A Variable Stepsize Sparsity Adaptive Matching Pursuit Algorithm

Yuehua Zhang, Yufeng Liu, and Xinhe Zhang

***Abstract*—The sparsity adaptive matching pursuit (SAMP) algorithm reconstructs the signal through multiple iterations with a fixed stepsize, which consumes too much time. To solve this problem, a variable stepsize sparsity adaptive matching pursuit (VSSAMP) algorithm is proposed. The sparsity is pre-estimated, and then the variable stepsize SAMP is used to reconstruct the signal. Different from the SAMP algorithm, VSSAMP algorithm determines the stepsize according to the energy difference of two consecutive residues. The experiment results show that the reconstruction probability of VSSAMP algorithm is slightly slower than SAMP algorithm in one-dimensional signal, and the reconstruction quality of VSSAMP is basically equal to SAMP algorithm in tow-dimensional image. Most notably, the reconstruction time of VSSAMP algorithm is significantly lower than SAMP in reconstruction of one-dimensional signal and two-dimensional image.**

***Index Terms*—compressed sensing, matching pursuit, reconstruction algorithm, sparsity adaption.**

I. Introduction

ACCORDING to the Nyquist-Shannon sampling theorem, perfect reconstruction of a signal is possible when the sampling frequency is greater than twice the maximum frequency of the signal being sampled. But generally speaking, the hardware devices are difficult to meet the requirement for some high-frequency signals. Compressed sensing (CS) has attracted much attention in recent years [1]-[5]. Compressed sensing has been a research hotspot in the fields of image compression [6], medical imaging [7], radar imaging [8], remote sensing satellite photography [9], quantum state tomography [10], and a large number of research achievements have emerged. Compressed sensing theory include three parts: signal sparse representation, observation matrix construction and reconstruction algorithm [11]-[12].

Consider a real-valued, finite-length, one-dimensional, discrete-time signal $\mathbf{x}$, which can be viewed as an $N \times 1$ column vector in $\mathbb{R}^N$ with entries $x[i]$, $i = 1, 2, ..., N$. If only K non-zero elements in $\mathbf{x}$, and other elements are zero, we can say the sparsity is K. In compressed operation, the K-sparse signal $\mathbf{x}$ is transformed into M-dimensional measurement $\mathbf{y}$, where $M < N$. The measurement system can be expressed as:

$$\mathbf{y} = \mathbf{\Phi x}, \tag{1}$$

where the observation vector $\mathbf{y} \in \mathbb{C}^M$, the observation matrix $\mathbf{\Phi} \in \mathbb{C}^{M\times N}$. The observation dimension M is smaller than sparse signal dimension N for most scenarios, that is, the number of unknowns is greater than the number of equations. Thus, it is impossible to obtain an accurate reconstruction of signal $\mathbf{x}$ using conventional inverse transform of observation matrix $\mathbf{\Phi}$. But due to the prior information of signal sparsity K, the signal $\mathbf{x}$ can be recovered by ℓ_0-minimization as follows [2]:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_0 \quad \textbf{s. t.} \quad \mathbf{y} = \mathbf{\Phi x}. \tag{2}$$

To the best of our knowledge, solving the problem essentially requires exhaustive searches over all subsets of column of observation matrix $\mathbf{\Phi}$, and the search procedure is exponentially complexity.

If the observation matrix $\mathbf{\Phi}$ satisfies the restricted isometry property (RIP) [2], the signal $\mathbf{x}$ can be recovered by solving the ℓ_1-minimization optimization problem [13]:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \textbf{s. t.} \quad \mathbf{y} = \mathbf{\Phi x}. \tag{3}$$

The observation matrix $\mathbf{\Phi}$ satisfies the RIP of order K if there exists a constant $\delta \in (0, 1)$ such that

$$(1-\delta)\|\mathbf{x}\|_2^2 \le \|\mathbf{\Phi x}\|_2^2 \le (1+\delta)\|\mathbf{x}\|_2^2 \tag{4}$$

for any K-sparse signal $\mathbf{x}$.

Up to now, the reconstruction methods of Eq. (1) mainly include: convex optimization algorithm and greedy reconstruction algorithm. The smoothed ℓ_0 norm method [14], alternating direction method of multipliers (ADMM) [15], interior-point method [16], iterative re-weighted least squares (IRLS) [17] method are typically convex optimization methods. Recently, some greedy algorithms have been proposed, include orthogonal matching pursuit (OMP) [18], generalized orthogonal matching pursuit (gOMP) [19], regularized orthogonal matching pursuit (ROMP) [20], subspace pursuit (SP) [21], compressive sampling matching pursuit (CoSaMP) [22], stagewise orthogonal matching pursuit (StOMP) [23], stagewise weak orthogonal matching pursuit (SWOMP) [24], sparsity adaptive matching pursuit (SAMP) [25], and so on. In above-mentioned algorithms, OMP, gOMP, ROMP, SP and CoSaMP algorithms need to know the sparsity as prior information, while StOMP, SWOMP and SAMP algorithms don't have to know the sparsity. Compared with some greedy algorithms which need to know the sparsity, the most prominent feature of SAMP lies in the fact that it doesn't have to know the sparsity.

In SAMP, the signal is recovered by adopting fixed stepsize in each iteration. It consumes too much iteration time. In order to reduce the running time, an improved SAMP

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algorithm, called variable stepsize sparsity adaptive matching pursuit (VSSAMP), is proposed. The primary contributions of this paper are twofold.

1) In order to speed up the running speed, we present a new sparsity pre-estimation strategy. In the first stage, the sparsity estimation strategy is used to per-estimate sparsity, which avoid the iteration step by step.
2) We present an improved SAMP algorithm, which the stepsize is changed according to the difference of two successive residues. If the difference is large than the preseted threshold, a large stepsize is adopted; otherwise a small stepsize is used.

The remainder of this paper is organized as follows. In Section II, we briefly introduce the sparsity adaptive matching pursuit algorithm. The proposed variable stepsize sparsity adaptive matching pursuit algorithm is presented in Section III. The simulation results and performance analysis are demonstrated in Section IV. Finally, Section V concludes this paper.

Notation: Boldface uppercase letters denote matrices, boldface lowercase letters represent vectors. $\|\cdot\|_0$, $\|\cdot\|_1$, $\|\cdot\|_2$ represent the zero-norm, one-norm and two-norm of a vector, respectively; $\lfloor\cdot\rceil$ is the operation of rounding a real number to the nearest integer; $(\cdot)^H$ and $(\cdot)^\dagger$ stand for the conjugate transpose and pseudo-inverse of a vector or a matrix, respectively; $|\cdot|$ denotes the module of a vector or a complex number; $A \cup B$ is the union of set A and set B, $card(\cdot)$ is the cardinality of a given set; $\mathbb{R}$ and $\mathbb{C}$ denote the field of real and complex numbers, respectively.

II. Sparsity Adaptive Matching Pursuit Algorithm

The greedy recovery algorithms can be grouped into top-down and down-top approaches by structure. Due to the backtracking strategy, the top-down methods such as SP and CoSaMP can identify the true support set accurately. The down-up method estimates the sparsity by step by step. The SAMP algorithm proposed in [25] is a combination of both bottom-up and top-down principles.

The SAMP adopts a stagewise approach to expand the real support set step by step. Meanwhile, it takes the advantage of the backtracking strategy in SP/CoSaMP to refine the estimation of true support set in each iteration. From practical perspective, the most prominent feature of SAMP lies in the fact that it does not require the sparsity K as an input parameter. In fact, SAMP provides a general framework for the OMP and SP/CoSaMP algorithms. When stepsize is 1, it can be regarded as OMP with backtracking strategy. When stepsize equals to sparsity, it becomes SP algorithm. TABLE I represents the pseudo code of SAMP algorithm.

TABLE I
PSEUDO CODE OF SAMP ALGORITHM

Input: observation matrix $\mathbf{\Phi}$, observation vector $\mathbf{y}$, stepsize s.
Initialization:
  $\hat{\mathbf{x}} = 0$ {Trivial initialization}
  $\mathbf{r}_0 = \mathbf{y}$ {Initial residue}
  $F_0 = \emptyset$ {Empty finalist}
  $\mathcal{I} = s$ {Stepsize of finalist}
  $k = 1$ {Iteration index}
  $j = 1$ {Stage index}
  repeat
    $S_k = max(|\mathbf{\Phi^H r}_{k-1}|, \mathcal{I})$ {Preliminary test}
    $C_k = F_{k-1} \cup S_k$ {Make candidate list}
    $F = max(|\mathbf{\Phi}^\dagger_{C_k}\mathbf{y}|, \mathcal{I})$ {Final test}
    $\mathbf{r} = \mathbf{y} - \mathbf{\Phi}_F \mathbf{\Phi}^\dagger_F \mathbf{y}$ {Compute residue}
    if halting condition true **then**
      quit the iteration;
    else if $||\mathbf{r}||_2 \geq ||\mathbf{r}_{k-1}||_2$ **then**
      $j = j + 1$ {Update the stage index}
      $\mathcal{I} = j \times s$ {Update the size of finalist}
    else
      $F_k = F$ {Update the finalist}
      $\mathbf{r}_k = \mathbf{r}$ {Update the residue}
      $k = k + 1$
    end if
  until halting condition true;
Output: $\hat{\mathbf{x}} = \mathbf{\Phi}^\dagger_F \mathbf{y}$ {The estimated signal $\hat{\mathbf{x}}$ of input signal}

III. Variable Stepsize Sparsity Adaptive Matching Pursuit Algorithm

SAMP algorithm adopts fixed stepsize to recover the signal through multiple iterations, it results in too much running time. In order to reduce the running time, we employ two strategies. Firstly, we estimate the sparsity before iteration operation. Next, the stepsize is dynamically changed according to the difference between two successive residues. If the difference is large than the predetermined threshold, a large stepsize is adopted, else a small one is applied. The implementation of these two strategies ensure a faster running speed.

In phase I, the sparsity is estimated. We compute the absolute value of the inner product of observation matrix $\mathbf{\Phi}$ and observation vector $\mathbf{y}$ by

$$\mathbf{u} = \left|\mathbf{\Phi}^H \mathbf{y}\right|, \tag{5}$$

where vector $\mathbf{u}$ has N entries. The threshold is calculated by

$$Th = a \times max(\mathbf{u}), \tag{6}$$

where a is a preseted parameter, which directly determines the threshold; where the function $max(\cdot)$ returns the largest element of a array or a vector. The pre-estimated sparsity can be expressed as:

$$\mathcal{I} = card(\{t||\mathbf{\Phi}_t^H \mathbf{y}| > Th\}), \tag{7}$$

where the function $card(\cdot)$ finds the number of elements in a vector.

In phase II, the revised SAMP algorithm is applied to recover the signal. Different from the fixed stepsize of SAMP, the revised algorithm adopts different stepsize according to the difference between two successive residues.

The pseudo code of VSSAMP algorithm is summarized in TABLE II.

IV. Simulation

In this section, the performance of the proposed algorithm is presented and compared with above-mentioned algorithms. In order to ensure that the per-estimated sparsity does not exceed the real sparsity, the parameter selection experiment is carried out. Next, we carry out the simulation in one-dimensional signal and two-dimensional image.

TABLE II
PSEUDO CODE OF VSSAMP ALGORITHM

Input: observation matrix $\mathbf{\Phi}$, observation vector $\mathbf{y}$, stepsize s, parameter a, the threshold ε.
Initialization:
$\hat{\mathbf{x}} = 0$ {Trivial initialization}
$\mathbf{r}_0 = \mathbf{y}$ {Initial residue}
$F_0 = \emptyset$ {Empty finalist}
$\mathcal{I} = 0$ {Stepsize of finalist}
$k = 1$ {Iteration index}
step 1:
$Th = a \times max(|\mathbf{\Phi^H y}|)$ {Estimate the threshold of sparsity}
$\mathcal{I} = card(\{t||\mathbf{\Phi}_t^H \mathbf{y}| > Th\})$ {Obtain pre-estimated sparsity}
step 2:
repeat
$S_k = max(|\mathbf{\Phi^H r}_{k-1}|, \mathcal{I})$ {Preliminary test}
$C_k = F_{k-1} \cup S_k$ {Make candidate list}
$F = max(|\mathbf{\Phi}_{C_k}^{\dagger} \mathbf{y}|, \mathcal{I})$ {Final test}
$\mathbf{r} = \mathbf{y} - \mathbf{\Phi}_F \mathbf{\Phi}_F^{\dagger} \mathbf{y}$ {Compute residue}
if halting condition true **then**
quit the iteration;
else if $||\mathbf{r}_k||_2 - ||\mathbf{r}_{k-1}||_2 > \varepsilon$ **then**
$\mathcal{I} = \mathcal{I} + s$ {Update the size of finalist}
$F_k = F$ {Update the finalist}
$\mathbf{r}_k = \mathbf{r}$ {Update the residue}
$k = k + 1$
else
$\mathcal{I} = \mathcal{I} + \lfloor \frac{s}{2} \rceil$ {Update the size of finalist}
$F_k = F$ {Update the finalist}
$\mathbf{r}_k = \mathbf{r}$ {Update the residue}
$k = k + 1$
end if
until halting condition true;
Output: $\hat{\mathbf{x}} = \mathbf{\Phi}_F^{\dagger} \mathbf{y}$ {The estimated signal $\hat{\mathbf{x}}$ of input signal}

A. Parameter Selection Simulation Experiment

Experiment 1: In order to determine the weak matching parameter a, it is necessary to carry out the sparsity pre-estimated experiment. A one-dimensional sparse signal with length $N = 256$ is generated randomly, and the sparsity is $K = 35$. The observation matrix $\mathbf{\Phi}$ is $M \times N$ Gaussian matrix. The observation vector can be obtained by the measurement system (1). The simulation curve is demonstrated in Fig. 1, where the horizontal axis represents the experimental repetition times, and the vertical axis denotes the pre-estimated sparsity.

It can be seen from Fig. 1 that when the parameter a is less than 0.5, the pre-estimated sparsity will exceed the real sparsity; when the parameter a is greater than 0.7, the pre-estimated sparsity is far less than the real sparsity. If the pre-estimated sparsity is too small, the iteration times will increase, which leads to an increase in running time. In the following experiments, we choose parameter a as 0.5.

B. One-dimensional Signal Simulation Experiments

Experiment 2: The reconstruction probability is a function of M and K. The reconstruction probability experiments are conducted with fixed M and varying K. A one-dimensional sparse signal with length $N = 512$ is generated randomly. The signal sparsity ranges from 10 to 55 with an interval of 5. If the error between the reconstructed signal $\hat{\mathbf{x}}$ and the original signal $\mathbf{x}$ is less than 10^{-6}, we consider that the signal is reconstructed successfully. The reconstruction

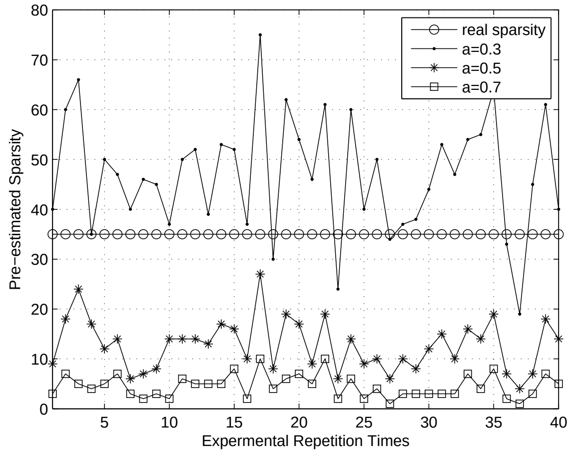

Fig. 1. Influence of parameter a on pre-estimated sparsity

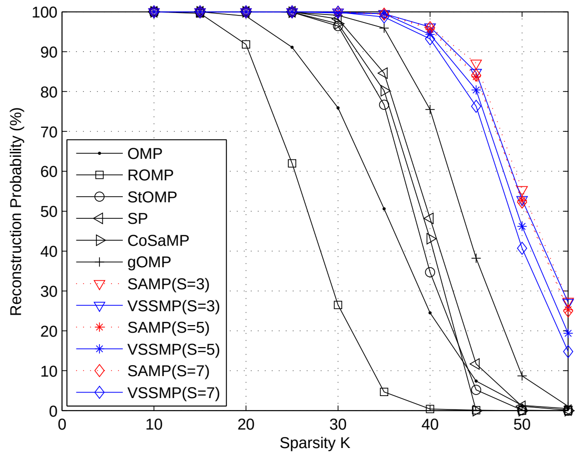

Fig. 2. The reconstruction probability vs the sparsity when the observation dimension $M = 128$.

probability is the percentage ratio of the the number of successful reconstructions to total experimental times. The stepsize of SAMP and VSSAMP is 3, 5, 7, respectively. The experiments are repeated 1000 times. When the observation dimension M is 128 and 140, the simulation curves are demonstrated in Fig. 2 and Fig. 3, respectively.

We can conclude from Fig. 2 and Fig. 3: 1) With the increase of sparsity K, the reconstruction probability of traditional algorithms which need to know the sparsity decreases rapidly. 2) When the sparsity K exceeds 40, the traditional algorithms has not reconstructed the signal. However, the SAMP and VSSAMP algorithms can still show better reconstruction performance. 3) The reconstruction probability of VSSAMP algorithm is slightly inferior to that of SAMP algorithm. But VSSAMP algorithm has obvious advantages over the traditional algorithms which need to know the sparsity.

Experiment 3: The reconstruction probability experiments are conducted with fixed K and varying M. A one-dimensional sparse signal with length $N = 256$ is generated randomly. The signal sparsity range from $2K$ to 160. The experiments are repeated 1000 times. When the signal sparsity K is 25 and 35, the simulation curves are demonstrated

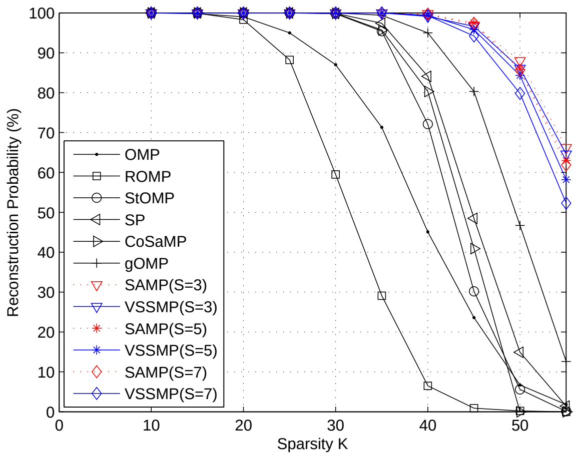

Fig. 3. The reconstruction probability vs the sparsity when the observation dimension $M = 140$.

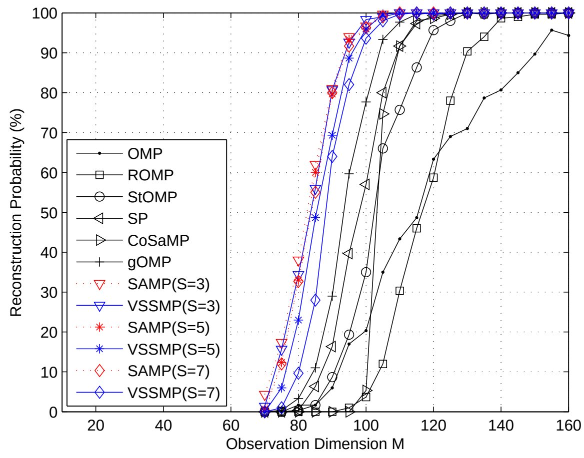

Fig. 5. The reconstruction probability vs the sparsity when the sparsity $K = 35$.

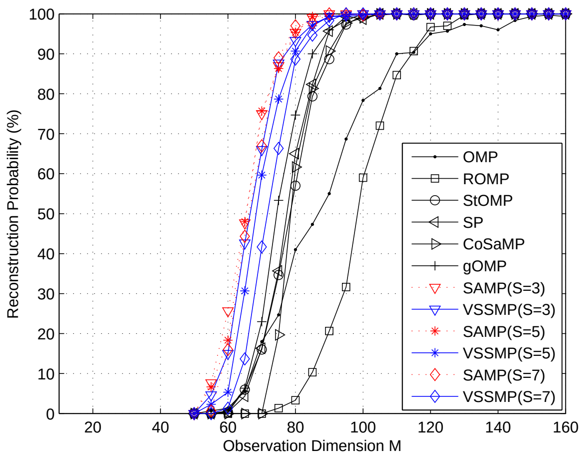

Fig. 4. The reconstruction probability vs the sparsity when the sparsity $K = 25$.

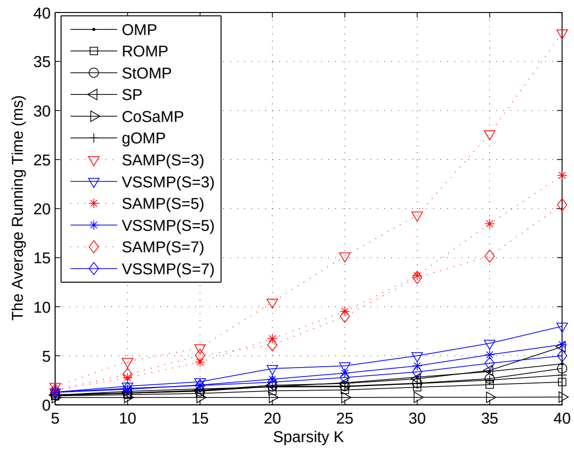

Fig. 6. The average running time as a function of sparsity K.

in Fig. 4 and Fig. 5, respectively.

From Fig. 4 and Fig. 5, we can draw the following conclusions: 1) The reconstruction performance of VSSAMP is slightly inferior to that of SAMP. 2) With the increase of observation dimension M, the reconstruction probability curves of SAMP and VSSAMP algorithms increase rapidly. However, the reconstruction probability cures of traditional algorithms increase slowly. That is, when the sparsity is a fixed value, the SAMP and VSSAMP algorithms can reconstruct the signal when the observation dimension is small.

Experiment 4: To verify the effectiveness of the proposed algorithm, the average running time of different algorithms is compared. The length of random signal is 256, the observation dimension is 128, and the sparsity K ranges from 5 to 40 with an interval of 5. The observation matrix $\mathbf{\Phi}$ is $M \times N$ Gaussian matrix. The simulation curves of average running time vs sparsity K are shown in Fig. 6.

The simulation results reveal that the average running time of the proposed VSSAMP algorithm is obviously lower than that of SAMP algorithm. The process of sparsity pre-estimation of VSSAMP algorithm reduces the whole iteration times, it makes the VSSAMP algorithm run faster than SAMP algorithm.

From the above experiments, we can conclude that compared with SAMP algorithm, the proposed VSSAMP algorithm obtains higher running speed at the cost of slightly drop in reconstruction probability. The construction probability of VSSAMP algorithm is better than traditional algorithms, but the running time is higher than traditional algorithms.

C. Two-dimensional Image Simulation Experiment

Experiment 5: Female image with 256×256 pixels is used to evaluate the peak signal-to-noise (PSNR) and the running time. Since the image is not sparse, it must be sparsely processed using wavelet transform matrix. TABLE III gives the PSNR comparison of SAMP and VSSAMP algorithms, TABLE IV demonstrates the time comparison.

We can conclude from TABLE III that the PSNR of VSSAMP and SAMP increases with the increase of compression ratio. That is, the larger the compression ratio, the better the reconstruction performance. Under the same compression ratio and the same stepsize, the PSNR of VSSAMP is basically the same to SAMP algorithm. TABLE IV shows that the running time of VSSAMP and SAMP increases with

TABLE III
PSNR COMPARISON OF DIFFERENT ALGORITHMS (UNIT:DB)

M/N	SAMP (S=3)	VSSAMP (S=3)	SAMP (S=5)	VSSAMP (S=5)	SAMP (S=7)	VSSAMP (S=7)
0.4	28.71	28.51	29.03	28.65	28.94	28.65
0.5	31.12	30.94	31.21	30.12	31.28	31.12
0.6	32.43	32.64	32.51	32.81	32.61	32.81
0.7	33.06	33.37	33.21	33.52	33.33	33.52

TABLE IV
TIME COMPARISON OF DIFFERENT ALGORITHMS (UNIT:MS)

M/N	SAMP (S=3)	VSSAMP (S=3)	SAMP (S=5)	VSSAMP (S=5)	SAMP (S=7)	VSSAMP (S=7)
0.4	1.58	0.56	1.11	0.39	1.17	0.37
0.5	1.79	0.57	1.32	0.40	1.33	0.43
0.6	2.43	0.60	1.58	0.42	1.35	0.44
0.7	2.61	0.68	1.73	0.46	1.44	0.50

the increase of compression ratio. When adopting the same stepsize and compression ratio, the running time of VSSAMP algorithm is obviously less than SAMP. For example, when the compression ratio is 0.5 and the stepsize is 5, the running time of VSSAMP algorithm is about 30% of that of SAMP algorithm; when the compression ratio is 0.6 and the stepsize is 5, the running time of of VSSAMP algorithm is about 26% of that of SAMP algorithm. The sparsity pre-estimation and the variable stepsize strategies of VSSAMP algorithm reduce the reconstruction time.

V. CONCLUSION

A variable stepsize sparsity adaptive matching pursuit algorithm is proposed. In the period I of VSSAMP, the signal sparsity is pre-estimated; the variable stepsize SAMP is used to reconstruct the signal in the period II. Different from the SAMP algorithm, VSSAMP algorithm determines the stepsize according to the energy difference of two consecutive residues. The experiment results show that the reconstruction probability of VSSAMP algorithm is slightly slower than SAMP algorithm in one-dimensional signal, and the reconstruction quality of VSSAMP is basically equal to SAMP algorithm in tow-dimensional image. What's more, the reconstruction time of VSSAMP algorithm is significantly lower than that of SAMP algorithm in one-dimensional signal and two-dimensional image.

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