Behavior of Stochastic Reaction-Diffusion Process in a Random Network with Dynamic Links

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***Abstract*—This paper analyzes the behavior of the discrete stochastic reaction-diffusion algorithm Greenberg-Hastings in a random network with dynamic links. The main research questions of this study are: 1. Can the stochastic Greenberg-Hastings reach an 'acceptable' steady state in a dynamic network with multiple topological and neighborhood variations? and 2. What is the critical value of the relevant model parameters?. To answer these questions, the model has been implemented by using multiple test scenarios and, as the main constraint is the network links variability, it has been defined in such a manner so that the parameters take low, medium, and high values. This paper presents some numerical experiments which demonstrate its robustness and stability since, in many of the scenarios experienced, the 'acceptable' steady state has been reached. Additionally, the critical values have been identified by the model parameters and iterations, and the behaviors for all the generated scenarios are described. This analysis allows for deducing from the results obtained that, given the robustness of the model, it is possible to conceive a stochastic control algorithm to modify, reach and maintain the 'acceptable' steady state in all analyzed scenarios.**

***Index Terms*—Reaction-Diffusion, Greenberg-Hastings, Complex System, Random Networks, Dynamic Networks, Simulation.**

I. Introduction

THE reaction-diffusion models are used to represent chemical, biological, or natural processes over a wide range of temporal and spatial scales [8]. There are some of them that are multi-scale [15] and are classified as a type of dynamic system [1], which, according to [7] are defined as a system whose states are specified only by a set of variables and whose behavior is described using predefined rules. They are generally divided into deterministic models and stochastic models [3]. These models can be implemented globally in continuous time and space along with differential equations [5] or in discrete time and space [6].

The advantage of stochastic models is that it enables better representation of systems with multiple favorable states given that, in those cases the deterministic models fail. The approximate solutions of stochastic models generate fluctuations and thus allow to alternate between several favorable states as in real life systems [3].

This article analyzes the stochastic Greenberg-Hastings (GH) algorithm, because these types of algorithms are efficient approximations for reaction-diffusion phenomena. The model is implemented on a partially connected network with random dynamic links. Moreover, it facilitates the description of dynamic social, and physical phenomena considering that it executes the mobility of the agents and the variable exchange between all of them.

This paper is organized as follows. The section II presents the fundamental concepts to understand the analyzed model. The section III describe the analyzed model. The results and discussions are presents in section IV, and finally the section V concludes the paper. The appendix show the generated statistical graphics.

II. Background

Greenberg-Hastings is a microscopic discrete non-conservative reaction diffusion model which is a simple model that presents a complex global behavior. The reaction-diffusion models explain mathematically the evolution of a system in which information is disseminated in an active medium; this means that there are local reactions that modify the phenomenon of propagation either by reinforcing it or attenuating it. The system evolves into two possible steady states: the first one is the total extinction of information, and the second one is know as 'acceptable' since it is when a 'wave' of information is generated along the network and does not disappear [1].

The GH is a model that illustrates a non-conservative reaction-diffusion system in a cellular automaton whose time and events are discrete. For example, a dynamic system modeled by variables that take values in a discrete domain and that evolve through discrete and instantaneous events [6], [2], [12], is a two-dimensional cellular automaton which imitates formation patterns in an excitable medium [12].

A cellular automaton is defined as a set of automatons that is organized in a regular-rectangular grid and whose states are simultaneously updated by a uniformly applied function which depends on the state of the local neighborhood [7]. The model converges into two possible stationary states: the 'acceptable' state where the information remains flowing in the network and the 'null' one where the information completely disappears from the network [16].

Apparently, it is a simple model; however due to the existent implicit dynamics, complex global behaviors are often presented [1]. They are also robust models and allow the comparison of the model with the system studied [13].

The model describes a reaction-diffusion process $A_M + B_0 \rightarrow B_M$, meaning that a node A in the excited

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state M reacts with a node B in the neutral state 0, then the node B changes its state from the neutral state 0 to the excited state M [3].

According to [2] and [6] the GH model is described mathematically as a matrix of nodes $C_{i,j}$. Each node has a set of nodes $C_{i',j'}$ that is know as a neighborhood; at each time $t = 0, 1, 2, ...$ the node $C_{i,j}$ has a state $S^t_{i,j} \in \{0, 1, 2\}$. The rules of evolution are defined as shown in equation 1.

$$S^{t+1}_{i,j} = \begin{cases} 0 & if\ S^t_{i,j} = 2 \\ 0 & if\ S^t_{i,j} = 0\ \wedge\ S^t_{i',j'} \neq 1 \\ 1 & if\ S^t_{i,j} = 0\ \wedge\ S^t_{i',j'} = 1 \\ 2 & if\ S^t_{i,j} = 1 \end{cases} \quad (1)$$

Generally, the GH algorithm is implemented in a static network defined as a cellular automaton. However, there are studies carried out on static small-world networks [10] and free scale networks [1], and it is well known that there is a close relationship between the stability of the reaction-diffusion models and the network topology on which they evolve [9]. On the other hand, dynamic networks allow the modeling of processes and systems where nodes interact in a temporary way. In those cases, temporary network models can improve the understanding and predictions made since many of the methods and models developed in static networks cannot be applied nor need non-trivial generalizations [14].

A dynamic network can be represented as a *stream graph* $S = (T, V, W, L)$, where V is a finite set of nodes, T a measurable set of time instants, W a set of time nodes $W \subseteq T \times V$ and a set of links $L = T \times V \bigotimes V$ [11].

III. Analyzed Model

The proposed model for the study is a microscopic reaction-diffusion system that evolves in relation to the stochastic GH algorithm in a space modeled as a random undirected graph $R = (V, L)$ and, according to the Erdös-Rényi [4] random network model, including within it the options of the 'null' link. The number of nodes V are static and the links L are dynamic, that is they change in time at random as proposed in [17]. Since this model implements a random network with dynamic links, the neighborhood for each node is dynamic, therefore the number of neighbors is variable. Each of the network nodes has a discrete set of possible states $\{0, ..., M\}$, where 0 is the neutral state, M is the excited state and if $M > 1$ (where the states between 1 and $M - 1$ are refractory states); a node can pass to the M state if it is in the neutral state 0 and connected directly to at least one node in the M state.

A. Partial Network with Random Links

Initially, a random partial network is created between the V configured nodes considering the connectivity limit RD, if the iteration i reaches $(i \mod C) = 0$ then all the links are deleted (tabula rasa) and new ones are created at random with uniform probability distribution, respecting the limit RD. The process can be graphically represented as shown in the figure 1. This type of configuration creates complex topological models combined with various types of neighborhoods within the same execution, including circular neighborhoods.

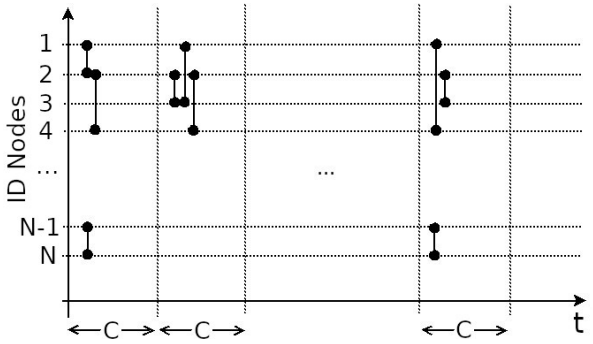

Fig. 1. Example of random dynamic network with N nodes and C period (Stream Graph)

B. Propagation

The implemented model is based on the Greenberg-Hastings stochastic algorithm proposed by [3], where the reaction-diffusion process is described as shown in equation 2. The node A in excited state M that reacts with a node B in neutral state 0 can change the state of node B depending on the individual threshold defined p_T and a random variable p_c.

$$A_M + B_0 \rightarrow \begin{cases} B_M & \text{if } p_c > p_T \\ B_0 & \text{if } p_c \leq p_T \end{cases} \quad (2)$$

Regarding the classic algorithm described in [2] and [6], the rules of evolution are changed as in [1], and is illustrated in equation 3, where σ^t_c corresponds to the value state for node c at time t and E^t_c are the number of neighbors of c in state M at time t.

$$\forall t, \forall c, \sigma^{t+1}_c = \begin{cases} M & \text{if } \sigma^t_c = 0,\ E^t_c > 0\ \wedge\ p_c > p_T \\ \sigma^t_c - 1 & \text{if } \sigma^t_c \in \{1, ..., M\} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

C. Model Parameters

The model receives the values that allow set the initial configuration of the simulation and describe the evolution of the network and the reaction-diffusion process, all of them are in the range of the natural numbers. The table I shows the parameters required by the model.

The parameter (I) represents the number of nodes that start in an active state (state=M) in the initial iteration. (It) is the number of iterations that are carried out in a single execution and when there is at least one node in an active or refractory state. (C) is the period of iterations with which the network structure is changed. (N) is the

TABLE I
TABLE OF MODEL PARAMETERS

ID	Description	Domain
I	Initially active nodes	$\mathbb{N}$
C	Network period (Iteration change of network)	$[1, It] \in \mathbb{N}$
N	Number of nodes (Static)	$\mathbb{N}$
It	Number of iterations	$\mathbb{N}$
NE	Number of runs	$\mathbb{N}$
M	Total number of states	$\mathbb{N}$
RD	Global Percentage of links (Dynamic Links)	$[1, 100] \in \mathbb{N}$
P	Maximum threshold of transmission probability	$[1, 100] \in \mathbb{N}$

number of nodes that constitute the network, this number is constant along a run. (NE) is the established number of runs that are made in order to analyze the stochastic results. (M) is the total number of states defined for an execution. (RD) represents the global total percentage of connections that must be within the network according to (N). (P) is the value over which the threshold p_c is selected.

```
initialization;
while i < It do
    Copy original network (links and states);
    for n ∈ node do
        p_c ← random;
        Propagation according to the equation 2 and based
        on the original network;
        Update state according to the equation 3 and
        based on the original network;
    end for
    Update network (links and states);
    if i mod C == 0 then
        Change Network as described in the section III.A
    end if
end while
```

Algorithm 1: analyzed model algorithm

IV. Results and Discussions

Multiple test scenarios have been set out for analyzing and determining the different behaviors of the model. A wide range of values has been selected for each parameter with the purpose of trying to obtain robustness in the variability to be analyzed by keeping a reasonable computing time. Table II shows the parameter values for each scenario.

TABLE II
PARAMETER VALUES FOR THE TEST SCENARIOS

Parameter ID	Set of values
I	{1}
C	{20}
N	{200}
It	{300}
NE	{50}
M	{1, 5, 10}
RD	{5, 20, 50, 80}
P	{20, 40, 60, 70, 80, 100}

The parameter (I) is equal to 1 for all test scenarios, the specific node is initialized by selecting at random from the set of defined nodes. Given that it is a stochastic and dynamic model, $NE = 50$ runs for each scenario have be made in order to estimate more accurately the overall behavior of the model.

For the total of runs, a set of figures of active nodes in M state has been generated in relation to the number of iterations grouped by the parameters M and RD. Considering that the model is stochastic, the figures generated are statistics that show the mean, maximum values and minimum, and outliers of all the runs performed.

For the sets 1M_xP_5RD (see figure 4), 1M_xP_20RD (see figure 5), 1M_xP_50RD (see figure 6), 1M_xP_80RD (see figure 7), with $P \neq 100$ before the iteration 30, the number of active nodes is 'stabilized' around a convergence value and the variance is smaller with a higher value of P. As the value of the parameter P is higher, the maximum, minimum and mean values tend to increase. With $P = 100$ the behavior is different, there is not a single convergence value. For 1M_xP_5RD there are large variance decreases, the variance is stable at 1M_xP_50RD and for 1M_xP_20RD and 1M_xP_80RD there are smaller variance and the separation of the convergence values is greater.

For the set 5M_xP_5RD (see figure 8) with $P < 80$ in all cases, the variance is higher and is unrelated to the value of P. As the value of the parameter P is higher, the maximum, minimum and mean values tend to increase. With $P = 80$ and $P = 100$, none of the execution reached the 'acceptable' steady state.

For the sets 5M_xP_20RD (see figure 9), 5M_xP_50RD (see figure 10), 5M_xP_80RD (see figure 11), with $P \neq 100$ before the iteration 30 the number of active nodes is 'stabilized' around a convergence value, hence the variance is smaller with a higher value of P. Since the value of the parameter P is higher, the maximum, minimum and mean values tend to increase. With $P = 100$ no execution reached the 'acceptable' steady state.

For the set 10M_xP_5RD (see figure 12) in all cases the variance is large and no execution reached the 'acceptable' steady state.

For the sets 10M_xP_20RD (see figure 13), 10M_xP_50RD (see figure 14), 10M_xP_80RD (see figure 15), the variance is large in all cases; before iteration 30, the number of active nodes is 'stabilized' around a convergence value, consequently the variance is smaller with a higher value of P. As the value of the parameter P is higher, the maximum, minimum and mean values tend to increase. For $P > 40$ no execution reached the 'acceptable' steady state.

The 'acceptable' steady state for all 50 runs and 300 iterations is reached within the following scenarios: 1M_xP_5RD only with $P = 70$, $P = 80$ and $P = 100$, 1M_xP_20RD in all cases, 1M_xP_50RD in all cases, 1M_xP_80RD in all cases, 5M_xP_5RD only with $P = 40$, 5M_xP_20RD, 5M_xP_50RD, 5M_xP_80RD with $P = 20$

and $P = 40$. The rest of the cases reach less of 100% or 0%, the table III presents all the values.

There are 63 (87.5%) cases within the evaluated scenarios where the generated distributions are single-modal and whose mean moves proportionally to the value of the parameter P, there are 9 (12.5%) cases where the distribution is bi-modal. On the unimodal distributions 25 (39.68%) reach 100% of the 'acceptable' steady state and 38 (60.31%) do not, for the case of the bimodal distributions 4 (44.44%) reach 100% of the 'acceptable' steady state and 5 (55.55%) do not obtain it.

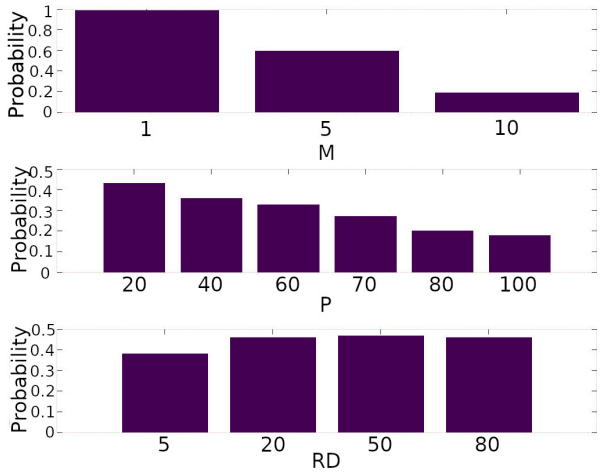

Fig. 2. Comparison of the probability of reaching the 'acceptable' steady state according to the parameters M, P, RD

Comparing the overall behavior of the evaluated scenarios, the critical iterations in which the first extinctions are presented occur between iterations $3(1\%)$ and $25(8.33\%)$, being the scenarios of the set 1M of lesser value and those of 10M of greater value, the identified critical iterations are presented according to a multimodal probability distribution as seen in the figure 3. The 'acceptable' steady state does not depend on the number of active nodes ranging from $1(0.5\%)$ to $150(75\%)$ in one iteration.

For each of the parameters M, P and RD individually, as shown in the figure 2, there is a higher probability of reaching the global 'acceptable' steady state with fewer states M, also when the transmission probability P is lower; whereas with the parameter RD, the higher probability is presented with the value 50 (25%) (this probability depends on the size of the network, that is the number of nodes. For this case, it is being evaluated with 200 nodes).

Table III shows that $28(38.88\%)$ of the 72 scenarios assessed achieve the 'acceptable' steady state in all runs, $23(31.94\%)$ of them do not achieve 'acceptable' steady state in all runs, and the remaining 21 scenarios (29.16%) can be considered as intermediate scenarios. For $PE = 1$, the minimum maximum is $46(23)\%$ of the 5M_20P_80RD scenario, and the maximum maximum is $173(86.5)\%$ of the 1M_100P_80RD scenario. In all tests performed, a maximum of 174 active nodes have been reached in one iteration. Therefore, the highest variance for both the minimum value (3884.4) and the maximum value (3869.9) is

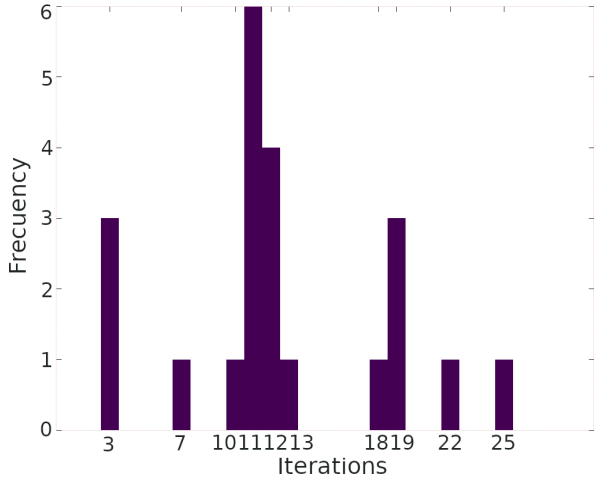

Fig. 3. Histogram for all runs. (critical iterations where the 'null' state is reached)

obtained with scenario 1M_100P_80RD, even in the 50 runs the 'acceptable' stationary state is reached. Additionally, most of probability distributions generated are unimodal (A) 80.55%, bimodal with peaks at both ends 18.05% (U) and bimodal with multiple peaks 1.38% (S).

V. Conclusion

This study has proposed, developed and researched the behavior of the stochastic Greenberg-Hastings (GH) model in a random network with dynamic links. The results confirmed the robustness of the stochastic GH algorithm confronted against the dynamic change of topology and the complex generated neighborhoods. For most scenarios, an emerging convergence value has been identified, which is reached before the 10% of iterations. The evaluated scenarios allow to conclude that the observed variability is not enough to determine the final steady state of system. It evidenced that the change in the values of the parameters modify some specific characteristics such as the number of active nodes, the convergence value for all the runs, the variance of active nodes, the critical iteration between the two stable states and the probability of reaching the 'acceptable' steady state. The analysis carried out has identified the relevant parameters M, P and RD and their incidence in the diffusion of a permanent information wave within a dynamic random network. Analyzing the total of the evaluated scenarios and considering the values of identified parameters independently, it is possible in all cases to reach the 'acceptable' stationary state by modifying the behavior of the individual nodes maintain low M, P and RD near 50% of N.

As future research could be based on the information obtained from the analysis of the experiments and parameters specified here for developing a static control local algorithm and maintaining the 'acceptable' steady state in most cases, it is possible to make each of the nodes that constitute the network act locally in order to modify the global behavior of the network and make the information remain in propagation. For example, regardless of the initial state of the system, global stability is achieved, and the system must

TABLE III
RESULTS FOR ALL SCENARIOS AND TOTAL RUNS. MIN - GLOBAL MINIMUM VALUE OF ACTIVE NODES, MAX - GLOBAL MAXIMUM VALUE OF ACTIVE NODES, VAR MIN - VARIANCE OF THE MINIMUM VALUE, VAR MAX - VARIANCE OF THE MAXIMUM VALUE, PE - PROBABILITY OF REACHING 'ACCEPTABLE' STEADY STATE, TD - PROBABILITY DISTRIBUTION TYPE (A - UNIMODAL, S - BIMODAL WITH MULTIPLE PICKS, U - BIMODAL WITH PEAKS AT BOTH ENDS)

Scenario Name	Min	Max	Var Min	Var Max	PE	TD
1M_20P_5RD	5	46	18.110	9.7395	0.76	A
1M_40P_5RD	8	79	40.556	27.419	0.96	A
1M_60P_5RD	14	101	60.406	45.085	0.98	A
1M_70P_5RD	12	111	65.528	47.192	1	A
1M_80P_5RD	13	121	75.916	58.378	1	A
1M_100P_5RD	17	140	206.27	185.19	1	S
1M_20P_20RD	15	56	17.91	8.4547	1	A
1M_40P_20RD	26	82	33.548	17.887	1	A
1M_60P_20RD	34	118	51.55	32.28	1	A
1M_70P_20RD	44	135	63.645	46.109	1	A
1M_80P_20RD	41	150	78.606	52.327	1	A
1M_100P_20RD	49	172	3684.8	3673.7	1	U
1M_20P_50RD	29	54	14.213	6.5679	1	A
1M_40P_50RD	53	85	27.736	13.589	1	A
1M_60P_50RD	73	102	38.754	21.760	1	A
1M_70P_50RD	81	110	42.904	25.330	1	A
1M_80P_50RD	94	120	48.457	28.612	1	A
1M_100P_50RD	114	116	44.317	24.518	1	U
1M_20P_80RD	40	55	14.632	6.2932	1	A
1M_40P_80RD	65	87	27.495	13.793	1	A
1M_60P_80RD	75	114	39.975	22.424	1	A
1M_70P_80RD	80	122	47.508	27.320	1	A
1M_80P_80RD	75	139	57.416	41.063	1	A
1M_100P_80RD	49	173	3884.4	3869.9	1	U
5M_20P_5RD	5	29	9.2785	4.3832	0.52	A
5M_40P_5RD	9	62	26.314	7.4620	1	A
5M_60P_5RD	10	91	111.30	18.011	0.8	A
5M_70P_5RD	5	102	215.15	21.976	0.38	U
5M_80P_5RD	0	124	872.06	660.01	0	A
5M_100P_5RD	0	136	2564.5	556.27	0	A
5M_20P_20RD	13	48	11.490	3.8665	1	A
5M_40P_20RD	17	84	26.362	10.979	1	A
5M_60P_20RD	7	117	77.672	39.702	0.98	A
5M_70P_20RD	4	141	155.90	70.791	0.56	U
5M_80P_20RD	2	142	265.02	152.28	0.1	U
5M_100P_20RD	0	170	3222.8	2496.4	0	A
5M_20P_50RD	21	51	11.398	4.2575	1	A
5M_40P_50RD	14	75	22.577	11.059	1	A
5M_60P_50RD	6	104	64.957	31.675	0.96	A
5M_70P_50RD	5	107	102.01	55.630	0.66	U
5M_80P_50RD	2	123	204.56	111.22	0.18	U
5M_100P_50RD	0	112	2632.7	1653.6	0	A
5M_20P_80RD	20	46	10.962	4.7177	1	A
5M_40P_80RD	13	78	23.595	12.290	1	A
5M_60P_80RD	5	114	69.796	38.824	0.82	A
5M_70P_80RD	3	131	114.68	59.169	0.54	U
5M_80P_80RD	1	142	190.83	112.31	0.18	U
5M_100P_80RD	0	171	3681.9	2575.1	0	A
10M_20P_5RD	0	28	36.336	10.158	0	A
10M_40P_5RD	0	61	267.88	126.39	0	A
10M_60P_5RD	0	93	869.42	0.05	0	A
10M_70P_5RD	0	106	902.26	271.15	0	A
10M_80P_5RD	0	120	1335.5	275.89	0	A
10M_100P_5RD	0	136	1851.7	402.83	0	A
10M_20P_20RD	9	48	13.929	5.2799	0.92	A
10M_40P_20RD	2	88	72.959	38.993	0.06	U
10M_60P_20RD	0	124	902.12	402.91	0	A
10M_70P_20RD	0	141	1210.4	572.85	0	A
10M_80P_20RD	0	148	1306	748.42	0	A
10M_100P_20RD	0	171	2179.3	1552.4	0	A
10M_20P_50RD	11	48	11.111	5.7232	0.94	A
10M_40P_50RD	3	80	48.813	22.320	0.14	U
10M_60P_50RD	0	96	736.26	331.27	0	A
10M_70P_50RD	0	104	942.26	460.85	0	A
10M_80P_50RD	0	111	1172.9	608.39	0	A
10M_100P_50RD	0	115	1846.1	1004.9	0	A
10M_20P_80RD	12	50	13.862	5.0096	0.98	A
10M_40P_80RD	3	82	51.428	23.907	0.18	U
10M_60P_80RD	0	112	905.57	397.04	0	A
10M_70P_80RD	0	125	1106.2	559.31	0	A
10M_80P_80RD	0	139	1345.9	822.49	0	A
10M_100P_80RD	0	174	2406.9	1705.9	0	A

be modified either globally or locally before the critical iteration identified. This model and this analysis can be used to generate a local distribution of probability for each node and find the incidence of number of neighbors and their states.

APPENDIX A
FIGURES FOR ALL SCENARIOS

All the figures generated for the proposed scenarios are shown in this section, the red points represents the outliers. All scenarios that complete the 300 iterations converge to a value, except the scenarios 1M-100P-20RD-1I, 1M-100P-80RD-1I, where a stable bifurcation occurs and the scenario 1M-100P-5RD-1I that produce a convergent bifurcation.

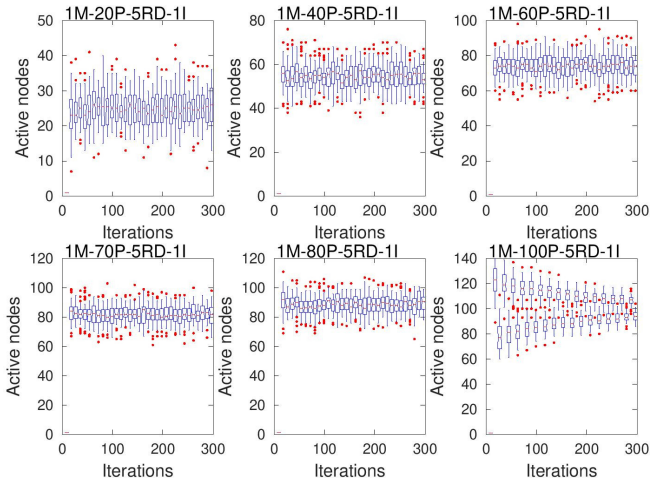

Fig. 4. Statistical behavior for all runs (active nodes for each iteration) for the set of parameters. 1M_xP_5RD_1I

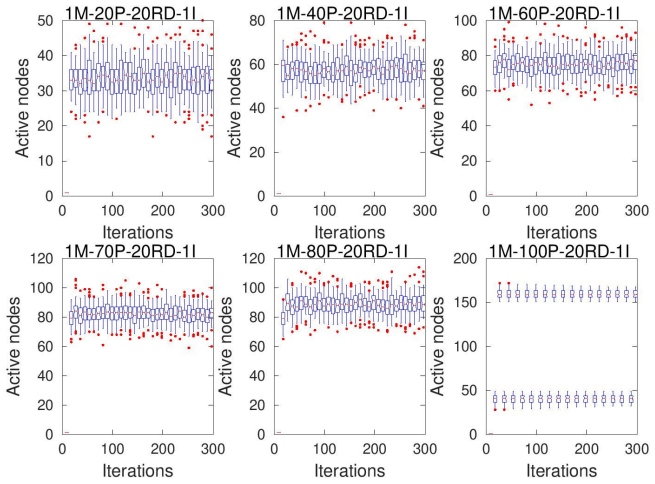

Fig. 5. Statistical behavior for all runs (active nodes for each iteration) for the set of parameters 1M_xP_20RD_1I

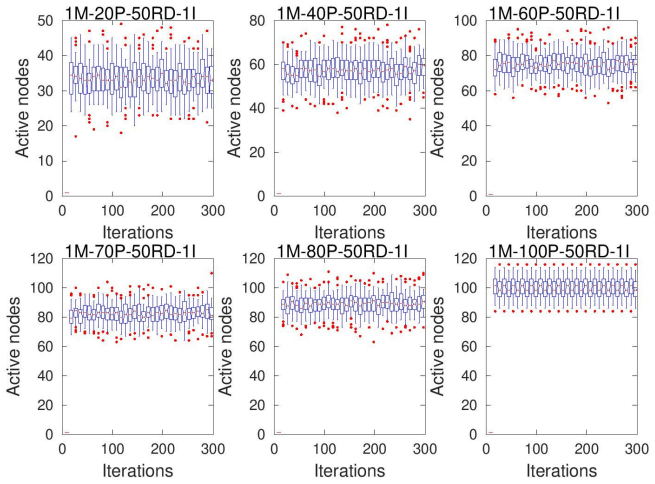

Fig. 6. Statistical behavior for all runs (active nodes for each iteration) for the set of parameters 1M_xP_50RD_1I

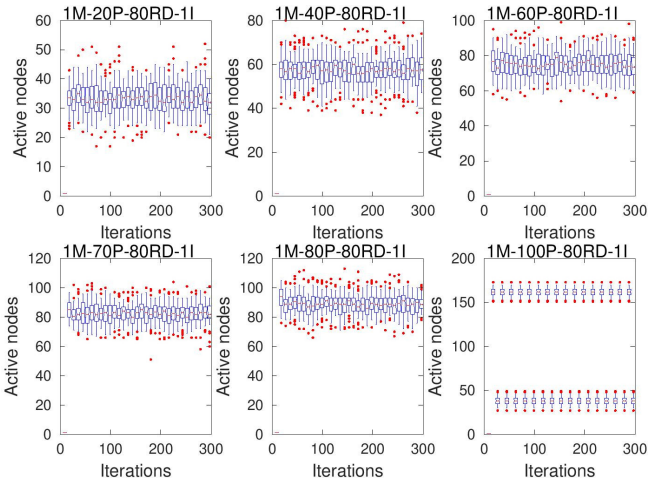

Fig. 7. Statistical behavior for all runs (active nodes for each iteration) for the set of parameters 1M_xP_80RD_1I

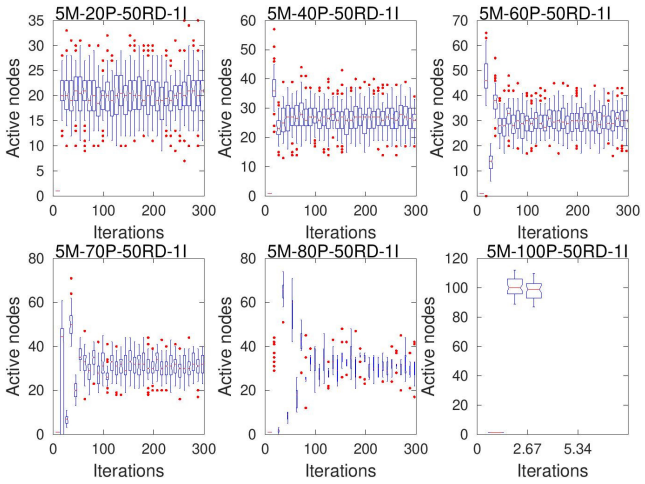

Fig. 10. Statistical behavior for all runs (active nodes for each iteration) for the set of parameters 5M_xP_50RD_1I

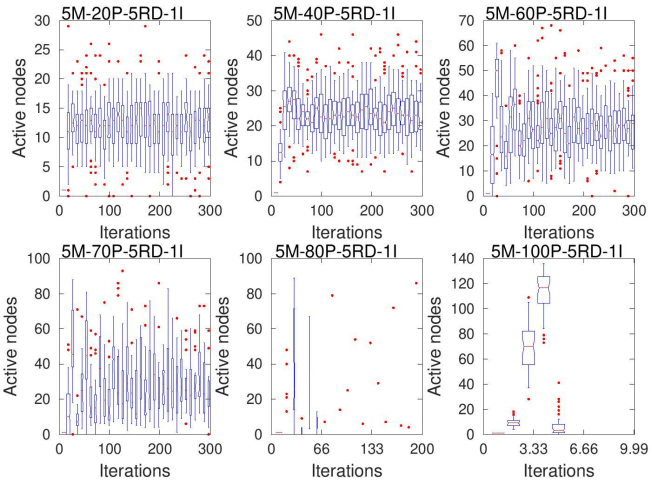

Fig. 8. Statistical behavior for all runs (active nodes for each iteration) for the set of parameters 5M_xP_5RD_1I

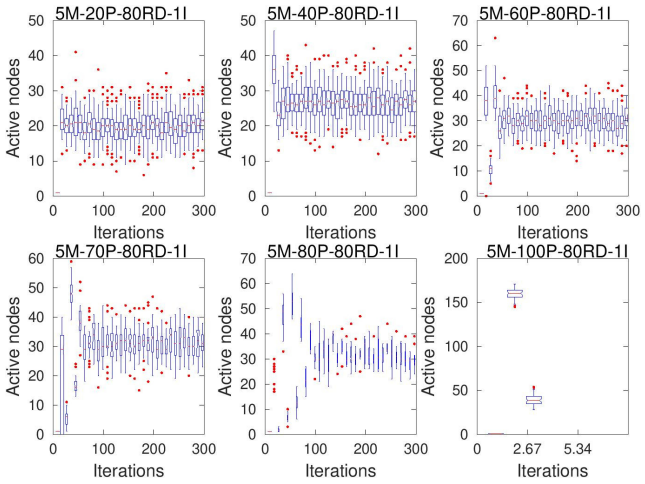

Fig. 11. Statistical behavior for all runs (active nodes for each iteration) for the set of parameters 5M_xP_80RD_1I

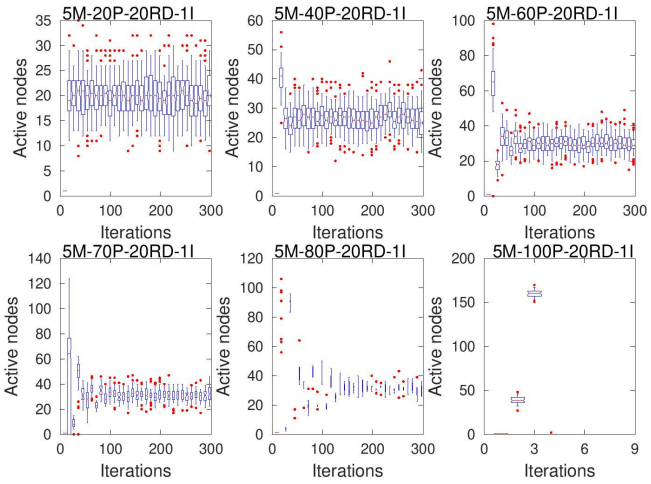

Fig. 9. Statistical behavior for all runs (active nodes for each iteration) for the set of parameters 5M_xP_20RD_1I

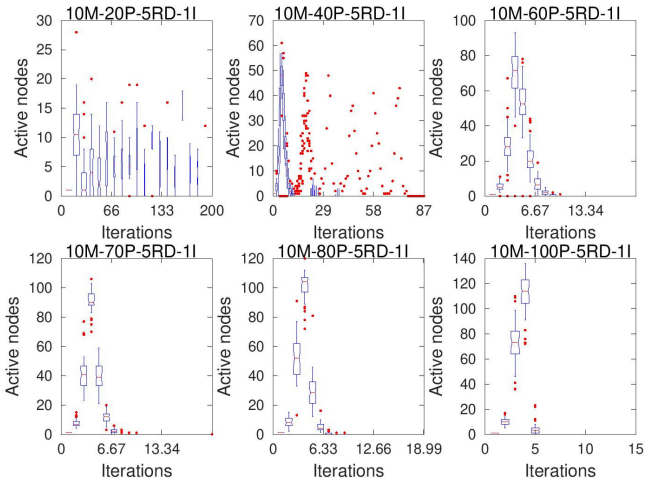

Fig. 12. Statistical behavior for all runs (active nodes for each iteration) for the set of parameters 10M_xP_5RD_1I

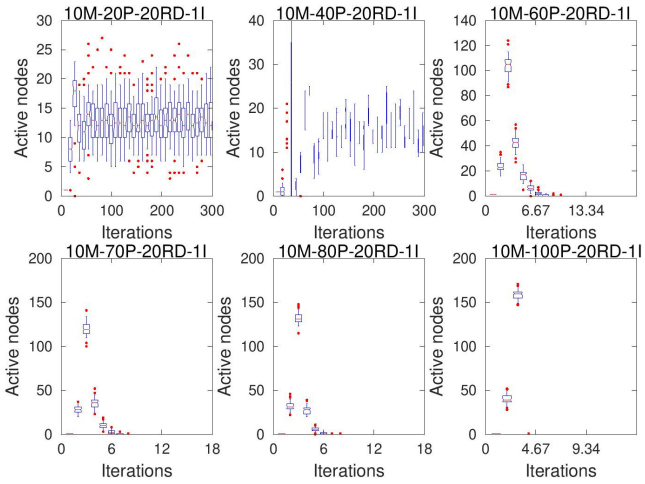

Fig. 13. Statistical behavior for all runs (active nodes for each iteration) for the set of parameters 10M_xP_20RD_1I

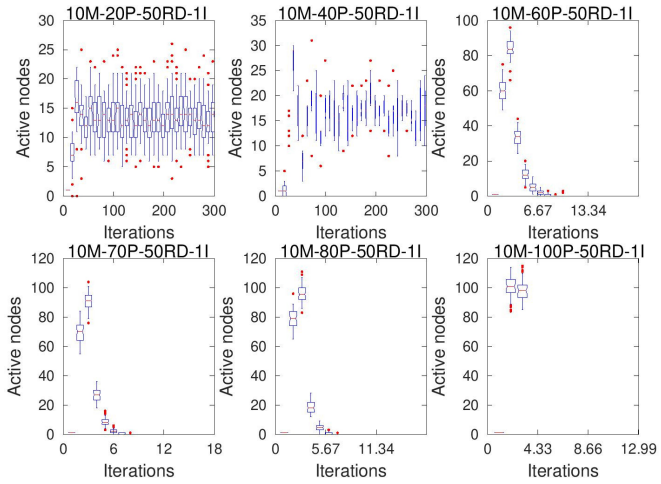

Fig. 14. Statistical behavior for all runs (active nodes for each iteration) for the set of parameters 10M_xP_50RD_1I

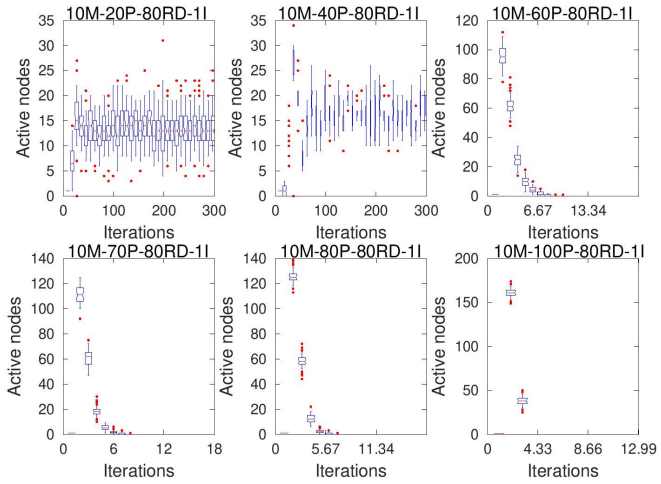

Fig. 15. Statistical behavior for all runs (active nodes for each iteration) for the set of parameters 10M_xP_80RD_1I

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