Fuzzy Observer-Based Finite-Time Prescribed Performance Control of Linear Stepping Motor

Yue Sun, Chuang Gao, and Xin Zhou

Abstract—**Due to the position tracking problem of linear stepping motor, a fuzzy controller is designed with observer and finite-time prescribed performance, which can improve the transient and steady-state performance of tracking error at finite time. Through the *d-q* state equations of linear stepping motor, the proposed control scheme is carried out by using backstepping method. To deal with the unmeasurable problem of motor states, an efficient observer is designed. In addition, it is not required to contain the known functions of motor state equations in the control law of the proposed scheme, which makes the control scheme more flexible. Based on Lyapunov stability analysis, it is proved that all the signals in the motor closed-loop system are uniformly ultimately bounded. Finally, the effectiveness of the proposed control scheme is verified by simulation.**

Index Terms—**Linear stepping motor, Backstepping, Finite-time control, Prescribed performance, Fuzzy observer**

I. INTRODUCTION

Linear stepping motor (LSM) is widely used in various fields with high precision requirements. It has the advantages of simple structure, small mechanical loss and good environmental adaptability. Recently, the control methods of LSM have been paid more and more attentions [1-3]. LSM system is a high-order and multivariable nonlinear system, and its controller is also affected by the changes of motor parameters and other uncertain factors. In order to solve above problems, a recursive design method called backstepping can be adopted [4-10]. It has also been widely used in the research of motor control problems [11-15]. With the continuous in-depth study of intelligent algorithm, fuzzy control schemes [16-20] can effectively solve the uncertainty problem of the system, and provide a new approach for the realization of high-performance control. From the existing literatures, it is found that there is no research to consider the unmeasurable motor states and optimize the transient performance of the motor system simultaneously. Therefore, a novel fuzzy observer-based controller is designed to estimate unmeasurable states of the LSM system, then a finite-time prescribed performance function is introduced to enhance the transient and steady-state performance of the LSM system. In addition, the proposed controller can ensure that the motor can track the trajectory of the reference signal.

Manuscript received October 10, 2020; revised January 21, 2021. This work was supported in part by Shandong Key Laboratory of Intelligent Buildings Technology (Grant No. SDIBT202002) and the Scientific Research Fund of Liaoning Provincial Education Department of China (Grant No. 2020LNZD05).

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II. STATE EQUATION OF LINEAR STEPPING MOTOR

Consider the following dynamic LSM model [14]:

$$\begin{cases} m\dfrac{d^2x}{dt^2}+B\dfrac{dx}{dt}+F_c\sin\left(\dfrac{8\pi}{p}x\right)=F_e, \\ F_e=-k_f i_a\sin\left(\dfrac{2\pi}{p}x\right)+k_f i_b\cos\left(\dfrac{2\pi}{p}x\right), \\ L\dfrac{di_a}{dt}+Ri_a-k_f\dot{x}\sin\left(\dfrac{2\pi}{p}x\right)=V_a, \\ L\dfrac{di_b}{dt}+Ri_b+k_f\dot{x}\cos\left(\dfrac{2\pi}{p}x\right)=V_b, \end{cases} \tag{1}$$

where x denotes the mover position, m is the mover mass, B denotes the viscous friction coefficient, p is the pitch, F_c is the cogging force constant, F_e is the electromagnetic thrust, $k_f = 2k_m/p$ and k_m is the back electromotive force constant, R is the winding resistance, L is the winding inductance, i_a, i_b, and V_a and V_b are the winding currents and voltages, respectively. By transforming i_a, i_b, V_a and V_b to i_d, i_q, V_d and V_q, the *d-q* system model of LSM is given in the following form [14]:

$$\begin{cases} \dot{x}_1=x_2, \\ \dot{x}_2=f_1(x_1,x_2)+b_1x_3, \\ \dot{x}_3=f_2(x_2,x_3,x_4)+b_2V_q, \\ \dot{x}_4=-b_3x_4+f_3(x_2,x_3)+b_2V_d, \\ y=x_1, \end{cases} \tag{2}$$

where $x_i \in R^i\ (i=1,2,3,4)$ are the state variables of motor and represent position, velocity, current i_d and i_q, respectively. V_q and V_d are input voltages, and y is the output of LSM system. The relative coefficients and functions are given below:

$$\begin{cases} b_1=k_f/m, \\ b_2=1/L, \\ b_3=R/L, \\ f_1(x_1,x_2)=-Bx_2/m-F_c\sin(8\pi x_1/p)/m, \\ f_2(x_2,x_3,x_4)=-k_fx_2/L-Rx_3/L-2\pi x_2x_4/p, \\ f_3(x_2,x_3)=2\pi x_2x_3/p. \end{cases} \tag{3}$$

For system (2), the control objective can be described as follows: when the motor states x_2, x_3 and x_4 are unmeasurable, an adaptive fuzzy tracking controller is designed based on observer to ensure that the motor position can follow the reference signal $y_d(t)$, and the tracking error can converge to a small neighborhood within tuning time, and all the signals of the motor closed-loop system are uniformly ultimately bounded. In order to achieve the proposed control objective, the following assumptions are needed:

Assumption 1 [16]**:** For any $X_1, X_2 \in R^i$, there exists a set of K_i satisfying the following inequality:

$$|f_i(X_1) - f_i(X_2)| \le K_i \| X_1 - X_2 \|,\ 1 \le i \le 4. \tag{4}$$

Assumption 2: The reference signal $y_d(t)$ and its k-th order derivatives $y_d^{(k)}(t)$ are continuous and bounded.

III. Design of fuzzy observer

In this section, a fuzzy observer is designed firstly. The advantages of the fuzzy observer are described as follows: 1) the controller does not contain the known function of the state equation; 2) the system state can be estimated when the system states are unmeasurable. Hence, the design method is more flexible. In the aspect of error estimation, the fuzzy approximation plays a role in ensuring good observation effect. Because the states of the motor are unmeasurable, then the fuzzy observer design method can be used to estimate the state variables of the motor. Generally, fuzzy logic system can be expressed by the following formula:

$$y(x) = \frac{\sum_{Z=1}^{N} \bar{y}_Z \Pi_{i=1}^{n} \varphi_{F_i^Z}(x_i)}{\sum_{Z=1}^{N} \left(\Pi_{i=1}^{n} \varphi_{F_i^Z}(x_i) \right)}, \tag{5}$$

where $x=[x_1,\ldots,x_n]^T \in R^n$, $x \in R$ and $y \in R$ are the input and output of fuzzy logic system, N denotes the number of IF-THEN rules, $\varphi_{F_i^Z}(x_i)$ and $\varphi_{G^Z}(x_i)$ are fuzzy membership functions related to fuzzy sets F_i^Z and G^Z, and $\bar{y}_Z = \max_{y\in R} \varphi_{G^Z}(y)$. Then, define the following function:

$$\phi_Z = \frac{\Pi_{i=1}^{n} \varphi_{F_i^Z}(x_i)}{\sum_{Z=1}^{N} \left(\Pi_{i=1}^{n} \varphi_{F_i^Z}(x_i) \right)}. \tag{6}$$

Hence, (6) can be expressed as follows:

$$y(x) = \theta^T \phi(x). \tag{7}$$

where $\theta^T = [\bar{y}_1,\ldots,\bar{y}_N]^T = [\theta_1,\ldots,\theta_N]^T$ and $\phi(x)=[\phi_1(x),\ldots,\phi_N(x)]^T$.

Lemma 1 [17]**:** If $f(x)$ is a continuous function defined on a compact set Ω, then for any constant, (7) can be approximated by $f(x)$, thus

$$\sup_{x\in\Omega} \left| f(x) - \theta^T \phi(x) \right| \leqslant \varepsilon. \tag{8}$$

where $\varepsilon > 0$ is an approximation error.

Define the estimated functions of $f_1(x_1,x_2)$, $f_2(x_2,x_3,x_4)$ and $f_3(x_2,x_3)$ as $f_1(\hat{X}_2) = f_1(\hat{x}_1,\hat{x}_2)$, $f_2(\hat{X}_4) = f_2(\hat{x}_2,\hat{x}_3,\hat{x}_4)$ and $f_3(\hat{X}_3) = f_3(\hat{x}_2,\hat{x}_3)$, respectively. Then, system (2) can be transformed into the following form:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = b_1 x_3 + f_1(\hat{X}_2) + \Delta F_1, \\ \dot{x}_3 = f_2(\hat{X}_4) + \Delta F_2 + b_2 V_q, \\ \dot{x}_4 = -b_3 x_4 + f_3(\hat{X}_3) + \Delta F_3 + b_2 V_d, \\ y = x_1, \end{cases} \tag{9}$$

where $\Delta F_1 = f_1(x_1,x_2) - f_1(\hat{X}_2)$, $\Delta F_2 = f_2(x_2,x_3,x_4) - f_2(\hat{X}_4)$ and $\Delta F_3 = f_3(x_2,x_3) - f_3(\hat{X}_3)$. Then, transform system (9) into the following matrix form:

$$\dot{X} = AX + Wy + F(\hat{X}) + \Delta F + b_2 u, \tag{10}$$

where $X = [x_1,x_2,x_3,x_4]^T$, $W = [w_1,\ldots,w_4]^T$ is called the gain vector, $A = \begin{bmatrix} -w_1 & 1 & 0 & 0 \\ -w_2 & 0 & b_1 & 0 \\ -w_3 & 0 & 0 & 0 \\ -w_4 & 0 & 0 & -b_3 \end{bmatrix}^T$, $u = [0,0,V_q,V_d]^T$, $F(\hat{X}) = [0, f_1(\hat{X}_2), f_2(\hat{X}_4), f_3(\hat{X}_3)]$ and $\Delta F = [0, \Delta F_1, \Delta F_2, \Delta F_3]^T$. Choose vector W to make matrix A be a strict Hurwitz matrix. If Q is a positive definite matrix and satisfies $Q = Q^T$, then there exists a positive definite matrix $P = P^T$ which satisfies the following condition:

$$A^T P + PA = -2Q. \tag{11}$$

According to **Lemma 1**, from (10), nonlinear function $f_i(\hat{X}_j)$ in $F(\hat{X})$ can be approximated by

$$\hat{f}_i(\hat{X}_j \mid \theta_i) = \theta_i^T \phi_i(\hat{X}_j), i = 1,2,3, j = 2,3,4. \tag{12}$$

Then, define the optimal vector θ_i^* of θ_i as

$$\theta_i^* = \arg\min_{\theta_i \in \Omega_i} \left\{ \sup_{\hat{X}_i \in U_j} \left| \hat{f}_i(\hat{X}_j \mid \theta_i) - f_i(\hat{X}_j) \right| \right\}, \tag{13}$$

where Ω_i is a bounded compact set of θ_i. Similarly, U_j is a bounded compact set of $\hat{X}_j$. In addition, the minimum approximation error can be defined by

$$\xi_i = f_i(\hat{X}_j) - \hat{f}_i(\hat{X}_j \mid \theta_i^*), \tag{14}$$

where $|\xi_i| \le \xi_i^* (i = 1,2,3)$ and $\xi^*_i > 0$. Then, the following fuzzy state observer is established to estimate the unmeasurable states:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + w_1(y - \hat{x}_1), \\ \dot{\hat{x}}_2 = b_1 \hat{x}_3 + w_2(y - \hat{x}_1) + \hat{f}_1(\hat{X}_2), \\ \dot{\hat{x}}_3 = w_3(y - \hat{x}_1) + \hat{f}_2(\hat{X}_4) + b_2 V_q, \\ \dot{\hat{x}}_4 = -b_3 \hat{x}_4 + w_4(y - \hat{x}_1) + \hat{f}_3(\hat{X}_3) + b_2 V_d, \\ \hat{y} = \hat{x}_1. \end{cases} \tag{15}$$

Furthermore, (16) can be obtained.

$$\begin{cases} \dot{\hat{X}} = A\hat{X} + Wy + \hat{F}(\hat{X}) + b_2 u, \\ \hat{y} = D\hat{X}, \end{cases} \tag{16}$$

where $\hat{F}\left(\hat{X}\right)=\left[0,\hat{f}_1\left(\hat{X}_2\right),\hat{f}_2\left(\hat{X}_4\right),\hat{f}_3\left(\hat{X}_3\right)\right]^T$, $D=[1,0,0,0]$ and $\hat{X}=\left[\hat{x}_1,\hat{x}_2,\hat{x}_3,\hat{x}_4\right]^T$.

Define the error vector e as follows:

$$e = X-\hat{X}, \tag{17}$$

where $e=\left[e_1,e_2,e_3,e_4\right]^T$.

From $\tilde{\theta}_i=\theta_i^*-\theta_i, i=1,...,3$, the time-derivative of e is

$$\dot{e}=\dot{X}-\dot{\hat{X}}=Ae+\xi+J+\Delta F, \tag{18}$$

where $J=\left[0,\tilde{\theta}_1^T\phi_1\left(\hat{X}_2\right),\tilde{\theta}_2^T\phi_2\left(\hat{X}_4\right),\tilde{\theta}_3^T\phi_3\left(\hat{X}_3\right)\right]^T$ and $\xi=[0,\xi_1,\xi_2,\xi_3]^T$.

Select the following Lyapunov function for fuzzy observer:

$$V_0=\frac{1}{2}e^TPe. \tag{19}$$

Then, the time-derivative of V_0 can be described as follows:

$$\dot{V}_0=e^TP\dot{e}. \tag{20}$$

According to **Assumption 1**, substituting (18) into (20) yields

$$\begin{aligned}\dot{V}_0&=e^TP\left(Ae+\xi+J+\Delta F\right)\\&=-e^TQe+e^TP\Delta F+e^TPJ+e^TP\xi.\end{aligned} \tag{21}$$

Generally, $\phi_i^T\left(\hat{X}_j\right)$ is chosen as Gaussian function, it gives $\phi_i^T\left(\hat{X}_j\right)^T\phi_i\left(\hat{X}_j\right)\le 1$. From Young's inequality, it produces

$$e^TPJ\leqslant\frac{1}{2}\lambda_{\max}\left(P\right)\|e\|^2+\sum_{i=1}^{3}\tilde{\theta}_i^T\tilde{\theta}_i, \tag{22}$$

$$e^TP\left(\Delta F+\xi\right)\leqslant\sum_{i=1}^{4}\frac{1}{2}\|P\|^2K_i^2\|e\|^2+\frac{1}{2}\|P\|^2\left\|\xi^*\right\|^2+\|e\|^2, \tag{23}$$

where $\lambda_{\max}\left(P\right)>0$ represents the maximum eigenvalue of matrix P and $\xi^*=\left[0,\xi_1^*,\xi_2^*,\xi_3^*\right]^T$.

Substituting (22) and (23) into (21) yields

$$\dot{V}_0\leqslant-\lambda_E\|e\|^2+\sum_{i=1}^{3}\tilde{\theta}_i^T\tilde{\theta}_i+q, \tag{24}$$

where $\lambda_E=\lambda_{\min}\left(Q\right)-\left(\frac{1}{2}\lambda_{\max}\left(P\right)+\sum_{i=1}^{4}\frac{1}{2}\|P\|^2K_i+1\right)$, $\lambda_{\min}(Q)$ represents the minimum eigenvalue of Q and $q=\frac{1}{2}\|P\|^2\|\xi^*\|^2$

IV. Design of Finite-Time Prescribed Performance Controller

The prescribed performance control method can effectively improve transient and steady-state performance of the nonlinear system. For the control problem of LSM, the following performance function is given as follows [8]:

$$v\left(t\right)=\begin{cases}\left(v_0-\dfrac{t}{T_f}\right)\exp\left(1-\dfrac{T_f}{T_f-t}\right)+v_{T_f}, & t\in\left[0,T_f\right),\\ v_{T_f}, & t\in\left[T_f,\infty\right),\end{cases} \tag{25}$$

where $v_0\ge 1.25$ and $v_{T_f}>0$ represent the initial and final values of the prescribed performance function, respectively, T_f is the tuning time. [8] proved that the j-th order derivative of the function $v^{(j)}\left(t\right)$ is continuous and smooth. The tracking error of the motor position is defined as $e_0\left(t\right)=x_1\left(t\right)-y_d\left(t\right)$, and the performance function control design can make the tracking error converge into a preset region at finite time. In this paper, the backstepping method is adopted. Firstly, the following coordinate transformation is selected:

$$e_0\left(t\right)=v\left(t\right)\tanh\left(\eta\right). \tag{26}$$

where $\tanh\left(\eta\right)=\left(e^{\eta}-e^{-\eta}\right)/\left(e^{\eta}+e^{-\eta}\right)$ and η is the transformation error. Then, the time-derivative of $e_0\left(t\right)$ is

$$\dot{e}_0\left(t\right)=\dot{v}\left(t\right)\tanh\left(\eta\right)+v\left(t\right)\frac{\partial\tanh\left(\eta\right)}{\partial\eta}\dot{\eta}\left(t\right). \tag{27}$$

Substituting $e_1\left(t\right)=x_1\left(t\right)-y_d\left(t\right)$ into (31) gives

$$\dot{\eta}\left(t\right)=\frac{\dot{x}_1\left(t\right)-\dot{y}_d\left(t\right)-\dot{v}\left(t\right)\tanh\left(\eta\right)}{v\left(t\right)\dfrac{\partial\tanh\left(\eta\right)}{\partial\eta}}. \tag{28}$$

Then, substituting $\dot{x}_1$ in (2) into (28) yields

$$\begin{aligned}\dot{\eta}\left(t\right)&=\frac{x_2\left(t\right)-\dot{y}_d\left(t\right)-\dot{v}\left(t\right)\tanh\left(\eta\right)}{v\left(t\right)\dfrac{\partial\tanh\left(\eta\right)}{\partial\eta}}\\&=\frac{\hat{x}_2+e_2-\dot{y}_d\left(t\right)-\dot{v}\left(t\right)\tanh\left(\eta\right)}{v\left(t\right)\dfrac{\partial\tanh\left(\eta\right)}{\partial\eta}}\\&=\gamma+G\left(\hat{x}_2+e_2-\dot{y}_d\right),\end{aligned} \tag{29}$$

where $\gamma=-\dot{v}\left(t\right)\tanh\left(\eta\right)/\left(v\left(t\right)\partial\tanh\left(\eta\right)/\partial\eta\right)$ and $G=1/\left(v\left(t\right)\partial\tanh\left(\eta\right)/\partial\eta\right)$.

From (29), (15) can be transformed into

$$\begin{cases}\dot{\eta}=\gamma+G\left(\hat{x}_2+e_2-\dot{y}_d\right),\\ \dot{\hat{x}}_2=b_1\hat{x}_3+w_2\left(y-\hat{x}_1\right)+\theta_1^T\phi_1(\hat{X}_2),\\ \dot{\hat{x}}_3=w_3\left(y-\hat{x}_1\right)+\theta_2^T\phi_2\left(\hat{X}_4\right)+b_2V_q,\\ \dot{\hat{x}}_4=-b_3\hat{x}_4+w_4\left(y-\hat{x}_1\right)+\theta_3^T\phi_3(\hat{X}_3)+b_2V_d.\end{cases} \tag{30}$$

According to (30), choose the following coordinate transformation:

$$\begin{cases}z_1=\eta\\ z_i=\hat{x}_i-\alpha_{i-1}, \quad i=2,3\end{cases} \tag{31}$$

where z_1 and z_i are transformation errors and α_{i-1} is the virtual control law.

Step 1:

From (31), the time-derivative of z_1 is

$$\dot{z}_1=\gamma+G\left(\hat{x}_2+e_2-\dot{y}_d\right). \tag{32}$$

Substituting $z_2=\hat{x}_2-\alpha_1$ into (32) produces

$$\dot{z}_1=\gamma+G\left(z_2+\alpha_1+e_2-\dot{y}_d\right). \tag{33}$$

For the 1st subsystem in (30), choose the following Lyapunov function:

$$V_1=\frac{1}{2}z_1^2. \tag{34}$$

Substitute (33) into the derivative of V_1 gives

$$\dot{V}_1 = z_1\dot{z}_1 = z_1\left(\gamma + G\left(z_2 + \alpha_1 + e_2 - \dot{y}_d\right)\right). \tag{35}$$

According to Young's inequality, it yields

$$z_1 G e_2 \le \frac{1}{2}z_1^2 G^2 + \frac{1}{2}\|e\|^2. \tag{36}$$

Substitute (36) into (35), the following result holds.

$$\dot{V}_1 \le z_1\left(\gamma + G\left(z_2 + \alpha_1 + \frac{1}{2}z_1 G - \dot{y}_d\right)\right) + \frac{1}{2}\|e\|^2. \tag{37}$$

Design the virtual control α_1 as

$$\alpha_1 = -\left(\frac{c_1}{G} + \frac{G}{2}\right)z_1 - \frac{\gamma}{G} + \dot{y}_d, \tag{38}$$

where, $c_1 > 0$ is a design parameter. Substituting α_1 into (38) gives

$$\dot{V}_1 \le -c_1 z_1^2 + \frac{1}{2}\|e\|^2 + G z_1 z_2. \tag{39}$$

Step 2:

Choose the following Lyapunov function for the 2nd subsystem:

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2r_1}\tilde{\theta}_1^T\tilde{\theta}_1, \tag{40}$$

where $r_1 > 0$ is a design parameter.

Substitute $\dot{\hat{x}}_2$ in (30), $\hat{x}_3 = z_3 + \alpha_2$ and $\tilde{\theta}_1 = \theta_1^* - \theta_1$ into the time-derivative of V_2, it produces

$$\begin{aligned}
\dot{V}_2 &= \dot{V}_1 + z_2\dot{z}_2 - \frac{1}{r_1}\tilde{\theta}_1^T\dot{\theta}_1. \\
&= \dot{V}_1 + z_2\left(b_1\left(z_3 + \alpha_2\right) + w_2 e_1 + \theta_1^T\phi_1\left(\hat{X}_2\right) - \dot{\alpha}_1\right) - \frac{1}{r_1}\tilde{\theta}_1^T\dot{\theta}_1 \\
&= \dot{V}_1 + z_2\left(b_1\left(z_3 + \alpha_2\right) + w_2 e_1 + \theta_1^T\phi_1\left(\hat{X}_2\right)_1\right) \\
&\quad - z_2\left(\tilde{\theta}_1^T\phi_1\left(\hat{X}_2\right) + \dot{\alpha}\right) + \frac{1}{r_1}\tilde{\theta}_1^T\left(r_1 z_2\phi_1\left(\hat{X}_2\right) - \dot{\theta}_1\right),
\end{aligned} \tag{41}$$

where $\dot{\alpha}_1 = \frac{\partial\alpha_1}{\partial y}\left(\hat{x}_2 + e_2\right) + \frac{\partial\alpha_1}{\partial\hat{x}_1}\left(\hat{x}_2 + w_1 e_1\right) + \sum_{j=1}^{2}\frac{\partial\alpha_1}{\partial y_d^{(j-1)}}y_d^{(j)} + \sum_{j=1}^{2}\frac{\partial\alpha_1}{\partial v^{(j-1)}}v^{(j)}$.

Furthermore, it yields

$$\begin{aligned}
\dot{V}_2 &= \dot{V}_1 + z_2\left(b_1\left(z_3 + \alpha_2\right) + w_2 e_1 + \theta_1^T\phi_1\left(\hat{X}_2\right)\right) \\
&\quad - z_2\left(\tilde{\theta}_1^T\phi_1\left(\hat{X}_2\right) + \frac{\partial\alpha_1}{\partial y}\left(\hat{x}_2 + e_2\right) + \frac{\partial\alpha_1}{\partial\hat{x}1}\left(\hat{x}_2 + w_1 e_1\right)\right) \\
&\quad - z_2\left(\sum_{j=1}^{2}\frac{\partial\alpha_1}{\partial y_d^{(j-1)}}y_d^{(j)} + \sum_{j=1}^{2}\frac{\partial\alpha_1}{\partial v^{(j-1)}}v^{(j)}\right) \\
&\quad + \frac{1}{r_1}\tilde{\theta}_1^T\left(r_1 z_2\phi_1\left(\hat{X}_2\right) - \dot{\theta}_1\right)
\end{aligned} \tag{42}$$

According to Young's inequality, it gives

$$-z_2\tilde{\theta}_1^T\phi_1(\hat{X}_2) \le \frac{1}{2}z_2^2 + \frac{1}{2}\tilde{\theta}_1^T\tilde{\theta}_1, \tag{43}$$

$$-z_2\frac{\partial\alpha_1}{\partial y}e_2 \le \frac{1}{2}z_2^2\left(\frac{\partial\alpha_1}{\partial y}\right)^2 + \frac{1}{2}\|e\|^2. \tag{44}$$

Substitute (43) and (44) into (42), the following result holds.

$$\begin{aligned}
\dot{V}_2 &\le z_2\left(b_1\left(z_3 + \alpha_2\right) + \frac{1}{2}z_2 + \frac{1}{2}z_2\left(\frac{\partial\alpha_1}{\partial y}\right)^2\right) \\
&\quad - c_1 z_1^2 + G z_1 z_2 + z_2 D_2 + \|e\|^2 \\
&\quad + \frac{1}{r_1}\tilde{\theta}_1^T\left(r_1 z_2\phi_1\left(\hat{X}_2\right) - \dot{\theta}_1\right),
\end{aligned} \tag{45}$$

where

$$\begin{aligned}
D_2 &= w_2 e_1 + \theta_1^T\phi_1\left(\hat{X}_2\right) - \frac{\partial\alpha_1}{\partial\hat{x}_1}\left(\hat{x}_2 + w_1 e_1\right) - \frac{\partial\alpha_1}{\partial y}\hat{x}_2 \\
&\quad - \sum_{j=1}^{2}\frac{\partial\alpha_1}{\partial y_d^{(j-1)}}y_d^{(j)} - \sum_{j=1}^{2}\frac{\partial\alpha_1}{\partial v^{(j-1)}}v^{(j)}.
\end{aligned} \tag{46}$$

Design the virtual control α_2 and adaptive law $\dot{\theta}_1$ as follows:

$$\alpha_2 = -\frac{1}{b_1}\left(c_2 z_2 + D_2 + \frac{1}{2}z_2 + \frac{1}{2}z_2\left(\frac{\partial\alpha_1}{\partial y}\right)^2 + G z_1\right) \tag{47}$$

and

$$\dot{\theta}_1 = r_1 z_2\phi_1(\hat{X}_2) - \kappa_1\theta_1, \tag{48}$$

where $c_2 > 0$ and $\kappa_1 > 0$ are design parameters.

Substitute (47) and (48) into (45), from $\tilde{\theta}_1^T\theta_1 \le \tilde{\theta}_1^T\left(\theta_1^* - \tilde{\theta}_1\right) \le -\tilde{\theta}_1^T\tilde{\theta}_1/2 + \|\theta_1^*\|^2/2$, it produces

$$\begin{aligned}
\dot{V}_2 &\le -\sum_{j=1}^{2}c_j z_j^2 + b_1 z_2 z_3 - \frac{\kappa_1}{2r_1}\tilde{\theta}_1^T\tilde{\theta}_1 + \frac{1}{2}\tilde{\theta}_1^T\tilde{\theta}_1 \\
&\quad + \|e\|^2 + \sigma_1,
\end{aligned} \tag{49}$$

where $\sigma_1 = \frac{\kappa_1}{2r_1}\|\theta_1^*\|^2$.

Step 3:

Choose the following Lyapunov function for the 3rd subsystem:

$$V_3 = V_2 + \frac{1}{2}z_3^2 + \frac{1}{2r_2}\tilde{\theta}_2^T\tilde{\theta}_2 + \frac{1}{2r_3}\tilde{\theta}_3^T\tilde{\theta}_3, \tag{50}$$

Substituting $\hat{x}_3 = z_3 + \alpha_2$ into the time-derivative of V_3 yields

$$\begin{aligned}
\dot{V}_3 &= \dot{V}_2 + z_3\dot{z}_3 - \frac{1}{r_2}\tilde{\theta}_2^T\dot{\theta}_2 - \frac{1}{r_3}\tilde{\theta}_3^T\dot{\theta}_3 \\
&= \dot{V}_2 + z_3\left(\dot{\hat{x}}_3 - \dot{\alpha}_2\right) - \frac{1}{r_2}\tilde{\theta}_2^T\dot{\theta}_2 - \frac{1}{r_3}\tilde{\theta}_3^T\dot{\theta}_3 \\
&= \dot{V}_2 + z_3\left(w_3 e_1 + b_2 V_q + \theta_2^T\phi_2\left(\hat{X}_4\right) - \tilde{\theta}_2^T\phi_2\left(\hat{X}_4\right)\right) \\
&\quad - z_3\left(\tilde{\theta}_3^T\phi_3\left(\hat{X}_3\right) + \dot{\alpha}_2\right) + \frac{1}{r_2}\tilde{\theta}_2^T\left(r_2 z_3\phi_2\left(\hat{X}_4\right) - \dot{\theta}_2\right) \\
&\quad + \frac{1}{r_3}\tilde{\theta}_3^T\left(r_3 z_3\phi_3\left(\hat{X}_3\right) - \dot{\theta}_3\right),
\end{aligned} \tag{51}$$

According to Young's inequality, it produces

$$-z_3\left(\tilde{\theta}_2^T\phi_2\left(\hat{X}_4\right) + \tilde{\theta}_3^T\phi_3(\hat{X}_3)\right) \le z_3^2 + \frac{1}{2}\tilde{\theta}_2^T\tilde{\theta}_2 + \frac{1}{2}\tilde{\theta}_3^T\tilde{\theta}_3, \tag{52}$$

$$-z_3\frac{\partial\alpha_2}{\partial y}e_2 \le \frac{1}{2}z_3^2\left(\frac{\partial\alpha_2}{\partial y}\right)^2 + \frac{1}{2}\|e\|^2. \tag{53}$$

From (47), $\dot{\alpha}_2$ is given as follows:

$$\dot{\alpha}_2 = \frac{\partial \alpha_2}{\partial y}\left(\hat{x}_2 + e_2\right) + \frac{\partial \alpha_2}{\partial \hat{x}_1}\left(\hat{x}_2 + w_1 e_1\right) + \sum_{j=1}^{3}\frac{\partial \alpha_2}{\partial y_d^{(j-1)}} y_d^{(j)} + \sum_{j=1}^{3}\frac{\partial \alpha_2}{\partial v^{(j-1)}} v^{(j)} + \frac{\partial \alpha_2}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial \alpha_2}{\partial \hat{x}_2}\left(b_1 x_3 + w_2 e_1 + \theta_1^T \phi_1\left(\hat{X}_2\right)\right). \tag{54}$$

Substitute (52), (53) and (54) into (51), $\dot{V}_3$ can be obtained.

$$\begin{aligned} \dot{V}_3 \le & -\sum_{j=1}^{2} c_j z_j^2 + b_1 z_2 z_3 - \frac{\kappa_1}{2r_1}\tilde{\theta}_1^T \tilde{\theta}_1 + \frac{1}{2}\tilde{\theta}_1^T \tilde{\theta}_1 \\ & + \| e \|^2 + \sigma_1 + z_3\left(b_2 V_q + D_3 + z_3 + \frac{1}{2} z_3 \left(\frac{\partial \alpha_2}{\partial y}\right)^2\right) \\ & + \frac{1}{2}\| e \|^2 + \frac{1}{2}\tilde{\theta}_2^T \tilde{\theta}_2 + \frac{1}{2}\tilde{\theta}_3^T \tilde{\theta}_3 \\ & + \frac{1}{r_2}\tilde{\theta}_2^T \left(r_2 z_3 \phi_2\left(\hat{X}_4\right) - \dot{\theta}_2\right) \\ & + \frac{1}{r_3}\tilde{\theta}_3^T \left(r_3 z_3 \phi_2\left(\hat{X}_3\right) - \dot{\theta}_3\right) \end{aligned} \tag{55}$$

where

$$D_3 = w_3 e_1 + \theta_2^T \phi_2\left(\hat{X}_4\right) - \frac{\partial \alpha_2}{\partial y}\hat{x}_2 - \frac{\partial \alpha_2}{\partial \hat{x}_2}\left(b_1 x_3 + w_2 e_1 + \theta_1^T \phi_1\left(\hat{X}_2\right)\right) - \frac{\partial \alpha_2}{\partial \hat{x}_1}\left(\hat{x}_2 + w_1 e_1\right) - \frac{\partial \alpha_2}{\partial \theta_1}\dot{\theta}_1 - \sum_{j=1}^{3}\frac{\partial \alpha_2}{\partial y_d^{(j-1)}} y_d^{(j)} - \sum_{j=1}^{3}\frac{\partial \alpha_2}{\partial v^{(j-1)}} v^{(j)}. \tag{56}$$

Choose the following actual control law V_q:

$$V_q = -\frac{1}{b_2}\left(c_3 z_3 + D_3 + z_3 + b_1 z_2 + \frac{1}{2} z_3 \left(\frac{\partial \alpha_2}{\partial y}\right)^2\right), \tag{57}$$

Then, select the following adaptive laws $\dot{\theta}_2$ and $\dot{\theta}_3$:

$$\begin{cases} \dot{\theta}_2 = r_2 z_3 \phi_2(\hat{X}_4) - \kappa_2 \theta_2, \\ \dot{\theta}_3 = r_3 z_3 \phi_3(\hat{X}_3) - \kappa_3 \theta_3, \end{cases} \tag{58}$$

where $c_1 > 0$, $\kappa_2 > 0$ and $\kappa_3 > 0$ are design parameters.

Substituting (57) and (58) into (55) yields

$$\begin{aligned} \dot{V}_3 \le & -\sum_{j=1}^{3} c_j z_j^2 - \frac{\kappa_1}{2r_1}\tilde{\theta}_1^T \tilde{\theta}_1 + \frac{\kappa_2}{r_2}\tilde{\theta}_2^T \theta_2 + \frac{\kappa_3}{r_3}\tilde{\theta}_3^T \theta_3 \\ & + \frac{1}{2}\sum_{j=1}^{3}\tilde{\theta}_i^T \tilde{\theta}_i + \frac{3}{2}\| e \|^2 + \sigma_1 \\ \le & -\sum_{j=1}^{3} c_j z_j^2 - \sum_{j=1}^{3}\frac{\kappa_j}{2r_j}\tilde{\theta}_i^T \tilde{\theta}_i + \frac{1}{2}\sum_{j=1}^{3}\tilde{\theta}_i^T \tilde{\theta}_i + \frac{3}{2}\| e \|^2 + \sigma_2, \end{aligned} \tag{59}$$

where $\sigma_2 = \sigma_1 + \frac{\kappa_2}{2r_2}\| \theta_2^* \|^2 + \frac{\kappa_3}{2r_3}\| \theta_3^* \|^2$.

Step 4:

For the 4th subsystem, design the following control voltage V_d:

$$V_d = -\frac{1}{b_2}\left(w_4 e_1 + \theta_3^T \phi_3(\hat{X}_3)\right), \tag{60}$$

then the state function $\dot{\hat{x}}_4 = -b_3 \hat{x}_4$ can be obtained. It can be seen that the state eventually tends to zero. Finally, combining the Lyapunov functions in the observer and the backstepping method gives

$$V = V_0 + V_3. \tag{61}$$

Therefore, the time-derivative of V is

$$\begin{aligned} \dot{V} = & \dot{V}_0 + \dot{V}_3 \\ \le & -\lambda_E \| e \|^2 + \sum_{j=1}^{3}\tilde{\theta}_i^T \tilde{\theta}_i + q - \sum_{j=1}^{3} c_j z_j^2 \\ & -\sum_{j=1}^{3}\frac{\kappa_j}{2r_j}\tilde{\theta}_i^T \tilde{\theta}_i + \sum_{j=1}^{3}\frac{1}{2}\tilde{\theta}_i^T \tilde{\theta}_i + \frac{3}{2}\| e \|^2 + \sigma_2 \\ \le & -\lambda \| e \|^2 - \sum_{j=1}^{3} c_j z_j^2 - \sum_{j=1}^{3}\beta \tilde{\theta}_i^T \tilde{\theta}_i + b, \end{aligned} \tag{62}$$

where $\lambda = \lambda_E - \frac{3}{2}$, $\beta = \frac{\kappa_j}{2r_j} - \frac{3}{2}$ and $b = q + \sigma_2$.

Select the appropriate design parameters to make $\lambda > 0$ and $\beta > 0$, and define

$$a = \min\left\{2\lambda / \lambda_{\min}(P), 2c_i, 2\beta r_i\right\}, i = 1,2,3. \tag{63}$$

Then, (62) can be written as

$$\dot{V} \leqslant -aV + b. \tag{64}$$

Therefore, from (64), it is concluded that

$$0 \leqslant V(t) \leqslant V(0) e^{-a(t-t_0)} + \frac{b}{a}, \tag{65}$$

where t_0 is the initial time. Inequality (65) illustrates that all the signals of the LSM system are uniformly ultimately bounded. From (26), it can be seen that $-1 < \tanh(\eta) < 1$ and $v(t) > 0$, thus $-v(t) < v(t)\tanh(\eta) < v(t)$ is obtained. Furthermore, it yields that $-v(t) < e_0(t) < v(t)$, hence the constraint by (26) can realize that the tracking error $e_0(t)$ of motor position can be limited in the range $\left(-v(t), v(t)\right)$. When the value of $v(t)$ is decreased, $e_0(t)$ can be limited in $\Delta = \left\{e_0(t) \in R : | e_0(t) | < v_{T_f}, t \ge T_f\right\}$ at finite tuning time T_f. Therefore, the control objective is achieved.

V. Simulation

In this section, the performance of the proposed control method is verified by simulation. The motor parameters and controller parameters are shown in TABLE I, where $r_1, r_2, r_3, \kappa_1, \kappa_2, \kappa_3$ are the parameters of adaptive laws (47) and (57), w_1, w_2, w_3, w_4 are the parameters of observer, c_1, c_2, c_3 denotes controller parameters in (38), (46) and (56), and v_0, v_{T_f}, T_f are the parameters of prescribed performance function (25). By selecting appropriate parameters, the motor can follow the reference signal $y_d(t) = \sin(t)$, and the tracking error converges to the predetermined region at finite time. The fuzzy membership function is selected as follows:

$$\varphi_{F_i^Z}(\hat{x}_i) = \exp\left(-\frac{\left(\hat{x}_i + 3 - Z\right)^2}{4}\right), \quad i = 1,2,3,4, \quad Z = 1,2,3,4,5. \tag{65}$$

Based on above design method, the simulation results are shown in Fig. 1-4. Fig. 1 describes the motor position and reference motion trajectory. From Fig. 1, it shows that the controller achieves a good tracking effect. Fig. 2 shows that the observer can estimate the position and velocity states of the electronic mover when the states are not measurable.

TABLE I
SELECTED PARAMETERS TABLE

Motor	Adaptive laws	Observer
$m = 0.65$ kg $B = 0.01$ N/m/s $F_c = 2.4$ N $p = 1.28$ mm $K_f = 27.83$ N/A $R = 3\ \Omega$ $L = 0.5$ mH	$r_1 = 1$ $r_2 = 1$ $r_3 = 1$ $\kappa_1 = 6$ $\kappa_2 = 10$ $\kappa_3 = 10$	$w_1 = 1$ $w_2 = 240$ $w_3 = 120$ $w_4 = 10$
Control Laws	Prescribed Performance Function	Initial state
$c_1 = 2$ $c_2 = 10$ $c_3 = 15$	$v_0 = 1.25$ $v_{Tf} = 0.25$ $T_f = 1$ s	$x_1(0) = 0.5$ m $x_2(0) = 0$ m/s $x_3(0) = 0$ A $x_4(0) = 0$ A

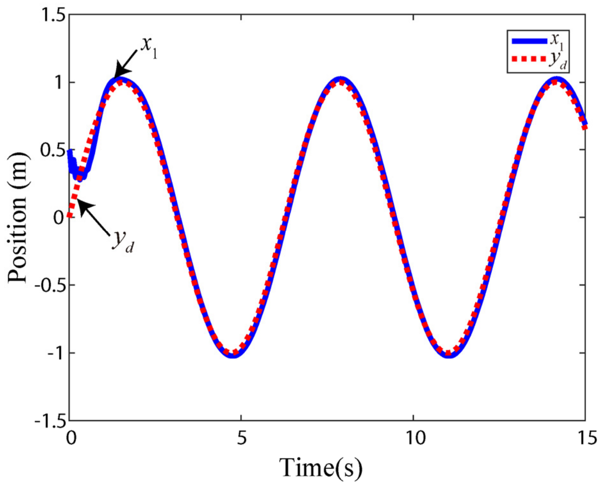

Fig. 1. Position tracking performance of linear stepping motor

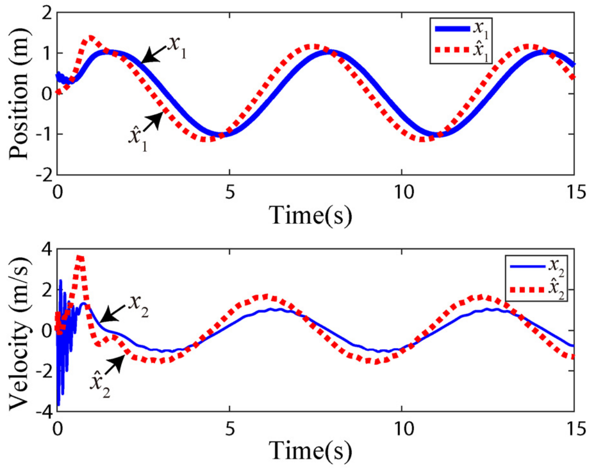

Fig. 2. Position and velocity estimations of linear stepping motor

Fig. 3 shows the tracking error of the motor position. Under the action of the prescribed performance function, the tracking error can quickly converge to the preset boundary in about 1 s, which shows that the control method can improve the transient and steady-state performance of the motor tracking error at finite time. In addition, the proposed controller ensures that all the signals in the closed-loop are uniformly ultimately bounded, and it can realize the motor tracking control when the motor states are unmeasurable. Under this effect, the curves of input voltage V_q and V_d are shown in Fig. 4. From the simulation analysis, It is concluded that the proposed controller can effectively improve the system performance and realize the tracking control of LSM more flexibly.

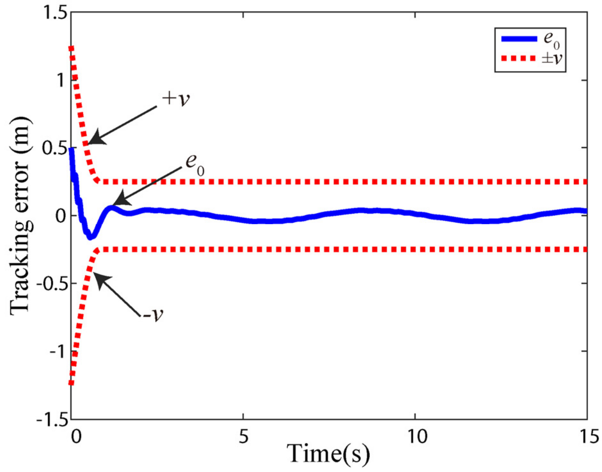

Fig. 3. Performance of tracking error

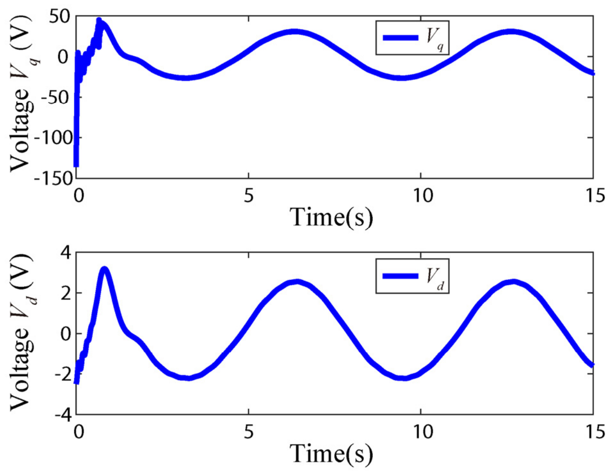

Fig. 4. Input voltage V_q and V_d

VI. CONCLUSION

Due to the position tracking problem of linear stepping motor, an adaptive fuzzy tracking control scheme is designed in this paper.

1. Compared with the previous motor control methods, it can improve the transient and steady-state performance of the position tracking error at finite time.
2. The proposed controller ensures that all the signals in the closed loop are uniformly ultimately bounded.
3. The backstepping method and fuzzy observer are used to solve the problem when the motor states are not measurable. The controller does not contain the known function in the state equations, which makes the control design more flexible.
4. The experimental results illustrate the feasibility and effectiveness of the control scheme.

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