Research on Mean-Variance-Efficiency Portfolio of Fuzzy DEA Based on Possibility Theory

Xue Deng, Cuirong Huang, Yusheng Liu

***Abstract*: the actual financial market is complicated and volatile, in which most investment portfolio models include the historical return of stock as an important indicator while overlooking the impact of stock efficiency. Meanwhile we have noticed that the specific indicators of fuzzy input and fuzzy output enable the fuzzy DEA model to effectively measure stock efficiency. Accordingly, this paper uses fuzzy DEA model to analyze the stock efficiency, with equity multiplier, price earnings ratio and beta value as fuzzy input, total assets turnover, earning per share and net profit growth rate as fuzzy output. Secondly, this paper takes the possibility mean and possibility variance as the measurement of portfolio return and risk, so as to build the mean-variance-efficiency portfolio model. And the genetic algorithm of exponential fitness function is applied to the model in order to satisfy the requirement of fitness values being non-negative. At the end, an empirical example is given to verify the feasibility of the model and improved algorithm, and we also prove that there is a certain correlation between stock efficiency and return. As shown by the results, it is quite indispensable to consider the portfolio efficiency in the actual financial market, which can provide investors with more comprehensive, scientific and effective decision-making scheme.**

***Index Terms*: Fuzzy DEA model, Genetic algorithm, Mean-Variance-Efficiency model, Fuzzy portfolio, Possibility theory**

I. Introduction

The portfolio selection theory proposed by Markowitz [1] is the origin of modern financial theories. Markowitz, on the frame of probability theory, proposed the mean-variance (M-V) model. The major matter of portfolio management, which concerns investors most, is "how to maximize the return while containing the risk" or "how to minimize the risk given a certain return". In this case, investors must comprehensively take into account lots of internal and external factors, which inspires researchers to discover a better portfolio model in actual consideration of fuzzy uncertainty and random uncertainty.

So far, distinct modified models and hybrid algorithms have developed from the M-V model to simulate the financial reality. For instance, Zheng and Yao [2] optimized the portfolio in view of risk measurement and disintegration of ensemble empirical pattern. Yu et al. [3] researched on fuzzy multi-objective portfolio with the application of hybrid genetic algorithm. Nazir [4] presented an efficient financial portfolio selection and improved the computational procedure of Anticor's algorithm.

Zadeh's [5] fuzzy set theory sets a mathematical foundation and establish an axiomatic system. The theory motivates scholars to study fuzzy portfolio model, such that Carlsson [6] and Fang [7] build portfolio model based on possibility theory; Li and Yi [8] proposed innovative trapezoidal fuzzy number and applied it to fuzzy mean-variance model and fuzzy mean-variance-skewness model. Ramli and Jaaman [9] proposed seven extended mean-variance fuzzy portfolio selection model. Chen and Wang [10] established a two-stage fuzzy portfolio selection model considering transaction costs.

The data envelopment analysis (DEA) is a valuable tool for measuring the relative efficiency of a decision-making unit (DMU). Now, it has raised a heated discussion among scholars. For example, Liu and Wang [11] proposed three secondary goal models, including the initial efficiency value of DMUs, based on DEA cross-evaluation. Mirdehghan et al. [12] studied the relations among technical efficiency, cost efficiency and revenue efficiency in the DEA.

In addition to the optimal asset allocation of a portfolio, selection of better-performing stocks also stands out for optimizing investment portfolios. The DEA has gradually come to the eyesight of scholars regarding portfolio selection, and therefore the portfolio performance has been considered in the model. Chen et al. [13] combined the mean-semi-variance model and the cross-efficiency model derived from DEA to construct a synthesized model of multi-objective fuzzy portfolio. Zhou et al. [14] constructed different evaluation models within the framework of fuzzy portfolio. Currently when compared to the large amount literature mentioned above, there exists few studies concentrating on combining DEA with fuzzy to make stock efficiency one of the objective functions.

In this paper, possibility theory and fuzzy set theory focus on exploring portfolio selection under the fuzzy circumstance. Combined with fuzzy number, the DEA introduces stock efficiency into the traditional mean-variance model. What's more, the genetic algorithm, by applying six different preference coefficients, can solve the portfolio model.

Manuscript received January 8, 2021; revised June 2, 2021. This research was supported by the "Humanities and Social Sciences Research and Planning Fund of the Ministry of Education of China, No. 18YJAZH014-x2lxY9180090", "Natural Science Foundation of Guangdong Province, No. 2019A1515011038", "Guangdong Province Characteristic Innovation Project of Colleges and Universities, No. 2019GKTSCX023", "Soft Science of Guangdong Province, No. 2018A070712006, 2019A101002118". The authors are highly grateful to the referees and editor in-chief for their very helpful comments.

Xue Deng is a Professor of the School of Mathematics, South China University of Technology, Guangzhou, 510640 China, (e-mail: dxue@scut.edu.cn).

Cuirong Huang is a Student of the School of Mathematics, South China University of Technology, Guangzhou, 510640 China, (corresponding author to provide e-mail: 472913094@qq.com).

Yusheng Liu is a Student of the Faculty of Economics and Business, University of Groningen, Groningen, 9747AE The Netherlands, (e-mail: yusheng_liu@hotmail.com).

In Section II, the preliminary knowledge on possibility theory and DEA is given. The fuzzy multi-objective mean-variance-efficiency portfolio model is constructed in the Section III. Section IV introduces the genetic algorithm used in this paper. In Section V, the feasibility of the model and algorithm is verified by an empirical example. Section VI summarizes the whole paper and draws the conclusion.

II. Preliminaries

A. Possibility Theory

Definition 1 [15]: Fuzzy number $A \in \mathrm{F}(\mathbb{R})$. For $\forall \gamma \in [0, 1]$, it holds that $[A]^\gamma = [\underline{a}(\gamma), \bar{a}(\gamma)]$. The possibility mean of A can be defined as

$$E(A) = \int_0^1 \gamma\left(\bar{a}(\gamma) + \underline{a}(\gamma)\right) d\gamma. \tag{1}$$

Definition 2 [16]: $A \in \mathrm{F}(\mathbb{R})$. For $\forall \gamma \in [0, 1]$, it stands that $[A]^\gamma = [\underline{a}(\gamma), \bar{a}(\gamma)]$, The possibility variance of A can be determined as

$$Var(A) = \int_0^1 \gamma\left[\left(\underline{a}(\gamma) - E(A)\right)^2 + \left(\bar{a}(\gamma) - E(A)\right)^2\right] d\gamma. \tag{2}$$

Definition 3 [16]: $A, B \in \mathrm{F}(\mathbb{R})$. Let $\forall \gamma \in [0, 1]$, it holds that $[A]^\gamma = [\underline{a}(\gamma), \bar{a}(\gamma)]$ and $[B]^\gamma = [\underline{b}(\gamma), \bar{b}(\gamma)]$, The possibility covariance of A and B is specified by

$$\begin{aligned} Cov(A, B) &= \int_0^1 \gamma\left(\underline{a}(\gamma) - E(A)\right)\left(\underline{b}(\gamma) - E(B)\right) d\gamma \\ &\quad + \int_0^1 \gamma\left(\bar{a}(\gamma) - E(A)\right)\left(\bar{b}(\gamma) - E(B)\right) d\gamma. \end{aligned} \tag{3}$$

Theorem 1 [15]: Provided that $\forall \lambda, \mu \in \mathbb{R}$, we have

$$E(\lambda A + \mu B) = \lambda E(A) + \mu E(B), \tag{4}$$

$$\begin{aligned} Var(\lambda A + \mu B) &= \lambda^2 Var(\phi(\lambda) A) + \mu^2 Var(\phi(\mu) B) \\ &\quad + 2|\lambda\mu| Cov(\phi(\lambda) A, \phi(\mu) B), \end{aligned} \tag{5}$$

where $\phi(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$.

Theorem 2 [15]: Let $A_1, A_2, \cdots, A_n \in \mathrm{F}(\mathbb{R})$ be fuzzy numbers. For any n real numbers $\lambda_1, \lambda_2, \cdots, \lambda_n \in \mathbb{R}$, we have

$$\begin{aligned} Var\left(\sum_{i=1}^n \lambda_i A_i\right) &= \sum_{i=1}^n \lambda_i^2 Var\left(\phi(\lambda_i) A_i\right) \\ &\quad + 2\sum_{i<j} |\lambda_i \lambda_j| Cov\left(\phi(\lambda_i) A_i, \phi(\lambda_j) A_j\right). \end{aligned} \tag{6}$$

The triangular fuzzy number A is denoted as $A = (a, \alpha, \beta)$, and its membership function is as follows:

$$\mu_A(x) = \begin{cases} 1 - \dfrac{a - x}{\alpha}, & a - \alpha \le x \le a, \\ 1 - \dfrac{x - a}{\beta}, & a \le x \le a + \beta, \\ 0, & \text{other} \end{cases} \tag{7}$$

The λ-cut set of A is given by $[A]^\gamma = [a - (1-\gamma)\alpha, a + (1-\gamma)\beta]$, where $\forall \gamma \in [0, 1]$. Based on Definition 1 and 2, the possibility mean and possibility variance of A are as follows:

$$E(A) = \int_0^1 \gamma\left(a + (1-\gamma)\beta + a - (1-\gamma)\alpha\right) d\gamma = a + \frac{\beta - \alpha}{6}, \tag{8}$$

$$\begin{aligned} Var(A) &= \int_0^1 \gamma\left(a - (1-\gamma)\alpha - a - \frac{\beta - \alpha}{6}\right)^2 d\gamma \\ &\quad + \int_0^1 \gamma\left(a + (1-\gamma)\beta - a - \frac{\beta - \alpha}{6}\right)^2 d\gamma \\ &= \left[\frac{\alpha + \beta}{6}\right]^2 + \frac{(\beta + \alpha)^2 + (\beta - \alpha)^2}{72}. \end{aligned} \tag{9}$$

B. Data Envelopment Analysis (DEA)

Data envelopment analysis (DEA) [17] is a method of evaluating the relative effectiveness of the same type of decision-making units (DMU) based on the multiple input- and output indicators. Specifically, the notations involved in DEA model are shown in TABLE I.

TABLE I
SYMBOLIC DESCRIPTION OF DEA MODEL

Notation	Description
j	Input indicator, $j = 1, 2, \cdots, m$
r	Output indicator, $r = 1, 2, \cdots, s$
i	Decision making unit (DMU), $i = 1, 2, \cdots, n$
x_{ji}	Input of DMU i in item j
y_{ri}	Output of DMU i in item r
v_j	A measurement of the input in item j, weight coefficient
u_r	A measurement of the output in item r, weight coefficient

Thereby, the efficiency evaluation index of each DMU is defined as

$$h_i = \frac{\sum_{r=1}^s u_r y_{ri}}{\sum_{j=1}^m v_j x_{ji}}, i = 1, 2, \cdots, n. \tag{10}$$

Here, h_i represents the relative efficiency of DMU i, proper weight coefficients u_r, v_j enable $h_i \le 1, i = 1, \cdots, n$. Regarding the DMU i_0, with the greater h_{i_0} normally comes the greater output while the input is equal to that of others. For this reason, we are in passionate pursuit of maximizing the efficiency evaluation index, and therefore found an objective function of maximizing the index under the condition that the indexes of all decision-making units are not larger than one. The model is structured by

$$\begin{cases} \max \ h_{i_0} = \sum_{r=1}^s u_r y_{ri_0} \Big/ \sum_{j=1}^m v_j x_{ji_0} \\ \text{s.t.} \ \sum_{r=1}^s u_r y_{ri} \Big/ \sum_{j=1}^m v_j x_{ji} \le 1 \\ \qquad u_r \ge 0, v_j \ge 0, i = 1, 2, \cdots, n \end{cases} \tag{11}$$

C. Fuzzy DEA Model

Normally, the commonly used model in the financial market is the standard DEA one, which requires high clarity and exact accuracy of input- and output indicators. However, in reality, it is impossible to fully describe the market with totally clear data. In this case, the fuzzy DEA utilizing imprecise and ambiguous data instead of clear one. The two models aforementioned vary from each other majorly regarding the input- and output data clarity (fuzzy number or clear number) of DMUs.

Kao and Tai [18] proposed to obtain the range of fuzzy numbers by means of $\lambda-$cut set, and cleverly transform the fuzzy DEA model into the standard DEA one with a certain range (upper- and lower limits of fuzzy numbers). This solution also applies to the current case. Furthermore, Mugera [19] once pointed out that triangular fuzzy number is most used, and thus the input and output indicators of DMUs are regarded as triangular fuzzy numbers.

Let the input indicator j of DMU i be $\tilde{x}_{ji}=\left(x_{ji}^m,\alpha_{ji},\beta_{ji}\right)$, and its output indicator r be $\tilde{y}_{ri}=\left(y_{ri}^m,\alpha_{ri}',\beta_{ri}'\right)$. The uncertain range of the fuzzy numbers $\tilde{x}_{ji},\tilde{y}_{ri}$ can be determined by different confidence intervals defined by the $\lambda-$cut set. That is, the $\lambda-$cut set $\left(0\le\lambda\le1\right)$ can specify the lower- and upper limits of the fuzzy numbers as follows:

$$\begin{aligned}\left(x_{ji}\right)_\lambda&=\left[\left(x_{ji}\right)_\lambda^L,\left(x_{ji}\right)_\lambda^U\right]\\&=\left[\lambda\alpha_{ji}+\left(x_{ji}^m-\alpha_{ji}\right),\left(x_{ji}^m+\beta_{ji}\right)-\lambda\beta_{ji}\right]\end{aligned}\tag{12}$$

and

$$\begin{aligned}\left(y_{ri}\right)_\lambda&=\left[\left(y_{ri}\right)_\lambda^L,\left(y_{ri}\right)_\lambda^U\right]\\&=\left[\lambda\alpha_{ri}'+\left(y_{ri}^m-\alpha_{ri}'\right),\left(y_{ri}^m+\beta_{ri}'\right)-\lambda\beta_{ri}'\right]\end{aligned}\tag{13}$$

When $\lambda=1$, the fuzzy numbers convert to the clear ones, meaning no uncertainty. When $\lambda=0$, the numbers drop between the full fuzzy range. That is, the uncertainty is adjusted by modifying λ. Further the efficiency score can be obtained in the concrete $\lambda-$cut set. By definition given by Kao and Tai [18], the output indicators can take the lower limits of the fuzzy numbers, and the input ones can take the upper limits instead when provided a specific value of the $\lambda-$cut set, which derives the lower limit of efficiency $\left(E_i\right)_\lambda^L$ of DMU i. The specific expression is as follows:

$$\left(E_i\right)_\lambda^L=\frac{\sum_{r=1}^{s}u_r\left(y_{ri}\right)_\lambda^L}{v_0+\sum_{j=1}^{m}v_j\left(x_{ji}\right)_\lambda^U}.\tag{14}$$

Conversely, the output indicators can take the upper limit of the fuzzy numbers while the input ones can take the lower limit of the numbers. Hereby the upper limit of the efficiency $\left(E_i\right)_\lambda^U$ of DMU i can be obtained by

$$\left(E_i\right)_\lambda^U=\frac{\sum_{r=1}^{s}u_r\left(y_{ri}\right)_\lambda^U}{v_0+\sum_{j=1}^{m}v_j\left(x_{ji}\right)_\lambda^L},\tag{15}$$

where v_0 is constant and u_r,v_j are weight coefficients.

When we maximize the lower limit of DMU i_0's efficiency, where all DMUs' efficiency is not greater than 1, the model (16) can be written as

$$\begin{cases}\max\quad \sum_{r=1}^{s}u_r\left(y_{ri_0}\right)_\lambda^L\Big/\left(v_0+\sum_{j=1}^{m}v_j\left(x_{ji_0}\right)_\lambda^U\right)\\ \text{s.t.}\quad \sum_{r=1}^{s}u_r\left(y_{ri}\right)_\lambda^L\Big/\left(v_0+\sum_{j=1}^{m}v_j\left(x_{ji}\right)_\lambda^U\right)\le1\\ \qquad \sum_{r=1}^{s}u_r\left(y_{ri}\right)_\lambda^U\Big/\left(v_0+\sum_{j=1}^{m}v_j\left(x_{ji}\right)_\lambda^L\right)\le1\\ \qquad i=1,\cdots,n,\ \ i\ne i_0,\ u_r,v_j>0,\ \ v_0 \text{ is constant}\end{cases}\tag{16}$$

Similarly, if the objective function aims to maximize the upper limit of DMU i_0's efficiency, the model (17) can be expressed as

$$\begin{cases}\max\quad \sum_{r=1}^{s}u_r\left(y_{ri_0}\right)_\lambda^U\Big/\left(v_0+\sum_{j=1}^{m}v_j\left(x_{ji_0}\right)_\lambda^L\right)\\ \text{s.t.}\quad \sum_{r=1}^{s}u_r\left(y_{ri}\right)_\lambda^L\Big/\left(v_0+\sum_{j=1}^{m}v_j\left(x_{ji}\right)_\lambda^U\right)\le1\\ \qquad \sum_{r=1}^{s}u_r\left(y_{ri}\right)_\lambda^U\Big/\left(v_0+\sum_{j=1}^{m}v_j\left(x_{ji}\right)_\lambda^L\right)\le1\\ \qquad i=1,\cdots,n,\ \ i\ne i_0,\ u_r,v_j>0,\ \ v_0 \text{ is constant}\end{cases}\tag{17}$$

By solving the models (16) and (17), we are able to have the upper- and lower limits of the efficiency corresponding to each value of λ. Assuming $\lambda=1$, the efficiency is the same as that obtained by the standard DEA. While $\lambda=0$, the efficiency range refers to that of the fuzzy numbers. Furthermore, the efficiency obtained with $0<\lambda<1$ does not exceed the efficiency range.

III. FUZZY MULTI-OBJECTIVE PORTFOLIO MODEL

Supposing that the investor plans to invest in n stocks, the return of the stock i is $r_i\left(i=1,2,\cdots,n\right)$, and its investment proportion is $x_i\left(i=1,2,\cdots,n\right)$. The return, possibility mean and variance of the portfolio R are as follows:

$$R=\sum_{i=1}^{n}x_ir_i.\tag{18}$$

$$E\left(R\right)=E\left(\sum_{i=1}^{n}x_ir_i\right),\tag{19}$$

$$Var\left(R\right)=\sum_{i=1}^{n}x_i^2Var\left(r_i\right)+2\sum_{i<j=1}^{n}x_ix_jCov\left(r_i,r_j\right),\tag{20}$$

where $Cov\left(r_i,r_j\right)$ represents the covariance between r_i and r_j. Next, we utilize the fuzzy DEA model to calculate the efficiency of stocks, so that the index drops within the range of fuzzy numbers. Provided the $\lambda-$cut set, let the efficiency interval of stock i be $\left[\left(E_i\right)_\lambda^L,\left(E_i\right)_\lambda^U\right]$. By averaging the efficiency, we have the efficiency evaluation index of the stock i as $\left(\left(E_i\right)_\lambda^L+\left(E_i\right)_\lambda^U\right)\Big/2$. Furthermore, the overall portfolio efficiency is a linear combination of investment proportion and stock efficiency. The expression is as follows:

$$e=\sum_{i=1}^{n}\frac{x_i\left[\left(E_i\right)_\lambda^L+\left(E_i\right)_\lambda^U\right]}{2}. \tag{21}$$

As mentioned above, the efficiency of such a portfolio is measured by efficiency evaluation index computed by the Fuzzy DEA model. Assuming investors tend to be rational, say, they seek to maximize the return and the efficiency, and minimize the risk of the portfolio, the corresponding model is specified by

$$\begin{cases}\max\ E(R)=E\left(\sum_{i=1}^{n}x_ir_i\right)\\ \min\ \ Var(R)=\sum_{i=1}^{n}x_i^2Var(r_i)+2\sum_{i<j=1}^{n}x_ix_jCov(r_i,r_j)\\ \max\ \ e=\sum_{i=1}^{n}0.5x_i\left[\left(E_i\right)_\lambda^L+\left(E_i\right)_\lambda^U\right]\\ \text{s.t.}\ \ \sum_{i=1}^{n}x_i=1,\ l_i\le x_i\le u_i,\ i=1,\cdots,n\end{cases} \tag{22}$$

where l_i and u_i are the upper- and lower limits. The model (22) is equivalent of the following model (23).

$$\begin{cases}\max\ E(R)=E\left(\sum_{i=1}^{n}x_ir_i\right)\\ \max\ \ -Var(R)=-\sum_{i=1}^{n}x_i^2Var(r_i)-2\sum_{i<j=1}^{n}x_ix_jCov(r_i,r_j)\\ \max\ \ e=\sum_{i=1}^{n}0.5x_i\left[\left(E_i\right)_\lambda^L+\left(E_i\right)_\lambda^U\right]\\ \text{s.t.}\ \ \sum_{i=1}^{n}x_i=1,\ l_i\le x_i\le u_i,\ i=1,\cdots,n\end{cases} \tag{23}$$

IV. GENETIC ALGORITHM

A. Basic Definition of Genetic Algorithm

The genetic algorithm is employed to solve the model (23). The algorithm initially encodes each solution from the problem domain, and establishes a fitness function based on the objective function. Later, all solutions are put into the fitness function to select the solutions with higher fitness values, which can form a new species. On this new species, we continue to perform selection, crossover and mutation operations iteratively so as to get the optimal solution with generality. Next are five basic steps of the algorithm.

(1) Initialization: the feasible solution $X=(x_1,x_2,\cdots,x_n)$ of the domain is generated randomly. It is then coded as chromosome $C=(c_1,c_2,\cdots,c_n)$ ($l_i\le c_i\le u_i$). The corresponding relation between x_i and c_i is as follows:

$$x_i=\frac{c_i}{\sum_{i=1}^{n}c_i},i=1,2,\cdots,n. \tag{24}$$

This practice can ensure that $\sum_{i=1}^{n}x_i=1$ holds true. Assume that the size of the entire species is *pop_siz*, where the number of all solutions meet the condition before. Repeat the equation (24) for *pop_size* times to obtain the chromosome labeled as $C_1,C_2,\cdots,C_{pop\_size}$.

(2) Fitness function: a function to judge the quality of an individual in a species. In order to prevent the unreasonable distribution of the fitness values, we adjust the scaling of the fitness function. The requirement of fitness value being non-negative should be satisfied as well. For the reasons above, the following exponential fitness function fits here well, that is

$$F(X)=e^{f(X)}, \tag{25}$$

where $f(X)$ is the weighted objective function by the preference coefficients of the model (23). Substituting the individuals $X_1,X_2,\cdots,X_{pop\_size}$ in the species into the equation (25), we have the corresponding fitness values recorded as $F_i\left(i=1,2,\cdots,pop\_size\right)$.

(3) Selection: The selection of individuals from the parent species to the next generation according to the values of the fitness function, reflecting a survival procedure of the species. The possibility of individual X_i being selected is

$$P(X_i)=\frac{F_i}{\sum_{i=1}^{pop\_size}F_i},i=1,2,\cdots,pop\_size. \tag{26}$$

(4) Crossover: Re-structing of the parent individuals. The algorithm performs this operation on each individual with probability P_c. For example, the convex combination of two parent individuals C_1,C_2 can derive two new individuals

$$C_1=\mu C_1+(1-\mu)C_2,C_2=\mu C_2+(1-\mu)C_1. \tag{27}$$

The algorithm will inspect whether the new individuals generated by crossover meet the constraint before. If so, the new individuals will enter the next generation.

(5) Mutation: the mutation is performed on individuals within the population with probability P_m. Suppose that the chromosome C from the parent generation is $(c_1,c_2,\cdots c_i,\cdots,c_n)$, where a gene c_i is randomly selected for mutation. Then we have a new individual C' $(c_1,c_2,\cdots,\overline{c}_i,\cdots,c_n)$, and the $\overline{c}_i$ is given by

$$\overline{c}_i=l_i+y\cdot(u_i-l_i), \tag{28}$$

within which y is a random number on the interval [0,1].

Similarly, the algorithm will also inspect whether the new individuals generated by mutation are limited by the condition before. If so, the new individuals will enter the next generation.

B. Calculation Steps

The specific flow of genetic algorithm is shown below:

Step 1: The feasible solutions of the problem domain are encoded to obtain the initial population *pop_size*(0);

Step 2: The fitness values of individuals in the population (*pop_size(k)*) are computed through the exponential fitness function;

Step 3: The selection of the superior individuals from the parent generation to the child generation;

Step 4: The crossover is performed on each of the new generation with probability P_c : $pop\_size(k) \leftarrow \text{Crossover}\left[pop\_size(k)\right]$;

Step 5: The mutation is conducted on each of the new generation to with probability P_m : $pop\_size(k) \leftarrow \text{Mutation}\left[pop\_size(k)\right]$;

Step 6: If the conditions are met, then export the solutions; if the conditions are not met, repeat the **step 2~6**.

V. EMPIRICAL EXAMPLE

A. *Fuzzy DEA Efficiency Analysis*

In this section, the effectiveness and applicability of the fuzzy DEA and portfolio model (23) are proved by an actual example. We select eight stocks that are traded in Shanghai stock market of China. The possibility distribution can be obtained by means proposed by Zhang et al. [20]. We collect the weekly closing prices of the eight stocks from October 2016 to October 2018, and therefore get the possibility distribution as shown in TABLE II.

TABLE II
THE POSSIBILITY DISTRIBUTION OF EIGHT STOCK RETURNS

Stock	1	2	3	4
Code	600801	600276	601933	600516
a_i	0.0112	0.0108	0.0061	0.0054
α_i	0.0820	0.0563	0.0533	0.0787
β_i	0.0836	0.0437	0.0562	0.0718
Stock	5	6	7	8
Code	600809	600487	600585	603799
a_i	0.0016	0.0089	0.0104	0.0054
α_i	0.0529	0.0743	0.0537	0.0871
β_i	0.0836	0.0558	0.0592	0.1016

For the purpose of working out the efficiency of the eight stocks, we select six indicators similar to those the reference [21]. The input indicators are equity multiplier, price earnings ratio and beta value. The output indicators are total assets turnover, earning per share and net profit growth rate. Similarly, this paper uses the historical data of the six indicators of the eight stocks from October 2016 to October 2018, and estimates their possibility distribution. The specific results are shown in TABLE III-VIII.

Using the fuzzy DEA model, combined with the data of the six indicators, we can get the efficiency of each stock given various $\lambda-$cut sets. The results are presented in TABLE IX.

TABLE III
DISTRIBUTION OF EQUITY MULTIPLIER OF THE EIGHT STOCKS

Stock	1	2	3	4
a_i	40.6000	20.9900	17.3600	137.6350
α_i	23.2360	0.8000	0.5220	88.9880
β_i	15.7840	0.8090	1.5210	94.8470
Stock	5	6	7	8
a_i	41.9900	38.3100	32.4800	75.5550
α_i	4.5180	0.5740	3.5720	8.6540
β_i	4.9320	2.7880	2.0150	3.2090

TABLE IV
DISTRIBUTION OF PRICE EARNINGS RATIO OF THE EIGHT STOCKS

Stock	1	2	3	4
a_i	17.7146	59.7610	48.7249	112.4947
α_i	2.7027	9.8068	5.2452	98.2008
β_i	9.8559	8.6875	4.7598	50.5034
Stock	5	6	7	8
a_i	44.0331	22.0589	12.2082	41.7184
α_i	7.3390	2.2151	1.3828	25.3441
β_i	4.3739	3.3220	0.9502	20.8615

TABLE V
DISTRIBUTION OF BETA VALUE OF THE EIGHT STOCKS

Stock	1	2	3	4
a_i	1.6255	0.5888	1.0835	1.3527
α_i	0.4842	0.5126	0.3694	0.4376
β_i	0.5087	0.5729	0.3947	0.5091
Stock	5	6	7	8
a_i	1.0798	1.5952	1.1302	1.5669
α_i	0.7686	0.6649	0.5182	0.8122
β_i	0.4962	0.5154	0.3918	0.8963

TABLE VI
DISTRIBUTION OF TOTAL ASSETS TURNOVER OF THE EIGHT STOCKS

Stock	1	2	3	4
a_i	162.5450	58.8750	108.3450	135.2650
α_i	48.4200	51.2620	36.9370	43.7640
β_i	50.8670	57.2880	39.4660	50.9110
Stock	5	6	7	8
a_i	107.9750	159.5200	113.0150	156.6850
α_i	76.8600	66.4940	51.8170	81.2150
β_i	49.6230	51.5360	39.1780	89.6260

TABLE VII
DISTRIBUTION OF EARNING PER SHARE OF THE EIGHT STOCKS

Stock	1	2	3	4
a_i	0.4201	0.6901	0.1039	0.6494
α_i	0.1131	0.1675	0.0175	0.5842
β_i	0.2594	0.1359	0.0240	0.4725
Stock	5	6	7	8
a_i	0.7595	0.7312	1.4386	1.2869
α_i	0.0642	0.1154	0.2981	0.7909
β_i	0.1598	0.3077	0.3882	0.5238

TABLE VIII
DISTRIBUTION OF NET PROFIT GROWTH RATE OF THE EIGHT STOCKS

Stock	1	2	3	4
a_i	40.6000	20.9900	17.3600	137.6350
α_i	23.2360	0.8000	0.5220	88.9880
β_i	15.7840	0.8090	1.5210	94.8470
Stock	5	6	7	8
a_i	41.9900	38.3100	32.4800	75.5550
α_i	4.5180	0.5740	3.5720	8.6540
β_i	4.9320	2.7880	2.0150	3.2090

TABLE IX
EFFICIENCY OF THE EIGHT STOCKS

Stock	λ =0.00		λ =0.10		λ =0.20	
	$(E_i)_\lambda^L$	$(E_i)_\lambda^U$	$(E_i)_\lambda^L$	$(E_i)_\lambda^U$	$(E_i)_\lambda^L$	$(E_i)_\lambda^U$
1	0.4943	1.0000	0.5153	1.0000	0.5376	1.0000
2	0.8145	1.0000	0.8910	1.0000	0.9391	1.0000
3	0.9197	1.0000	1.0000	1.0000	1.0000	1.0000
4	0.2011	1.0000	0.3001	1.0000	0.4279	1.0000
5	0.4589	1.0000	0.4897	1.0000	0.5209	1.0000
6	0.5289	1.0000	0.5560	1.0000	0.5870	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	0.3175	1.0000	0.3534	1.0000	0.3936	1.0000

TABLE IX

EFFICIENCY OF THE EIGHT STOCKS (CONTINUED)

Stock	λ =0.30		λ =0.40		λ =0.50	
	$(E_i)_\lambda^L$	$(E_i)_\lambda^U$	$(E_i)_\lambda^L$	$(E_i)_\lambda^U$	$(E_i)_\lambda^L$	$(E_i)_\lambda^U$
1	0.5613	1.0000	0.5866	1.0000	0.6355	1.0000
2	0.9598	1.0000	0.9816	1.0000	1.0000	1.0000
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	0.5639	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.5596	1.0000	0.6024	1.0000	0.6497	1.0000
6	0.6234	1.0000	0.6713	1.0000	0.7200	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	0.4389	1.0000	0.4901	1.0000	0.5481	1.0000

TABLE IX

EFFICIENCY OF THE EIGHT STOCKS (CONTINUED)

Stock	λ =0.60		λ =0.70		λ =0.80	
	$(E_i)_\lambda^L$	$(E_i)_\lambda^U$	$(E_i)_\lambda^L$	$(E_i)_\lambda^U$	$(E_i)_\lambda^L$	$(E_i)_\lambda^U$
1	0.6979	1.0000	0.7856	1.0000	0.9276	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.7024	1.0000	0.7617	1.0000	0.8291	1.0000
6	0.7539	1.0000	0.7948	1.0000	0.8466	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	0.6205	1.0000	0.7034	1.0000	1.0000	1.0000

According to TABLE X, we can find that when λ = 0.00, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70 or 0.80, the upper limit of the efficiency of the eight stocks is always 1. Then we draw the efficiency lower limits of the eight stocks regarding distinct the values of λ-cut set, as shown in Fig. 1.

B. Preference Coefficient

The portfolio model in this paper proposes a preference-weighting method for investors, with a preference coefficient $r=(r_E,r_V,r_{eff})$, in which r_E,r_V,r_{eff} represent their preference for return, risk and efficiency separately, reflecting their attitudes towards different portfolios. In this paper, six preference coefficients are introduced. Specifically, the first one (2,1,2) fits radical investors because this cluster prefers return and efficiency to risk; the second one (1,2,1) is suitable for conservative investors because their preference focuses on risk rather than return and efficiency; and the third one is (4,3,1). The fourth preference coefficient is (1,0,1), indicating that investors only pay attention to return and efficiency without considering risk; the fifth preference coefficient is (1,1,1), stating that investors give equal preference to the three objectives; the sixth preference coefficient is (1,1,0), which is equivalent of the traditional mean-variance model.

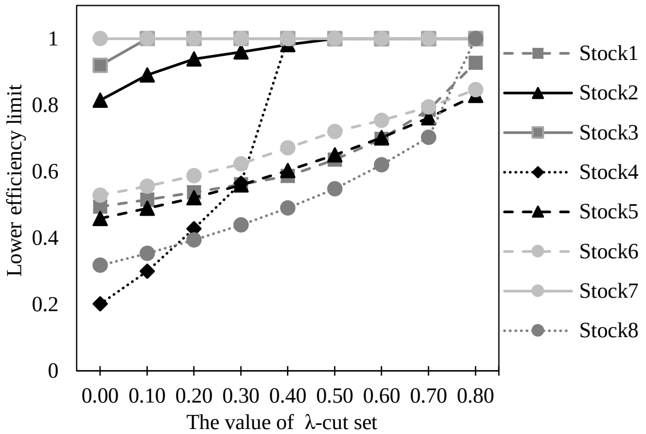

Fig. 1. The lower efficiency limit of the eight stocks

C. Analysis of Solution Results of the Model

Using the algorithm designed in Section IV, we solve the model (23) based on the six preference coefficients, and then get the proportions of the stocks with $\lambda = 0.1, 0.3, 0.5, 0.7$. The specific results are shown in TABLE X-XIII. The TABLE X-XIII show us that the investment proportions of stock 1 and 8 in the mean-variance model are non-zero while the figures are zero in the mean-variance-efficiency model. This is because stock 1 and 8 have lower efficiency and return. Such a phenomenon can lead to a conclusion that the efficiency and return of the portfolio have a certain correlation, and hence investors could consider the efficiency of the portfolio when making decisions.

TABLE X

INVESTMENT PROPORTION CORRESPONDING TO DIFFERENT PREFERENCE COEFFICIENTS WHEN $\lambda=0$

Weight	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
(2,1,2)	0.0000	0.0294	0.7051	0.0000	0.0000	0.0000	0.2655	0.0000
(1,2,1)	0.0000	0.0000	0.5288	0.0000	0.0000	0.0000	0.4712	0.0000
(4,3,1)	0.0000	0.0045	0.1292	0.0039	0.0000	0.0000	0.8624	0.0000
(1,0,1)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
(1,1,1)	0.0000	0.2116	0.4462	0.0000	0.0000	0.0000	0.3423	0.0000
(1,1,0)	0.0065	0.9379	0.0016	0.0000	0.0000	0.0000	0.0525	0.0015

TABLE XI

INVESTMENT PROPORTION CORRESPONDING TO DIFFERENT PREFERENCE COEFFICIENTS WHEN $\lambda=0.3$

Weight	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
(2,1,2)	0.0000	0.0968	0.2685	0.0000	0.0000	0.0000	0.6346	0.0000
(1,2,1)	0.0000	0.1612	0.3127	0.0000	0.0000	0.0000	0.5261	0.0000
(4,3,1)	0.0000	0.2532	0.2570	0.0000	0.0000	0.0000	0.4897	0.0000
(1,0,1)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
(1,1,1)	0.0000	0.0482	0.1879	0.0000	0.0000	0.0014	0.7625	0.0000
(1,1,0)	0.0065	0.9379	0.0016	0.0000	0.0000	0.0000	0.0525	0.0015

TABLE XII

INVESTMENT PROPORTION CORRESPONDING TO DIFFERENT PREFERENCE COEFFICIENTS WHEN $\lambda = 0.5$

Weight	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
(2,1,2)	0.0000	0.5892	0.0481	0.0664	0.0000	0.0000	0.2963	0.0000
(1,2,1)	0.0000	0.5499	0.1751	0.0000	0.0000	0.0000	0.2749	0.0000
(4,3,1)	0.0000	0.4093	0.1217	0.1209	0.0000	0.0000	0.3482	0.0000
(1,0,1)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
(1,1,1)	0.0000	0.4008	0.0875	0.1191	0.0000	0.0000	0.3926	0.0000
(1,1,0)	0.0065	0.9379	0.0016	0.0000	0.0000	0.0000	0.0525	0.0015

TABLE XIII

INVESTMENT PROPORTION CORRESPONDING TO DIFFERENT PREFERENCE COEFFICIENTS WHEN $\lambda = 0.7$

Weight	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
(2,1,2)	0.0000	0.4646	0.0438	0.0436	0.0000	0.0000	0.4480	0.0000
(1,2,1)	0.0000	0.6246	0.0712	0.0000	0.0000	0.0000	0.3042	0.0000
(4,3,1)	0.0000	0.5420	0.1408	0.0704	0.0000	0.0000	0.2468	0.0000
(1,0,1)	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
(1,1,1)	0.0000	0.3902	0.1795	0.0000	0.0000	0.0000	0.4303	0.0000
(1,1,0)	0.0065	0.9379	0.0016	0.0000	0.0000	0.0000	0.0525	0.0015

VI. Conclusions

This paper proposes a multi-objective fuzzy portfolio model considering return, variance and efficiency. Its specific goal is to maximize return and efficiency, and minimize variance. The efficiency of a portfolio is calculated by the fuzzy DEA using six fuzzy indicators of stocks. Equity multiplier, price earnings ratio and Beta value are regarded as three input indicators of the fuzzy DEA. Total assets turnover, earning per share and net profit growth rate are the three output indicators of the model. Finally, an actual example is given to solve the model by using genetic algorithm under different preference coefficients. The actual results indicate that it is necessary to consider portfolio efficiency and offer more decision-making options for investors.

References

[1] H.M. Markowitz, “Portfolio selection,” Journal of Finance, vol. 7, pp77-91, 1952

[2] C. Zheng, Y. Yao, “Portfolio optimization based on risk measures and Ensemble Empirical Mode Decomposition,” IAENG International Journal of Computer Science, vol. 46, no.1, pp76-82, 2019

[3] Y. Yu, X. Deng, C. Chen, K. Cheng, “Research on fuzzy multi-objective multi-period portfolio by hybrid genetic algorithm with wavelet neural network,” Engineering Letters, vol. 28, no.2, pp594-600, 2020

[4] A. Nazir, “Accelerated Anticor Online portfolio selection on multi-core CPUs and GPU with OpenCL,” IAENG International Journal of Computer Science, vol. 45, no.3, pp390-402, 2018

[5] L.A. Zadeh, “Fuzzy Sets,” Information and Control, vol. 8, no.3, pp338-353, 1965.

[6] C. Carlsson, R. Fullér, P. Majlender. “A possibilistic approach to selecting portfolios with highest utility score,” Fuzzy Sets and Systems, vol. 131, no.1, pp13-21, 2002

[7] Y. Fang, K.K. Lai, S.Y. Wang. “Portfolio rebalancing model with transaction costs based on fuzzy decision theory,” European Journal of Operational Research, vol. 172, no.2, pp879-893, 2006

[8] H.Q. Li, Z.H. Yi, “Portfolio Selection with Coherent Investor's Expectations Under Uncertainty,” Expert Systems with Applications, vol. 133, pp49-58, 2019

[9] S. Ramli, S.H. Jaaman. “Several Extended Mean-variance Fuzzy Portfolio Selection Models Based on Possibility Theory,” Journal of Physics: Conference Series, vol. 1212, no.1, pp1-8, 2019

[10] Y. Chen, Y. Wang. “Two-Stage Fuzzy Portfolio Selection Problem with Transaction Costs,” Mathematical Problems in Engineering, vol. 2015, pp1-12, 2015

[11] P. Liu, L.F. Wang, “Determination of cross-efficiency considering the original efficiency value of DMUs in DEA cross-evaluation,” IAENG International Journal of Applied Mathematics, vol. 49, no.2, pp181-187, 2019

[12] S.M. Mirdehghan, M. Nazaari A., J. Vakili, “Relations among technical, cost and revenue efficiencies in data envelopment analysis,” IAENG International Journal of Applied Mathematics, vol. 45, no.4, pp249-258, 2015

[13] W. Chen, S.S. Li, J. Zhang, M.K. Mehlawat, “A comprehensive model for fuzzy multi-objective portfolio selection based on DEA cross-efficiency model,” Soft Computing, vol. 24, no.4, pp2515-2526, 2020

[14] Z.B. Zhou, E.M. Chen, H.L. Xiao, T.T. Ren, Q.Y. Jin, “Performance evaluation of portfolios with fuzzy returns,” RAIRO-Operations Research, vol. 53, no.5, pp1581-1600, 2019

[15] C. Carlsson, R. Fullér, “On Possibilistic Mean Value and Variance of Fuzzy Numbers,” Fuzzy Sets & Systems, vol. 122, no.2, pp315-326, 2001.

[16] W.G. Zhang, Y.L. Wang, Z.P. Chen, “Possibilistic mean-variance models and efficient frontiers for portfolio selection problem,” Information Sciences an International Journal, vol. 177, no.13, pp2787-2801, 2007

[17] Q.L. Wei, Data envelopment analysis model for evaluating relative effectiveness-DEA and network DEA. Beijing: China Renmin University Press, 2012 (Chinese)

[18] C. Kao, L.S. Tai, “Fuzzy efficiency measures in data envelopment analysis,” Fuzzy Sets and Systems, vol. 113, no.3, pp427-437, 2000

[19] A.W. Mugera. “Measuring technical efficiency of dairy farms with imprecise data: a fuzzy data envelopment analysis approach,” Australian Journal of Agricultural and Resource Economics, vol. 57, no.4, pp501-520, 2013

[20] W.G. Zhang, X.L. Zhang, W.J. Xu. “A risk tolerance model for portfolio adjusting problem with transaction costs based on possibilistic moments,” Insurance Mathematics & Economics, vol. 46, no.3, pp493-499, 2010

[21] M.K. Mehlawat, A. Kumar, S. Yadav, et al. “Data envelopment analysis based fuzzy multi-objective portfolio selection model involving higher moments,” Information Sciences, vol. 460, pp128-150, 2018