Event-based Reliable Control for Switched Systems with Actuator Failures and Nonlinear Perturbation

Xiaona Wang, Yongzhao Wang, Wenqiong Hou

***Abstract*—This paper considers the issue of the reliable H_∞ control for switched systems with nonlinear perturbation and actuator failures based on event-triggered scheme. The actuator failures and nonlinear perturbation are considered, then a closed-loop state feedback switched system is established under proposed the event-triggered scheme. Furthermore, by means of Lyapunov stability theory, some stability criteria and satisfactory performance of the switched with actuator failures and nonlinear perturbation are obtained. In addition, the reliable feedback controller can be designed through a special matrix transformation. Finally, the rationality of the method is given through a simulation example.**

***Index Terms*—Reliable, event-triggered, switched systems, average dwell time, nonlinear perturbation.**

I. Introduction

OVER the last few years, with the progress of science and technology and the development of economy, the normal operation of engineering system has higher and higher requirements on the efficiency and maintainability of system model. As the representative of the system, the switched system has been widely used in engineering systems due to its characteristics of high efficiency and simple maintenance. E.g, chemical processing [1], redundant manipulator [2], networked systems [3], fault detection systems [4], aero-engine model [5] and the references cited therein. In particular, the normal operation of the system, that is, the stability of the system, is essential for the study of switched systems and have attracted significant academic interest. For instance, In [6], the slowly switched systems are considered, and the issue of new results on stability are addressed under dwell time switching strategy. In [7], another switching method is adopted, that is, based on the persistent time control strategy, and the stability criteria of switched Takagi-Sugeno fuzzy systems are obtained under Lyapunov stability theory and special inequality transformation technique. Consider the inconsistency between the subsystem and the control, in [8], the author discusses the output tracking control problem of switched systems, and the explicit expressions are provided for the designed controllers based on special matrix deformation method.

On the other hand, In engineering practice system, the existence of external disturbance often disturbs the normal operation of the system, because it usually makes the running system suddenly appear unexpected situation, that is, system collapse and abnormal performance, which often disturbs researchers. [9-12]. Therefore, the above mentioned difficulties have attracted wide attention from researchers in the past few years. For instance, in [13], the robust stabilization issue of nonlinear switched systems is addressed based on a novel switching Lyapunov function, and when the switched systems does not have a stable subsystem, some less conservative results are obtained by considered switching strategy. [14] deals with the switched systems subject to all modes unstable, then admissible dwell time is proposed to exploit the stabilization property. With the renewal of research methods and the comprehensive upgrade of technology, researchers have begun to consider the influence of factors such as random noise and nonlinear perturbation on the system in a lot of work. E.g, in [15], the author considers the situation where the controller has a delay, and the state feedback controllers are designed by using a special matrix transformation. In [16], the discrete-time switched systems is studied, and some stability criteria and satisfactory performance of resulting closed-loop system are obtained. However, in actual engineering system, it is possible to encounter the worst cases, such as potential process abnormalities and component failures. The above results only give some simplified methods to deal with exogenous disturbance, and the authors do not consider the situation of actuator failure. In order to solve this shortcoming, it is necessary to consider a reliable controller that ensures the closed-loop system stability. Moreover, it's worth pointing out that H_∞ control as effective methods can provide disturbance rejection capability. Based on the above analysis, we learned that the actuator faults and nonlinear disturbances were rarely considered in the system they considered. This leads to our current research motivation.

It should be pointed out that, in the general environment of network hardware development, sampled-data mechanism is superior in flexibility, maintainability and simpler installation than traditional control mechanism. Such mechanisms are often applied in many modern industrial control applications over the past decades. Specifically, periodical sampling

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mechanism (or time-triggered mechanism) is often investigated to obtain the instantaneous sampling information of physical plants states. Fruitful results on sampled-data-based control have been achieved in the past few decades [17-19]. However, under limited network resources, the method based on periodic sampling may cause network breakdown. As a result, with the development of modern network communication technology and the improvement of data communication reliability requirements in work and life, the event-triggered mechanism is introduced to deal with or improve the problems arising from time-triggered scheme while retaining a satisfactory performance.[20-22]. For example, in [23], the uncertain factors are considered. By using linear matrix inequalities technique, some stability criteria of switched systems with an event-triggered scheme are derived. [24] considers the hardware endurance of the system in practical applications, that is, the limited quantization range, and provides a method to solve such problems through special switching rules and event control method. Under this scenario, a positive lower bound is obtained to eliminate the Zeno behavior in [24]. [25] studies quasi-time-dependent event-triggered filtering problem of switched systems under the limited quantization range. It should be pointed out that the work of the above results only considers the simple form switched system model. However, the system in real life is quite complicated, and when more influencing factors are considered, the above methods may not be used. Particularly, the effects of actuator failures and exogenous disturbance on switched systems have not been fully revealed so far.

In order to fill such a gap, the purpose of this article is to study the reliable event-triggered control for switched systems affected by nonlinear perturbation and actuator failure. The main novelties are as follows: (i) the method of different triggering thresholds is proposed to reduce the utilization of limited communication resource. Moreover, the influence of actuator failures and exogenous disturbance are taken into consideration simultaneously, then the state feedback switched system model is established. (ii) By using Lyapunov stability theory, some stability crite- ria and satisfactory H_∞ performance of the switched subject to nonlinear perturbation and actuator failure are obtained. (iii) based on a skillful matrix decoupling approach, The reliable feedback controller can be designed under a proficient matrix transformation.

II. Problem formulation and preliminaries

The switched systems considered is as follows:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u^f(t) + C_{\sigma(t)}\omega(t) \\ \qquad + D_{\sigma(t)}f(t,x(t)), \\ z(t) = E_{\sigma(t)}x(t) + F_{\sigma(t)}\omega(t), \end{cases} \tag{1}$$

The state vector, the actuator fault and disturbance, respectively, are represented by $x(t) \in R^n$, $u^f(t) \in R^m, \omega(t)$. The measured output is represented by $z(t) \in R^n$. Specifically, The switching law function is represented by $\sigma(t)$, and it is assumed that there are s subsystems. t_k is recorded as the switching instant. For convenience of expression, we denote $\sigma(t) = i$.

It is worth mentioning that with the increasing complexity of industrial system models and the uncertainty of the working environment, the actuator failure usually occurs in the process of controlling the dynamic system, and it will work abnormally under certain actual conditions. Therefore, we design the reliability controller based on the practical application and the comprehensive control method of the model.

$$u^f(t) = M_i u(t) + g(u(t)), \forall t \in [t_0, \infty) \tag{2}$$

The model of actuator fault matrix $M_i, i \in N$ is given below:

$$M_i = diag\{m_{i1}, m_{i2}, \dots, m_{im}\}, \tag{3}$$

where $0 \le \underline{m}_{ik} \le m_{ik} \le \overline{m}_{ik} \le 1, k = 1, 2, \dots, l$. $\underline{m}_{ik}$ and $\overline{m}_{ik}$ are known constants. Given the following matrix relationship.

$$\begin{aligned} L_i &= diag\{l_{i1}, l_{i2}, \dots, l_{il}\} \\ J_i &= diag\{j_{i1}, j_{i2}, \dots, j_{il}\} \\ M_{i0} &= diag\{\tilde{m}_{i1}, \tilde{m}_{i2}, \dots, \tilde{m}_{il}\}, \end{aligned} \tag{4}$$

where $l_{ik} = \frac{m_{ik} - \tilde{m}_{ik}}{\tilde{m}_{ik}}, j_{ik} = \frac{\overline{m}_{ik} - \underline{m}_{ik}}{\overline{m}_{ik} + \underline{m}_{ik}}, \tilde{m}_{ik} = \frac{1}{2}(\underline{m}_{ik} + \overline{m}_{ik})$. According to (4), then

$$M_i = M_{i0}(I + L_i), \quad |L_i| \le J_i \le I \tag{5}$$

where $|L_i| = diag\{|l_{i1}|, |l_{i2}|, \dots, |l_{im}|\}$.
$g(u(t))$ satisfies

$$g^T(u(t))g(u(t)) \le u^T(t)\Lambda_i u(t) \tag{6}$$

where $\Lambda_i = \mathrm{diag}\{\lambda_{1i}, \lambda_{2i}, \dots, \lambda_{mi}\}$.

$f(t, x(t))$ is a nonlinear perturbation and satisfies the following Lipschitz condition:

$$f^T(t, x(t))f(t, x(t)) \le x^T(t)N_i^T N_i x(t). \tag{7}$$

Remark 1. It is well known that actuators are suffers from failures by limited system properties and operating performance. In addition, when the system suffers from external interference, the existence of this factor will cause stability of system to be damaged and performance degradation. Taking into account the actual situation and the application in engineering practice, we have considered all the above situations. The model considered in this paper is more comprehensive and more practical.

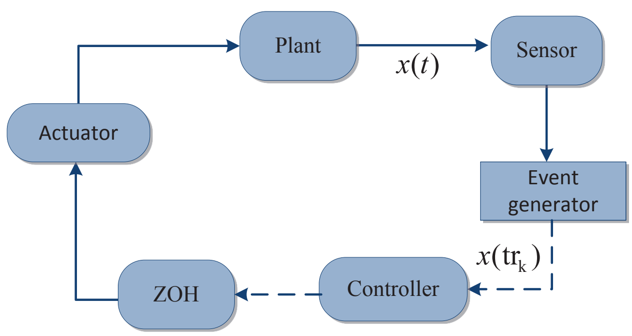

Fig. 1: Event-triggered control structure

With the development of computer technology and technological hardware, technology that relies on digital platforms came into being, that is, based on event-triggered strategies, through this method to achieve system performance design.

Under the event trigger mechanism, we will show the entire process framework of the switched systems operation through Figure 1. In particular, an event detector is introduced in Figure 1. The significance of the event detector is to determine newly sampled information by using the event trigger condition, and then pass the data to the controller. If the trigger condition is satisfied, a new trigger instant will be generated; if not, the next trigger instant will be judged by the trigger condition. We consider the following mode-dependent event-triggered conditions:

$$\mathrm{tr}_{k+1} = \inf\{t > \mathrm{tr}_k | e^T(t)\phi_i e(t) > \epsilon_i x^T(t)\phi_i x(t)\} \quad (8)$$

The measurement error is represented by $e(t) = x(t) - x(\mathrm{tr}_k)$, and the event-triggered weighting matrix are represented by Φ_i. The event-triggered constant threshold is represented by ρ_i and $\rho_i \in [0, 1)$. m sampling data generated on the interval $[t_k, t_{k+1})$ in the following analysis, and the first sampling time in the above interval is represented by $x(\mathrm{tr}_{k+1})$, then the piecewise non-fragile control inputs $u(t)$ can be given by

$$u(t) = u_i(t) = \begin{cases} K_i x(\mathrm{tr}_k) & t \in [t_k, \mathrm{tr}_{k+1}), \\ K_i x(\mathrm{tr}_{k+1}) & t \in [\mathrm{tr}_{k+1}, \mathrm{tr}_{k+2}), \\ \cdots, \\ K_i x(\mathrm{tr}_{k+m}) & t \in [\mathrm{tr}_{k+m}, t_{k+1}), \end{cases} \quad (9)$$

where K_i is the controller gain, the state $x(\mathrm{tr}_k)$ is sampled and transmitted at event-triggered sampling instant tr_k, the system can keep continuous via a zero-order holder (ZOH).

Then, by combining (1) and (9), the following system is generated:

$$\begin{cases} \dot{x}(t) = (A_i + B_i M_i K_i)x(t) - B_i M_i K_i e(t) + B_i g(u(t)) \\ \qquad\qquad + C_i\omega(t) + D_i f(t, x(t)), \\ z(t) = E_i x(t) + F_i \omega(t). \end{cases} \quad (10)$$

Before proving the theorem, we give some definitions and lemmas that have an effect on the calculation process.

Definition 1.([22]) *For $T > t \geq 0$, $N_\sigma(t, T)$ represents the number of switching an interval (t, T). If exist $N_0 \geq 1, \tau_\alpha \geq 0$, the following inequality*

$$N_\sigma(t, T) \leq N_0 + \frac{(T - t)}{\tau_\alpha} \quad (11)$$

holds, then, the constant τ_α is called the average dwell time.

Definition 2.([23]) *When $\omega(t) = 0$, the closed-loop (8) with is exponentially stable for any $\sigma(t)$, if the following inequality*

$$\|x(t)\|^2 \leq \eta e^{-\delta(t-t_0)}\|x(t_0)\|, \quad \forall t \geq t_0, \eta \geq 1, \delta > 0, \quad (12)$$

holds.

Definition 3.([9]) *Given $\gamma > 0$, the closed-loop (8) is said to be exponential stabilization and and satisfies passive performance, if (1) and (2) hold:*
(1) When $\omega(t) = 0$, the closed-loop (8) is exponentially stabilizable .
(2) There is

$$\int_0^\infty e^{-\beta t} z^T(t)z(t)dt \leq \gamma^2 \int_0^\infty \omega^T(t)\omega(t)dt \quad (13)$$

holds under any nonzero exogenous disturbance.

Lemma 1([15]) *Given matrix $\Lambda = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} \\ * & \Lambda_{22} \end{pmatrix}$ with $\Lambda_{11} = \Lambda_{11}^T, \Lambda_{22} = \Lambda_{22}^T$, then the following conditions are equivalent:*

(1) $\Lambda < 0$,
(2) $\Lambda_{11} < 0, \Lambda_{22} - \Lambda_{12}^T\Lambda_{11}^{-1}\Lambda_{12} < 0$,
(3) $\Lambda_{22} < 0, \Lambda_{11} - \Lambda_{12}\Lambda_{22}^{-1}\Lambda_{12}^T < 0$.

Lemma 2([10]) *For matrices P, Q and S with appropriate dimensions with $S^T = S$. Then, for all $S^T S \leq I$, there exists $\theta > 0$, such that*

$$PSQ + Q^T S^T P^T \leq \theta PP^T + \theta^{-1}Q^T Q.$$

III. Main results

In the following, the influence of actuator failures and nonlinear perturbation are taken into consideration simultaneously under the proposed event-triggered mechanism. Theorem 1 and Theorem 2 respectively study whether the system is affected by disturbances. In Theorem 1, we consider the first case, that is, when $\omega(t) = 0$.

A. Stability analysis

Theorem 1. *Given positive scalar $\beta, \varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}$ and $\lambda \geq 1$, if exist positive definite matrices $X_i, \bar{\phi}_i$ and matrices Y_i, such that the following matrix inequalities hold*

$$X_i \leq \mu X_j, \quad i, j \in M \quad (14)$$

$$\Sigma_i = \begin{bmatrix} \Sigma_{11}^i & -B_i M_{i0} Y_i & B_i & D_i & Y_i^T\Lambda_i & \sqrt{\varepsilon_{2i}}X_i N_i^T & Y_i^T J_i^{\frac{1}{2}} \\ * & -\bar{\phi}_i & 0 & 0 & -Y_i^T\Lambda_i & 0 & -Y_i^T J_i^{\frac{1}{2}} \\ * & * & -\varepsilon_{1i}I & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_{2i}I & 0 & 0 & 0 \\ * & * & * & * & -\Lambda_i & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -\varepsilon_{3i}I \end{bmatrix} < 0 \quad (15)$$

where $\Sigma_{11}^i = A_i X_i + X_i A_i^T + B_i M_{i0} Y_i + Y_i^T M_{i0}^T B_i^T + \beta X_i + \epsilon_i \bar{\phi}_i + \varepsilon_{3i} B_i M_{i0} J_i M_{i0} B_i$
then, the resulting closed-loop system (10) when $\omega(t) = 0$ is exponentially stabilizable for $\tau_a > \tau_a^* = \frac{\ln \lambda}{\beta}$. Moreover, the controller gains are given by $K_i = Y_i X_i^{-1}$.

Proof: For $[t_k, t_{k+1})$, we assume that the ith subsystem is activated. The following multiple L-K functional is considered:

$$V(t) = V_i(t) = x^T(t)P_i x(t) \quad (16)$$

the time derivative of (16) along solutions of the system (10) yields

$$\begin{aligned}\dot{V}_i(t) =&2x^T(t)P_i\dot{x}(t)\\ =&x^T(t)[P_i(A_i+B_iM_iK_i)+(A_i+B_iM_iK_i)^TP_i]x(t)\\ &-x^T(t)P_iB_iM_iK_ie(t)-e^T(t)K_i^TM_i^TB_i^TP_ix(t)\\ &+x^T(t)P_iB_ig(u(t))+g^T(u(t))B_i^TP_ix(t)\\ &+x^T(t)P_iD_if_i(t,x(t))+f_i^T(t,x(t))D_iP_ix(t)\end{aligned} \tag{17}$$

Further, we have

$$\begin{aligned}&\varepsilon_{1i}(u^T(t)\Lambda_iu(t)-g^T(u(t))g(u(t)))\geq 0\\ &\varepsilon_{2i}(x^T(t)N_i^TN_ix(t)-f^T(t,x(t))f(t,x(t)))\geq 0\end{aligned} \tag{18}$$

Form (8), we have

$$\epsilon_ix^T(t)\phi_ix(t)-e^T(t)\phi_ie(t)\geq 0 \tag{19}$$

By (17)-(19), then

$$\begin{aligned}&\dot{V}_i(t)+\beta V_i(t)\\ &\leq x^T(t)[P_i(A_i+B_iM_iK_i)+\beta P_i+\epsilon_i\phi_i+\varepsilon_{2i}N_i^TN_i\\ &\quad+(A_i+B_iM_iK_i)^TP_i]x(t)-x^T(t)P_iB_iM_iK_ie(t)\\ &\quad-e^T(t)\phi_ie(t)-e^T(t)K_i^TM_i^TB_i^TP_ix(t)\\ &\quad+x^T(t)P_iB_ig(u(t))+g^T(u(t))B_i^TP_ix(t)\\ &\quad+x^T(t)P_iD_if(t,x(t))+f^T(t,x(t))D_iP_ix(t)\\ &\quad-\varepsilon_{1i}g^T(u(t))g(u(t))-\varepsilon_{2i}f^T(t,x(t))f(t,x(t))\\ &\quad+\varepsilon_{1i}u^T(t)\Lambda_iu(t)\\ &=\eta^T(t)\Theta_i\eta(t)\end{aligned} \tag{20}$$

where

$$\Theta_i=\begin{bmatrix}\Theta_{11}^i & \Theta_{12}^i & P_iB_i & P_iD_i\\ * & -\phi_i & 0 & 0\\ * & * & -\varepsilon_{1i}I & 0\\ * & * & * & -\varepsilon_{2i}I\end{bmatrix}+\begin{bmatrix}K_i^T\Lambda_i\\ -K_i^T\Lambda_i\\ 0\\ 0\end{bmatrix}\Lambda_i^{-1}\begin{bmatrix}K_i^T\Lambda_i & -K_i^T\Lambda_i & 0 & 0\end{bmatrix},$$

$$\eta^T(t)=\begin{bmatrix}x^T(t) & e^T(t) & g^T(u(t)) & f^T(t,x(t))\end{bmatrix},$$

$$\begin{aligned}\Theta_{11}^i=&P_i(A_i+B_iM_iK_i)+(A_i+B_iM_iK_i)^TP_i+\beta P_i\\ &+\varepsilon_{2i}N_i^TN_i+\epsilon_i\phi_i,\end{aligned}$$

$$\Theta_{12}^i=-P_iB_iM_iK_i.$$

Applying Lemma 1, we understand that $\Theta_i<0$ is equivalent to

$$\begin{bmatrix}\bar{\Theta}_{11}^i & \Theta_{12}^i & P_iB_i & P_iD_i & K_i^T\Lambda_i & \sqrt{\varepsilon_{2i}}N_i^T\\ * & -\phi_i & 0 & 0 & -K_i^T\Lambda_i & 0\\ * & * & -\varepsilon_{1i}I & 0 & 0 & 0\\ * & * & * & -\varepsilon_{2i}I & 0 & 0\\ * & * & * & * & -\Lambda_i & 0\\ * & * & * & * & * & -I\end{bmatrix}<0 \tag{21}$$

where $\bar{\Theta}_{11}^i=P_i(A_i+B_iM_iK_i)+(A_i+B_iM_iK_i)^TP_i+\beta P_i+\epsilon_i\phi_i$

Multiply both sides of (21) by Υ_i. Denote $\Upsilon_i=diag\{P_i^{-1},P_i^{-1},I,I,I,\ I\}$. $P_i^{-1}=X_i,Y_i=K_iX_i,\bar{\Phi}_i=X_i\Phi_iX_i$, we have

$$\begin{bmatrix}\tilde{\Theta}_{11}^i & \tilde{\Theta}_{12}^i & B_i & D_i & Y_i^T\Lambda_i & \sqrt{\varepsilon_2}X_iN_i^T\\ * & -\bar{\phi}_i & 0 & 0 & -Y_i^T\Lambda_i & 0\\ * & * & -\varepsilon_{1i}I & 0 & 0 & 0\\ * & * & * & -\varepsilon_{2i}I & 0 & 0\\ * & * & * & * & -\Lambda_i & 0\\ * & * & * & * & * & -I\end{bmatrix}<0 \tag{22}$$

where
$\tilde{\Theta}_{11}^i=A_iX_i+X_iA_i^T+B_iM_iY_i+Y_i^TM_i^TB_i^T+\beta X_i+\epsilon_i\bar{\phi}_i$,
$\tilde{\Theta}_{12}^i=-B_iM_iY_i$.

Substituting (5) into (22), the following formula can be obtained:

$$\Xi_i+\begin{bmatrix}B_iM_{i0}\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}L_i\begin{bmatrix}Y_i\\ -Y_i\\ 0\\ 0\\ 0\\ 0\end{bmatrix}^T+\begin{bmatrix}Y_i^T\\ -Y_i^T\\ 0\\ 0\\ 0\\ 0\end{bmatrix}L_i^T\begin{bmatrix}M_{i0}^TB_i^T\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}^T \tag{23}$$

where

$$\Xi_i=\begin{bmatrix}\Xi_{11}^i & \Xi_{12}^i & B_i & D_i & Y_i^T\Lambda_i & \sqrt{\varepsilon_{2i}}X_iN_i^T\\ * & -\bar{\phi}_i & 0 & 0 & -Y_i^T\Lambda_i & 0\\ * & * & -\varepsilon_{1i}I & 0 & 0 & 0\\ * & * & * & -\varepsilon_{2i}I & 0 & 0\\ * & * & * & * & -\Lambda_i & 0\\ * & * & * & * & * & -I\end{bmatrix} \tag{24}$$

$\Xi_{11}^i=A_iX_i+X_iA_i^T+B_iM_{i0}Y_i+Y_i^TM_{i0}^TB_i^T+\beta X_i+\epsilon_i\bar{\phi}_i$,
$\Xi_{12}^i=-B_iM_{i0}Y_i$.

From (5), we can obtain

$$\begin{aligned}&\Xi_i+\begin{bmatrix}B_iM_{i0}\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}L_i\begin{bmatrix}Y_i\\ -Y_i\\ 0\\ 0\\ 0\\ 0\end{bmatrix}^T+\begin{bmatrix}Y_i^T\\ -Y_i^T\\ 0\\ 0\\ 0\\ 0\end{bmatrix}L_i^T\begin{bmatrix}M_{i0}^TB_i^T\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}^T\\ &\leq\Xi_i+\begin{bmatrix}B_iM_{i0}\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}J_i\begin{bmatrix}Y_i\\ -Y_i\\ 0\\ 0\\ 0\\ 0\end{bmatrix}^T+\begin{bmatrix}Y_i^T\\ -Y_i^T\\ 0\\ 0\\ 0\\ 0\end{bmatrix}J_i^T\begin{bmatrix}M_{i0}^TB_i^T\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}^T\end{aligned} \tag{25}$$

By Lemma 2, (25) is satisfied if the following inequality holds

$$\Xi_i + \varepsilon_{3i} \begin{bmatrix} B_i M_{i0} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} J_i \begin{bmatrix} B_i M_{i0} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \varepsilon_{3i}^{-1} \begin{bmatrix} Y_i^T \\ -Y_i^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} J_i^T \begin{bmatrix} Y_i & -Y_i & 0 & 0 & 0 & 0 \end{bmatrix} < 0 \quad (26)$$

Then, we have

$$\dot{V}_i(t) + \beta V_i(t) < 0. \quad (27)$$

So, we can get

$$(e^{\beta t} V_i(t))' = \beta e^{\beta t} V_i(t) + e^{\beta t} \dot{V}_i(t) \le 0. \quad (28)$$

Integrating both sides of (28) from t_k to t, we have

$$V_{\sigma(t)}(t) \le V_{\sigma(t_k)}(t_k) e^{-\beta(t-t_k)}, \quad t_k \le t < t_{k+1}. \quad (29)$$

Setting $\sigma(t_k^-) = j$ at the switching time t_k, and combining (14), we obtain

$$V_{\sigma(t_k)}(t_k) \le \lambda V_{\sigma(t_k^-)}(t_k^-). \quad (30)$$

Then

$$\begin{aligned} V_{\sigma(t)}(t) &\le \lambda V_{\sigma(t_k^-)}(t_k{}^-) e^{-\beta(t-t_k)} \\ &\le \cdots \\ &\le e^{-(t-t_0)(\beta - \ln\lambda/\tau_a)} V_{\sigma(t_0)}(t_0). \end{aligned} \quad (31)$$

According to (16), so

$$V_{\sigma(t)}(t) \ge a\|x(t)\|^2, \quad V_{\sigma(t_0)}(t_0) \quad \le b\|x(t_0)\|^2, \quad (32)$$

where

$$a = \min_{i\in\bar{N}} \lambda_{\min}(P_i), \quad b = \max_{i\in\bar{N}} \lambda_{\max}(P_i).$$

So

$$\|x(t)\| \le \sqrt{\frac{b}{a}} \|x(t_0)\| e^{-\frac{1}{2}(\beta - \frac{ln\lambda}{\tau_a})(t-t_0)}. \quad (33)$$

Therefore, through the above proof process, we can draw the conclusion of the Theorem 1 from Definition 2.

B. H_∞ control

In the following subsection, we considered the issue of reliable event-triggered control for the resulting system (10) with respect to the exogenous disturbance input $\omega(t) \neq 0$.

Theorem 2. *Given positive scalar $\beta, \varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}$ and $\lambda \ge 1$, if exist positive definite matrices $X_i, \bar{\phi}_i$ and Y_i, such that the following matrix inequalities hold*

$$X_i \le \mu X_j, \quad i, j \in M \quad (34)$$

$$\bar{\Sigma}_i = \begin{bmatrix} \Sigma_{11}^i & -B_i M_{i0} Y_i & B_i & D_i & C_i & Y_i^T \Lambda_i & \sqrt{\varepsilon_2} X_i N_i^T & X_i E_i^T & Y_i^T J_i^{\frac{1}{2}} \\ * & -\bar{\phi}_i & 0 & 0 & 0 & -Y_i^T \Lambda_i & 0 & 0 & -Y_i^T J_i^{\frac{1}{2}} \\ * & * & -\varepsilon_{1i} I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_{2i} I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & F_i^T & 0 \\ * & * & * & * & * & -\Lambda_i & 0 & 0 & \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_{3i} I \end{bmatrix} < 0 \quad (35)$$

where $\Sigma_{11}^i = A_i X_i + X_i A_i^T + B_i M_{i0} Y_i + Y_i^T M_{i0}^T B_i^T + \beta X_i + \epsilon_i \bar{\phi}_i + \varepsilon_{3i} B_i M_{i0} J_i M_{i0} B_i$
Then, the resulting system (10) is exponentially stabilizable with attenuation performance γ for $\tau_a > \tau_a^* = \frac{\ln\lambda}{\beta}$. Moreover, the controller gains are given by $K_i = Y_i X_i^{-1}$.

Proof: For $\omega(t) \neq 0$, we choose Lyapunov-Krasovskii functional (16) of Theorem 1, thus

$$\begin{aligned} &\dot{V}_i(t) + \beta V_i(t) + z^T(t) z(t) - \gamma^2 \omega^T(t)\omega(t) \le x^T(t)[P_i \\ &\quad (A_i + B_i M_i K_i) + (A_i + B_i M_i K_i)^T P_i + \varepsilon_{2i} N_i^T N_i \\ &\quad + \beta P_i + \epsilon_i \phi_i + E_i^T E_i] x(t) + x^T(t) E_i^T F_i \omega(t) \\ &\quad + \omega^T(t) F_i^T E_i x(t) + \omega^T(t) F_i^T F_i \omega(t) - \gamma^2 \omega^T(t)\omega(t) \\ &\quad - x^T(t) P_i B_i M_i K_i e(t) - e^T(t) K_i^T M_i^T B_i^T P_i x(t) \\ &\quad + x^T(t) P_i B_i g(u(t)) + x^T(t) P_i C_i \omega(t) - e^T(t)\phi_i e(t) \\ &\quad + g^T(u(t)) B_i^T P_i x(t) + x^T(t) P_i D_i f(t, x(t)) \\ &\quad + f^T(t, x(t)) D_i P_i x(t) - \varepsilon_{2i} f^T(t, x(t)) f(t, x(t)) \\ &\quad - \varepsilon_{1i} g^T(u(t)) g(u(t)) + \omega^T(t) C_i P_i x(t) \\ &\quad + \varepsilon_{1i} u^T(t) \Lambda_i u(t) \\ &\quad = x^T(t) \Delta_i x(t) \end{aligned} \quad (36)$$

where $\zeta^T(t) = \begin{bmatrix} \eta^T(t) & \omega^T(t) \end{bmatrix}$

$$\Delta_i = \begin{bmatrix} \Delta_{11}^i & -P_i B_i M_i K_i & P_i B_i & P_i D_i & \Delta_{15}^i \\ * & -\phi_i & 0 & 0 & 0 \\ * & * & -\varepsilon_{1i} I & 0 & 0 \\ * & * & * & -\varepsilon_{2i} I & 0 \\ * & * & * & * & \Delta_{55}^i \end{bmatrix} + \begin{bmatrix} K_i^T \Lambda_i \\ -K_i^T \Lambda_i \\ 0 \\ 0 \\ 0 \end{bmatrix} \Lambda_i^{-1} \begin{bmatrix} K_i^T \Lambda_i \\ -K_i^T \Lambda_i \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \quad (37)$$

$$\begin{aligned} \Delta_{11}^i &= P_i(A_i + B_i M_i K_i) + (A_i + B_i M_i K_i)^T P_i + \beta P_i \\ &\quad + \varepsilon_2 N_i^T N_i + \epsilon_i \phi_i + E_i^T E_i, \\ \Delta_{15}^i &= P_i C_i + E_i^T F_i, \\ \Delta_{55}^i &= F_i^T F_i - \gamma^2 I. \end{aligned}$$

From (35), we have

$$\begin{bmatrix} \tilde{\Delta}_{11}^{i} & -B_iM_iY_i & B_i & D_i & C_i & Y_i^T\Lambda_i & \sqrt{\varepsilon_2}X_iN_i^T & X_iE_i^T \\ * & -\bar{\phi}_i & 0 & 0 & 0 & -Y_i^T\Lambda_i & 0 & 0 \\ * & * & -\varepsilon_{1i}I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_{2i}I & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & F_i^T \\ * & * & * & * & * & -\Lambda_i & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad (38)$$

where $\tilde{\Delta}_{1i} = A_iX_i + X_iA_i^T + B_iM_iY_i + Y_i^TM_i^TB_i^T + \beta X_i + \epsilon_i\bar{\phi}_i$ So, we can get,

$$\dot{V}(t) + \beta V(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \leq 0. \quad (39)$$

Integrating from t_k to t on both sides of (39), then

$$V(t) \leq e^{-\beta(t-t_k)}V(t_k) - \int_{t_k}^{t} e^{-\beta(t-s)}\Upsilon(s)ds. \quad (40)$$

where $\Upsilon(t) = -\gamma^2\omega^T(t)\omega(t) + z^T(t)z(t)$.

Combining (11) and (40), we have

$$\begin{aligned} V(t) &\leq e^{-\beta(t-t_k)}V(t_k) - \int_{t_k}^{t} e^{-\beta(t-s)}\Upsilon(s)ds \\ &\leq e^{-\beta t + N_\sigma(0,t)\ln\lambda}V(0) - \int_{0}^{t} e^{-\beta t + N_\sigma(s,t)\ln\lambda}\Upsilon(s)ds. \end{aligned} \quad (41)$$

Then,

$$0 \leq -\int_{0}^{t} e^{-\beta(t-s)+N_\sigma(s,t)\ln\lambda}\Upsilon(s)ds. \quad (42)$$

Multiply both ends of (42) by $e^{-N_\sigma(0,t)\ln\lambda}$, so

$$\begin{aligned} &\int_{0}^{t} e^{-\beta(t-s)-N_\sigma(0,s)\ln u}z^T(s)z(s)ds \\ &\quad \leq \int_{0}^{t} e^{-\beta(t-s)-N_\sigma(0,s)\ln u}\gamma^2\omega^T(s)\omega(s)ds. \end{aligned} \quad (43)$$

When $N_\sigma(0,s) \leq \frac{s}{\tau_a}, \tau_a > \tau_a^* = \frac{\ln u}{\beta}$, it easy to obtain $N_\sigma(0,s)\ln u \leq \beta s$. So,

$$\int_{0}^{t} e^{-\beta(t-s)-\alpha s}z^T(s)z(s)ds \leq \int_{0}^{t} e^{-\beta(t-s)}\gamma^2\omega^T(s)\omega(s)ds. \quad (44)$$

Integrating (44) from 0 to ∞, we have

$$\int_{0}^{\infty} e^{-\beta s}z^T(s)z(s)ds \leq \int_{0}^{\infty} \gamma^2\omega^T(s)\omega(s)ds.$$

Therefore, through the above proof process, we can draw the conclusion of the Theorem 2 from Definition 3.

Remark 2. In practice, we have gradually learned that the communication bandwidth in the network is limited, and the system sample and update signals in a periodicity may cause network breakdown. In order to handle this issue, an event-triggered mechanism with different triggering thresholds is introduced to improve the problems arising from time-triggered mechanism. In terms of cost saving and efficiency, the method considered in this paper is more comprehensive and practical than the [26] and [27].

Remark 3. It is clear that (34) and (35) are mutually dependent. First, we obtain matrix X_i by solving linear matrix inequalities (34) and (35). Besides this, a feasible solution is obtained through constant adjustment of the parameters. Finally, the feedback controllers K_i can be designed through a special matrix transformation.

Remark 4. It is well known that the switching strategy is necessary to improve poor system performance since a single system or controller can not guarantee the normal operation of the switched systems in some complex situations. In response to the above mentioned problems, the Lyapunov stability theory is considered, which is essential to obtain less conservative result. Moreover, this paper uses Lyapunov stability theory as an effective method for studying the stability of switched systems.

Remark 5. The weighted term $e^{-\beta s}$ cannot be canceled when an inappropriate switching strategy is used to derive the exponentially stable conditions and satisfactory performance. Therefore, the H_∞ performance index is weighted term in this paper. Generally, through some special transformation techniques, the weighted performance can be degenerated into non-weighted performance. In fact, in terms of efficiency and time cost, the proposed approach has a wider application range than existing results of non-weighted term.

IV. Numerical examples

Based on the proof and analysis of the theorem, in order to obtain effective results, we will give an example.

Example Consider switched system (1) composed of two subsystems with the following parameters:

Mode 1:

$$A_1 = \begin{bmatrix} -2.7 & -1 \\ 0 & -4.3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 2.1 & -1.2 \\ -1.3 & 3.4 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$
$$M_{10} = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$J_1 = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.6 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

Mode 2:

$$A_2 = \begin{bmatrix} -2.5 & 0 \\ -1 & -5.4 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2.1 & -3.5 \\ -1.1 & 2.3 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix},$$
$$E_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.2 \\ -0.3 \end{bmatrix},$$
$$M_{20} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix}, \quad \Lambda_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$J_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

The actuator failures matrices are as follows:

$$0.1 \leq m_{11} \leq 0.7, \quad 0.2 \leq m_{12} \leq 0.8,$$
$$0.1 \leq m_{21} \leq 0.9, \quad 0.2 \leq m_{22} \leq 0.6.$$

According to (6) ,we can get

$$M_{10} = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix}, J_1 = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.6 \end{bmatrix},$$
$$M_{20} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix}, J_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.5 \end{bmatrix},$$

Let $\beta = 0.6, \mu = 1.15, \gamma = 0.8, \varepsilon_{1i} = 1.4, \varepsilon_{2i} = 1.3, \varepsilon_{1i} = 1.5, \epsilon_1 = 0.2, \epsilon_1 = 0.3$, $\omega(t) = e^{-2t}$, $f(t, x(t)) = (\sin(x_1(t)) \quad \sin(x_2(t)))^T$. By $\tau_a > \tau_a^* = \frac{\ln \lambda}{\beta}$, we can get $\tau_a > 0.2329$.

By solving (34) and (35), we have

$$X_1 = \begin{bmatrix} 0.1136 & 0.0405 \\ 0.0405 & 0.2386 \end{bmatrix},$$
$$X_2 = \begin{bmatrix} 0.1133 & 0.0394 \\ 0.0394 & 0.2349 \end{bmatrix},$$
$$Y_1 = \begin{bmatrix} 0.5315 & -0.1072 \\ -0.2454 & 0.5198 \end{bmatrix},$$
$$Y_2 = \begin{bmatrix} 0.1124 & -0.1541 \\ -0.1962 & 0.5911 \end{bmatrix}$$

Then, the controller gains are

$$K_1 = \begin{bmatrix} 0.0560 & -0.0041 \\ -0.0068 & 0.1141 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 0.0067 & -0.0318 \\ 0.0011 & 0.1311 \end{bmatrix}.$$

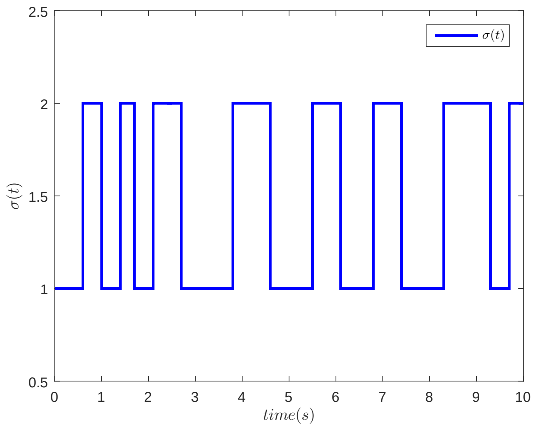

Fig. 2: The switching law.

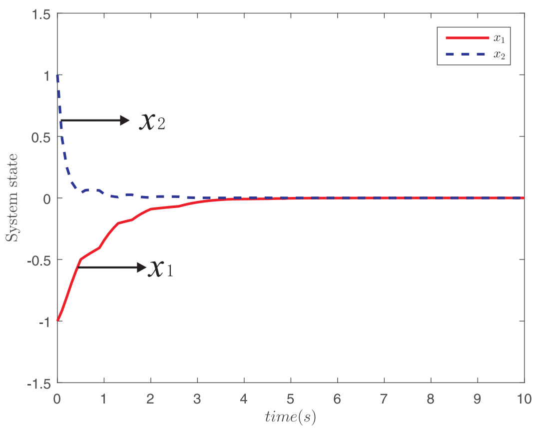

Fig. 3: State response.

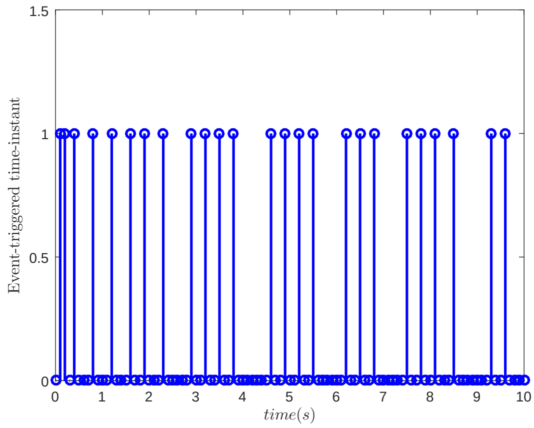

Fig. 4: Event-triggered time-instant.

The system switched signal $\sigma(t)$ and the state trajectories during time interval $[0, 10]$ are given in Fig. 2 and Fig. 3, respectively. Fig.4 represents the event triggering instants. In the figure, 1 is used to indicate that new transmission data is generated at the current moment, and 0 is not violated. Moreover, only a few instants in Figure 4 violate the event trigger condition. Obviously, from the analysis of the data results, the trigger condition we considered has a good effect in avoiding network congestion due to the reduction of data transmission. Then, the effectiveness of the method is verified. After the above analysis and discussion, as well as comparison with existing methods and results, our results are more advantageous. In fact, our model will be more practical when faced with more complex factors

V. Conclusions

The event-based reliable H_∞ control for switched systems subject to nonlinear perturbation and actuator failures has been investigated. By taking the effects of actuator failures, nonlinear perturbation and event-triggered control into consideration, a closed-loop state feedback switched system model is established. Based on Lyapunov stability theory, some stability criteria and satisfactory performance of the switched with actuator failures and nonlinear perturbation are obtained under the considered method. Compare existing results and methods, the reliable feedback controller can be designed through a special matrix transformation. It is worth noting that the present investigation only focuses on the synchronization, the asynchronous issue is not studied here. Moreover, with the increase of sampled data, network delay will follow, and this situation will also cause asynchronous behavior. We will study this issue next.

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