Research on Mean-Variance-Efficiency Portfolio of Fuzzy DEA Based on Possibility Theory

Xue Deng, Cuirong Huang, Yusheng Liu

Abstract: The actual financial market is complicated and volatile, in which most investment portfolio models include the historical return of stock as an important indicator while overlooking the impact of stock efficiency. Meanwhile we have noticed that the specific indicators of fuzzy input and fuzzy output enable the fuzzy DEA model to effectively measure stock efficiency. Accordingly, this paper uses fuzzy DEA model to analyze the stock efficiency, with equity multiplier, price earnings ratio and beta value as fuzzy input, total assets turnover, earning per share and net profit growth rate as fuzzy output. Secondly, this paper takes the possibility mean and possibility variance as the measurement of portfolio return and risk, so as to build the mean-variance-efficiency portfolio model. And the genetic algorithm of exponential fitness function is applied to the model in order to satisfy the requirement of fitness values being non-negative. At the end, an empirical example is given to verify the feasibility of the model and improved algorithm, and we also prove that there is a certain correlation between stock efficiency and return. As shown by the results, it is quite indispensable to consider the portfolio efficiency in the actual financial market, which can provide investors with more comprehensive, scientific and effective decision-making scheme.

Index Terms: Fuzzy DEA model, Genetic algorithm, Mean-Variance-Efficiency model, Fuzzy portfolio, Possibility theory

I. INTRODUCTION

The portfolio selection theory proposed by Markowitz [1] is the origin of modern financial theories. Markowitz, on the frame of probability theory, proposed the mean-variance (M-V) model. The major matter of portfolio management, which concerns investors most, is "how to maximize the return while containing the risk" or "how to minimize the risk given a certain return". In this case, investors must comprehensively take into account lots of internal and external factors, which inspires researchers to discover a better portfolio model in actual consideration of fuzzy uncertainty and random uncertainty.


The data envelopment analysis (DEA) is a valuable tool for measuring the relative efficiency of a decision-making unit (DMU). Now, it has raised a heated discussion among scholars. For example, Liu and Wang [11] proposed three secondary goal models, including the initial efficiency value of DMUs, based on DEA cross-evaluation. Mirdehghan et al. [12] studied the relations among technical efficiency, cost efficiency and revenue efficiency in the DEA.

In addition to the optimal asset allocation of a portfolio, selection of better-performing stocks also stands out for optimizing investment portfolios. The DEA has gradually come to the eyesight of scholars regarding portfolio selection, and therefore the portfolio performance has been considered in the model. Chen et al. [13] combined the mean-semi-variance model and the cross-efficiency model derived from DEA to construct a synthesized model of multi-objective fuzzy portfolio. Zhou et al. [14] constructed different evaluation models within the framework of fuzzy portfolio. Currently when compared to the large amount literature mentioned above, there exists few studies concentrating on combining DEA with fuzzy to make stock efficiency one of the objective functions.

In this paper, possibility theory and fuzzy set theory focus on exploring portfolio selection under the fuzzy circumstance. Combined with fuzzy number, the DEA introduces stock efficiency into the traditional mean-variance model. What’s more, the genetic algorithm, by applying six different preference coefficients, can solve the portfolio model.
In Section II, the preliminary knowledge on possibility theory and DEA is given. The fuzzy multi-objective mean-variance-efficiency portfolio model is constructed in the Section III. Section IV introduces the genetic algorithm used in this paper. In Section V, the feasibility of the model and algorithm is verified by an empirical example. Section VI summarizes the whole paper and draws the conclusion.

II. PRELIMINARIES

A. Possibility Theory

Definition 1 [15]: Fuzzy number \( A \in F(\mathbb{R}) \). For \( \forall \gamma \in [0, 1] \), it holds that \( [A^\gamma] = [\bar{a}(\gamma), \tilde{a}(\gamma)] \). The possibility mean of \( A \) can be defined as

\[
E(A) = \int_0^1 \gamma (\bar{a}(\gamma) + \tilde{a}(\gamma)) d\gamma.
\]

Definition 2 [16]: \( A \in F(\mathbb{R}) \). For \( \forall \gamma \in [0, 1] \), it stands that \( [A^\gamma] = [\bar{a}(\gamma), \tilde{a}(\gamma)] \). The possibility variance of \( A \) can be determined as

\[
Var(A) = \int_0^1 \gamma \left( (\tilde{a}(\gamma) - E(A))^2 + (\bar{a}(\gamma) - E(A))^2 \right) d\gamma.
\]

Definition 3 [16]: \( A, B \in F(\mathbb{R}) \). Let \( \forall \gamma \in [0, 1] \), it holds that \( [A^\gamma] = [\bar{a}(\gamma), \tilde{a}(\gamma)] \) and \( [B^\gamma] = [\bar{b}(\gamma), \tilde{b}(\gamma)] \). The possibility covariance of \( A \) and \( B \) is specified by

\[
Cov(A, B) = \int_0^1 \gamma \left( (\bar{a}(\gamma) - E(A))(\bar{b}(\gamma) - E(B)) + (\tilde{a}(\gamma) - E(A))(\tilde{b}(\gamma) - E(B)) \right) d\gamma.
\]

Theorem 1 [15]: Provided that \( \forall \lambda, \mu \in \mathbb{R} \), we have

\[
E(\lambda A + \mu B) = \lambda E(A) + \mu E(B),
\]

\[
Var(\lambda A + \mu B) = \lambda^2 Var(\phi(\lambda) A) + \mu^2 Var(\phi(\mu) B) + 2\lambda \mu Cov(\phi(\lambda) A, \phi(\mu) B),
\]

where \( \phi(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases} \)

Theorem 2 [15]: Let \( A_1, A_2, \ldots, A_n \in F(\mathbb{R}) \) be fuzzy numbers. For any \( n \) real numbers \( \lambda_1, \lambda_2, \ldots, \lambda_n \in \mathbb{R} \), we have

\[
Var\left( \sum_{i=1}^{n} \lambda_i A_i \right) = \sum_{i=1}^{n} \lambda_i^2 Var(\phi(\lambda_i) A_i) + 2 \sum_{i<j} \lambda_i \lambda_j Cov(\phi(\lambda_i) A_i, \phi(\lambda_j) A_j).
\]

The triangular fuzzy number \( A \) is denoted as \( A = (a, \alpha, \beta) \), and its membership function is as follows:

\[
\mu_a(x) = \begin{cases} 1 - \frac{a-x}{\alpha}, & a \leq x \leq a, \\ \frac{1 - x-a}{\beta}, & a \leq x \leq a + \beta, \\ 0, & \text{other} \end{cases}
\]

The \( \lambda \)-cut set of \( A \) is given by \([A^\lambda] = [a - (1-\gamma)\alpha, a + (1-\gamma)\beta]\), where \( \forall \gamma \in [0, 1] \).

Based on Definition 1 and 2, the possibility mean and possibility variance of \( A \) are as follows:

\[
E(A) = \int_0^1 \gamma (a + (1-\gamma)\beta + a - (1-\gamma)\alpha) d\gamma = a + \beta - \frac{\alpha}{6},
\]

\[
Var(A) = \int_0^1 \gamma \left( (a - (1-\gamma)\alpha - a + \beta - \frac{\alpha}{6})^2 \right) d\gamma
\]

\[
+ \int_0^1 \gamma \left( (a + (1-\gamma)\beta - a - \frac{\beta}{6})^2 \right) d\gamma
\]

\[
= \left[ \frac{\alpha + \beta}{6} \right]^2 + \left[ \frac{\beta + \alpha}{2} + \frac{\beta - \alpha}{2} \right]^2.
\]

B. Data Envelopment Analysis (DEA)

Data envelopment analysis (DEA) [17] is a method of evaluating the relative effectiveness of the same type of decision-making units (DMU) based on the multiple input and output indicators. Specifically, the notations involved in DEA model are shown in TABLE I.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j )</td>
<td>Input indicator, ( j = 1, 2, \ldots, m )</td>
</tr>
<tr>
<td>( r )</td>
<td>Output indicator, ( r = 1, 2, \ldots, s )</td>
</tr>
<tr>
<td>( i )</td>
<td>Decision making unit (DMU), ( i = 1, 2, \ldots, n )</td>
</tr>
<tr>
<td>( x_s )</td>
<td>Input of DMU ( i ) in item ( j )</td>
</tr>
<tr>
<td>( y_r )</td>
<td>Output of DMU ( i ) in item ( r )</td>
</tr>
<tr>
<td>( v_j )</td>
<td>A measurement of the input in item ( j ), weight coefficient</td>
</tr>
<tr>
<td>( u_r )</td>
<td>A measurement of the output in item ( r ), weight coefficient</td>
</tr>
</tbody>
</table>

Thereby, the efficiency evaluation index of each DMU is defined as

\[
h_i = \frac{\sum_{j=1}^{m} u_j y_{ri}}{\sum_{j=1}^{s} v_j x_{ji}}, i = 1, 2, \ldots, n.
\]

Here, \( h_i \) represents the relative efficiency of DMU \( i \), proper weight coefficients \( u_j, v_j \) enable \( h_i \leq 1, i = 1, \ldots, n \).

Regarding the DMU \( h_i \), with the greater \( h_i \), normally comes the greater output while the input is equal to that of others. For this reason, we are in passionate pursuit of maximizing the efficiency evaluation index, and therefore found an objective function of maximizing the index under the condition that the indexes of all decision-making units are not larger than one. The model is structured by

\[
\begin{array}{ll}
\text{max} & \sum_{j=1}^{m} u_j y_{ri} / \sum_{j=1}^{s} v_j x_{ji} \\
\text{s.t.} & \sum_{j=1}^{m} u_j y_{ri} / \sum_{j=1}^{s} v_j x_{ji} \leq 1 \\
& u_r \geq 0, v_j \geq 0, i = 1, 2, \ldots, n
\end{array}
\]

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C. Fuzzy DEA Model

Normally, the commonly used model in the financial market is the standard DEA one, which requires high clarity and exact accuracy of input- and output indicators. However, in reality, it is impossible to fully describe the market with totally clear data. In this case, the fuzzy DEA utilizing imprecise and ambiguous data instead of clear one. The two models aforementioned vary from each other majorly regarding the input- and output data clarity (fuzzy number or clear number) of DMUs.

Kao and Tai [18] proposed to obtain the range of fuzzy numbers by means of $\lambda$ – cut set, and cleverly transform the fuzzy DEA model into the standard DEA one with a certain range (upper and lower limits of fuzzy numbers). This solution also applies to the current case. Furthermore, Mugera [19] once pointed out that triangular fuzzy number is most used, and thus the input and output indicators of DMUs are regarded as triangular fuzzy numbers.

Let the input indicator $j$ of DMU $i$ be $\tilde{x}_i = \left( x_{i}^{\alpha}, x_{i}^{\beta}, \alpha_{i}, \beta_{i} \right)$, and its output indicator $r$ be $\tilde{y}_r = \left( y_{r}^{\alpha}, y_{r}^{\beta}, \alpha_{r}, \beta_{r} \right)$. The uncertain range of the fuzzy numbers $\tilde{x}_i, \tilde{y}_r$ can be determined by different confidence intervals defined by the $\lambda$ – cut set. That is, the $\lambda$ – cut set ($0 \leq \lambda \leq 1$) can specify the lower- and upper limits of the fuzzy numbers as follows:

$$(x_{i})_{\lambda} = \left[ \left( x_{i}^{\alpha} - \alpha_{i}, x_{i}^{\beta} + \beta_{i} \right), \left( \lambda x_{i}^{\alpha} + \left( x_{i}^{\alpha} - \alpha_{i} \right), \left( x_{i}^{\beta} + \beta_{i} - \lambda \beta_{i} \right) \right) \right]$$

and

$$(y_{r})_{\lambda} = \left[ \left( y_{r}^{\alpha} - \alpha_{r}, y_{r}^{\beta} + \beta_{r} \right), \left( \lambda y_{r}^{\alpha} + \left( y_{r}^{\alpha} - \alpha_{r} \right), \left( y_{r}^{\beta} + \beta_{r} - \lambda \beta_{r} \right) \right) \right]$$

When $\lambda = 1$, the fuzzy numbers convert to the clear ones, meaning no uncertainty. When $\lambda = 0$, the numbers drop between the full fuzzy range. That is, the uncertainty is adjusted by modifying $\lambda$. Further the efficiency score can be obtained in the concrete $\lambda$ – cut set. By definition given by Kao and Tai [18], the output indicators can take the lower limits of the fuzzy numbers, and the input ones can take the upper limits instead when provided a specific value of the $\lambda$ – cut set, which derives the lower limit of efficiency ($E_i^L$) of DMU $i$. The specific expression is as follows:

$$E_i^L = \frac{\sum_{r=1}^{n} u_r \left( y_{r} \right)_{\lambda}^L}{v_0 + \sum_{j=1}^{m} v_j \left( x_{j} \right)_{\lambda}^L}$$

(14)

Conversely, the output indicators can take the upper limit of the fuzzy numbers while the input ones can take the lower limit of the numbers. Hereby the upper limit of the efficiency ($E_i^U$) of DMU $i$ can be obtained by

$$E_i^U = \frac{\sum_{r=1}^{n} u_r \left( y_{r} \right)_{\lambda}^U}{v_0 + \sum_{j=1}^{m} v_j \left( x_{j} \right)_{\lambda}^U}$$

(15)

where $v_0$ is constant and $u_r, v_j$ are weight coefficients.

When we maximize the lower limit of DMU $i$’s efficiency, where all DMUs’ efficiency is not greater than 1, the model (16) can be written as

$$\max \sum_{r=1}^{n} u_r \left( y_{r} \right)_{\lambda}^L / \left( v_0 + \sum_{j=1}^{m} v_j \left( x_{j} \right)_{\lambda}^L \right)$$

s.t. \[
\sum_{r=1}^{n} u_r \left( y_{r} \right)_{\lambda}^L / \left( v_0 + \sum_{j=1}^{m} v_j \left( x_{j} \right)_{\lambda}^L \right) \leq 1
\]

$$i = 1, \ldots, n, \; i \neq i_0, \; u_r, v_j > 0, \; v_0 \text{ is constant}$$

Similarly, if the objective function aims to maximize the upper limit of DMU $i$’s efficiency, the model (17) can be expressed as

$$\max \sum_{r=1}^{n} u_r \left( y_{r} \right)_{\lambda}^U / \left( v_0 + \sum_{j=1}^{m} v_j \left( x_{j} \right)_{\lambda}^U \right)$$

s.t. \[
\sum_{r=1}^{n} u_r \left( y_{r} \right)_{\lambda}^U / \left( v_0 + \sum_{j=1}^{m} v_j \left( x_{j} \right)_{\lambda}^U \right) \leq 1
\]

$$i = 1, \ldots, n, \; i \neq i_0, \; u_r, v_j > 0, \; v_0 \text{ is constant}$$

By solving the models (16) and (17), we are able to have the upper- and lower limits of the efficiency corresponding to each value of $\lambda$. Assuming $\lambda = 1$, the efficiency is the same as that obtained by the standard DEA. While $\lambda = 0$, the efficiency range refers to that of the fuzzy numbers. Furthermore, the efficiency obtained with $0 < \lambda < 1$ does not exceed the efficiency range.

III. FUZZY MULTI-OBJECTIVE PORTFOLIO MODEL

Supposing that the investor plans to invest in $n$ stocks, the return of the stock $i$ is $r_i (i = 1, 2, \ldots, n)$, and its investment proportion is $x_i (i = 1, 2, \ldots, n)$. The return, possibility mean and variance of the portfolio $R$ are as follows:

$$R = \sum_{i=1}^{n} x_i r_i$$

(18)

$$E(R) = E \left( \sum_{i=1}^{n} x_i r_i \right)$$

(19)

$$\text{Var}(R) = \sum_{i=1}^{n} x_i^2 \text{Var}(r_i) + 2 \sum_{i<j}^{n} x_i x_j \text{Cov}(r_i, r_j)$$

(20)

where $\text{Cov}(r_i, r_j)$ represents the covariance between $r_i$ and $r_j$. Next, we utilize the fuzzy DEA model to calculate the efficiency of stocks, so that the index drops within the range of fuzzy numbers. Provided the $\lambda$ – cut set, let the efficiency interval of stock $i$ be $\left( \left( E_i^L \right)_\lambda, \left( E_i^U \right)_\lambda \right)$. By averaging the efficiency, we have the efficiency evaluation index of the stock $i$ as $\left( \left( E_i^L + \left( E_i^U \right)_\lambda \right) / 2 \right)$. Furthermore, the overall portfolio efficiency is a linear combination of investment proportion and stock efficiency. The expression is as follows:
\[ e = \sum_{i=1}^{n} \left( \frac{(E_i)^{l_i} + (E_i)^{u_i}}{2} \right) \]  
(21)

As mentioned above, the efficiency of such a portfolio is measured by efficiency evaluation index computed by the Fuzzy DEA model. Assuming investors tend to be rational, say, they seek to maximize the return and the efficiency, and minimize the risk of the portfolio, the corresponding model is specified by

\[
\begin{align*}
\max & \quad E(R) = E \left( \sum_{i=1}^{n} x_i r_i \right) \\
\min & \quad \text{Var}(R) = \sum_{i=1}^{n} x_i ^2 \text{Var}(r_i) + 2 \sum_{i<j} x_i x_j \text{Cov}(r_i, r_j) \\
\text{s.t.} & \quad x_i = 1, \quad l_i \leq x_i \leq u_i, \quad i = 1, \ldots, n
\end{align*}
\]

(22)

where \(l_i\) and \(u_i\) are the upper- and lower limits. The model (22) is equivalent of the following model (23).

\[
\begin{align*}
\max & \quad E(R) = E \left( \sum_{i=1}^{n} x_i r_i \right) \\
\max & \quad -\text{Var}(R) = -\sum_{i=1}^{n} x_i ^2 \text{Var}(r_i) - 2 \sum_{i<j} x_i x_j \text{Cov}(r_i, r_j) \\
\text{s.t.} & \quad x_i = 1, \quad l_i \leq x_i \leq u_i, \quad i = 1, \ldots, n
\end{align*}
\]

(23)

\(IV.\) GENETIC ALGORITHM

A. Basic Definition of Genetic Algorithm

The genetic algorithm is employed to solve the model (23). The algorithm initially encodes each solution from the problem domain, and establishes a fitness function based on the objective function. Later, all solutions are put into the fitness function to select the solutions with higher fitness values, which can form a new species. On this new species, we continue to perform selection, crossover and mutation operations iteratively so as to get the optimal solution with generality. Next are five basic steps of the algorithm.

1) Initialization: the feasible solution \(X = (x_1, x_2, \ldots, x_n)\) of the domain is generated randomly. It is then coded as chromosome \(C = (c_1, c_2, \ldots, c_n)\) \((l_i \leq c_i \leq u_i)\). The corresponding relation between \(x_i\) and \(c_i\) is as follows:

\[ x_i = \frac{c_i - l_i}{u_i - l_i}, \quad i = 1, 2, \ldots, n. \]  
(24)

This practice can ensure that \(\sum_{i=1}^{n} x_i = 1\) holds true. Assume that the size of the entire species is \(\text{pop}_\text{size}\), where the number of all solutions meet the condition before. Repeat the equation (24) for \(\text{pop}_\text{size}\) times to obtain the chromosome labeled as \(C_1, C_2, \ldots, C_{\text{pop}_\text{size}}\).

2) Fitness function: a function to judge the quality of an individual in a species. In order to prevent the unreasonable distribution of the fitness values, we adjust the scaling of the fitness function. The requirement of fitness value being non-negative should be satisfied as well. For the reasons above, the following exponential fitness function fits here well, that is

\[ F(X) = e^{f(X)}, \]  
(25)

where \(f(X)\) is the weighted objective function by the preference coefficients of the model (23). Substituting the individuals \(X_1, X_2, \ldots, X_{\text{pop}_\text{size}}\) in the species into the equation (25), we have the corresponding fitness values recorded as \(F_i (i = 1, 2, \ldots, \text{pop}_\text{size})\).

3) Selection: The selection of individuals from the parent species to the next generation according to the values of the fitness function, reflecting a survival procedure of the species. The possibility of individual \(X_i\) being selected is

\[ P(X_i) = \frac{F_i}{\sum_{j=1}^{\text{pop}_\text{size}} F_j}, i = 1, 2, \ldots, \text{pop}_\text{size}. \]  
(26)

4) Crossover: Re-structuring of the parent individuals. The algorithm performs this operation on each individual with probability \(P_c\). For example, the convex combination of two parent individuals \(C_1, C_2\) can derive two new individuals

\[ C_1 = \mu C_1 + (1-\mu) C_2, C_2 = \mu C_2 + (1-\mu) C_1. \]  
(27)

The algorithm will inspect whether the new individuals generated by crossover meet the constraint before. If so, the new individuals will enter the next generation.

5) Mutation: the mutation is performed on individuals within the population with probability \(P_m\). Suppose that the chromosome \(C\) from the parent generation is \(C = (c_1, c_2, \ldots, c_n)\), where a gene \(c_i\) is randomly selected for mutation. Then we have a new individual \(C' = (c'_1, c'_2, \ldots, c'_n)\), and the \(c'_i\) is given by

\[ c'_i = l_i + Y \cdot (u_i - l_i), \]  
(28)

within which \(Y\) is a random number on the interval \([0,1]\).

Similarly, the algorithm will also inspect whether the new individuals generated by mutation are limited by the condition before. If so, the new individuals will enter the next generation.

B. Calculation Steps

The specific flow of genetic algorithm is shown below:

Step 1: The feasible solutions of the problem domain are encoded to obtain the initial population \(\text{pop}_\text{size}(0)\).

Step 2: The fitness values of individuals in the population \((\text{pop}_\text{size}(k))\) are computed through the exponential fitness function.

Step 3: The selection of the superior individuals from the parent generation to the child generation;
Step 4: The crossover is performed on each of the new generation with probability $P_c$:

$$pop\_size(k) \leftarrow \text{Crossover}[pop\_size(k)];$$

Step 5: The mutation is conducted on each of the new generation with probability $P_m$:

$$pop\_size(k) \leftarrow \text{Mutation}[pop\_size(k)];$$

Step 6: If the conditions are met, then export the solutions; if the conditions are not met, repeat the step 2–6.

V. EMPIRICAL EXAMPLE

A. Fuzzy DEA Efficiency Analysis

In this section, the effectiveness and applicability of the fuzzy DEA and portfolio model (23) are proved by an actual example. We select eight stocks that are traded in Shanghai stock market of China. The possibility distribution can be obtained by means proposed by Zhang et al. [20]. We collect the weekly closing prices of the eight stocks from October 2016 to October 2018, and therefore get the possibility distribution as shown in TABLE II.

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>THE POSSIBILITY DISTRIBUTION OF EIGHT STOCK RETURNS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>1</td>
</tr>
<tr>
<td>Code</td>
<td></td>
</tr>
<tr>
<td>$a_i$</td>
<td></td>
</tr>
<tr>
<td>$\beta_i$</td>
<td></td>
</tr>
<tr>
<td>$a_i$</td>
<td></td>
</tr>
<tr>
<td>$\beta_i$</td>
<td></td>
</tr>
</tbody>
</table>

For the purpose of working out the efficiency of the eight stocks, we select six indicators similar to those the reference [21]. The input indicators are equity multiplier, price earnings ratio and beta value. The output indicators are total assets turnover, earning per share and net profit growth rate. Similarly, this paper uses the historical data of the six indicators of the eight stocks from October 2016 to October 2018, and estimates their possibility distribution. The specific results are shown in TABLE III–VIII.

Using the fuzzy DEA model, combined with the data of the six indicators, we can get the efficiency of each stock given various $\lambda$–cut sets. The results are presented in TABLE IX.

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>DISTRIBUTION OF EQUITY MULTIPLIER OF THE EIGHT STOCKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>1</td>
</tr>
<tr>
<td>Code</td>
<td></td>
</tr>
<tr>
<td>$a_i$</td>
<td></td>
</tr>
<tr>
<td>$\beta_i$</td>
<td></td>
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</tbody>
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<thead>
<tr>
<th>TABLE IV</th>
<th>DISTRIBUTION OF PRICE EARNINGS RATIO OF THE EIGHT STOCKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>1</td>
</tr>
<tr>
<td>$a_i$</td>
<td></td>
</tr>
<tr>
<td>$\beta_i$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE V</th>
<th>DISTRIBUTION OF BETA VALUE OF THE EIGHT STOCKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>1</td>
</tr>
<tr>
<td>$a_i$</td>
<td></td>
</tr>
<tr>
<td>$\beta_i$</td>
<td></td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>TABLE VI</th>
<th>DISTRIBUTION OF TOTAL ASSETS TURNOVER OF THE EIGHT STOCKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>1</td>
</tr>
<tr>
<td>$a_i$</td>
<td></td>
</tr>
<tr>
<td>$\beta_i$</td>
<td></td>
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<tr>
<th>TABLE VII</th>
<th>DISTRIBUTION OF EARNING PER SHARE OF THE EIGHT STOCKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>1</td>
</tr>
<tr>
<td>$a_i$</td>
<td></td>
</tr>
<tr>
<td>$\beta_i$</td>
<td></td>
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<table>
<thead>
<tr>
<th>TABLE VIII</th>
<th>DISTRIBUTION OF NET PROFIT GROWTH RATE OF THE EIGHT STOCKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>1</td>
</tr>
<tr>
<td>$a_i$</td>
<td></td>
</tr>
<tr>
<td>$\beta_i$</td>
<td></td>
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<table>
<thead>
<tr>
<th>TABLE IX</th>
<th>EFFICIENCY OF THE EIGHT STOCKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>$\lambda = 0.00$</td>
</tr>
<tr>
<td>$(E_i^y)$</td>
<td>1</td>
</tr>
<tr>
<td>$a_i$</td>
<td></td>
</tr>
<tr>
<td>$\beta_i$</td>
<td></td>
</tr>
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<table>
<thead>
<tr>
<th>TABLE X</th>
<th>EFFICIENCY OF THE EIGHT STOCKS</th>
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<tbody>
<tr>
<td>Stock</td>
<td>$\lambda = 0.00$</td>
</tr>
<tr>
<td>$(E_i^y)$</td>
<td>1</td>
</tr>
<tr>
<td>$a_i$</td>
<td></td>
</tr>
<tr>
<td>$\beta_i$</td>
<td></td>
</tr>
</tbody>
</table>
According to TABLE X, we can find that when \( \lambda = 0.00, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70 \) or 0.80, the upper limit of the efficiency of the eight stocks is always 1. Then we draw the efficiency lower limits of the eight stocks regarding the different values of \( \lambda \)-cut set, as shown in Fig. 1.

### B. Preference Coefficient

The portfolio model in this paper proposes a preference-weighting method for investors, with a preference coefficient 

\[
    r = (r_X, r_Y, r_Z)
\]

in which \( r_X, r_Y, r_Z \) represent their preference for return, risk and efficiency separately, reflecting their attitudes towards different portfolios. In this paper, six preference coefficients are introduced. Specifically, the first one (2,1,2) fits radical investors because this cluster prefers return and efficiency to risk; the second one (1,2,1) is suitable for conservative investors because their preference focuses on risk rather than return and efficiency; and the third one is (4,3,1). The fourth preference coefficient is (1,0,1), indicating that investors only pay attention to return and efficiency without considering risk; the fifth preference coefficient is (1,1,1), stating that investors give equal preference to the three objectives; the sixth preference coefficient is (1,1,0), which is equivalent to the traditional mean-variance model.

### C. Analysis of Solution Results of the Model

Using the algorithm designed in Section IV, we solve the model (23) based on the six preference coefficients, and then get the proportions of the stocks with \( \lambda = 0.1, 0.3, 0.5, 0.7 \). The specific results are shown in TABLE X-XIII. The TABLE X-XIII show us that the investment proportions of stock 1 and 8 in the mean-variance model are non-zero while the figures are zero in the mean-variance-efficiency model. This is because stock 1 and 8 have lower efficiency and return. Such a phenomenon can lead to a conclusion that the efficiency and return of the portfolio have a certain correlation, and hence investors could consider the efficiency of the portfolio when making decisions.
VI. CONCLUSIONS

This paper proposes a multi-objective fuzzy portfolio model considering return, variance and efficiency. Its specific goal is to maximize return and efficiency, and minimize variance. The efficiency of a portfolio is calculated by the fuzzy DEA using six fuzzy indicators of stocks. Equity multiplier, price earnings ratio and Beta value are regarded as three input indicators of the fuzzy DEA. Total assets turnover, earning per share and net profit growth rate are the three output indicators of the model. Finally, an actual example is given to solve the model by using genetic algorithm under different preference coefficients. The actual results indicate that it is necessary to consider portfolio efficiency and offer more decision-making options for investors.

REFERENCES


| TABLE XII | INVESTMENT PROPORTION CORRESPONDING TO DIFFERENT PREFERENCE COEFFICIENTS WHEN $\lambda = 0.5$
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<td>$x_3$</td>
<td>$x_4$</td>
<td>$x_5$</td>
<td>$x_6$</td>
<td>$x_7$</td>
<td>$x_8$</td>
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<td>0.0000</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0525</td>
<td>0.0015</td>
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</table>

| TABLE XIII | INVESTMENT PROPORTION CORRESPONDING TO DIFFERENT PREFERENCE COEFFICIENTS WHEN $\lambda = 0.7$
<table>
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</thead>
<tbody>
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<td>$x_2$</td>
<td>$x_3$</td>
<td>$x_4$</td>
<td>$x_5$</td>
<td>$x_6$</td>
<td>$x_7$</td>
<td>$x_8$</td>
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