Single-image Super-resolution based on Non-local Means and Double-sparsity Dictionaries

Xiuxiu Liao, Kejia Bai*, Qian Zhang, Xiping Jia, Jia Ouyang, Yunzhi Jiang

Abstract—Sparse representation models have been widely used in single-image super-resolution reconstruction. The construction of dictionaries is especially important in these models. Basically, approaches of dictionary construction in sparse representation can be divided into two categories: analytical and learning-based approaches. Analytical approaches are effective and fast, but they are unable to fit different types of data; learning-based approaches are adaptive, but their implementation takes a significant amount of time. In this study, an image super-resolution reconstruction algorithm based on double-sparsity dictionaries is proposed. The algorithm combines the efficiency of analytical approaches and adaptability of learning-based approaches. In addition to the sparsity prior, the non-local self-similarity prior is also considered in the algorithm. Non-local means filtering is used to be the constraints on regularized super-resolution reconstruction procedures, which improves the quality of super-resolution reconstruction results further, while the runtime of the algorithm is still acceptable. Experimental results demonstrate the advantage of the proposed algorithm.

Index Terms—Double-sparsity Dictionaries, Non-local Means, Sparse Representation, Super-resolution

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I. INTRODUCTION

In recent years, single-image super-resolution (SR) has gained increasingly attention from researchers. In image processing, obtaining larger and clearer images from the original images is a priority. SR reconstruction not only can improve the visual effect of an image but also has very important meaning for further applications, such as feature extraction, automatic recognition, and image analysis.

Single-image SR refers to the problem of using signal-processing techniques to estimate a high-resolution (HR) image $X$ with better quality from an observed low-resolution (LR) image $Y$. The image-observation model is usually described as $Y = SHX + V$, where $H$ is a blurring filter, $S$ a down-sampling operator, and $V$ additive noise. The above image-degradation model shows that image SR reconstruction is an inverse problem and ill-posed. Many HR images $X$ may satisfy the model obtained from a given LR image $Y$.

During the reconstruction processing, introducing an effective prior or constraint (regularization) is very efficacious in helping people obtain a well-defined solution. The Maximum a posterior (MAP) estimation was first introduced into the image SR reconstruction algorithm by Schultz et al. [1], and the Huber-Markov field (MRF) was used as the prior knowledge to improve the image resolution. Based on the MAP framework, Belekos et al. [2] utilized the state-of-the-art single-channel image prior and observation models, and defined a new multichannel image prior model. Mofidi et al. [3] introduced a set of weighting coefficients that control the contribution between the regularization and data-error term in each of the estimated HR pixels. The employed coefficients were defined according to the information of neighbors of the estimated pixel. Li et al. [4] obtained SR images of real beam scanning radar using an accelerated MAP method. Based on the first and second orders of difference information, a prediction vector was constructed before each iteration operation, and the convergence speed was improved without performance loss.

Another widely used image prior is the non-local self-similarity prior [5]. The self-similarity of non-local patches is an essential feature of natural images. Based on this observation, small image patches are redundantly reproduced on the same scale and between different scales. Buades et al. [6] established a mathematical framework for non-local means (NLM) filtering. The idea of the framework was rather simple:
image patches with similar patterns may be far apart in space, so it was possible to search for image patches with similar patterns throughout the entire image. It compared the geometrical configuration in an entire neighborhood and obtained a more robust comparison than neighborhood filters. NLM was used to be the constraints on regularized SR reconstruction problems, which further improved the quality of reconstructed images, while the time consumption was still acceptable. Dong et al. [7] utilized both the non-local and local priors to optimize the objective function. Zhang et al. [8] incorporated the global reconstruction constraint, non-local similarity, and local structural regularity into a unified iterative framework. Here, the NLM was used to learn a non-local prior. This algorithm achieved a good reconstruction effort, but the burden of calculation was rather heavy, and the running speed was rather slow. In this paper, NLM is regarded as prior information and tries to acquire the initial value of the MAP estimation in the learning process. The desire of the proposed algorithm is to accelerate the convergence process and obtain better quality of reconstruction and time performance.

Apart from the above image priors, sparse-representation models have been widely used in single-image SR tasks. Yang et al. [9] first proposed an image SR reconstruction scheme based on sparse representation. An image patch was regarded as the sparse representation for an over-complete dictionary. The sparsity of an image patch was used as a priori information with which to regularize the SR reconstruction problems. The algorithm was improved in their later paper [10], not directly using the HR and LR image-patch pairs as dictionaries, but using a sparse-coding algorithm to learn more compact dictionary pairs, which greatly improved the calculation speed. Zeyde et al. [11] applied the k singular value decomposition (K-SVD) dictionary training procedure for the LR training patches, and orthogonal matching pursuit (OMP) algorithm to solve parse-coding. Experiments results showed that the algorithm can make both visual and peak signal-to-noise ratio (PSNR) improvements. Huang and Dragotti [12] proposed a deep dictionary model in which the dictionaries consist of $L$ layers, with the first $L−1$ layers being analysis dictionaries and the last layer a synthesis dictionary, and the dictionary was updated iteratively in a backward fashion.

In sparse-representation-based SR, the construction of dictionaries is especially important for the performance of the reconstruction model. In general, these dictionary construction methods can be divided into two categories: analytical and learning based methods [13]. Dictionaries created from analytical approaches are “implicit” dictionaries described by algorithms such as wavelet, complex wavelet, contourlet, and bandelet algorithms. In contrast, learning-based dictionaries offer more convenience and power to handle specific signal data. These learning-based approaches usually use machine-learning techniques to infer dictionaries from training datasets, which are typically represented by concrete matrices. Dictionaries obtained via learning can fit the data better, but they make the system more complex and require a high calculation cost. Complexity constraints limit the learning of the dictionaries, especially the size of the atom. From a more practical standpoint, it would be desirable to effectively combine dictionaries of both types. A double-sparsity dictionary algorithm is proposed by Rubinstein et al. [14] that used both the analysis-based and learning-based methods. This algorithm had not only the efficiency of an analytical dictionary but also the adaptability of a learning-based dictionary. The structure was based on a sparsity model of the dictionary atoms over a known base dictionary. Ai et al. [15] used wavelet coefficients of LR images as the futures to train dictionary, so the trained dictionary pair had the property of double sparsity, and the training dataset can be small. In their later work [16], the bootstrapping method was used to construct training set, and four wavelet sub-bands of the two difference images were used as extra information to train the dictionary.

The high time efficiency of sparsity dictionaries meets the requirements for fast learning, so they are introduced here into the SR reconstruction, which can help obtain better time performance while ensuring reconstruction quality. Our double-sparsity dictionaries are different from those of Ai et al. [15], and the double-sparsity dictionaries are trained directly, while Ai et al. first performed a wavelet transform on the image, and the dictionaries were trained on the wavelet coefficients to achieve “double sparsity.”

In this paper, an image SR reconstruction algorithm based on double-sparsity dictionaries is proposed. The algorithm combines both the advantages of analytical and learning-based dictionaries, which can ensure the quality of reconstruction and improve speed. The algorithm is suitable for applications with high-time-performance requirements. On this basis, non-local self-similar constraints are used in the regularization of SR reconstruction, which further improves the quality of reconstructed images while keeping runtimes within an acceptable range.

The main contributions of this paper are the following.

1) Combining MAP estimation and NLM in SR reconstruction problems.

2) Use of double-sparsity dictionaries in the image SR reconstruction process, which combines the efficiency of an analytical dictionary and the adaptability of a learned dictionary, and acquires better time performance while ensuring the reconstruction quality.

3) Use of the non-local self-similar constraints in the reconstruction process, which further improves the quality of reconstructed images while keeping runtime within an acceptable range. The algorithm offers a desirable compromise between low computational complexity and high reconstruction quality.

The rest of this paper is organized as follows. In Section II, a novel image SR reconstruction algorithm based on double-sparsity dictionaries is proposed, which considers both the sparsity prior and non-local self-similarity prior. The theory of sparse representation of signals is discussed in Section II.A and the theory of a double-sparsity dictionary in Section II.B. In Section II.C, a SR based on double-sparsity dictionaries (DSD-SR) algorithm is proposed that can be divided into an offline training-data pre-processing part and an online testing-data SR reconstruction part. The proposed DSD-SR algorithm is then developed with NLM constraints (NLM-DSD-SR) in Section II.D. Experimental results and
analysis are shown in Section III.

II. PROPOSED APPROACH

A. Sparse Representation of Signals

Scientists have put a significant amount of effort on sparse representation of signals over redundant dictionaries. They suppose natural signals can be represented and estimated by a sparse linear combination of pre-determined atom signals, chosen from a group of data called a dictionary.

For a given signal \( \mathbf{z} \in \mathbb{R}^{n \times 1} \), the sparse-representation model can be expressed as \( \mathbf{Z} = \mathbf{D} \mathbf{a} \), where \( \mathbf{D} \in \mathbb{R}^{n \times K} \) is a dictionary of \( K \) atoms of length \( n \), and \( \mathbf{a} \in \mathbb{R}^{K \times 1} \) is the sparse coefficient. The sparse representation of the signal \( \mathbf{z} \) can be estimated by the following formula:

\[
\text{min} \| \mathbf{a} \|_0 \quad \text{subject to} \quad \| \mathbf{z} - \mathbf{D} \mathbf{a} \|_F^2 \leq \varepsilon ,
\]

where \( \| \mathbf{a} \|_0 \) is the \( L_0 \)-norm which represents a count of the number of non-zeros in the vector, and \( \varepsilon \) is the error allowed for accuracy.

Given the signal set \( \{ \mathbf{z}_i \}_{i=1}^N, \mathbf{Z} = \{ \mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_N \} \), the learning problem can be described as

\[
\text{min}_{\mathbf{D}, \mathbf{a}} \left\{ \| \mathbf{Z} - \mathbf{D} \mathbf{a} \|_F^2 \right\} \quad \text{subject to \( \forall i, \| \mathbf{a}_i \|_0 \leq T_0 \) ,}
\]

or

\[
\text{min}_{\mathbf{D}, \mathbf{a}} \sum_i \| \mathbf{a}_i \|_0 \quad \text{subject to} \quad \| \mathbf{Z} - \mathbf{D} \mathbf{a} \|_F^2 \leq \varepsilon .
\]

where \( \mathbf{a}_i \) is the column \( i \) of \( \mathbf{A} \) indicating the coefficient corresponding to the signal \( \mathbf{z}_i \), and \( T_0 \) is a sparse constraint.

B. Double-Sparsity Dictionaries

SR reconstruction is an ill-posed problem since, for a given LR input image \( \mathbf{Y} \), there are many HR images \( \mathbf{X} \) that satisfy the above reconstruction constraint. The sparsity of image patches is used as prior information to regularize the ill-posed problem in reconstruction. Supposing that \( \mathbf{x}_i \) and \( \mathbf{y}_i \) are the HR and LR image patches, respectively, which have the same sparse representation under the HR dictionary \( \mathbf{D}_h \) and LR dictionary \( \mathbf{D}_l \), the problem can be regularized via the sparse-representation prior on small patches \( x \) of \( \mathbf{X} \): The patches \( x \) of the HR image \( \mathbf{X} \) can be represented as a sparse linear combination in a dictionary \( \mathbf{D}_h \) of HR patches sampled from training images:

\[
\mathbf{x} \approx \mathbf{D}_h \mathbf{a} \quad \text{for some} \quad \mathbf{a} \in \mathbb{R}^K \quad \text{with} \quad \| \mathbf{a} \|_0 = K ,
\]

and \( \mathbf{x} \) can be reconstructed by \( \mathbf{y}_i \) under the sparse priors:

\[
\text{min}_{\mathbf{a}, \mathbf{x}} \| \mathbf{a} \|_0 \quad \text{subject to} \quad \| \mathbf{F}_d \mathbf{a} - \mathbf{F}_x \mathbf{x} \|_2 \leq \varepsilon_1,
\]

\[
\| \mathbf{F}_d \mathbf{a} - \mathbf{F}_y \mathbf{y} \|_2 \leq \varepsilon_2 ,
\]

\[
\| \mathbf{S}_H \mathbf{X} - \mathbf{Y} \|_2 \leq \varepsilon_3
\]

where \( \mathbf{F} \) is a (linear) feature-extraction operator.

Double-sparsity dictionaries combine the efficiency of analytical dictionaries and adaptability of learning-based dictionaries. The structure is based on the sparse model of the dictionary atom on the selected base dictionary:

\[
\mathbf{D} = \Phi \mathbf{W} ,
\]

where \( \Phi \) is the base dictionary, and \( \mathbf{W} \) the atomic representation matrix, which is assumed to be sparse, and each column of which has a fixed number of non-zero values, \( \| \mathbf{w}_i \|_0 \leq P_0 \). The selection of the base dictionary \( \Phi \) has great influence on the performance of the double-sparsity dictionary model. Usually, analytical dictionaries with fast implementation are selected, such as wavelets, complex wavelets, and contourlets.

Compared with analytical dictionaries, the double-sparsity dictionary model provides adaptability through the modification of \( \mathbf{W} \). In addition, one can use different types of dictionaries as \( \Phi \). Compared to learning-based dictionaries, the sparse structure of double-sparsity dictionaries is more efficient, which mainly depends on the choice of \( \Phi \).

The training of double-sparsity dictionaries can be described as solving the following optimization problem:

\[
\min_{\mathbf{W}, \mathbf{A}} \left\{ \| \mathbf{Z} - \Phi \mathbf{WA} \|_F^2 \right\} \quad \text{subject to}
\]

\[
\forall i, \| \mathbf{a}_i \|_0 \leq T_0,
\]

\[
\forall j, \| \mathbf{w}_j \|_0 \leq P_0, \quad \| \Phi \mathbf{w}_j \|_2 = 1
\]

where \( \mathbf{Z} \) is the signal to be represented, and \( \mathbf{A} \) the sparse coefficient.

C. Super-Resolution based on Double-Sparsity Dictionaries
Zeyde et al. [11] applied K-SVD dictionary training and OMP sparse-coding to obtain good SR performance on learned dictionaries. Rubinstein et al. [14] were the first to introduce double-sparsity dictionaries. They proposed a sparse dictionary based on a sparsity model of the dictionary atoms over a base dictionary and demonstrated the advantages of the double-sparsity dictionary structure for computed tomography (CT) denoising. Inspired by these works, an image SR reconstruction algorithm based on double-sparsity dictionaries is proposed herein. The algorithm is divided into two parts: training-data pre-processing and testing-data SR reconstruction. The first part can be done offline.

**Part 1 Training Data Pre-processing**

1. The HR image in the training set is blurred and down-sampled according to the degraded model to obtain a corresponding LR image.

2. LR feature vectors \( \mathbf{Y}_s \) are extracted, and the LR image is divided into \( n \times n \) patches and features extracted. A total of four one-dimensional filters are used in this step:

\[
\begin{align*}
    f_1 &= [-1, 1], \\
    f_2 &= f_1^T, \\
    f_3 &= [1, -2, 1] / 2, \\
    f_4 &= f_3^T.
\end{align*}
\]

These four filters are applied to LR images, and four eigenvectors are obtained for each image patch, which are concatenated as a characteristic representation of an image patch. Therefore, the length of the feature vector corresponding to each LR image patch is \( 4n^2 \), which is denoted by \( \{ \mathbf{y}_i \}_{i=1}^N \), \( \mathbf{Y}_s = \{ \mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_N \} \).

3. LR dictionaries \( \mathbf{D}_i \) are trained, and an overcomplete discrete cosine transform (DCT) dictionary is selected as the base dictionary \( \Phi \). That is to solve

\[
\min_{\mathbf{w}, \mathbf{A}} \left\{ \| \mathbf{Y}_s - \Phi \mathbf{WA} \|_F^2 \right\}
\]

subject to

\[
\begin{align*}
    \forall i, \| \mathbf{a}_i \|_0 &\leq T_0, \\
    \forall j, \| \mathbf{w}_j \|_0 &\leq P_0, \quad \| \Phi \mathbf{w} \|_2 = 1
\end{align*}
\]

where the LR dictionary \( \mathbf{D}_i = \Phi \mathbf{W} \).

4. HR feature vectors \( \mathbf{X}_s \) are extracted. After interpolating LR images to the size of HR images (interpolation of the images), the corresponding interpolated image is subtracted from the HR image, and the high-frequency part can be obtained. The high-frequency image is divided into \( (R_n) \times (R_n) \) image patches and the patches converted to vectors, which are regarded as the feature vectors of HR image patches \( \{ \mathbf{x}_i \}_{i=1}^N \), \( \mathbf{X}_s = \{ \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N \} \). \( R \) is the SR zoom factor.

5. The HR dictionary \( \mathbf{D}_h \) is calculated. Suppose that HR-LR image patches have the same sparse-representation coefficient \( \mathbf{A} \) under the HR-LR dictionaries pair, then \( \mathbf{D}_h \) can be calculated by minimizing the following approximation:

\[
\mathbf{D}_h = \arg \min_{\mathbf{D}_h} \left\{ \| \mathbf{X}_s - \mathbf{D}_h \mathbf{A} \|_F^2 \right\},
\]

Using the pseudo-inverse solution,

\[
\mathbf{D}_h = \mathbf{X}_s \mathbf{A}^+ = \mathbf{X}_s \mathbf{A}^T (\mathbf{A} \mathbf{A}^T)^{-1},
\]

where “+” represents pseudo-inverse operator.

The framework of Part 1 of DSD-SR is illustrated in Figure 1.

**Part 2 Testing Data SR Reconstruction**

1. The LR test image \( \mathbf{Y} \) is interpolated to the size of the target HR image.

2. \( \mathbf{Y} \) is divided into \( n \times n \) image patches and features extracted. For each feature vector \( \mathbf{y}_i \), the following steps are
The self-similarity of the local patch mode is an important feature of natural images. Based on this observation, small image patches are redundantly reproduced in the same scale and between different scales. For each local image patch $x_i$, the patches that are similar to it throughout the whole image are searched for (actually the searching is done in a large enough area around $x_i$). The top $L$ most similar patches $x_i^j$ for $x_i$ are selected. Letting $p_i$ and $p_i^j$ be the center pixels of $x_i$ and $x_i^j$, respectively, the weighted average of $p_i^j$ can be used to predict $p_i$:

$$p_i = \sum_{j=1}^{L} w_j p_i^j,$$  \hspace{1cm} (15)

where the weight $w_j$ is defined as

$$w_j = \frac{\exp(-e_i^j / h)}{\sum_{i=1}^{L} \exp(-e_i^j / h)}, \hspace{1cm} e_i^j = \|x_i - x_i^j\|_2^2,$$  \hspace{1cm} (16)

and $h$ is the global filter parameter, controlling the decline of exponential expression. Considering the non-local redundancy of natural images, the expected estimation error $\|p_i - \sum_{j=1}^{L} w_j p_i^j\|_2^2$ is very small. Letting $\omega_i$ be the column vector containing all weights $w_i^j$, $\beta_i$ is the column vector containing all center pixels. Introducing non-local self-similarity regularization terms into formula (14), one obtains

$$X^* = \underset{X}{\arg \min} \{\|Y - SHX\|_2^2 + \lambda \sum_{i} \|x_i - \omega_i^T \beta_i\|_2^2\},$$  \hspace{1cm} (17)

where $X^*$ is solved using the gradient-descent method.
Inspired by the work of Zhang et al. [8] and Rubinstein et al. [14], an image SR reconstruction algorithm based on double-sparsity dictionaries and NLM constraint is proposed (denoted NLMDSD-SR). The diagram of the algorithm is shown in Figure 3.

### III. EXPERIMENTAL RESULTS AND ANALYSIS

Several benchmarks, including Set14 [11], Set5 [17] and B100 [18], were used as the testing set in the present work. Approximately 100,000 training patch pairs were collected from the training image set used in [10].

In the experiments, the size of the LR image patches was $3 \times 3$, and the number of filters was four, so the length of the feature vectors was $3 \times 3 \times 4 = 36$. An overcomplete DCT dictionary was selected as the base dictionary $\Phi$. Supposing the number of atoms in the dictionary is $K$, then the scale of $\Phi$ is $36 \times K$; the scale of the dictionary representation matrix $W$ is $K \times K$. Sparse constraint $P_0$ was set to be 6, and the sparse constraint $T_0$ of the signal representation matrix $A$ was set to 5. Experimental results show that the algorithm is not sensitive to the choice of $K$, which was finally set to be 576. All the simulations were conducted in MatLab R2016a (MathWorks, USA) on a PC with an Intel(R) Core(TM) i7 processor running at 3.6 GHz with 4GB of RAM.

#### A. Proposed Double-Sparsity Dictionary versus Learned Dictionary

The results of SR reconstruction with the proposed double-sparsity dictionaries (denoted DSD-SR) were evaluated compared with a learning-based dictionary in [11] (denoted LD-SR). To determine the contribution of the double-sparsity dictionary independently, NLM was not applied to enforce the global reconstruction constraint in the experiments for this part.

Seven images were selected from Set14 as the testing set. For each test image, the PSNR between the reconstructed image and original one was calculated, and the reconstruction time given, as shown in Table I (magnification ×2) and Table II (magnification ×4). As can be seen, the proposed DSD-based method has a slightly higher average PSNR than the LD-based method in [11], but the time-performance improvements are greater, i.e., approximately 60% less than the average reconstruction time of [11].

### B. Proposed versus Other Algorithms

In this experiment, 2× magnification and 4× magnification of SR reconstruction were performed on the test images. The proposed DSD-SR and NLMDSD-SR methods were compared with the bi-cubic interpolation method [19], Kim’s method using sparse regression and a natural image prior [20], Zhang’s SR with non-local means and steering kernel regression [8], and Yang’s method by simple functions [21]. The results were compared by visual quality subjectively and by numerical measurements of PSNR, mean structural similarity (MSSIM) [22], and feature similarity (FSIM) [23]. The higher the value PSNR, MSSIM, and FSIM, the more similar the reconstructed image is to the original image.

The results of 2× magnification are shown in Table III (PSNR), Table IV (MSSIM), Table V(FSIM).

### Table I

**Magnification ×2 Performance in Terms of PSNR(dB) and Running Time(s)**

<table>
<thead>
<tr>
<th>Images</th>
<th>LD-SR</th>
<th>DSD-SR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSNR(dB)</td>
<td>Time(s)</td>
</tr>
<tr>
<td>Baboon</td>
<td>22.609</td>
<td>4.064</td>
</tr>
<tr>
<td>Barbara</td>
<td>26.564</td>
<td>7.303</td>
</tr>
<tr>
<td>Bird</td>
<td>34.916</td>
<td>1.355</td>
</tr>
<tr>
<td>Bridge</td>
<td>26.961</td>
<td>4.573</td>
</tr>
<tr>
<td>Face</td>
<td>31.400</td>
<td>1.238</td>
</tr>
<tr>
<td>Lenna</td>
<td>32.552</td>
<td>4.631</td>
</tr>
<tr>
<td>Pepper</td>
<td>30.662</td>
<td>4.366</td>
</tr>
<tr>
<td>Zebra</td>
<td>30.923</td>
<td>3.735</td>
</tr>
<tr>
<td>Average</td>
<td>29.573</td>
<td>3.883</td>
</tr>
</tbody>
</table>

### Table II

**Magnification ×4 Performance in Terms of PSNR(dB) and Running Time(s)**

<table>
<thead>
<tr>
<th>Images</th>
<th>LD-SR</th>
<th>DSD-SR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSNR(dB)</td>
<td>Time(s)</td>
</tr>
<tr>
<td>Baboon</td>
<td>20.210</td>
<td>1.463</td>
</tr>
<tr>
<td>Barbara</td>
<td>23.593</td>
<td>2.613</td>
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<tr>
<td>Bird</td>
<td>27.828</td>
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<tr>
<td>Bridge</td>
<td>23.288</td>
<td>1.618</td>
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<tr>
<td>Face</td>
<td>28.821</td>
<td>0.451</td>
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<tr>
<td>Lenna</td>
<td>28.518</td>
<td>1.611</td>
</tr>
<tr>
<td>Pepper</td>
<td>27.265</td>
<td>1.626</td>
</tr>
<tr>
<td>Zebra</td>
<td>23.490</td>
<td>1.549</td>
</tr>
<tr>
<td>Average</td>
<td>25.377</td>
<td>1.405</td>
</tr>
</tbody>
</table>

### Table III

**PSNR(dB) of the Reconstructed Images by Different Methods (2×Magnification)**

(4) Yang [21] (5) DSD-SR (6) NLMDSD-SR

<table>
<thead>
<tr>
<th>Images</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bird</td>
<td>31.914</td>
<td>33.560</td>
<td>35.483</td>
<td>33.771</td>
<td>34.942</td>
<td>35.485</td>
</tr>
<tr>
<td>Bridge</td>
<td>25.316</td>
<td>26.377</td>
<td>27.250</td>
<td>26.713</td>
<td>27.081</td>
<td>27.468</td>
</tr>
<tr>
<td>Face</td>
<td>30.287</td>
<td>30.705</td>
<td>31.573</td>
<td>30.676</td>
<td>31.488</td>
<td>31.653</td>
</tr>
<tr>
<td>Lenna</td>
<td>30.646</td>
<td>31.627</td>
<td>32.865</td>
<td>31.736</td>
<td>32.631</td>
<td>32.932</td>
</tr>
<tr>
<td>Pepper</td>
<td>29.469</td>
<td>30.239</td>
<td>30.665</td>
<td>29.661</td>
<td>30.767</td>
<td>30.877</td>
</tr>
<tr>
<td>Average</td>
<td>27.730</td>
<td>28.790</td>
<td>29.844</td>
<td>28.918</td>
<td>29.659</td>
<td>30.010</td>
</tr>
</tbody>
</table>

### Table IV

**MSSIM of the Reconstructed Images by Different Methods (2×Magnification)**

(4) Yang [21] (5) DSD-SR (6) NLMDSD-SR

<table>
<thead>
<tr>
<th>Images</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baboon</td>
<td>0.602</td>
<td>0.671</td>
<td>0.728</td>
<td>0.700</td>
<td>0.717</td>
<td>0.743</td>
</tr>
<tr>
<td>Barbara</td>
<td>0.787</td>
<td>0.827</td>
<td>0.849</td>
<td>0.836</td>
<td>0.846</td>
<td>0.860</td>
</tr>
<tr>
<td>Bird</td>
<td>0.948</td>
<td>0.969</td>
<td>0.978</td>
<td>0.974</td>
<td>0.974</td>
<td>0.977</td>
</tr>
<tr>
<td>Bridge</td>
<td>0.725</td>
<td>0.785</td>
<td>0.831</td>
<td>0.811</td>
<td>0.822</td>
<td>0.840</td>
</tr>
<tr>
<td>Face</td>
<td>0.809</td>
<td>0.834</td>
<td>0.857</td>
<td>0.845</td>
<td>0.854</td>
<td>0.863</td>
</tr>
<tr>
<td>Lenna</td>
<td>0.869</td>
<td>0.888</td>
<td>0.905</td>
<td>0.896</td>
<td>0.902</td>
<td>0.907</td>
</tr>
<tr>
<td>Pepper</td>
<td>0.870</td>
<td>0.883</td>
<td>0.894</td>
<td>0.889</td>
<td>0.893</td>
<td>0.895</td>
</tr>
<tr>
<td>Zebra</td>
<td>0.848</td>
<td>0.895</td>
<td>0.929</td>
<td>0.917</td>
<td>0.923</td>
<td>0.933</td>
</tr>
<tr>
<td>Average</td>
<td>0.807</td>
<td>0.844</td>
<td>0.871</td>
<td>0.858</td>
<td>0.866</td>
<td>0.877</td>
</tr>
</tbody>
</table>
From the perspective of numerical measures, the NLMDSR-SR algorithm has the best SR reconstruction quality, Zhang’s method is second-best, and the DSD-SR algorithm is third-best.

Table IX shows the 2× magnification reconstruction time used to investigate the time complexity, and Table X shows the 4× results. As can be seen in Table IX, the proposed DSD-SR method has the best time efficiency, with an average time of 1.635s, which is approximately 7 times faster than the average speed of the Kim’s method. Next is the NLMDSR-SR method, which improves the reconstructed-image quality, while the time performance remains within an acceptable range.
Table XII

| Average FSIM of the Reconstructed Images of Set4, Set14 and B100
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicubic[19]</td>
<td>0.804</td>
<td>0.810</td>
<td>0.706</td>
</tr>
<tr>
<td>Kim[20]</td>
<td>0.836</td>
<td>0.847</td>
<td>0.731</td>
</tr>
<tr>
<td>Zhang[8]</td>
<td>0.864</td>
<td>0.865</td>
<td>0.796</td>
</tr>
<tr>
<td>Yang[21]</td>
<td>0.862</td>
<td>0.851</td>
<td>0.740</td>
</tr>
<tr>
<td>DSD-SR</td>
<td>0.867</td>
<td>0.856</td>
<td>0.780</td>
</tr>
<tr>
<td>NLMDS-SR</td>
<td>0.880</td>
<td>0.868</td>
<td>0.801</td>
</tr>
</tbody>
</table>

Table XI (PSNR), Table XII (MSSIM) and Table XIII (FSIM) shows the numerical metrics for reconstructing images by different methods. The experiment was performed on 4× magnification of SR reconstruction on benchmarks, including Set4, Set5, and B100.

From the experimental results, the average performance of the NLMDS-SR method is the best on all datasets, followed by the DSD-SR and Zhang’s methods. The reconstruction results on Set4 and B100 of DSD-SR are better than those of Zhang’s method, and the results on Set5 of Zhang’s method are better than those of DSD-SR.

Considering all factors, including subjective visual quality, objective assessment, and time complexity, the proposed NLMDS-SR method obtains good performance for image SR reconstruction. In addition, the proposed DSD-SR algorithm significantly improves reconstruction speed while guaranteeing reconstruction quality, so it is suitable for applications with high-time-performance requirements.

IV. CONCLUSIONS

In this paper, an image SR reconstruction algorithm based on double-sparsity dictionaries is proposed. The algorithm combines both the advantages of analytical and learning-based dictionaries, which can ensure the quality of reconstruction and improve speed. The algorithm is suitable for applications with high-time-performance requirements. On this basis, non-local self-similar constraints are used in regularization of SR reconstruction (donated as NLMDS-SR), which further improve the quality of reconstructed images while keeping the runtime within an acceptable range. The algorithm offers a desirable compromise between low computational complexity and reconstruction quality.

An overcomplete DCT dictionary is chosen as a base dictionary. In fact, there are a variety of base dictionaries to choose from, such as wavelets, complex wavelets, contourlets, and bandelets. How to choose a more appropriate base dictionary is one of our planned future research directions.

The non-local self-similarity constraint improves the quality of SR reconstruction, but the time complexity of the regularization reconstruction is relatively high. Finding the regularization constraint that reduces the time complexity and has certain anti-noise performance under the premise of ensuring the reconstruction quality is another planned research direction.

REFERENCES


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