*-th spectrum line, so the corresponding target distance is $r=k*cTr/(2B*Ns*Ts)$. While the spectrum interval of velocity transformation is $1/(Tr*Nr)$, and each spectrum line corresponds to a Doppler bin. If the peak value appears at *l*-th spectrum line, the corresponding target velocity is $v=l*c/(2f_0*Tr*N_r)$.

Then, through the algorithm of 2D constant false alarm ratio (CFAR), the range and velocity information of each target can be obtained. And then the distribution of the targets in range-velocity space can be known, so as to lay the foundation for dividing the target space. By dividing the data space according to the targets' distribution in range-velocity space, the number of targets in each subspace can be accurately known. In this way, the step of estimating the number of signal sources can be eliminated and the reliability of the DOA estimation algorithm can be improved.

Assume that the target distribution in range-velocity space is shown in Fig. 3. Sort the targets according to the distance or velocity from small to large, and then the target subspaces are divided from near to far. And whether the division of each subspace is completed depends on whether the number of targets in the subspace reaches N-1. Assume the target space is divided into *Q* sub-spaces, and the number of targets in each subspace is G_q. For the *n*-th IF echo signal, the subspace data matrix is:

$$w_{nq}[(k_q:k_q+Ns_q),(l_q:l_q+Nr_q)]\in F^{Ns_q\times Nr_q} \quad (13)$$

where $q=1,2,\cdots Q$, (k_q,l_q) represents the starting coordinate of the *q*-th subspace. Ns_q represents the length of the data taken by the *q*-th subspace in the distance dimension. Nr_q represents the length of the data taken in the velocity dimension. The symbol (a : b) denotes continues value from a to b.

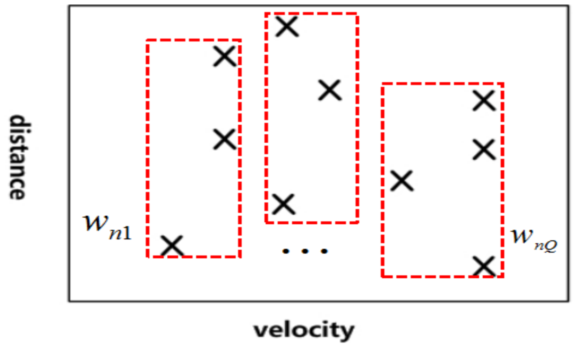

Fig. 3. Target space partitioning according to the distribution of targets

Then vectorize the submatrix w_{nq}, and have $z_{nq}(h)=rvec(w_{nq})$, $z_{nq}\in F^{1\times(Ns_q*Nr_q)}$. $rvec(\cdot)$ means vectorization into a row vector. At last, combine the corresponding row vector of each array element into array-received data sub-matrix:

$$Z_q(h)=\begin{bmatrix} z_{1q}\\ z_{2q}\\ \vdots\\ z_{Nq}\end{bmatrix}=\begin{bmatrix} \sum_{g=1}^{G_q} e^{j*2\pi f_{q\cdot 1g}}\bullet s_{q\cdot g}(h)+n_q(h)\\ \sum_{g=1}^{G_q} e^{j*2\pi f_{q\cdot 2g}}\bullet s_{q\cdot g}(h)+n_q(h)\\ \vdots\\ \sum_{g=1}^{G_q} e^{j*2\pi f_{q\cdot Ng}}\bullet s_{q\cdot g}(h)+n_q(h)\end{bmatrix} \quad (14)$$

where $s_{q\cdot g}(h)$ represents the echo signal of the *g*-th target in the *q*-th subspace.

The above formula can be written in matrix form:

$$Z_q(h)=A_qS_q(h)+N_q(h) \quad (15)$$

Then for each $Z_q\in F^{N\times(Ns_q*Nr_q)}$, DOA estimation is performed using the MUSIC algorithm, and at last, M targets can be estimated. The steps can be summarized as follows:

1) Perform 2D FFT on N echo signals respectively and then select one signal to execute the 2D CFAR algorithm to obtain the range and velocity information.

2) Sort the targets according to the distance or velocity from small to large

3) The target space is divided from near to far into Q target subspaces. And whether the division of each subspace is completed depends on whether the number of targets in the subspace reaches N-1, so that each subspace meets the overdetermined condition.

4) Restore each sub-matrix to a one-dimensional vector form.

5) Recombine the corresponding vector of each channel into the sub-matrix Z_q.

6) DOA estimation is performed on the *Q* sub-matrices, thereby estimating the DOA of the targets in each subspace.

C. Discussion on the division method of targets space

In this section, several methods for partitioning target space are given. Through comparison, we try to find a better method for dividing the space of the targets. Simplifying the Eq. (12), we can get:

$$|w_n(l,k)| = \sum_{m=1}^{M} \left| \frac{\sin[\pi(k-(f_{dm}+f_{Rm})Ts*Ns)]}{\sin[\pi(k-(f_{dm}+f_{Rm})Ts*Ns)/Ns]} \right| * \left| \frac{\sin[\pi(l-(f_{dm})Tr*Nr)]}{\sin[\pi(l-(f_{dm})Tr*Nr)/Nr]} \right| + |n_n(l,k)| \quad (16)$$

Assume that there is only one target. The distance is 80*m* and the velocity is 15*m/s*. It can be known from Eq. (16) and Fig. 4 that there is spectrum leakage in both the distance dimension and the velocity dimension where the peak position is located. This will affect the DOA estimation of the surrounding targets. Therefore, it is necessary to find a suitable method of dividing targets to minimize this effect.

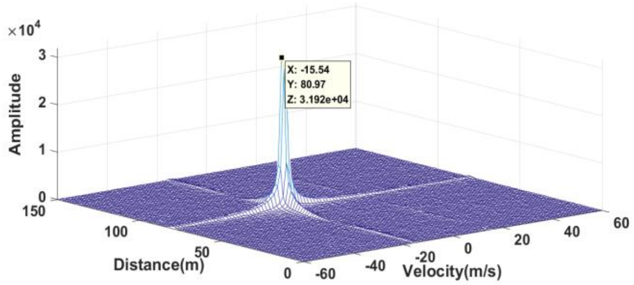

Fig. 4. Spectrum map of target for LFMCW

As shown in Fig. 5, several feasible partition methods for target space are given below.

In Fig. 5(a), the subspace is partitioned according to the distribution of the targets in the distance dimension. First sort the targets according to the distance from near to far, and then the targets subspace is divided from near to far. Considering that the maximum number of targets that each subspace can resolve is limited by the number of array elements, so whether the division of each subspace is completed depends on the number of targets in the subspace reaching N-2. This is to leave a margin for dividing the targets that are close or even the same in distance into the same subspace.

In Fig. 5(b), similar to the method of Fig. 5(a), it just divides the target space in the velocity dimension. Considering that the number of sampling periods is short, this results in the target's velocity resolution being worse than the range resolution. In order to research its influence on the DOA estimation in each space, the method of Fig. 5(b) is proposed.

In Fig. 5(c), we combine the distance dimension and velocity dimension together to further reduce the data range in each subspace. In order to investigate whether the division of targets with the same velocity or distance into the same subspace will affect the DOA estimation, this method is proposed. The target’s range-velocity space is further divided so that the distance and velocity of the targets in each subspace are different.

In Fig. 5(d), to reduce the effect of spectrum leakage of other targets as much as possible, each subspace only takes data points around the peak position. Compared with other methods, the advantage of this method is that it can estimate the angle-distance-velocity of the target simultaneously, but the disadvantage is that the amount of calculation is larger.

The other three methods mainly discuss the DOA estimation problem, so the angle-distance-velocity pairing problem is not considered. Suppose that there is more than one target in the same subspace. Two of these targets have the same angle but different distances or speeds. At this time, only one target angle can be detected for the other three methods. Of course, this is a common problem of subspace-like DOA estimation algorithms when there are multiple targets. But for the method of Fig. 5(d), the targets which have the same angle but different distances or speeds can be distinguished.

The comparison of the simulation results of the above four methods will be given in the next section.

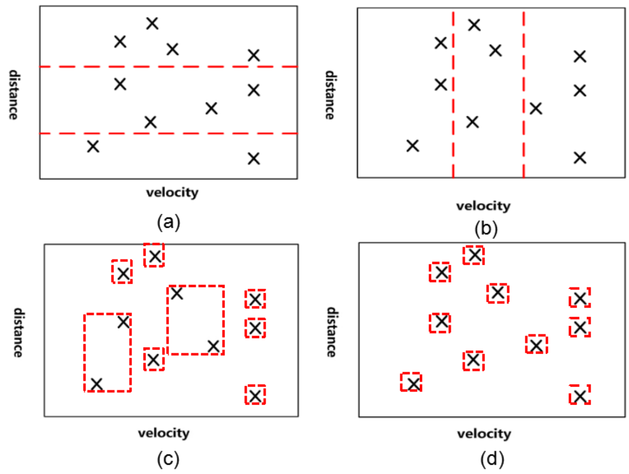

Fig. 5. The methods of dividing target space

III. Simulation Analysis

The numerical simulations are given below to verify the feasibility of the algorithm in solving the underdetermined DOA estimation problem. Meanwhile, the performance comparison of various partition methods for target space is given. Then explain the time complexity of the algorithm.

A. Simulation Results

The simulation parameters are set as follows: the number of antenna elements N is 6, the element spacing is $d=\lambda/2$, the carrier frequency f_0 is 9.1GHz, the bandwidth is B=100MHz, the sawtooth wave sweep period is *Tr*=0.1ms. The sampling frequency f_s is 10MHz. A total of $Nr = 128$ repetition periods is sampled, and $Ns = 1000$ points are sampled for each period. Therefore, the number of range transformation FFT points is 1024, and the velocity transformation is 128 points. The distance, velocity and angle information of the targets are shown in Table 1. The signal-to-noise ratio (SNR) is 15dB.

TABLE 1
DISTANCE AND SPEED INFORMATION PARAMETERS OF THE TARGET

sequence	Angle(degree)	Distance(m)	velocity(m/s)
1	-50	60	15
2	50	80	-15
3	5	95	0
4	20	110	10
5	-18	135	15
6	12	160	-10
7	30	180	15
8	40	208	5
9	-1	238	-10
10	-25	280	0

After the 2D FFT and 2D CFAR, the distribution of the targets in range-velocity space can be obtained, as shown in Fig. 6.

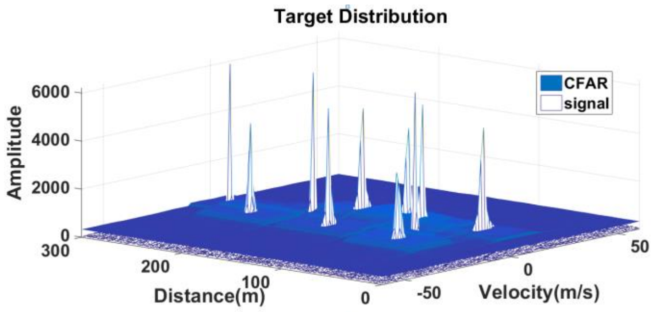

Fig. 6. Spectrum map of target for LFMCW

Here, the method of Fig. 5(d) is used to divide the target space, and the result of underdetermined DOA estimation is shown in Fig. 7. In order to make the result displayed clearly, the DOA power spectrum of multiple target subspaces are added together. And then a DOA spatial spectrum as shown in Fig. 7 is obtained, in fact, it should be one DOA spatial spectrum for each target subspace. It can be seen that the method proposed in this paper can realize the underdetermined DOA estimation.

Next, compare the performance of the underdetermined DOA estimation algorithms of various subspace division methods. The methods of the target space division adapt the four methods proposed in Sec. 2.3. The root mean square error (RMSE) is used as the performance measure：

$$RMSE = \frac{1}{M}\sum_{m=1}^{M}\sqrt{\frac{1}{G}\sum_{g=1}^{G}(\hat{\theta}_{gm}-\theta_m)^2} \quad (17)$$

where $\hat{\theta}_{gm}$ is the estimate of the angle θ_m of the g-th Monte Carlo trial. M is the number of targets and G is the number of Monte Carlo trial.

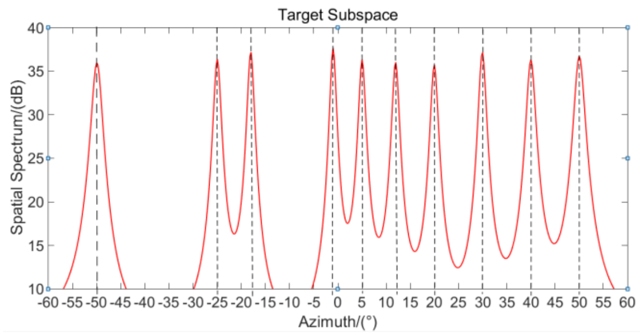

Fig. 7. Simulation results of underdetermined DOA estimation based on target space diversity

As the SNR changes, the algorithm performance of different methods for partitioning targets is shown in Fig. 8.

As shown in Fig.8, we can know that the finer the subspace division, the more uncorrelated among the targets in each subspace, the better the performance of the algorithm. The method proposed in Fig. 5(d), that is, each subspace takes only the data around the peak position, has the best performance. Comparing the method of dividing the subspace in the distance dimension with the method of dividing the subspace in the velocity dimension, it can be found that the method of dividing the target space in the distance dimension has better performance. The reason is that distance resolution is better than velocity resolution, and it is easier to distinguish targets.

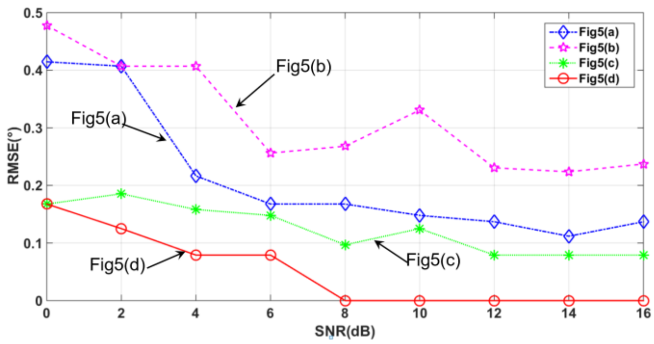

Fig.8 Mean square error performance for different methods of target space

B. *Algorithm complexity comparison*

This section gives the time complexity of the underdetermined DOA estimation algorithm proposed in this paper. The time complexity of this paper is $Q*[O(N^2 \times knp)+O(N^3)+O(N^2 \times kth)]$. knp represents the number of snapshots for each subspace. Q is the number of subspaces. *kth* depends on the interval used to search for spectral peaks. The smaller the interval, the greater the *kth*. It can be seen that the algorithm time mainly focuses on three aspects: estimating the covariance matrix, eigenvalue decomposition, and searching the spectrum peaks. For the partitioning method which each subspace only takes the data around the peak position, the time to estimating the covariance matrix is greatly reduced due to the small number of snapshots in each subspace. But compared with other methods, this method greatly increases the time of searching the spectrum peaks due to the larger number of divided subspaces. ESPIRT [16] algorithm can be used instead of MUSIC algorithm to save the time of spectrum peak search, thereby reducing the time complexity of the algorithm. However, the ESPIRT algorithm is only suitable for uniform linear arrays, which reduces the application scope of the proposed method. Although this method of Fig. 5(d) has higher time complexity, it has better performance and can estimate the angle-distance-velocity of the targets simultaneously. It is equivalent to using algorithm time complexity in exchange for more accurate DOA estimation results.

Therefore, the DOA estimation based on partitioning the target space proposed in this paper can not only realize the underdetermined DOA estimation, but also have a wider application range, especially in the short-range target detection. It is suitable for various array structures. For the division method which each subspace only takes the data around the peak position, it can achieve the angle-distance-velocity pairing of the target in the condition of sacrificing the time complexity of the algorithm. And it also has better algorithm performance.

IV. Conclusion

The focus of this article is to provide a new idea to solve the underdetermined DOA estimates. The innovation is to solve the difficult problem of underdetermined DOA estimation through the clever fusion of two existing technologies. Processing the IF signals received by the

LFMCW radar to get the distribution information of the targets and dividing the target rang-velocity space into multiple subspaces. Ensure that each subspace can meet the overdetermined conditions so that makes it suitable for various DOA estimation algorithms. Therefore, the spatial freedom of the algorithm proposed in this paper depends on the radar's range, speed resolution and target detection capabilities. As long as the resolution of the radar is large enough, theoretically the number of DOA that can be estimated by the method proposed in this article will approach infinite. Simulation results show that the algorithm is feasible and simple to operate. The partition method that only takes data near the peak position in each subspace can also estimate the angle-distance-velocity of the target simultaneously. Therefore, it is conducive to the application and implementation in practical engineering and has a broad application prospect. By selecting other types of DOA algorithms, it is more helpful to improve the accuracy of DOA estimation and saving the time of algorithm.

REFERENCES

[1] Yongliang Wang. *Spatial spectrum estimation theory and algorithm*. 2004.11:1-3

[2] Hao Chen, Zhaohui Wu, Lu Gan, Xiaoming Liu, and Jinji Ma. “Fast estimation of range and bearing for a single near-field source without eigenvalue decomposition,” *IAENG International Journal of Applied Mathematics*, vol.48, no.4, pp499-503,2018.

[3] Van Veen B D , Buckley K M. “Beamforming: a versatile approach to spatial filtering,” *IEEE ASSP Magazine*, 1988, 5(2):4-24.

[4] Mingyan Cao.“Underdetermined super-resolution direction of arrival estimation with uniform circular array,” dissertation, 2014.

[5] Jie Hu. “The Study on the Underdetermined DOA Estimation Based on Acoustic Vector Sensor,” dissertation, 2015

[6] Pal P, Vaidyanathan P. “Nested arrays in two dimensions, Part II: Application in two dimensional array processing,” *IEEE Transactions on Signal Processing*. 2012, 60(9): 4706–4718.

[7] Pal P, Vaidyanathan P. “Coprime sampling and Music algorithm,” *IEEE Digital Signal Processing Workshop*. 2010, 54:289-294.

[8] Ma W K , Hsieh T H , Chi C Y. “DOA estimation of Quasi-Stationary signals with less sensors than sources and unknown spatial noise covariance: A Khatri–Rao subspace approach,” *IEEE Transactions on Signal Processing*, 2010, 58(4):2168-2180.

[9] Ma W K, Hsieh T H, and Chi C Y. “DOA estimation of Quasi-Stationary signals via Khatri-Rao subspace,” *Proc. Int. Conf. Acoust.Speech Signal Process.(ICASSP)*, 2009: 2165–2168.

[10] Ben W , Wei W , Yujie G . “Underdetermined DOA estimation of Quasi -Stationary signals using a Partly-calibrated array,” *Sensors*, 2017, 17(4):702.

[11] Yue Zhao. “Research of practical super-resolution algorithm based on NC-MUSIC algorithm,” dissertation, 2010.

[12] Jiang G , Mao X , Wang M. “Underdetermined DOA estimation with unknown source number in nonuniform noise.” *2018 IEEE Radar Conference (RadarConf18)*. IEEE, 2018.

[13] Wu C X , Zhang M , Wang K R . “Underdetermined DOA estimation for coprime array based on noncircular signal,” *Bjing Ligong Daxue Xuebao/Transaction of Bjing Institute of Technology*, 2018, 38(3):313-319.

[14] Wu H, Shen Q, Liu W. “Underdetermined low-complexity wideband DOA estimation with uniform linear arrays,” *IEEE 11th Sensor Array and Multichannel Signal Processing Workshop*. IEEE, 2020.

[15] Schmidt R O. “A signal subspace approach to multiple emitter location and spectral estimation,” dissertation, USA: Stanford Univ, 1981.

[16] Paulraj A,Roy R and Kailath T. “Estimation of signal parameters via rotational invariance techniques-ESPIRT,” *Proc. 19th Asilomar Conf., Pacific Grove, CA*, Nov. 1985

[17] Zhu Siqiao. “Study and simulation of signal processing algorithm for LFMCW radar,” *Fire Control Radar Technology*, 2007, 36(3): 41- 45 .

[18] Di Zhu. “Application of digital beamforming technology in millimeter wave automotive radar,” dissertation, 2017

[19] Hui X, Weidong H, Wenxian Y. “Multi-cycle Integration of Weak Target Signals in LFMCW Radar,” *Radar, CIE '06. International Conference on. IEEE*, 2006.