Novel Intelligent Data Processing Technology, based on Nonstationary Nonlinear Wavelet Bispectrum, for Vibration Fault Diagnosis

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Abstract: A novel integrated data processing algorithm for vibration fault testing for electromechanical devices, energy systems and engineering structures, based on the spectral kurtosis and the nonstationary nonlinear higher order wavelet bispectrum (WB), is proposed and investigated. A novel adaptive systematic approach for identification of frequency ranges for the WB is also proposed, investigated and successfully experimentally validated. Experimental validation of the proposed data processing technology is performed, using measured data, related to non-faulty rolling bearings and bearings at an early fault stage. The high effectiveness of early bearing diagnostics by the proposed nonlinear data processing technology has been experimentally demonstrated, using the Fisher criterion and probability of correct identification. Important advantage of the proposed technology is that it could be employed for data processing and identification of electromechanical devices and structures with unknown a priori frequency characteristics. The technology is generic and, therefore, has a potential to cover multiple applications for electromechanical devices, energy systems and engineering structures.

Keywords: digital data processing; vibration fault identification; local defect detection

I. INTRODUCTION

Rolling element bearings are widely used in almost all complex electromechanical devices, (e.g. electrical motors, generators, etc.) energy systems and engineering structures. A failure of bearings could lead to a failure of a whole device and a structure.

Therefore, an early defect identification of bearings is of a great importance for integrity of electromechanical devices and complex engineering structures.

It is known, that a local defect in bearings and gears creates low intensity shock pulses [1-29]. Data processing technique, the spectral kurtosis (SK), based on the Fourier transform [1-9] and on the wavelet transform [6], is an effective tool for identification of characteristic frequencies, related to non-stationary shock pulses, spikes, etc.

Another data processing technique, the wavelet transform, is also an effective tool for processing of non-stationary shock pulses, spikes, etc. [11, 14, 15-16].Therefore, the wavelet transform (i.e. the second order technique) is widely employed for vibration condition monitoring of bearings and gears, A data processing technique in [11] for vibration condition monitoring of bearings, is based on the covariance spectra of the correlation of the wavelet transform. Condition monitoring for gearboxes in [15] employed the integrated diagnostic feature, based on the wavelet transform.

It is known, that the advanced data processing technique, the nonlinear higher order spectral techniques [30-42], are more effective for an early fault identification, than the second order data processing. The classical Fourier bispectrum is widely employed for bearing identification.

Works [30-42] are related to usage of multiple spectral narrowband components, generated by a defect, for defect identification. Further investigation on application of the HOS for defect identification for bearings and gearboxes is resulted in development of a novel approach for gear and bearing monitoring, that is not employing narrowband components, generated by a defect. In a new approach, presented in this paper, higher order technique is employing spectral components, that are, normally, not narrowband, and are contained in multiple frequency ranges, where non-stationary processes, related to machinery faults, are present. This approach is employing the wavelet transform as a HOS kernel function, as the Fourier transform is not suitable for data processing of non-stationary shock pulses, generated by bearing and gear local defect.

Therefore, the novel data processing, the nonlinear non-stationary WB [43-48], is more suitable and more effective for an early condition monitoring for gears and bearings, than the classical bispectrum and the second order wavelet transforms. Initially, the locally averaged WB is proposed [46] for a turbulence analysis. The novel non-stationary data analysis, the instantaneous WB, is proposed by L. Gelman [43], developed and experimentally validated for condition monitoring of gearboxes and bearings [43-44, 47-48]. The WB for condition monitoring of gearboxes is integrated [43-44, 47-48] via multiple frequency ranges.

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One of the main problems, associated with monitoring technologies, based on the WB, is an estimation of linked frequency ranges for shock pulses for a WB estimation and integration. This problem has a limited investigation in the existing literature. The absence of an automatic method of frequency band selection for identification technologies, based on the WB, is one of the main reasons of why, currently, these technologies have a limited application for monitoring of gears and bearings.

In order to overcome this problem, a novel proposition here is to use the SK as a data processing tool for systematic adaptive frequency range estimation for the WB technologies. The use of the SK for identifying frequency ranges, related to shock pulses, spikes, etc. is common data analysis tool. The SK could automatically and adaptively identify multiple frequency ranges, related to shock pulses, generated by bearing/gear local defect. The traditional integrating diagnostic approach [1-9] of using these multiple frequency ranges is to create an optimal Wiener filter, that is based on all these frequency ranges, and to make a filtering of gear/bearing waveforms, based on the created filter.

However, valuable diagnostic information could be obtained if a novel differential data processing approach, that is proposed here, will be employed: i.e. to differentially extract, via the SK, gear/bearing waveforms, related to multiple linked frequency ranges, and investigate these waveforms simultaneously, using the WB. It is clear, that gear and bearing waveforms in these multiple linked frequency ranges are statistically dependent, as they are generated by bearing/gear local defect. Therefore, the linked frequency ranges, identified by the SK, could be used for identification technology, based on the WB. Another important advantage of the proposed approach is that the SK could be employed for adaptive frequency band identification for gears and bearings with unknown a priori frequency characteristics of shock pulses.

This novel proposition is generic and could be widely applied to all vibration condition monitoring tasks, that are related to identification of shock pulses, generated by a defect: e.g.condition monitoring of bearings, gearboxes, complex electro-mechanical devices, etc. as it allows an automatic optimum identification of frequency ranges for the WB. However, the proposition has even a wider implementation: e. g. ultrasound non-destructive testing of structures via testing by pulses, etc. The main aims of this study are to:

- propose and develop a novel data processing technology for adaptive condition monitoring of electromechanical devices and structures, based on combined use of the SK and the nonlinear non-stationary WB
- perform novel investigation by experimental trials of the proposed technology for bearing identification

II. THE SPECTRAL KURTOSIS AND THE WAVELET BISPECTRUM

The SK identifies how shock pulses in waveforms are distributed/located over frequency ranges [1-9]. Therefore, the SK is normally employed for development of the optimal adaptive filters for identification of frequency ranges of shock pulses and shock pulse detection. In order to estimate the SK, a waveform should be divided into multiple segments. The SK expression is as follows [1]:

$$K(f) = \frac{\langle s^4(t,f)\rangle}{\langle s^2(t,f)\rangle} - 2 \qquad (1)$$

where $s(t, f)$ is the short time Fourier transform of a waveform segment, $\langle \ldots \rangle$ is mean value symbol; the functions in the nominator and the denominator of expression (1) should be averaged over a sequence of the selected time segments.

The novel instantaneous WB is determined as follows [43]:

$$B(a_1, a_2) = \int_T W_\psi(a_1, t) W_\psi(a_2, t) W_\psi^*(a, t)\, dt \qquad (2)$$

where W is the wavelet transform; a and t are a dilation (i.e. a scale) and a translation (i.e. a time shift) respectively, * is the complex conjugate symbol, the scale (frequency) rule $a^{-1} = a_1^{-1} + a_2^{-1}$ should be satisfied for the WB.

$$W(a,t) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{+\infty} x(t')\psi^*\left(\frac{t'-t}{a}\right) dt' \qquad (3)$$

For identification of defect related shock pulses, amplitude and phase characteristics of a waveform should be exploited. Therefore, complex mother wavelet functions should be employed for the technology. The Morlet mother wavelet function is employed here as follows:

$$\psi(t') = \frac{1}{\sqrt{\pi f_b}} \left(e^{j2\pi f_c t'} - e^{-f_b(\pi f_c)^2}\right) e^{-t'^2/f_b} \qquad (4)$$

where f_b is the bandwidth parameter; j is the imaginary quantity; f_c is the central frequency of the Morlet mother wavelet function.

The main approach of wavelet function selection is that a function should have a shape, similar to shock pulse shapes, generated by a defect. The Morlet wavelet is selected because : (i) it has a shape similar to typical impulse shapes, generated by local faults (ii) it is employing the Gaussian window that provides reasonable trade-off between time and frequency resolutions (iii) it minimizes the spectral leakage (iv) it allows efficient trade-off between time and frequency resolutions by the centre frequency and bandwidth parameter.

The instantaneous normalized wavelet bispectrum (NWB) is defined as [43]:

$$b_{W,T}(a_1,a_2,t) = \frac{E\left\{\left\langle W_\psi(a_1,t)W_\psi(a_2,t)W_\psi^*(a,t)\right\rangle_T\right\}}{E\left\{\left\langle \left|W_\psi(a_1,t)W_\psi(a_2,t)\right|^2\right\rangle_T\right\}E\left\{\left\langle \left|W_\psi(a,t)\right|^2\right\rangle_T\right\}} \quad (5)$$

The normalization of the WB allows avoiding a misleading interpretation of the WB due to intensity changes of data. The clear physical sense of the NWB technology is that complex frequency components, that have appeared in the spectrum of measured data due to device/structure local defect, exhibit non-zero spectral coherence. Therefore, the bispecttrum is sensitive to the appearance of defect related spectral components. If the proposed technology is being applied to health monitoring of electromechanical devices and structures, then interval for NWB is (0-1). NWB magnitude values, closed to 0 are related to no defect case, while magnitude values closed to unity (i.e. a high cross coherence between spectral components) are related to a defect.

The advantages of the non-stationary NWB, comparing with the stationary classical Fourier bispectrum, are preservation of time information and an exploitation of the coherence between multiple non-stationary shock pulses, generated by a defect. This is true for both the locally averaged NWB [46] and the novel instantaneous NWB [43]. The NWB modulus, integrated and normalized by frequency ranges, is normally used for bearing and gear identification [43-44, 47-48]:

$$I_{b_{W,T}}(t) = \frac{1}{B_1 B_2}\int_{B_1}\int_{B_2}\left|b_{W,T}(f_1,f_2,t)\right|^2 df_1 df_2 \quad (6)$$

The integrated feature (6) can be analyzed in time/angle domain, that allows an efficient fault identification. The ranges B_1 and B_2 need to be selected according to linked frequencies, related to bearing and gear shock pulses, produced by defects, as well the frequencies, related to coherence between these shock pulses. It is proposed here to employ the SK for automatic adaptive selection of linked frequency ranges for the NWB technology. Thus, the proposed differential approach of using the SK is non-traditional, as the traditional integrating approach of SK usage is to create an integral optimal Wiener filter, based on all frequency ranges, defined by the SK.

III. EXPERIMENTAL INVESTIGATION OF THE MONITORING TECHNOLOGY

Bearing test rig has a coupled VSD motor driving a shaft, supported on three identical bearings. Tests made at full speed/load: i.e. 60 Hz rotational frequency and 196 N load. Two experiments were performed, with a non-defective bearing and with a defective bearing (i.e. an inner ring fatigue defect). The relative defect size is 1.2% of the circumference. Vibrations were collected from an accelerometer, that was mounted on housing, by NI data acquisition card. Speed data were collected by the same NI data acquisition card.

It is well known that vibrations, related to a defect of inner ring consist of multiple harmonics of the inner ring defect frequency. These waveforms are modulated by rotational speed f_r. Time duration $1/(f_r - FTF)$ consists all bearing ball impulses, where the FTF is the fundamental train frequency (cage frequency). Thus, the final sampling frequency of vibrations is $(f_r - FTF)$. Vibrations are divided into segments (periods); segment duration is $1/(f_r - FTF)$.

The SK values are evaluated for realizations, that have five time segments (periods). The shock pulse linked frequencies are identified by the SK filter [1-4]:

$$\widehat{W}(f) = \begin{cases}\sqrt{K(f)} for K(f) > s \\ 0\ otherwise\end{cases} \quad (7)$$

where s is a SK threshold

The SK identifies linked frequencies, that are linked to high SK values, i.e. $K(f) > s$. Multiple frequency ranges, that are identified for defective bearings, are *differentially used* for the proposed technology. For no-defect case, the SK values are lower than a threshold; therefore, no frequency ranges are identified.

For the NWB, defect related shock pulses should be identified in the frequency domain. Each period $1/(f_r - FTF)$ contains eight shock pulses due to defect in inner ring, as the tested bearing has eight balls. Dividing the period into 24 subranges is sufficiently localizing these shock pulses. Averaging is made within a time range equal to 1/24 of the period.

Identification of 8 shock pulses by averaging of 'n' periods is performed for defect identification. Due to a possible ball slippage, n = 5 is selected.

The NWB diagnostic feature should be estimated in multiple frequency ranges (Eq. 6) for identification purposes. Bearing identification is based on correlation between multiple shock pulses, produced by a defect. Non-defective bearings are identified by low correlation values; defective bearings are identified by high correlation values. Therefore, bearing identification should be performed as following: (i) identification of all linked frequency ranges, related to a defect, by the SK; (ii) identification of all frequency pairs for the integrated NWB feature; (iii) evaluation of the integral NWB diagnostic feature by integrating features from multiple frequency pairs.

For evaluation of the linked frequencies for the NWB feature, the SK of multiple realizations is estimated. The bearing shock pulse time duration is less, than time domain distance between the shock pulses. A time window of 10% of the time distance between bearing impulses was selected for evaluation of the SK. The SK for the bearing inner ring defect is shown in Figure 1.

The linked frequencies, that are selected by the SK, are concentrated at the following central frequencies: 4.5kHz, 11kHz and 15.5 kHz. The determined peaks of the SK are essentially separated. Ranges in the frequency domain were selected, using [(f_p – i×BI) (f_p + i×BI)], were f_p is a peak frequency, BI is the inner ring defect frequency, $i = 4$. Selection of 'i' was made to leave essential frequencies in the vicinity of frequency peaks.

The NWB is estimated for frequency ranges, that are identified by the SK. All pairs of frequency ranges (i.e. B1 and B2), including self-pairing, were considered. Finally, the following two frequency band pairs are be selected: (1) [(3350; 5650) (3350; 5650)] and (2) [(3350; 5650) (9850; 12150)]. All frequencies are in Hz. The first selected pair is a self-pairing.

The partial NWB features are evaluated for these two pairs. Then, these partial NWB features are summed in order to obtain the integral NWB feature (6).

The integral NWB for non-defective bearings and defective bearings (i.e. the inner ring defect) for 50 bearing vibration realizations are shown in Fig 2. 8 shock pulses, that are clearly linked to 8 bearing balls for defective bearing, are visualized in Figure 2.

The clear separation between the integral diagnostic features for defective and non- defective bearings is seen in Figure 2. The Fisher criterion (FC) of identification effectiveness is evaluated as follows [49]:

$$FC(\theta) = \frac{(\mu_D(\theta) - \mu_{ND}(\theta))^2}{\sigma_D^2(\theta) + \sigma_{ND}^2(\theta)} \quad (8)$$

where μ and σ are the mean and the standard deviation of the integral wavelet bicoherence, evaluated for angles θ; D and ND are for defective and non-defective bearings. The FC (Figure 3) has 8 maxima, related to 8 rolling elements of the tested bearing. The averaged FC is 6.

For evaluation of the probability of correct identification for defective case, based on NWB related shock pulses, the maximum of the integral NWB feature for non-defective is selected, as an identification threshold, and, then, the NWB features for shock pulses are compared against the identification threshold. Estimates of the probability of correct identification, based on the NWB pulses, are shown in Figure 4. The averaged probability of correct identification is 99%. These estimates justify the effective bearing fault identification at early stage of defect development.

IV. CONCLUSIONS

The novel combined vibration fault identification technology for electromechanical devices (e.g. electrical motors, generators, etc.) and structures, that integrates two data processing techniques, the SK and the NWB, is proposed, developed and investigated. The integrated technology employs a novel differential approach via the SK for the optimal frequency range identification for the NWB. The proposed technology also employs a non-traditional method of using spectral components, that are wideband, and are contained in multiple frequency ranges, where non-stationary waveforms, related to machinery faults, are present.

The technology is generic and, therefore, has a potential to cover multiple identification applications for electromechanical devices and structures.

One of the main implementations of the technology is vibration identification of local defects of bearings and gears. However, the proposed technology has even wider implementation: e. g. vibration and ultrasound non-destructive testing of structures via testing by pulses.

The proposed novel differential approach of using the SK for the NWB estimation is in contrast to the traditional integral approach of SK usage, that is to create an integral optimal Wiener filter, based on all frequency ranges, defined by the SK. The NWB uses the SK's identified frequency ranges for local defect identification. Local defect identification is performed by the integral NWB feature over the identified frequency ranges.

Experimental testing of the proposed technology was made via test rig trials with non- defective bearing and bearing with early stage of a fault. Estimate of the averaged probability of correct identification, based on NWB shock pulses, related to bearing defect, is 99%. This result confirm that the technology was successfully experimentally validated for identification of early stage of bearing fault.

The proposed novel concept, that combines data processing by the SK and the NWB, will make a major influence for the NWB implementation for electromechanical device and structure identification. It opens a wide usage of the NWB for condition monitoring of local defects, that generates shock pulses, by allowing an automatic selection of frequency ranges for the NWB. Another important advantage of the proposed concept is that the NWB could be employed for adaptive identification of electromechanical devices and structures with unknown a priori frequency characteristics of shock pulses.

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