Time-Varying Model Predictive Control for Train Regulation and Passenger Flow in Metro Lines with Sinusoidal Disturbance

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Abstract—**This research intended to create a control strategies at each time step to optimize train regulation, and passenger flows with existing constraints to improve the regularity of headway and commercial speed on metro lines. Additionally, inevitable disturbances on the metro lines are considered as a periodic sine function, and the uncertainty of fluctuating passenger arrival flow was handled using time-varying MPC. The best solution was sought as a quadratic programming problem by using time-varying MPC to issues of train regulation and passenger flow control in which the systems were time-dependent. Moreover, time-varying MPC was utilized to predict future outputs and calculate optimal inputs for the objective function. Numerical examples were provided to illustrate the effectiveness of the proposed method.**

Index Terms—**Train regulation; Passenger Flow Control; Quadratic Programming; Time Varying MPC.**

I. Introduction

DUE to their inherent characteristics of speed, efficiency, and safety, metro systems have become an essential source of public transportation for passengers in urban centers ([1],[2]). However, metro systems frequently experience minor disruptions due to irregular occurrences of passenger demand fluctuations, equipment failure, and crises. They can significantly impact the service quality of service with a short headway. Variations in passenger demand result in an unanticipatedly crowded passenger arrival flow, which impacts dwell time ([3],[4]). Specifically, as the number of arriving passengers during peak hours increases, train delays caused by random disruptions will spread from one station to the next, making the system unstable. Periodic passenger arrival can be assumed to be a disruption and is considered a sine function [5]. Train regulation, which involves changing the operating duration and dwell time of each train, is necessary to recover from delays and prevent unstable operations ([6],[7]).

Various train regulation mechanisms have been proposed for metro lines. In order to guarantee system stability and the reduction of a particular performance index, a state feedback control strategy built on the linear quadratic controller was used in [6]. In [8], a genetic algorithm was successfully applied to the problem of optimal train regulation. However, train regulation, which controls each train's operating duration and dwells time, can not handle the overloaded passenger flow during peak hours. The joint dynamic train regulation and passenger flow control design problem for metro lines was established in [9] to enhance commercial speed and headway regularity.

The amount of passengers boarding and departing each train has an impact on dwell time [9]. It is assumed that the number of passengers boarding the train is proportional to the time between trains ([3], [10]) and that the passenger arrival rate is uncertain ([11]), both of which are time-dependent. Therefore, there should be a proportionate relationship between the number of passengers entering and exiting trains [12], which depends on the time change. This study assumes that the uncertain passenger arrival rate is different at each station and train and that the sine-shaped passenger arrival rate causes disturbances. This is more relevant to factual problems. However, as the number of variables and constraints increases, the calculation time of conventional linear and nonlinear programming methods increases, making them unsuitable for calculating daily activity schedules.

One of the most promising subfields of modern control, model predictive control (MPC), is capable of effectively handling large-scale optimization problems with complex constraints. [13]. There are numerous MPC types, such as distributed MPC [18], time-varying MPC [15], SPF-MPC [19], and Nonlinear MPC [20]. MPC is an effective solver for real-time metro traffic regulation and passenger load due to its high predictive ability. A linear programming-based MPC approach was published in [14] to compute optimal train schedules on metro lines, which can successfully generate a daily timetable. In addition, [9] addressed a challenge in predictive design for metro lines, including dynamic train regulation and passenger flow control.

In addition, the unpredictability of passenger flow and periodic sinusoidal disturbances that cause train delays must be consider. Time-varying MPC can solve problems involving time-dependent parameters, such as uncertain passenger arrival rates, sinusoidal disturbances, and proportional factors, among others. Using time-varying algorithms, autonomous cars have been developed [15]. In order to deal with unpredictable changes in passenger flow, various proportional factors, and the sine disturbance, we examine the optimal train regulation and passenger load control within the framework of time-varying MPC.

Based on the time-varying MPC scheme, this research suggests a novel approach to train regulation and passenger flow management approaches in metro lines. A constrained state space model was utilized to take safety, passenger, and control constraints into account for the joint dynamic model of train regulation and passenger flow on the metro lines.

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Furthermore, time-varying MPC regulates train operation and passenger flow. Using dynamic models of train traffic and passenger load on metro lines, time-varying MPC predicts future outcomes. The train departure time and the passenger load error are the system outputs. While passenger load error refers to the difference between the factual and nominal passenger load, train departure time error relates to the factual and nominal departure time.

The suggested approaches convert the optimization problem into an easily-solvable convex quadratic programming problem at each time step. In addition, the quadratic programming issue might have constraints for control, load, and safety headway. Therefore, the suggested technique offers a less computationally expensive solution and is easier to implement. This paper is organized as follow : the next section discuses a coupled relationship between the dynamics of train traffic and passenger load. The following sections describe the time-varying model predictive control method for train regulation and passenger flow. Numerical examples to illustrate the effectiveness of the propose strategies is presented in the next section. Last section states the conclusion.

II. Problem Formulation

We considered a metro-typetrain line with one terminal station, N stations, and an ordered train set that stops between stations to allow passengers to enter and leave. Stations, trains, and passengers are components of the metro-typerail system. The metro line's mission is to transport every passengers from their starting point to their final destination safely and efficiently.

Disruptions, such as equipment failure or non-compliant driver/passenger activity, are unavoidable in the real-time operation of metro lines. The optimal train schedule was no longer required when a disruption occurred, thus a train control plan was necessary to implemented to decrease on delays. Some stations typically have a high number of passengers. Passenger demand is relatively high at several stations, especially during peak hours. At such a station, the number of passengers will exceed the train's nominal passenger load. If passenger flow is not controlled, train delays will increase significantly. When a train deviates from its usual schedule due to a disruption, a train regulation and passenger flow control strategy must be implemented to improve the safety and efficiency of the metro line systems.

A train traffic dynamics model and a train passenger load dynamics model were created to solve this issue. This model integrated the two relationships between train traffic and passenger load dynamics to generate a train traffic and passenger flow dynamics model. This study applied a dynamic model of changes in passenger load between stations, as characterized by the number of people entering and exiting the train at each station. Previous research described passenger demand using a time-dependent origin-destination (OD) matrix ([16],[17]). The number of passengers boarding the train was thought to be proportionate to the duration between trains ([3], [10]), and the passenger arrival rate was unpredictable [1], both of which were time-dependent. It was anticipated that the number of people boarding and departing the train would equal the number of passengers on board [12], which depends on the time change. Few trains were affected by the system's disruption [9]. In this study, the disturbance was modeled as a periodic sine function occurring on every train. It was considered that the passenger arrival rate varied by train and station and was uncertain.

A. Train traffic dynamic model

Based on [6], the dynamics of train traffic for high-frequency metro lines were presented. Let t_j^i was the departure time of train i from station j. The departure time of train i from station $j+1$ was stated as

$$t_{j+1}^i = t_j^i + r_j^i + s_{j+1}^i \tag{1}$$

where s_{j+1}^i was the dwell time for the train i at station $j+1$. The running time for the train i from station j to station $j+1$, r_j^i was

$$r_j^i = R_j^i + {u_1}_j^i + {w_1}_j^i \tag{2}$$

where R_j^i was the nominal running time of train i from station j to station $j+1$, ${u_1}_j^i$ was the control to adjust the running time of train i between station j to station $j+1$, and ${w_1}_j^i$ were uncertain disturbances occured when the i train ran from j station to $j+1$ station. If ${u_1}_j^i > 0$ it means that the running time was increased, while if ${u_1}_j^i < 0$ it means that the running time was decreased.

Suppose that the dwell time of the trains at the station was affected by both the number of entering and leaving passengers [9]. According to this, the dwell time s_{j+1}^i was modeled as

$$s_{j+1}^i = \alpha\left(m_{j+1}^i + n_{j+1}^i\right) + D_{j+1} + {u_2}_j^i + {w_2}_{j+1}^i. \tag{3}$$

where D_{j+1} was the minimum dwell time at the station $j+1$ when there were no passengers, α is the delay rate which represents the time it takes to get on or off the train when the train stops , $\alpha \in [0.01.0.06]$. The dwell time adjustment of train i at station $j+1$ denoted as ${u_2}_j^i$. If ${u_2}_j^i > 0$ it means that the dwell time was increased, while if ${u_2}_j^i < 0$ it means that the dwell time was decreased. Furthermore, ${w_2}_{j+1}^i$ was a disturbance occured when the train i stopped at station $j+1$. From the Equation (1)-(3), the train traffic dynamic model is

$$t_{j+1}^i = t_j^i + R_j^i + \alpha\left(m_{j+1}^i + n_{j+1}^i\right) + D_{j+1} + u_j^i + w_j^i. \tag{4}$$

with $u_j^i = {u_1}_j^i + {u_2}_j^i$ and $w_j^i = {w_1}_j^i + {w_2}_{j+1}^i$.

B. The passenger load dynamic model

When a train arrives at a station, there are passengers enter the train, and there are others leaving it. According to [9], the dynamic change of the passenger load on the train at the station was

$$l_{j+1}^i = l_j^i + m_{j+1}^i - n_{j+1}^i + p_{j+1}^i. \tag{5}$$

where m_{j+1}^i and n_{j+1}^i were respectively the numbers of passengers entering and leaving the train i at station $j+1$, and p_{j+1}^i was a control to increase the number of passengers entering the ith train at the $j+1$ station. This control was implemented during peak hours or on holidays, especially for the sudden arrival of passengers in which the value was non-positive to reduce passenger load.

The number of passengers entering train i at station $j+1$ or m_{j+1}^{i} was supposed to be proportional to the waiting time between consecutive trains and it satisfies that

$$m_{j+1}^{i} = \gamma_{j+1}^{i}\left(t_{j+1}^{i} - t_{j+1}^{i-1}\right) \tag{6}$$

where γ_{j+1}^{i} was the passenger arrival rate at station $j+1$ for train i. According to [10], the passenger arrival rate would change with time and it was assumed that the value of γ_{j+1}^{i} varies in a symmetrical range around γ with half the length of d was

$$\gamma_{j+1}^{i} = \gamma + \lambda_{j+1}^{i} d, \quad -1 \leq \lambda_{j+1}^{i} \leq 1 \tag{7}$$

with λ_{j+1}^{i} different in each station. For simplicity, the parameter γ and half the length of d was assumed to be similar for each station. In this study, the parameter of λ_{j+1}^{i} varied at each station $j+1$ it was more general and realistic compared [10] which assumed that the average passenger arrival rate was the same for all stations. We might ensure a maximum allowable passengers arrival rate by modifying the parameter γ and the half-length d, which would satisfy the trains limited capacity for transporting passengers. The p_{j+1}^{i} control approach minimized the number of people entering the train in satisfying the train's limited passenger capacity. The passenger flow control, in particular, induced a change in train dwell time from s_{j+1}^{i} to s_{j+1}^{i} was

$$s_{j+1}^{i} = \alpha\left(m_{j+1}^{i} + n_{j+1}^{i} + p_{j+1}^{i}\right) + D_{j+1} + u_{2}{}_{j}^{i} + w_{2}{}_{j+1}^{i}. \tag{8}$$

The number of passengers leaving the train i at station $j+1$ was assumed to be proportional to the number of passengers on the train which satisfy

$$n_{j+1}^{i} = \beta_{j+1}^{i} l_{j}^{i} \tag{9}$$

where l_{j}^{i} was the passenger load of train i between stations j and $j+1$, and β_{j+1} was a proportional factor for passengers leaving the train. From the Equation (5)-(9), the dynamic model of passenger load on the train was

$$l_{j+1}^{i} = l_{j}^{i} + \gamma_{j+1}^{i}\left(t_{j+1}^{i} - t_{j+1}^{i-1}\right) - \beta_{j+1}^{i} l_{j}^{i} + p_{j+1}^{i} \tag{10}$$

which indicated that the dynamic model of passenger loads on the train was influenced by the dynamic model of train traffic.

C. The joint dynamic model

From the Equation (4) and (10), we obtained a joint dynamic model of the factual departure time and passenger load on the train as

$$\begin{cases} t_{j+1}^{i} = & t_{j}^{i} + R_{j}^{i} + \alpha\left(m_{j+1}^{i} + n_{j+1}^{i} + p_{j+1}^{i}\right) + D_{j+1} \\ & + u_{j}^{i} + w_{j}^{i}. \\ l_{j+1}^{i} = & l_{j}^{i} + \gamma_{j+1}^{i}\left(t_{j+1}^{i} - t_{j+1}^{i-1}\right) - \beta_{j+1}^{i} l_{j}^{i} + p_{j+1}^{i}. \end{cases} \tag{11}$$

This demonstrated how the train's factual departure time and passenger load interact. Based on Equation (11), it was able to determine that if one train is delayed, the train delay would increase from one station to the next, as would be the aggregation of passengers potentially causing metro line instability.

By substituting Equation (6) and (9) to the first Equation (11), and take $x_{j}^{i} = \left[t_{j}^{i}, l_{j}^{i}\right]^{T}$ and $\bar{u}_{j}^{i} = \left[u_{j}^{i}, p_{j+1}^{i}\right]^{T}$ we acquired a joint dynamic model with departure time and load passengers on the train was

$$x_{j+1}^{i} = A_{j}^{i} x_{j}^{i} + B_{j}^{i} x_{j+1}^{i-1} + C_{j}^{i} \bar{u}_{j}^{i} + G_{j}^{i}\left(D_{j+1} + R_{j}^{i} + w_{j}^{i}\right). \tag{12}$$

with

$$x_{j}^{0} = [0,0]^{T}, A_{j}^{i} = \begin{bmatrix} \frac{1}{(1-\alpha\gamma_{j+1}^{i})} & \frac{\gamma_{j+1}^{i}}{(1-\alpha\gamma_{j+1}^{i})} \\ \frac{\alpha\beta_{j+1}^{i}}{(1-\alpha\gamma_{j+1}^{i})} & (1-\beta_{j+1}^{i}) + \frac{\alpha\beta_{j+1}^{i}\gamma_{j+1}^{i}}{(1-\alpha\gamma_{j+1}^{i})} \end{bmatrix},$$

$$B_{j}^{i} = \begin{bmatrix} \frac{-\alpha\gamma_{j+1}^{i}}{(1-\alpha\gamma_{j+1}^{i})} & 0 \\ \frac{-\gamma_{j+1}^{i}}{1-\alpha\gamma_{j+1}^{i}} & 0 \end{bmatrix}, C_{j}^{i} = \begin{bmatrix} \frac{1}{1-\alpha\gamma_{j+1}^{i}} & \frac{\alpha}{1-\alpha\gamma_{j+1}^{i}} \\ \frac{\gamma_{j+1}^{i}}{1-\alpha\gamma_{j+1}^{i}} & \frac{1}{1-\alpha\gamma_{j+1}^{i}} \end{bmatrix},$$

$$G_{j}^{i} = \begin{bmatrix} \frac{1}{1-\alpha\gamma_{j+1}^{i}} \\ \frac{\gamma_{j+1}^{i}}{1-\alpha\gamma_{j+1}^{i}} \end{bmatrix}.$$

Equation (12) was a standard model for metro lines system operation management under disturbance and it described the change in train departure time and passenger load. Furthermore, the dynamic model for the nominal departure time and passenger load of the nominal train was

$$T_{j+1}^{i} = T_{j}^{i} + R_{j}^{i} + \alpha\left(\gamma_{j+1}^{i}\left(T_{j+1}^{i} - T_{j+1}^{i-1}\right) + \beta_{j+1}^{i} L_{j}^{i}\right) + D_{j+1}. \tag{13}$$

and

$$L_{j+1}^{i} = L_{j}^{i} + \gamma_{j+1}^{i}\left(T_{j+1}^{i} - T_{j+1}^{i-1}\right) - \beta_{j+1}^{i} L_{j}^{i}. \tag{14}$$

The constant time difference between two successive trains determined the nominal departure time, denoted by $H = T_{j+1}^{i} - T_{j+1}^{i-1}$. In terms of service demands, train capacity, and passenger flow during operating hours, the H headway schedule corresponded to operational hours. In particular, headway scheduling was reduced during peak hours.

The error vector is $e_{j}^{i} = \left[t_{j}^{i} - T_{j}^{i}, l_{j}^{i} - L_{j}^{i}\right]^{T}$, from Equation (11), we found the error dynamics for the joint dynamic model as

$$e_{j+1}^{i} = A_{j}^{i} e_{j}^{i} + B_{j}^{i} e_{j+1}^{i-1} + C_{j}^{i} \bar{u}_{j}^{i} + G_{j}^{i} w_{j}^{i} \tag{15}$$

with $A_{j}^{i}, B_{j}^{i}, C_{j}^{i}$, and G_{j}^{i} taken from Equation (12). For the dynamic model error Equation (15), the difference between the factual departure time and the nominal departure time was represented by e_{j}^{i}, as well as the difference between the factual passenger load on the train and its nominal passenger load. Minimizes e_{j}^{i} referred to improving the metro lines operating efficiency in order to recover train delays caused by disturbances. Furthermore, if $e_{j}^{i} \to \mathbf{0}$ then $t_{j}^{i} \to T_{j}^{i}$ and $l_{j}^{i} \to L_{j}^{i}$ which prevent instability on metro lines.

According to Equation (12), information for x_{j+1}^{i} was generated from x_{j}^{i} and x_{j+1}^{i-1} for each train i and j station. Let X_k is the state vector of the joint dynamic model with $X_k = [x_1^{k-1}, x_2^{k-2}, \ldots, x_N^{kN}]^T$, $k > N$ which displayed the factual departure time of the train and the passenger load on the train at all stations. The dimension of the vector state is $2N$. It was assumed that every elements of the state X_k vector lied in the same time interval. Furthermore, by using Equation (12) we obtained the form state space for the joint dynamic model as

$$X_{k+1} = \bar{A}_k X_k + \bar{B}_k U_k + \bar{G}_k\left(w_k + R_k + D\right) \tag{16}$$

with $X(k)$ as the state vector, the input vector $U_k = \left[\bar{u}_0^k, \bar{u}_1^{k-1}, \ldots, \bar{u}_{N-1}^{k-N+1}\right]^T$, the disturbance vector $w_k = [w_0^k, w_1^{k-1}, \ldots, w_{N-1}^{k-N+1}]^T$, $R_k = \left[R_0^k, R_1^{k-1}, \ldots, R_{N-1}^{k-N+1}\right]^T$, $D = [D_1, D_2, \ldots, D_N]^T$, and matrix $\bar{A}_k, \bar{B}_k$, and $\bar{G}_k$ was

$$\bar{A}_k = \begin{bmatrix} B_0^k & 0 & 0 & \ldots \\ A_1^{k-1} & B_1^{k-1} & 0 & \ldots \\ & \ldots & \ldots & \ldots \\ 0 & \ldots & A_{N-1}^{k-N+1} & B_{N-1}^{k-N+1} \end{bmatrix},$$

$$\bar{B}_k = \begin{bmatrix} C_0^k & 0 & 0 & \ldots \\ 0 & C_1^{k-1} & 0 & \ldots \\ & \ldots & \ldots & \ldots \\ 0 & \ldots & \ldots & C_{N-1}^{k-N+1} \end{bmatrix},$$

$$\bar{G}_k = \begin{bmatrix} G_0^k & 0 & 0 & \ldots \\ 0 & G_1^{k-1} & 0 & \ldots \\ & \ldots & \ldots & \ldots \\ 0 & \ldots & \ldots & G_{N-1}^{k-N+1} \end{bmatrix}$$

with the dimensions of matrix $\bar{A}_k, \bar{B}_k$, and $\bar{G}_k$ were $2N \times 2N, 2N \times 2N$, and $2N \times N$, respectively.

According to Equation (16), $2N$ was the number of stations, not trains. The matrices $\bar{A}_k$ and $\bar{B}_k$ represented the dynamic relationship between train traffic and passenger load, and $\bar{G}_k$ represented the system disturbance parameter. Moreover, we obtained the state space model of the joint error dynamic

$$E_{k+1} = \bar{A}_k E_k + \bar{B}_k U_k + \bar{G}_k w_k \tag{17}$$

with $E_k = \left[e_1^{k-1}, e_2^{k-2}, \ldots, e_N^{k-N}\right]^T$ which consisted of the departure time error and the passenger load error and with the matrix $\bar{A}_k, \bar{B}_k$ and $\bar{G}_k$ was same as in Equation (16).

D. Objective Function and System Constraints

To address this issue, metro lines that integrate train regulation and passenger flow control were designed to improve commercial speed and headway regularity. The cost function of the joint dynamic model of metro lines was defined to solve this problem

$$J = \sum_{i,j} \left\{ {e_j^i}^T P_j^i e_j^i + \left(e_j^i - e_j^{i-1}\right)^T Q_j^i \left(e_j^i - e_j^{i-1}\right) + \left(\bar{u}_j^i\right)^T R_j^i \bar{u}_j^i \right\}. \tag{18}$$

with positive definite weighted matrix P_j^i, Q_j^i, R_j^i. The first term in (18) was used to reduce train delays by comparing factual and nominal timetables and passenger loads. The second term improved headway regularity and reduced average passenger waiting time by summing train headway deviations from the nominal value. The third term saved cost. The weight matrix P_j^i, Q_j^i, and R_j^i were related to departure time deviations, headway deviations, and control action amplitude, respectively.

From the state space for the joint dynamic model (17), the objective function matrix (18) was formulated as

$$J = \sum_{k=k_0}^{k_f} \left\{ E_k^T P E_k + \left(E_{k+1} - E_k\right)^T Q \left(E_{k+1} - E_k\right) + U_k^T R U_k \right\} \tag{19}$$

where k_0 and k_f were the initial and final stages, respectively. P, Q, and R were positive definite weighted matrix.

In additon, to ensure safety on metro lines, we considered the following constraints.

1) State constraints for the departure time
To ensure that a safe distance exists between two adjacent trains is satisfied $t_j^i - t_j^{i-1} \geq t_{min}$, where t_{min} represented the minimum allowed headway. Furthermore, the state constraint for each train's departure time could be changed into an error state constraint for each train's departure time that satisfy

$$\left(t_j^i - T_j^i\right) - \left(t_j^{i-1} - T_j^{i-1}\right) \geq t_{min} - H \tag{20}$$

with t_{min} and H were provided. Furthermore, from Equation (17) the constraint for departure time could be written as

$$H_1 \left(E_{k-1} - E_k\right) \leq (H - t_{min}) I_{N \times 1} \tag{21}$$

with H_1 as a matrix of dimension $N \times 2N$ where

$$H_1 = [h_{ij}], \quad h_{ij} = \begin{cases} 1, & j = 2i - 1 \\ 0, & otherwise. \end{cases}$$

2) State constraints for the passenger load
To meet the train's capacity, the passenger load constraint was $l_j^i \leq l_{max}$, where l_{max} was the maximum capacity of the train. To meet the capacity of the train, the passenger load constraint was $l_j^i \leq l_{max}$, where l_{max} was the train's maximum capacity for passengers. In addition, the state constraint for the train's passenger load could be transformed into the state error constraint for the passenger load on each train that satisfy

$$l_j^i - L_j^i \leq l_{maks} - L_j^i \tag{22}$$

with l_{maks} and L_j^i were provided. Furthermore, from Equation (17) the constraint for passenger loads could be written as

$$H_2 E_k \leq L_k \tag{23}$$

with H_2 as a matrix dimension of $N \times 2N$ where

$$H_2 = [h_{ij}], \quad h_{ij} = \begin{cases} 1, & j = 2i \\ 0, & otherwise \end{cases}$$

and $L_k = \left[l_{maks} - L_1^{k-1}, l_{maks} - L_2^{k-2}, \ldots, l_{maks} - L_N^{k-N}\right]^T$.

3) The input constraint was

$$\left[u_{min}, p_{min}\right]^T \leq \bar{u}_j^i \leq \left[u_{max}, p_{max}\right]^T. \tag{24}$$

where $[u_{min}, p_{min}]$ was the minimum allowed input vector and $[u_{max}, p_{max}]$ is the maximum allowed input vector. Furthermore, from Equation (17) the input constraint could be written as

$$U_{min} \leq U_k \leq U_{maks} \tag{25}$$

where U_{max} was a column vector of dimension $2N$ whose odd row elements were equal to u_{max} and even rows were equal to p_{max}. Similarly, for U_{min} was a column vector of dimension $2N$ whose odd row elements were equal to u_{min} and even rows were equal to p_{min}.

III. Time-Varying MPC design

In this section, we developed a time-varying MPC algorithm for train regulation based on the model predictive control (MPC) method. In time-varying MPC methods, the optimal control input that minimized the specified cost function over a predetermined prediction horizon was calculated at each k step. In this case, the value of state E_k calculated along the H_p horizon prediction step $(k+1,\ldots,k+H_p)$, and the set of prediction sequences input was calculated as $U_k, U_{k+1}, \ldots, U_{k+H_p-1}$. The state E_{k+i} prediction was calculated using the state change of the System (17). Only the first element in control U_k was applied to the system in order to calculate for changes in disturbance and system parameters at each step of k. The process will be repeated until the horizon prediction is reached.

The optimization issue across a particular prediction horizon was solved directly at each step k based on the dynamic model in the system by calculating the optimal control sequence. According to the most recent information, optimization was performed on the metro lines system to determine control over the problem of train regulation and passenger flow on the train in order to increase headway regularity and commercial speed of high-frequency metro lines with constraints.

The objective function for each step k optimization problem to determine the control input was

$$\begin{aligned}\min_{U_{k+j}} \quad & \sum_{j=0}^{H_p-1} \Big\{ E_{k+j+1}^T P E_{k+j+1} + \left(E_{k+j+1} - E_{k+j}\right)^T Q \\ & \left(E_{k+j+1} - E_{k+j}\right) + U_{k+j}^T R U_{k+j} \Big\} \qquad (26) \\ \text{s.t.} \quad & E_{k+j+1} = \bar{A}_{k+j} E_{k+j} + \bar{B}_{k+j} U_{k+j} + \bar{G}_{k+j} w_{k+j}, \\ & H_1 \left(E_{k+j} - E_{k+j+1}\right) \le \left(H - t_{min}\right) I_{N\times 1}, \\ & H_2 E_{k+j+1} \le L_{k+j+1}, \\ & -U_{k+j} \le -U_{min}, \\ & U_{k+j} \le U_{maks}, \quad j = 0, 1, \ldots, H_p - 1.\end{aligned}$$

For each step of k, the optimization problem in Equation (26) could be converted into a quadratic programming problem. Furthermore, let $E = \left[E_{k+1}^T, E_{k+2}^T, \ldots, E_{k+H_p}^T\right]^T$, $U = \left[U_k^T, U_{k+1}^T, \ldots, U_{k+H_p-1}^T\right]^T$, and $W = \left[w_k^T, w_{k+1}^T, \ldots, w_{k+H_p-1}^T\right]^T$ at each step k for the prediction of state E_k until the prediction horizon H_p was calculated from Equation (26) as

$$E = FE_k + \Phi U + \Gamma W \qquad (27)$$

with

$$F = \begin{bmatrix} \bar{A}_k \\ \bar{A}_{k+1}\bar{A}_k \\ \cdots \\ \bar{A}_{k+H_p-1}\bar{A}_{k+H_p-2}\cdots\bar{A}_k \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} \bar{G}_k & 0 & 0 & \cdots \\ \tilde{A}_{k+1}\bar{G}_k & \bar{G}_{k+1} & 0 & \cdots \\ & \cdots & \cdots & \cdots \\ \mathbb{Z}_1\bar{G}_k & \mathbb{Z}_2\bar{G}_{k+1} & \cdots & \bar{G}_{k+H_p-1} \end{bmatrix}$$

where $\mathbb{Z}_j = \prod_{i=k+j}^{k+H_p-1} \tilde{A}_i$, j was the column number in matrix Γ.

The following theorem provided the corresponding quadratic programming formulation at step k for the optimization problem (26) associated with state prediction E.

Theorem 3.1: For $E = FE_k + \Phi U + \Gamma W$, the simplified quadratic programming formulation at step k for the optimization problem (26) was provided as

$$\begin{aligned}\min_{U} \quad & J = \frac{1}{2} U^T H U + U^T f + \Psi. \qquad (28) \\ \text{s.t.} \quad & \begin{bmatrix} H_3 H_4 \Phi \\ H_6 \Phi \\ I_{2H_pN} \\ -I_{2H_pN} \end{bmatrix} U \le \begin{bmatrix} \bar{Z} \\ \bar{O} \\ \bar{U}_{maks} \\ -\bar{U}_{min} \end{bmatrix},\end{aligned}$$

where the weight matrix $\bar{P}, \bar{Q}$, and $\bar{R}$ could be directly found from the objective function (26),

$$\begin{aligned}\Psi = & E_k^T F^T \left[\bar{P} + \bar{Q}\right] F E_k + W^T \Gamma^T \left[\bar{P} + \bar{Q}\right] \Gamma W \\ & + E_k F^T \left[\bar{P} + \bar{Q}\right] \Gamma W + W^T \Gamma \left[\bar{P} + \bar{Q}\right] F E_k \\ & + U^T \Phi^T \left[\bar{P} + \bar{Q}\right] \Gamma W\end{aligned}$$

constant.

Matrix
$H = 2\left[\Phi^T \bar{P} \Phi + \Phi^T \bar{Q} \Phi + \bar{R}\right]$,
$f = 2\left[\Phi^T \bar{P} F E_k + \Phi^T \bar{Q} F E_k\right]$, and
$\bar{Z} = (H - t_{min}) I_{H_pN\times 1} - H_3 H_4 F E_k - H_3 H_4 \Gamma W$
$-H_3 H_5 E_k$.

Matrix
$\bar{O} = L - H_6 F E_k - H_6 \Gamma W$,
$L = [L_{k+1}^T, L_{k+2}^T, \ldots, L_{k+H_p}^T]^T$,
$\bar{U}_{maks} = \left[U_{maks}^T, U_{maks}^T, \ldots, U_{maks}^T\right]^T$,
$\bar{U}_{min} = [U_{min}^T, U_{min}^{,} \ldots, U_{min}^T]^T$, and
matrix H_3, H_4, H_5, and H_6 respectively were

$$H_3 = [h_{ij}]_{H_pN\times 2H_pN}, \quad h_{ij} = \begin{cases} 1, & j = 2i-1 \\ 0, & otherwise \end{cases},$$

$$H_6 = [g_{ij}]_{H_pN\times 2H_pN}, \quad g_{ij} = \begin{cases} 1, & j = 2i \\ 0, & otherwise \end{cases},$$

$$H_4 = \begin{bmatrix} -I_{2N} & 0_{2N} & 0_{2N} & 0_{2N} & \cdots \\ I_{2N} & -I_{2N} & 0_{2N} & 0_{2N} & \cdots \\ & \cdots & \cdots & \cdots & \\ 0_{2N} & \cdots & 0_{2N} & I_{2N} & -I_{2N} \end{bmatrix},$$

and

$$H_5 = \begin{bmatrix} I_{2N} \\ 0_{2N} \\ \vdots \\ 0_{2N} \end{bmatrix}.$$

Proof: Recalling $E = \left[E_{k+1}^T, E_{k+2}^T, \ldots, E_{k+H_p}^T\right]^T$, $U = \left[U_k^T, U_{k+1}^T, \ldots, U_{k+H_p-1}^T\right]^T$, and $W = \left[w_k^T, w_{k+1}^T, \ldots, w_{k+H_p-1}^T\right]^T$, the objective function (26) could be written as

$$
\begin{aligned}
&E^T\bar{P}E + E^T\bar{Q}E + U^T\bar{R}U \\
&\quad = (FE_k + \Phi U + \Gamma W)^T\bar{P}(FE_k + \Phi U + \Gamma W) \\
&\qquad + (FE_k + \Phi U + \Gamma W)^T\bar{Q}(FE_k + \Phi U + \Gamma W) \\
&\qquad + U^T\bar{R}U \\
&\quad = U^T\left[\Phi^T\bar{P}\Phi + \Phi^T\bar{Q}\Phi + \bar{R}\right]U + 2U^T\left[\Phi^T\bar{P}FE_k\right. \\
&\qquad \left. + \Phi^T\bar{Q}FE_k\right] + E_k^TF^T\bar{P}FE_k + E_k^TF^T\bar{Q}FE_k \\
&\qquad + W^T\Gamma^T\bar{P}\Gamma W + W^T\Gamma^T\bar{Q}\Gamma W \\
&\quad = U^T\left[\Phi^T\bar{P}\Phi + \Phi^T\bar{Q}\Phi + \bar{R}\right]U + 2U^T\left[\Phi^T\bar{P}FE_k\right. \\
&\qquad \left. + \Phi^T\bar{Q}FE_k\right] + \Psi. \qquad (29)
\end{aligned}
$$

Minimizing the objective function in (26) was equivalent with minimize

$$\min_{U} \quad J = \frac{1}{2}U^THU + U^Tf + \Psi. \qquad (30)$$

The first constraint of (26) was equivalent to

$$H_3H_4E + H_3H_5E_k \le (H - t_{min})\, I_{H_pN\times 1}, \qquad (31)$$

with $E = FE_k + \Phi U + \Gamma W$, could be written as

$$
\begin{aligned}
H_3H_4\Phi U \le (H - t_{min})\, I_{H_pN\times 1} - H_3H_4FE_k \\
- H_3H_4\Gamma W - H_3H_5E_k. \qquad (32)
\end{aligned}
$$

The second constraint of (26) was equivalent to

$$H_6\Phi U \le L - H_6FE_k - H_6\Gamma W, \qquad (33)$$

Similarly, the last two constraint (26) were respectively, identical with

$$I_{2H_pN}U \le \bar{U}_{maks}, \quad -I_{2H_pN}U \le -\bar{U}_{min}. \qquad (34)$$

The proof has been complete. ■

According to Theorem 3.1, the primary method for joint optimal train regulation and passenger flow control on metro lines with sinusoidal disturbances was given below. **Algorithm 3.2 :**

- The measured state E_k for the error joint dynamic model (17) was calculated at each sample step k using the updated system parameters $\bar{A}_k$, $\bar{B}_k$, and $\bar{G}_k$, as well as disturbances.
- Calculate F and Φ for a selected prediction horizon H_p and formulate the quadratic programming problem (28) based on Theorem 3.1.
- The following E_{k+1}would be computed by solving the quadratic programming problem efficiently (28), computing the optimal train regulation and passenger flow control U, and applying it to the joint dynamic model (17).
- Steps 1-4 should be repeated based on the measured value E_{k+1} until the step horizon k_f is reached.

In the time-varying MPC algorithm, the metro line system's stability was a complex function parameters, namely $\bar{P}, \bar{Q}, \bar{R}, \bar{A}_k, \bar{B}_k, L_k, U_{max}, U_{min}$. Based on [15], for the proposed time-varying MPC algorithm in this study, we constructed a Lyapunov function with state and control constraints (26). For stability analysis, the following theorem was utilized.

Theorem 3.3: Consider the joint error dynamic model (17), which is based on the following optimization problem and is subject to a time-varying MPC algorithm.

$$
\begin{aligned}
\min_{U_{k+j}} \quad & \sum_{j=0}^{H_p-1} \left\{E_{k+j+1}^TPE_{k+j+1} + (E_{k+j+1} - E_{k+j})^T Q\right. \\
& \left.(E_{k+j+1} - E_{k+j}) + U_{k+j}^TRU_{k+j}\right\} \qquad (35) \\
\text{s.t.} \quad & E_{k+j+1} = \bar{A}_{k+j}E_{k+j} + \bar{B}_{k+j}U_{k+j} + \bar{G}_{k+j}w_{k+j}, \\
& H_1\left(E_{k+j} - E_{k+j+1}\right) \le (H - t_{min})\, I_{N\times 1}, \\
& H_2E_{k+j+1} \le L_{k+j+1}, \\
& -U_{k+j} \le -U_{min}, \\
& U_{k+j} \le U_{maks}, \quad j = 0, 1, \ldots, H_p - 1.
\end{aligned}
$$

Assume that the optimization problem $k = k_0$ was feasible from the start, that the system parameters $\bar{A}_k$ and $\bar{B}_k$ were provided, and that $E_{k+H_p} = 0$. Then, for all $P > 0, Q > 0$, and $R > 0$, it held that $\lim_{k\to\infty} E_k = 0$, implying that the proposed time-varying MPC algorithm's joint error dynamic model (17) was stable at zero under constraints, and the factual timetable converged to the nominal timetable.

Proof: First, for the joint error dynamic model of Equation (17) under time-varying MPC, the function of the optimization problem (35) is chosen as the lyapunov function, i.e.

$$V(k) = J(U^*(k), E_k), \qquad (36)$$

with $U^*(k) = \left\{U_k^*, U_{k+1}^*, \ldots, U_{k+H_p-1}^*\right\}$ as the optimal control sequence for the problem (35). It is clear that $V(k)$ is non-negative.

The state vector for the optimal control solution $U^*(k)$ will be obtained in step k, $E(k) = \left[E_{k+1}^T, E_{k+2}^T, \ldots, E_{k+H_p}^T\right]$. The constraints are clearly satisfied by $U^*(k)$and $E(k)$. As a result, a control sequence $U(k+1) = \left\{U_{k+1}^*, U_{k+2}^*, \ldots, U_{k+H_p-1}^*, 0\right\}$ was formed for the next step $k+1$. At step $k+1$, it was obvious that $U(k+1)$ is feasible for the problem (35). By substituting $U(k+1)$ into the objective function, we get $J(U(k+1), E_{k+1})$. Then, using the assumption that $E_{k+H_p} = 0$, we get

$$
\begin{aligned}
V(k+1) &= J(U^*(k+1), E_{k+1}) \\
&\le J(U(k+1), E_{k+1}) \\
&= V(k) - E_{k+1}^TPE_{k+1} \\
&\quad - (E_{k+1} - E_k)^TQ(E_{k+1} - E_k) - U_k^TRU_k
\end{aligned} \qquad (37)
$$

which means that $V(k+1) - V(k) \le 0$, and $V(k)$ is decreasing and lower-bounded by 0. Then, using Lyapunov's theory of stability, it was claimed that $\lim_{k\to\infty} E_k = 0$, i.e. the joint error dynamic model (17) under the proposed time-varying MPC algorithm was stable at zero subject to the constraints, and the factual timetable converged to the nominal timetable. ■

IV. Numerical Experiment

Consider the problem of train regulation and passenger flow on a metro lines consisting of 12 stations ($N = 12$) and 20 trains ($Z = 20$). The disturbance occured on train j at station i was denoted by w_j^i and was assumed as a sine periodic function, $w_j^i = asin(\beta\gamma_j^i)$ for $i = 1, 2, \ldots, Z, j =$

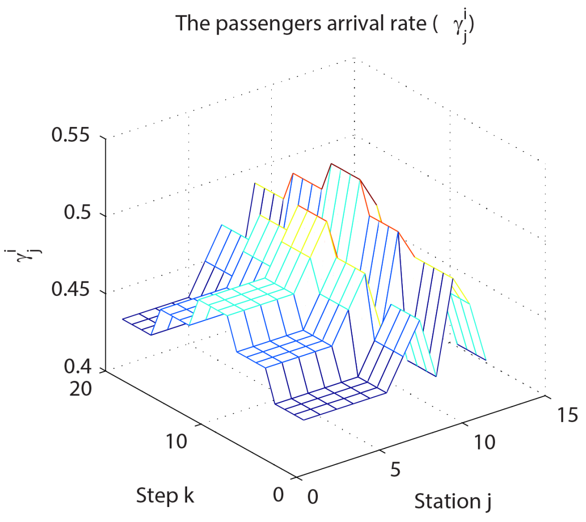

Fig. 1. The passengers arrival rate (γ_j^i)

$1, 2, \ldots, N$, $a > 0$ constant. In this study, $\beta = \frac{\pi}{4}$, $a = 5$, and β_j^i was different at each station.

Given a delay rate (α) 0.03, the passenger arrival rate at station j for trains i or γ_j^i varied within a range of values symmetrically around $\gamma = 0.35$ with half the length of $d = 0.2$ at each station. The passenger arrival rates at station j for trains i or γ_j^i are presented in Table I.

TABLE I
PASSENGERS ARRIVAL RATE (γ_j^i)

Index j	1	2	3	4	5	6
$i = 1:4$	0.43	0.43	0.43	0.43	0.43	0.43
$i = 5:8$	0.45	0.45	0.45	0.45	0.45	0.47
$i = 9:12$	0.47	0.47	0.47	0.47	0.47	0.49
$i = 13:16$	0.45	0.45	0.45	0.45	0.45	0.47
$i = 17:20$	0.43	0.43	0.43	0.43	0.43	0.45
Index j	7	8	9	10	11	12
$i = 1:4$	0.45	0.47	0.43	0.49	0.47	0.43
$i = 5:8$	0.49	0.45	0.51	0.49	0.45	0.45
$i = 9:12$	0.51	0.47	0.53	0.51	0.47	0.47
$i = 13:16$	0.49	0.45	0.51	0.49	0.45	0.45
$i = 17:20$	0.47	0.43	0.49	0.47	0.43	0.43

Based on Table I, the passengers arrival rate increased from time $k = 1$ to $k = 8$ and maximum at $k = 9$ to $k = 12$, then decreased at $k = 13$ to $k = 20$. Furthermore, the passengers arrival rate (γ_j^i) was illustrated in Fig. 1.

Due to the state constraint for the departure time on the Inequality (21), the minimum allowable headway (t_{min}) was $125s$ and headway scheduling (H) of 150 seconds, therefore we obtained $H - t_{min} = 25$ seconds. Next, for the constraint state passenger load on Inequality (23) the maximum capacity of the train for passengers (l_{max}) was 2000 and it was assumed that $l_{max} - L_j^i \leq 50$. Table II contains the initial error for the departure time and the passenger load. Table II shows the initial error fordeparture time and passenger load. The maximum train delay and maximum number of overloaded passenger are $30s$ and 40, respectively, both of which are greater than the maximum timetable and passenger load capacity adjustments. Delays require multiple stations to keep trains on a nominalschedule.

TABLE II
THE INITIAL CONDITIONS OF THE ERROR TIMETABLE AND PASSENGER LOAD AT EACH STATION

Station i	1	2	3	4	5	6
Timetable	40	45	40	35	40	30
Passenger Load	35	50	45	60	40	30
Station i	7	8	9	10	11	12
Timetable	50	40	40	30	50	30
Passenger Load	60	50	50	0	25	40

From the initial conditions in Table II, it can be simulated the condition when the metro lines system was not regulated or in other words $\bar{u}_j^i = \mathbf{0}$. The error of train departure time at stations 1-12 are illustrated in Fig. 2, which demonstrates that the disturbance causes large fluctuations in the nominal state and has a negative impact on passenger waiting times. The passenger load error on the train at stations 1-12 is illustrated in Fig. 3, which indicates that the passenger load fluctuated greatly from the nominal state. The fluctuations in departure time errors and passenger load errors have a negative impact on reducing train operational efficiency and passenger service levels.

In this study, time-varying MPC was performed with time step $T = 20$. The simulation was carried out with the aim that the departure time error and the passenger load become zero, which means that there is no delay in train departure time and passenger overload. The input control constraint on Inequality (25) are $u_{min} = -20$ and $u_{max} = 20$ which means it satisfies the constraint (24) . For given $p_{min} = -25$ and $p_{max} = 0$ which satisfies the constraint (24).

The weights P, Q, and R, respectively, were $P = \text{diag}\{0.5, 0.5, \ldots, 0.5\}, Q = \text{diag}\{0.5, 0, \ldots, 0.5,\ 0\}$, and $R = \text{diag}\{0.3, 0.3, \ldots, 0.3\}$. Let $H_u = H_p = 5$. With MATLAB, the simulation applied during the peak hour period at 07.00-09.00 with time interval is 6 minutes, therefore the rush hour period was equivalent to a 20 time step.

Using the initial conditions of the departure time error, parameter γ_j^i, and parameter β_j^i, we acquired the simulation results and input for the departure time error on each station can be seen in Fig. 4-7.

From Fig. 4-7, it can be concluded that the error in the departure time of the train at station 1 to station 12 converge to zero in several steps , it means that the control provided in the adjustment of waiting time and train travel time was successfully implemented efficiently. The input in Fig. 4-7 at each station in the k time step is less than zero or $u_j^i < 0$, which means the running time and dwell time are reduced to reduce train delays.

Furthermore, by using the initial conditions of the passenger load error on the train, the parameters γ_j^i in Table I and parameter β_j^i . The simulation result and input for the passenger load error at each station are displayed in Fig. 8-11.

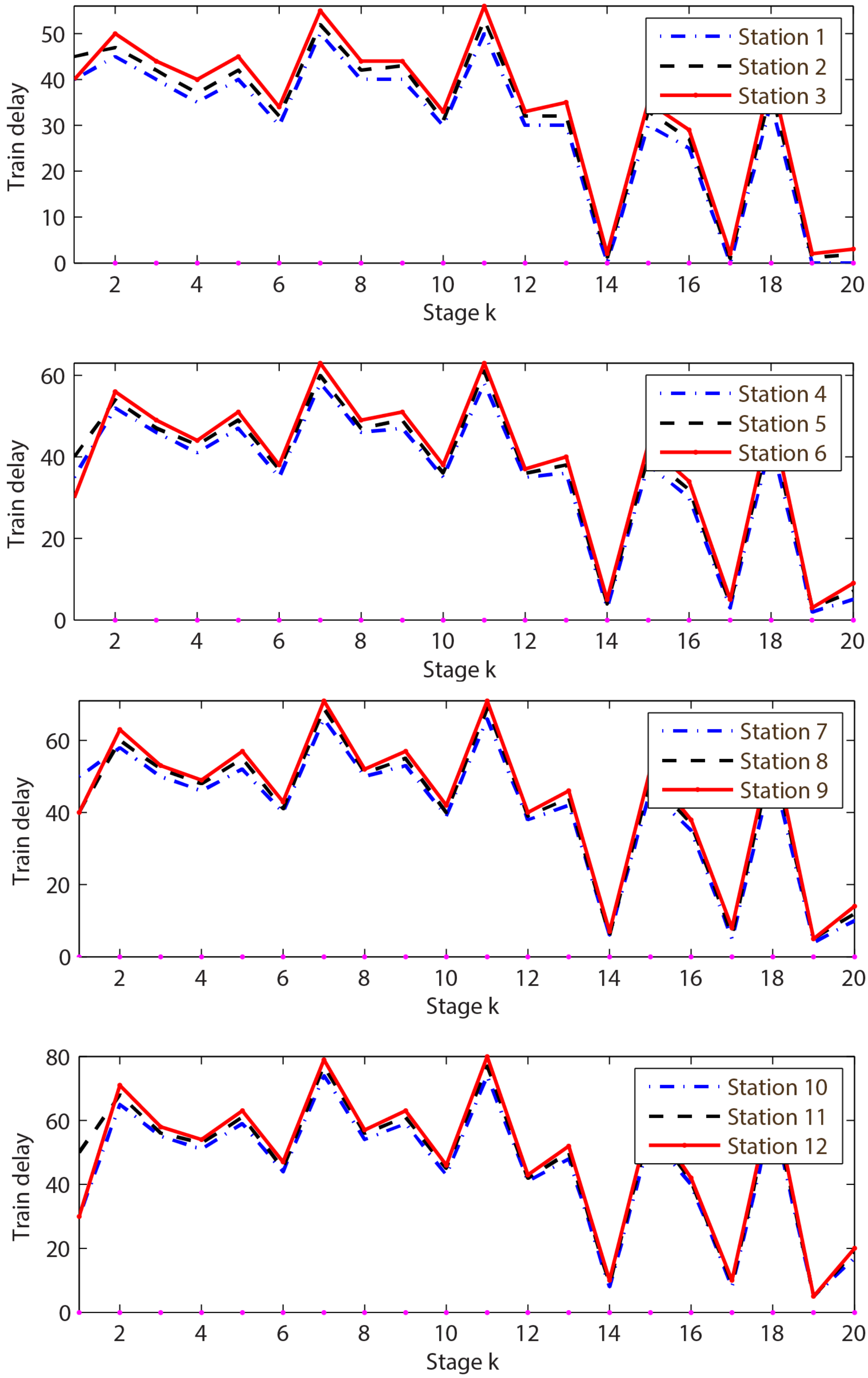

Fig. 2. The headway deviations of metro lines without train regulation.

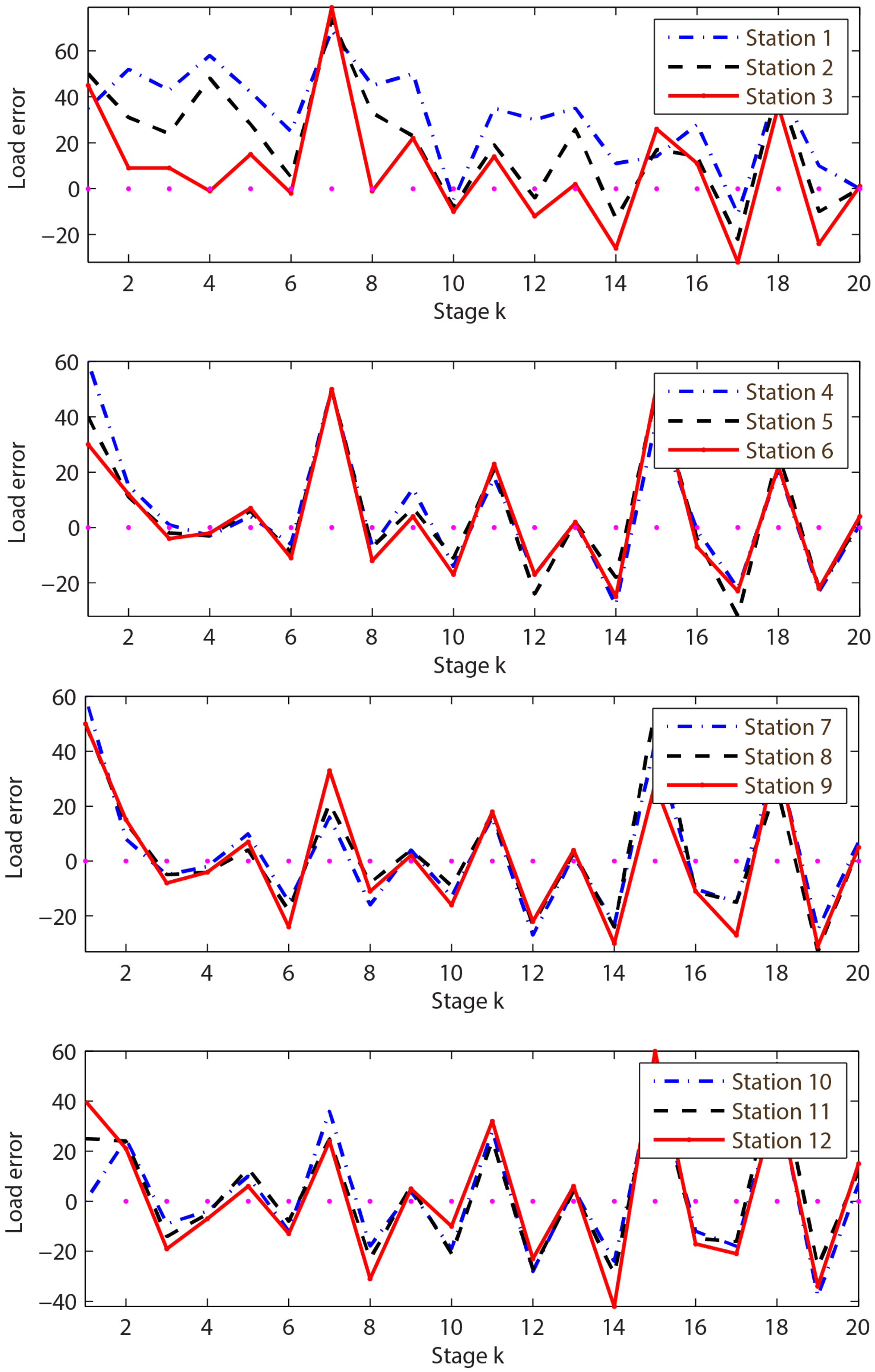

Fig. 3. The passenger load errors of metro lines without train regulation.

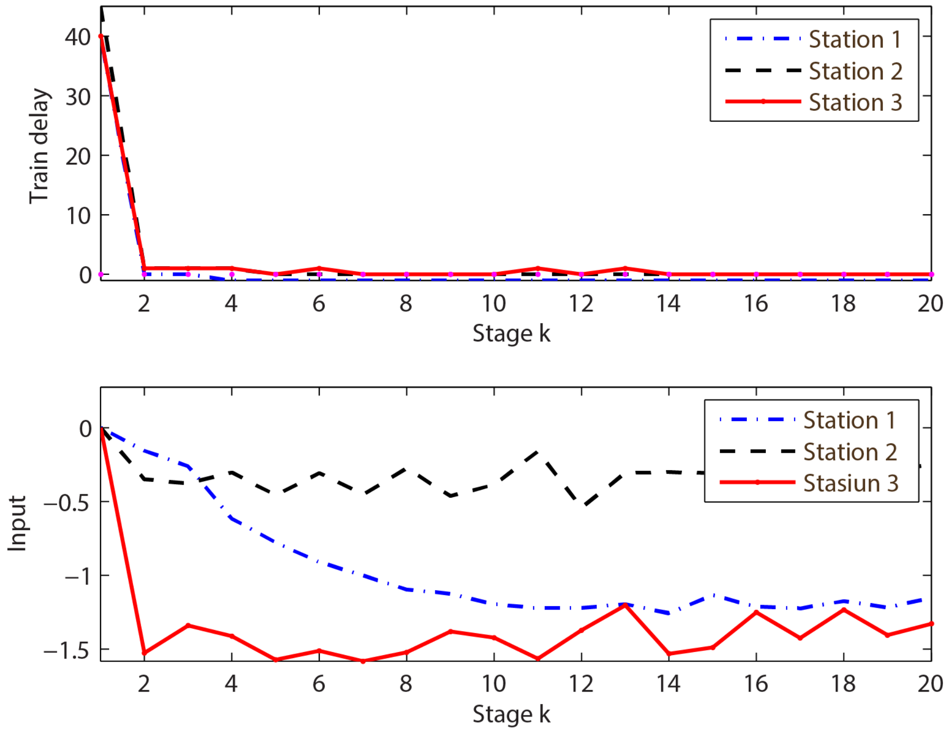

Fig. 4. Train delay at different time k in station 1,2, and 3 under the time-varying MPC.

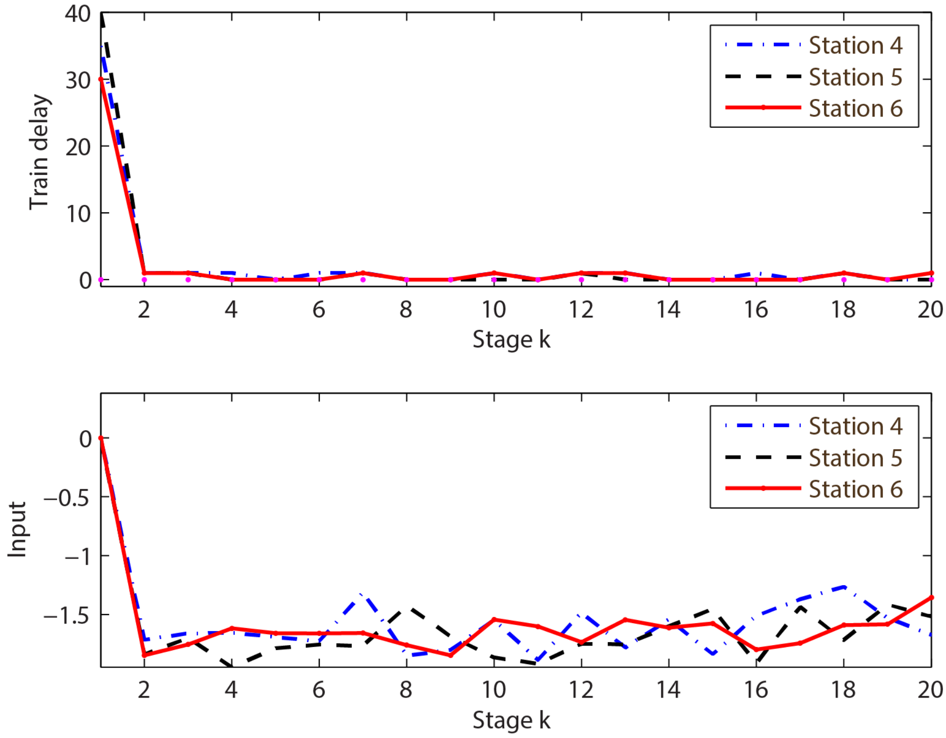

Fig. 5. Train delay at different time k in station 4,5, and 6 under the time-varying MPC.

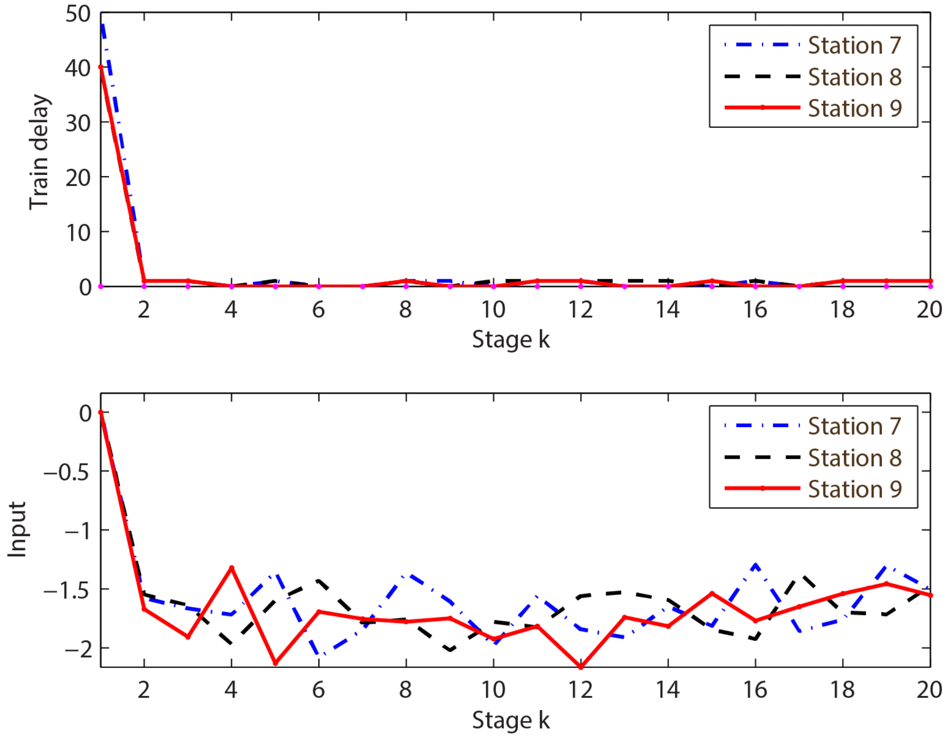

Fig. 6. Train delay at different time k in station 7,8, and 9 under the time-varying MPC.

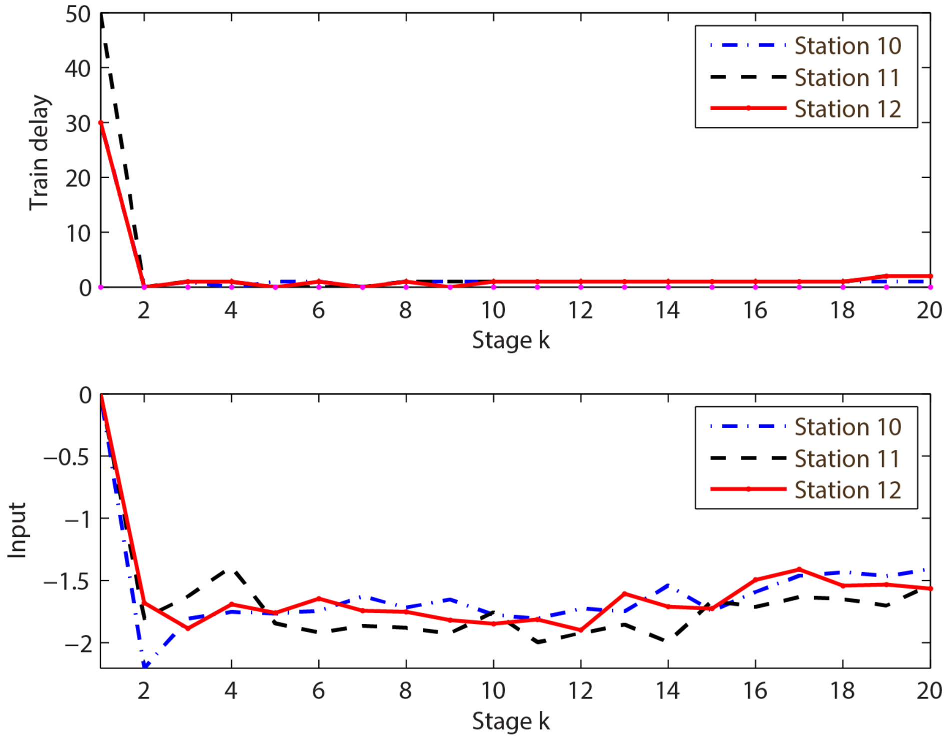

Fig. 7. Train delay at different time k in station 10,11, and 12 under the time-varying MPC.

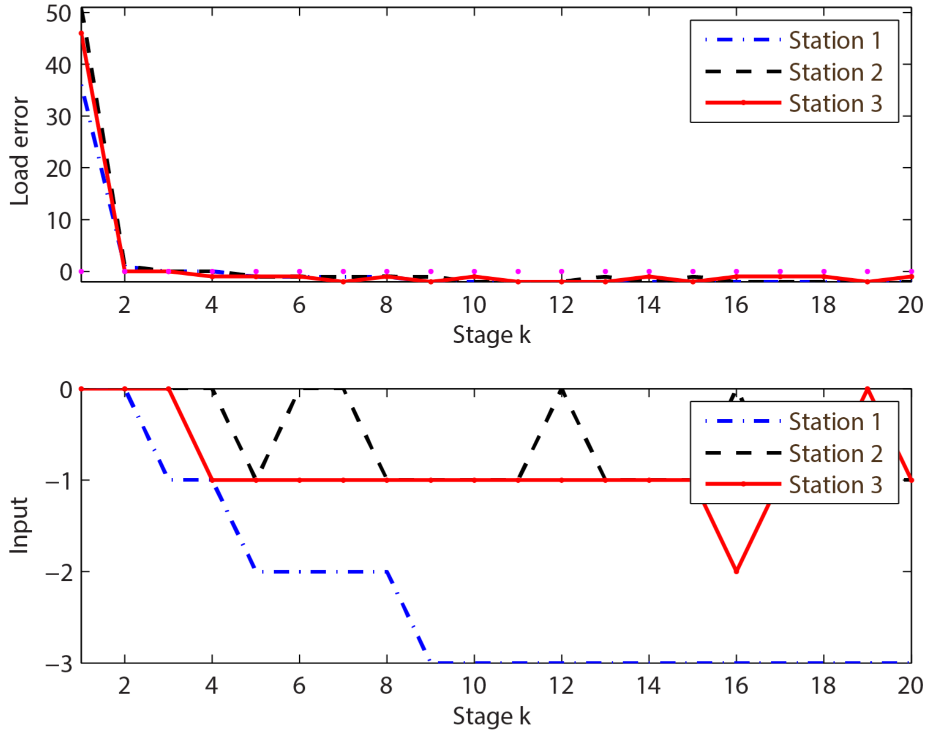

Fig. 8. Passenger load error at different time k in station 1,2, and 3 under the time-varying MPC.

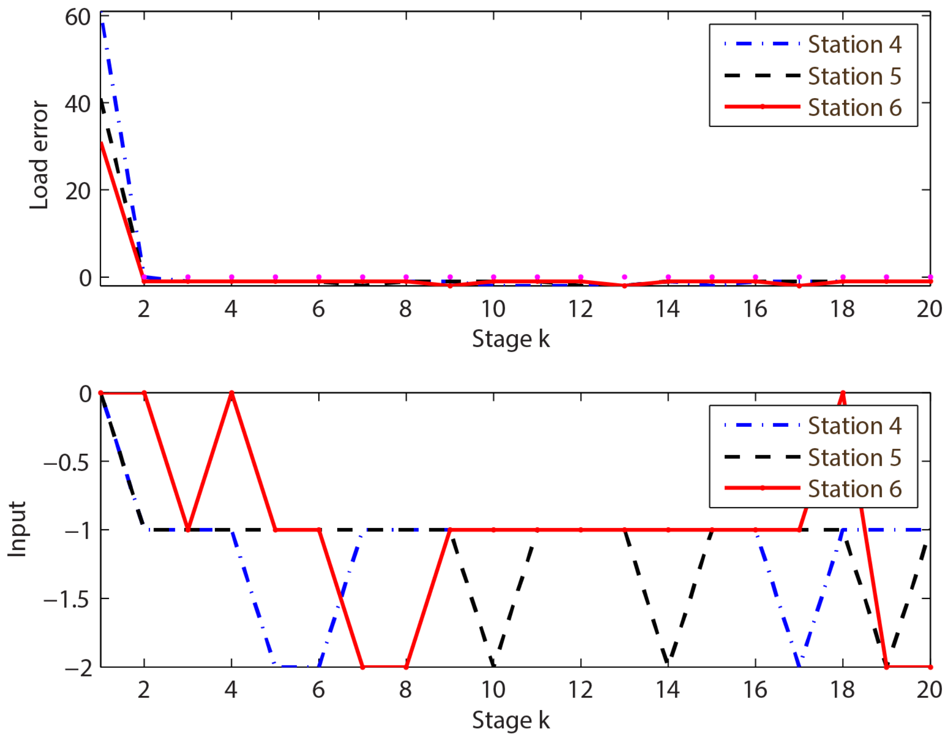

Fig. 9. Passenger load error at different time k in station 4,5, and 6 under the time-varying MPC.

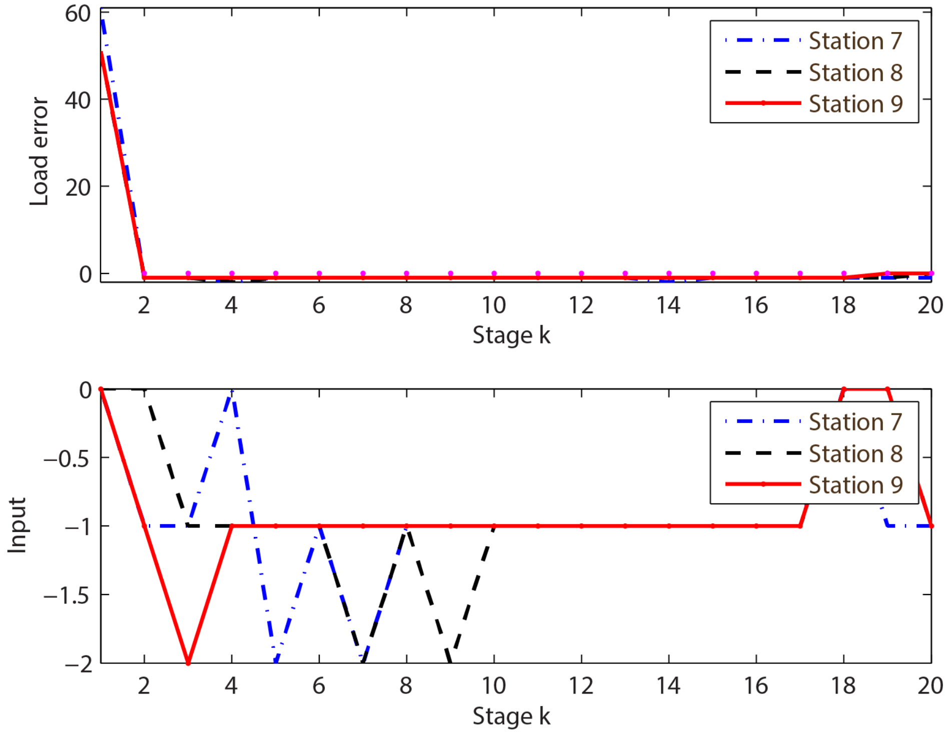

Fig. 10. Passenger load error at different time k in station 7,8, and 9 under the time-varying MPC.

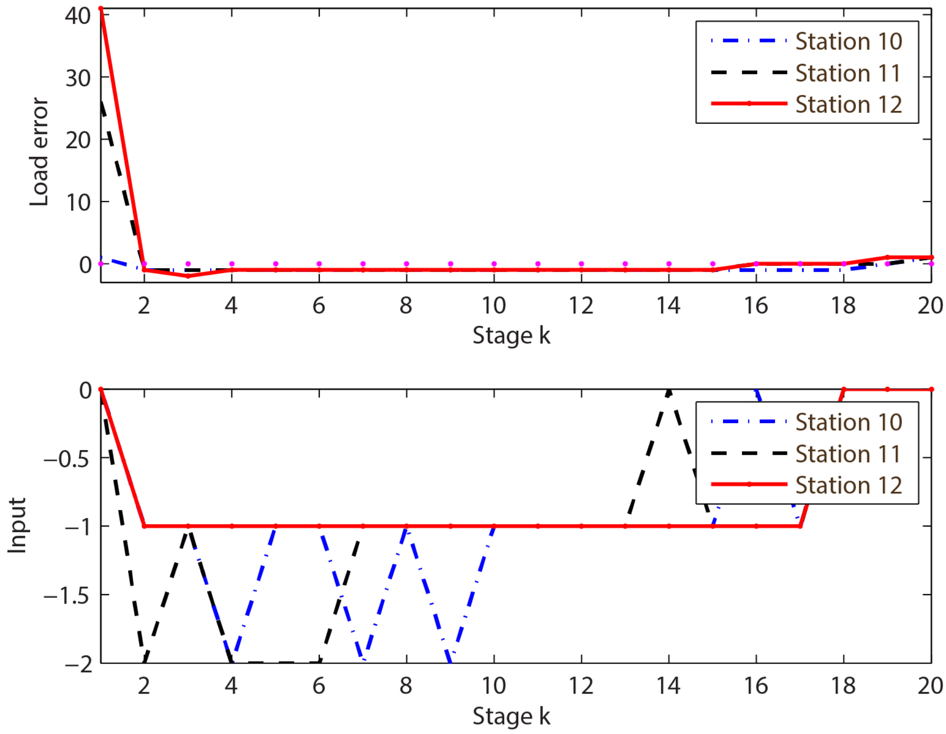

Fig. 11. Passenger load error at different time k in station 10,11, and 12 under the time-varying MPC.

From Fig. 8-11 , it can be concluded that the error of passenger load on the train at station 1 to station 12 convergent to zero in several time steps. It means the control provided was successfully implemented efficiently. The input in Fig. 4-7 at each station in the k time step is less than zero or $p^i_j < 0$, which indicates that there was a decrease in the number of arriving passengers and that the train's limited passenger capacity was met. Furthermore, the optimization problem in Equation (28), was solved using quadratic programming by *quadprog* in MATLAB. The result of objective function was 630.131.

V. Conclusion

The joint optimum train control and passenger flow strategy were investigated in this article to optimize headway regularity and comercial speed. The time-varying MPC approach was used to design an optimal control problem for the combined dynamic train regulation and passenger flow management strategy, and it was addressed by considering the headway regularity and commercial speed of the cost function. The numerical solution of a set of quadratic programming problems provided an optimal control strategy for the joint dynamic train regulation and passenger flow control method.

The suggested method offered a real-time train control and management technique for passenger flow that could be efficiently applied to real-time metro lines. The recommended joint optimum control strategy reduced train delays, passenger load errors, and train headway deviations, according to numerical experiments. Additionally, it improves passenger service standards and train operating efficiently.

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