A One-dimensional Salinity Measurement Model in the Chao Phraya River with the Chao Phraya Barrage Dam Using a Shooting Method

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Abstract—**Sea water receding from the Gulf of Thailand causes salinity diffusion in the Chao Phraya River, Thailand, and the amount of northern water is reduced in the dry season. It has an impact on people, particularly the generation of tap water. The Samlae raw water pumping station is the major pumping station. The Ban Krachaeng subdistrict is located in Mueang district, Pathum Thani province, and is affected by saltwater intrusion, which causes the salinity level to exceed the recommended threshold. In order for the Metropolitan Waterworks Authority (MWA)'s water supply system to achieve the standard, the salinity index at the Samlae raw water pumping station is controlled to not exceed the surveillance threshold of 0.25 g/l in this research. A barrage dam consists of a number of large gates that can be opened or closed to control the amount of fresh water passing through. There is a Chao Phraya barrage dam which is across the Chao Phraya River at Chai Nat, the northern part of the focused area. The objective of this research is to demonstrate a one-dimensional steady-state salinity measurement model in a river with a barrage dam. Irrigation is done in rivers with dams using the shooting method to estimate the solution. The results obtained from simulating simulated salinity measurements from Phra-Nakhon Tai Power Plant Station to Samlae Station demonstrated that the shooting method can be used to accurately estimate the solution. Freshwater flow velocity and salinity dissolving efficiency were discovered to be the most important elements controlling salinity levels. The suggested salinity control approach may help to regulate the salinity level until it reaches a normal level.**

Index Terms— **Shooting Method, Salinity, Measurement Model, Barrage dam, Chao Phraya River**

I. Introduction

THE models for water contamination were solved using the finite element technique. The closed uniform reservoir hydrodynamic model with constant coefficients was solved in [6] using the finite difference method. [7] presented an analytical answer to the hydrodynamic model in a closed, uniform reservoir. To estimate the water height and flow velocity, the Lax-Wendroff finite difference approach was also put forth in [8].

The fourth-order method for a one-dimensional water quality model in a stream with nonuniform flow was put out in [9]. For defining the height of water waves in an open uniform reservoir, a non-dimensional variant of a two-dimensional hydrodynamic model with generalized boundary conditions and beginning conditions was developed in [10].

[11] proposes a one-dimensional mathematical model for calculating salinity in a river. Also presented is a modified model of salinity control in a river with a barrage dam. However, they are not comparing their numerical answer with the real salinity measurements that have been gathered.

In this study, a non-dimensional mathematical model for calculating salinity is put forward. The river in South Chaophraya, Thailand, is taken into account. Techniques for setting the physical parameters are also suggested. Additionally, we will contrast our numerical conclusion with the salinity data that was really gathered.

II. Salinity Measurement Models

A. Salinity Measurement in a River

In a salinity measurement model is a river with a barrage dam, a one-dimensional advection-diffusion equation will be introduced. Modeling several environmental problems using the advection-diffusion equation can be done for [14], [15], [16], [17], [18], [19], [20], and [21].

The equation is averaged over the depths in a simplified form in [2],

$$\frac{\partial C}{\partial T}+u\frac{\partial C}{\partial X}=D\frac{\partial^2 C}{\partial X^2}-Q, \qquad (1)$$

for all $(X,T)\in\Omega=[0,L]\times[0,\tau]$, u is the flow velocity, D is a given diffusion coefficient (m^2/s) L is a length of a considered river segment (km), τ is a stationary of simulation time and Q is the sink rate function (m^3/s) . Assuming that the freshwater dilutes the salinity, the velocity of the freshwater reduces the salinity advection level. It is assumed that freshwater can diluted salinity by a factor of, $0\le k\le 1$.

B. Salinity Measurement Model in a River with a Barrage Dam

The following [11], are representations of a one-dimensional salinity water pollution measuring model in a

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river with a barrage dam by,

$$\frac{\partial C}{\partial T}+\left(u_s-ku_w\right)\frac{\partial C}{\partial X}=D\frac{\partial^2 C}{\partial X^2}-Q, \quad (2)$$

where $C(X,T)$ is the salinity concentration (kg/m^3), u_s is advective velocity of salinity water (m/s), k is water salinity removal efficiency rate (m/s) and u_w is the fresh water flow velocity (m/s).

C. Initial Condition and Boundary Conditions Setting

1) *The initial condition*: An interpolation function of measured raw salinity data defines the initial condition. It runs parallel to the river's length from the estuary to the furthest point of the region under consideration. The initial condition is assumed to be

$$C(X,0)=F(X), \quad (3)$$

for all $X\in[0,L]$, where $F(X)$ is an interpolation function of measured salinity data.

2) *The left boundary condition:* An interpolation function of the measured raw data is used for the left boundary condition. It is based on a river's salinity at the first station near the estuary. The boundary condition is assumed to be

$$C(0,T)=G(T), \quad (4)$$

for all $T\in[0,\tau]$, where $G(T)$ is a given interpolation function by measured salinity data at the first monitoring station.

3) The right boundary condition: The right boundary condition is a function of the measured raw data interpolated. It is predicated on a river's salinity at the end station. The boundary condition is assumed to be

$$C(L,T)=H(T), \quad (5)$$

for all $T\in[0,\tau]$, where $H(T)$ is a given interpolation function by measured salinity data at the end monitoring station.

D. A Non-dimensional Salinity Measurement Model with a Barrage Dam

Taking non-dimensional technique [12] into (2), we get the following discretization:

$$\frac{\partial c}{\partial t}=\frac{\partial^2 c}{\partial x^2}-\left(\left(u_s-ku_w\right)\frac{L}{D}\right)\frac{\partial c}{\partial x}-\frac{QL^2}{DC_0}, \quad (6)$$

where L represent the length of river, C_0 some salinity at zero time,

$$c=\frac{C}{C_0}, \quad (7)$$

$$t=\frac{DT}{L^2}, \quad (8)$$

$$x=\frac{X}{L}. \quad (9)$$

E. Initial and Boundary Condition of the Non-dimensional Model

1) *The initial condition:* An interpolation function of measured raw salinity data defines the initial condition. It runs parallel to the river's length from the estuary to the furthest point of the region under consideration. The initial condition is assumed to be

$$c(x,0)=f(x), \quad (10)$$

for all $x\in[0,1]$, where $f(x)$ is an interpolation function of measured salinity data.

2) *The left boundary condition:* An interpolation function of the measured raw data is used for the left boundary condition. It is based on a river's salinity at the first station near the estuary. The boundary condition is assumed to be

$$c(0,t)=g(t), \quad (11)$$

for all $t\in[0,\Gamma]$, where $g(t)$ is a given interpolation function by measured salinity data at the first monitoring station.

3) The right boundary condition: A function of interpolation using measured raw data creates the boundary condition on the right-hand side. It is predicated on a river's salinity at the end station. The boundary condition is assumed to be

$$c(L,t)=h(t), \quad (12)$$

for all $t\in[0,\Gamma]$, where $h(t)$ is a given interpolation function by measured salinity data at the end monitoring station.

III. A Steady-State Salinity Measurement Model in the Chao Phraya River with a Barrage Dam

A non-dimension steady salinity measurement model is obtained by

$$\left(u-Ku_w\right)\frac{dC}{dX}=\frac{D}{L}\frac{d^2C}{dX^2}.$$

A. The left boundary condition letting that

$$C(0)=c_0, \quad (13)$$

where c_0 is the salinity at the first monitored station.

B. The right boundary condition

$$C(L)=c_L, \quad (14)$$

where c_L is the salinity at the last monitored station.

IV. Shooting Method

Corollary 1 The boundary value problem

$y''=p(x)y'+q(x)y+r(x)$, for all $a\le x\le b$,
$y(a)=\alpha$, $y(b)=\beta$, where α and β are given, satisfies

I. $p(x)$, $q(x)$ and $r(x)$ are continuous on $[a,b]$,
II. $q(x)>0$ on $[a,b]$,
then the problem has a unique solution.

To approximate the unique solution to this linear boundary value problem, we first consider the initialvalue problems

$$y'' = p(x)y' + q(x)y + r(x), \quad (15)$$

with $a \le x \le b$, $y(a) = \alpha$ and $y'(a) = 0$,
and

$$y'' = p(x)y' + q(x)y, \quad (16)$$

with $a \le x \le b$, $y(a) = 0$ and $y'(a) = 0$.

Letting that $y_1(x)$ is denoted the solution to (15) and let $y_2(x)$ be the solution to (16). Assuming that $y_2(b) \ne 0$.

Define $y(x) = y_1(x) + \frac{\beta - y_1(b)}{y_2(b)} y_2(x)$.

Then $y(x)$ is the solution to the linear boundary value problem. To see this, first note that

$$y'(x) = y'_1(x) + \frac{\beta - y_1(b)}{y_2(b)} y'_2(x), \quad (18)$$

and

$$y''(x) = y''_1(x) + \frac{\beta - y_1(b)}{y_2(b)} y''_2(x). \quad (19)$$

Substituting for $y''_1(x)$ and $y''_2(x)$ in this equation gives

$$y'' = p(x)y'_1 + q(x)y_1 + r(x)$$

$$+ \frac{\beta - y_1(b)}{y_2(b)}\left(p(x)y'_2 + q(x)y_2\right), \quad (20)$$

$$= p(x)\left(y'_1 + \frac{\beta - y_1(b)}{y_2(b)} y'_2\right)$$

$$+ q(x)\left(y_1 + \frac{\beta - y_1(b)}{y_2(b)} y_2\right) + r(x), \quad (21)$$

$$= p(x)y'(x) + q(x)y(x) + r(x). \quad (22)$$

Moreover,

$$y(a) = y_1(a) + \frac{\beta - y_1(b)}{y_2(b)} y_2(a), \quad (23)$$

$$= \alpha + \frac{\beta - y_1(b)}{y_2(b)} \cdot 0 = \alpha, \quad (24)$$

and

$$y(b) = y_1(b) + \frac{\beta - y_1(b)}{y_2(b)} y_2(b), \quad (25)$$

$$= y_1(b) + \beta - y_1(b) = \beta. \quad (26)$$

Next, we will employ the fourth-order Runge-Kutta method to approximate their solutions of both initial value problems Eqs(15)-(16). The fourth-order Runge-Kutta method can be described by an algorithm as below.

INPUT Endpoints a, b;

boundary conditions α, β;

number of subintervals N.

OUTPUT Approximations $w_{1,i}$ to $y(x_i)$ and $w_{2,i}$ to $y'(x_i)$ for each $i = 0,1,\ldots,N$.

Step 1. Set $h = (b-a)/N$, (17)

$$u_{1,0} = \alpha, \quad u_{2,0} = 0,$$
$$v_{1,0} = 0, \quad v_{2,0} = 1.$$

Step 2. For $i = 0,1,\ldots,N-1$ do step 3 and 4.
(The Runge-Kutta method for systems is used in Steps 3 and 4.)
Step 3. Set $x = a + ih$.
Step 4. Set $k_{1,1} = hu_{2,i}$.

$$k_{1,2} = h\left[p(x)u_{2,i} + q(x)u_{1,i} + r(x)\right],$$

$$k_{2,2} = h\left[p(x+h/2)\left(u_{2,i} + \frac{1}{2}k_{1,2}\right)\right.$$

$$\left. + q(x+h/2)\left(u_{1,i} + \frac{1}{2}k_{1,1}\right) + r(x+h/2)\right],$$

$$k_{3,1} = h\left[u_{2,i} + \frac{1}{2}k_{2,2}\right],$$

$$k_{3,2} = h\left[p(x+h/2)\left(u_{2,i} + \frac{1}{2}k_{2,2}\right)\right.$$

$$\left. + q(x+h/2)\left(u_{1,i} + \frac{1}{2}k_{2,1}\right) + r(x+h/2)\right],$$

$$k_{4,1} = h\left[u_{2,i} + k_{3,2}\right],$$

$$k_{4,2} = h\left[p(x+h)\left(u_{2,i} + \frac{1}{2}k_{3,2}\right)\right.$$

$$\left. + q(x+h)\left(u_{1,i} + k_{3,1}\right) + r(x+h)\right],$$

$$u_{1,i+1} = u_{1,i} + \frac{1}{6}\left[k_{1,1} + 2k_{2,1} + 2k_{3,1} + k_{4,1}\right],$$
$$u_{2,i+1} = u_{2,i} + \frac{1}{6}\left[k_{1,2} + 2k_{2,2} + 2k_{3,2} + k_{4,2}\right],$$

$$k'_{1,1} = hv_{2,i},$$

$$k'_{1,2} = h\left[p(x)v_{2,i} + q(x)v_{1,i}\right],$$

$$k'_{2,1} = h\left[v_{2,i} + \frac{1}{2}k'_{1,2}\right],$$

$$k'_{2,2} = h\left[p(x+h/2)\left(v_{2,i} + \frac{1}{2}k'_{1,2}\right) + q(x+h/2)\left(v_{1,i} + \frac{1}{2}k'_{1,1}\right)\right],$$

$$k'_{3,1} = h\left[v_{2,i} + \frac{1}{2}k'_{2,2}\right],$$

$$k'_{3,2} = h\left[p(x+h/2)\left(v_{2,i} + \frac{1}{2}k'_{2,2}\right) + q(x+h/2)\left(v_{1,i} + \frac{1}{2}k'_{2,1}\right)\right],$$

$$k'_{4,1} = h\left[v_{2,i} + k'_{3,2}\right],$$

$$k'_{4,2} = h\left[p(x+h)\left(v_{2,i} + k'_{3,2}\right) + q(x+h)\left(v_{1,i} + \frac{1}{2}k'_{3,1}\right)\right],$$

$$v_{1,i+1} = v_{1,i} + \frac{1}{6}\left[k'_{1,1} + 2k'_{2,1} + 2k'_{3,1} + k'_{4,1}\right],$$

$$v_{2,i+1} = v_{2,i} + \frac{1}{6}\left[k'_{1,2} + 2k'_{2,2} + 2k'_{3,2} + k'_{4,2}\right].$$

Step 5. Set $w_{1,0} = \alpha$,

$$w_{2,0} = \frac{\beta - u_{1,N}}{v_{1,N}},$$

OUTPUT $(a, w_{1,0}, w_{2,0})$.

Step 6. For $i = 1, \ldots, N$,

Set $w1 = u_{1,i} + w_{2,0}v_{1,i}$,

$$w2 = u_{2,i} + w_{2,0}v_{2,i},$$

Step 7. STOP.

V. Numerical Simulations

5.1 Simulation 1: The salinity measurement in the Chao Phraya river for a dry season from the first station to the controlled station.

Considered from the South Bangkok Power Plant, which is the first salinity measurement station, to the Samlae Station which is the last station of the measurement. The total distance from the first station to the last station is 84 km. The convection velocity of saltwater is 0.05 m/s, and the freshwater flow velocity is 0.3 m/s. The diffusion coefficient of the saltwater was set to 1.68 m^2/s, and the salinity solubility efficiency was 0.1 under the dry season northern water flow assumption. The average salinity in 7 days of the Southern Bangkok Power Plant Station was set to 18.9 g/l and the Samlae Station to 0.15 g/l. In the non-dimensional salinity model, we can obtain that a $c(0)=1$ and $c(1)=0.007936$.

In this case we choose $h = 0.05$. The proposed numerical techniques are used to approximate their salinity along the segment of the river. The non-dimensional solution is approximated as shown in TABLE I.

TABLE I
Approximated Results of Non-dimensional Solutions

X	$u_{1,i+1} \approx C_1(X_i)$	$v_{1,i+1} \approx C_2(X_i)$	$C(X_i)$
0.00	1	0.000000	1.000000
0.05	1	0.051271	0.970398
0.10	1	0.105171	0.939279
0.15	1	0.161834	0.906564
0.20	1	0.221403	0.872171
0.25	1	0.284025	0.836016
0.30	1	0.349859	0.798006
0.35	1	0.419068	0.758048
0.40	1	0.491825	0.716041
0.45	1	0.568312	0.671880
0.50	1	0.648721	0.625455
0.55	1	0.733253	0.576650
0.60	1	0.822119	0.525343
0.65	1	0.915541	0.471405
0.70	1	1.013753	0.414702
0.75	1	1.117000	0.355091
0.80	1	1.225541	0.292424
0.85	1	1.339647	0.226544
0.90	1	1.459603	0.157286
0.95	1	1.585710	0.084478
1.00	1	1.718282	0.007936

The values shown in TABLE I will be calculated in the unit scale which is the actual distance from the Southern Bangkok Power Plant Station to Samlae Station is 84 km. The actual salinity will be converted as shown in TABLE II and Fig.1.

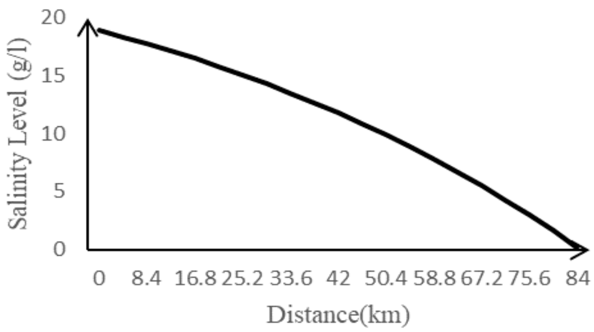

Fig. 1 The approximated salinity in the Chao Phraya river in a dry season.

TABLE II
THE APPROXIMATED SALINITY IN THE CHAO PHRAYA RIVER IN A DRY SEASON

Distance(km)	Salinity Level (g/l)
0.00	18.90000
4.20	18.34053
8.40	17.75237
12.60	17.13405
16.80	16.48404
21.00	15.80070
25.20	15.08232
29.40	14.32711
33.60	13.53317
37.80	12.69854
42.00	11.82111
46.20	10.89869
50.40	9.928983
54.60	8.909555
58.80	7.837861
63.00	6.711219
67.20	5.526813
71.40	4.281681
75.60	2.972710
79.80	1.596627
84.00	0.149990

5.2 Simulation 2: The salinity measurement in the Chao Phraya river for a rainy season from the first monitored station to the last monitored station

From the South Bangkok Power Plant, which is the first salinity measurement station, to the last stop at Wat Phai Lom station, the total distance from the first station to the last station is 90 km. The convection velocity of saltwater is 0.05 m/s, and the freshwater flow velocity is 0.3 m/s. The diffusion coefficient of the saltwater was set to 1.8 m^2/s, and the salinity solubility efficiency was 0.6 under the rainy season northern water flow assumption. The average salinity in 7 days of the Southern Bangkok Power Plant Station was set to 18.9 g/l, and the Wat Phai Lom Station.

The values shown in TABLE III will be calculated in the unit scale which is the actual distance from the Southern Bangkok Power Plant Station to the Wat Phai Lom Station is 90 km. The actual salinity will be converted as shown in TABLE IV and Fig. 2.

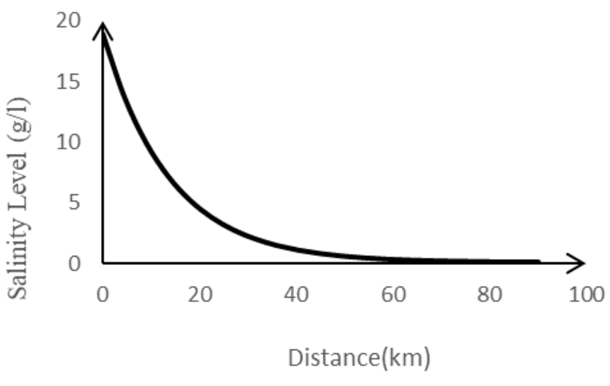

Fig. 2 The approximated salinity in the Chao Phraya river in a rainy season.

TABLE III
THE APPROXIMATED NON-DIMENSIONAL SOLUTION OF SIMULATION 2

X	$u_{1,i+1} \approx C_1(X_i)$	$v_{1,i+1} \approx C_2(X_i)$	$C(X_i)$
0.00	1	0.000000	1.000000
0.05	1	0.042684	0.724196
0.10	1	0.073525	0.524912
0.15	1	0.095810	0.380919
0.20	1	0.111912	0.276875
0.25	1	0.123546	0.201698
0.30	1	0.131953	0.147378
0.35	1	0.138027	0.108129
0.40	1	0.142416	0.079769
0.45	1	0.145587	0.059278
0.50	1	0.147879	0.044472
0.55	1	0.149534	0.033773
0.60	1	0.150731	0.026043
0.65	1	0.151595	0.020458
0.70	1	0.152220	0.016422
0.75	1	0.152671	0.013506
0.80	1	0.152997	0.011399
0.85	1	0.153233	0.009876
0.90	1	0.153403	0.008776
0.95	1	0.153526	0.007981
1.00	1	0.153615	0.007407

TABLE IV
THE APPROXIMATED SALINITY IN THE CHAO PHRAYA RIVER IN A RAINY SEASON

Distance(km)	Salinity Level (g/l)
0.00	18.900000
4.50	13.687310
9.00	9.920840
13.50	7.199360
18.00	5.232938
22.50	3.812088
27.00	2.785444
31.50	2.043636
36.00	1.507638
40.50	1.120350
45.00	0.840512
49.50	0.638314
54.00	0.492215
58.50	0.386649
63.00	0.310373
67.50	0.255258
72.00	0.215435
76.50	0.186661
81.00	0.165870
85.50	0.150847
90.00	0.139992

VI. DISCUSSION

Salinity was measured from the Southern Bangkok Power Plant Station to Samlae Station. The salinity was determined over a distance of 84 kilometers using the shooting method. The water flow velocity is 0.3 m/s, while the velocity of saltwater convection is 0.05 m/s. The salinity diffusion coefficient was set to 1.68 m2/s, and the salinity dissolving efficiency was adjusted to 0.1. In seven days, the average salinity at the Southern Bangkok Power Plant Station was 18.9 g/l and at the Samlae Station it was 0.15 g/l. In TABLE

IV, we can see that the simulated salinity at 85.50 km. is 0.150847 g/l which is closed to the actual field salinity 0.15 g/l at the Samlae pumping station. There results demonstrate that the proposed numerical model produces good agreement results. According to this research, salinity had entered the river. As a result, the salinity dissolution in the water area before reaching the Samlae station is limited.

VII. Conclusion

The approximated salinity in the Chao Phraya river in a dry season, because of the problem of salinity intrusion in the Chao Phraya River, the salinity concentration of the river was measured using the shooting method with the corresponding boundary conditions during both the flood and dry seasons. However, in the future, if a boundary condition is unknown and there is no constant water quality monitoring station on site, measurements can be made simply by computing the mean salinity around the area. The most important elements affecting salinity levels were identified to be freshwater flow velocity and salinity dissolving efficiency. The suggested procedure for salinity control may help to control the salinity level until it reaches a normal level. Additional research can be used to create realistic salinity control scenarios in rivers with barrage dam systems, as well as fresh water usage reductions.

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