Analysis of Queueing System of Two Service Desks for Customer Classification Service

Shengli Lv, *Member, IAENG*, Xiaodong Ren and Jingbo Li

***Abstract*—The formulation of service rules is an important issue in queueing theory research. Different queueing rules have a direct impact on the efficiency of the queueing system and customer satisfaction. This paper mainly studies the queueing system index of two identical service desks of customer classification service system. Based on the queueing system theory of single service desk and multiple service desks, the main queueing index of the system such as average queue length and average waiting time is obtained. Through numerical analysis, the two service modes of customer classification service and customer non-classification service are compared, and it is found that there are obvious differences in system operation indexes of different service modes. Under a reasonable classification strategy, customer classification service can improve system operation efficiency.**

***Index Terms*—Customer sorting Service, Multi-desk queueing system, Average queue length, Average waiting time.**

I. Introduction

IN the queueing system, we can reduce latency time and queue length of customers by adding the number of service desks, but this obviously increases the operating costs. If the number of open service desks is small, the waiting time of customers and queue length will increase, and customer satisfaction will be affected. Therefore, maximizing the benefits of both the customer and the service system is the key problem of the queueing system.

At present, a large number of theoretical studies focus on how to construct more reasonable service and queueing rules to optimize the queueing system, and then queueing service efficiency and customer satisfaction have been improved. The reference [1] optimizes the bank service window based on queueing model $\mathrm{M/M/}c$ and marginal analysis method, and then improves the service efficiency of the bank. Considering the queueing problem of banks, the reference [2] has established a banking service system model based on $\mathrm{M/M/}c$ queueing system. This provides decision-making reference for banks to reasonably optimize the service system.

Many scholars optimized supermarket cashier system based on the queueing theory [3–6]. The reference [3] have constructed the queue model $\mathrm{M/M/1}$ for supermarket cashier system. By analyzing a cash register system, the results show that improving service efficiency of the cashier costs a lot, but queuing time and queuing length are superior to increasing a cash register. The reference [4] firstly takes the longest queue length and longest waiting time acceptable to customers as constraints, then establishes a multi-service desk negative exponential distribution queue system model, and finally optimizes the cashier queue system of supermarkets. Some studies have considered the impact of customer waiting time differences on customer satisfaction and fairness under different queueing modes. The reference [7] has consider that waiting time is a key factor for customers to evaluate service quality, and puts forward that time value, sense of control and ability of control are important factors that affect customers' waiting time. There are differences in customers' perceived fairness under different queueing modes in the reference [8] has been proposed. The reference [9] has consider variable waiting time from the perspective of cognitive calculation, and at the same time, added the variation factor of special customers to explore the differences in customer service fairness perception under different queueing modes. There are also some studies to optimize queueing systems from different queueing models. The flexible service policy of $\mathrm{M/M/2}$ queue system in the reference [12] has been proposed. The reference [13] has consider an $\mathrm{M/M/1}$ retrial queue with working vacation, orbit search and balking. The sufficient and necessary conditions for system stability are obtained by matrix analysis. The reference [14] has consider a discrete-time $\mathrm{Geo/Geo/1}$ queue with server breakdowns and repairs. The sufficient and necessary conditions for system stability are derived by matrix analysis. An $\mathrm{M/G/1}$ G-queue with server breakdown, working vacations and Bernoulli vacation interruption in the reference [15] has been proposed. The reference [16] according to the different service rate of each service desk, the configuration strategy of the queueing system is optimized.

Within the same service system, the service hours of different customers may vary greatly. For example, some customers buy only one item in the supermarket, while others buy multiple items. If the queueing rules treat all customers equally, the purchase of a commodity of the customer's service time is short, but waiting in line for a long time. This queueing method makes customers who buy a small number of items queue for a long time and occupies the queueing space of the mall, it will cause congestion, so this way of queueing is fairness and mall management efficiency. If a dedicated service desk is set up separately for customers with short service time, customers can pass through the service desk quickly, thus shortening the waiting time of customers and improving the efficiency of the queueing space in the shopping mall, which is conducive to improve customer satisfaction and business benefits of the shopping

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mall. Therefore, it is a predictable and feasible strategy to improve the service quality of service system by classifying customers. In the multi-service desk service system, the strategy of customer classification service is implemented, and the queuing theory is used to analyze the quantity of the system, so as to achieve the best operating effect of the service system, improve the customer satisfaction and the benefit of the service system.

This paper studies the queueing system with two identical service desks. Based on the queueing rule of "first come, first served", taking into account the different service hours of customers, we classify customers accordingly. Based on the supermarket cashier service system with two cash registers, the customers in the system are classified according to the different number of goods purchased, and the system adopts multi-queue queueing mode according to the classification of customers. Based on the M/M/1 and M/M/c queueing theories, corresponding queueing models are established for different queueing modes. Different queueing models are analyzed according to the arrival rate and service rate of different types of customers, the average queue length and average waiting time of different queueing modes is calculated and numerical comparison is made. The results can be used as a reference for general supermarket and bank service outlets to formulate service strategies.

II. Description of the system

This paper studies a queueing system with two identical service desks and classifies customers considering their service time. Based on the supermarket cashier service system with two cash registers, it classifies the number of goods purchased by customers. The two service desks serve two types of customers. Different queueing service methods are adopted according to customer classification. The two methods are as follows:

Queueing mode 1: Considering the difference in the number of goods purchased by customers, the customers who buy less than or equal to m items are called the first type customers. The first type customers are placed in a separate row, and the first cashier desk is dedicated to serving the first type customers. Customers who buy more than m items of goods are called the second type of customers, and the second type of customers are separately arranged in another column. The second cashier is dedicated to serving the second type of customers. Customers are not allowed to switch between two queues while waiting for service. It is first come, first served in the same queue.

Queueing mode 2: The two service desks work in parallel, and the customer does not realize the classified service, that is, the system is a single-queue queuing system with two service desks.

Model assumption: The arrival of customers is independent of each other, and the customer source and queueing space are infinite. In queueing mode 1, customers who buy less than or equal to m items arrive at Poisson flow with parameter λ_{11}. Customers who purchase more than m items arrive at the Poisson flow of parameter λ_{12}. The customer service time for purchasing less than or equal to m items is the exponential distribution of parameter μ_{11}, and the customer service time for purchasing more than m items is the exponential distribution of parameter μ_{12}. The total customer arrival rate in the system is λ_2, which is the sum of λ_{11} and λ_{12}. The customer arrival rate in queueing mode 2 is the Poisson flow of parameter λ_2, and the service time is the exponential distribution of parameter μ_2. Customers in the same queue are served on a first-come-first-served basis, and their service hours are independent of each other. According to the hypothesis, queueing mode 1 is two relatively independent queueing systems of M/M/1, and queueing mode 2 is a queueing system of M/M/2 single queue.

III. System analysis

A. Fundamental Theory

The paper have studied the M/M/c queueing model [10]. If the customer arrival is the Poisson flow of parameter λ, the service time is the exponential distribution of parameter μ, and there are c service desks in the system, the service intensity of the system is $\rho = \frac{\lambda}{c\mu}$. When $\rho < 1$, the system has steady-state probability.

The birth rate and death rate of the system are respectively:

$$\lambda_n = \lambda, n = 0, 1, 2, \ldots, \quad \mu_n = \begin{cases} n\mu, 1 \leq n \leq c-1, \\ c\mu, n \geq c. \end{cases}$$

The system state transition rate matrix is

$$Q = \begin{pmatrix} -\lambda & \lambda & 0 & & & \\ \mu & -(\lambda+\mu) & \lambda & & & \\ & \ddots & \ddots & \ddots & & \\ & & (c-1)\mu & -(\lambda+(c-1)\mu) & \lambda & \\ & & & c\mu & -(\lambda+c\mu) & \lambda \\ & & & & \ddots & \ddots \ddots \end{pmatrix} \quad (1)$$

The steady-state probability equilibrium equation of the system is

$$\begin{cases} \mu p_1 = \lambda p_0, \\ (n+1)\mu p_{n+1} + \lambda p_{n-1} = (\lambda + n\mu) p_n, 1 \leq n \leq c-1, \\ c\mu p_{n+1} + \lambda p_{n-1} = (\lambda + c\mu) p_n, n \geq c, \\ \sum_{n=0}^{+\infty} p_n = 1. \end{cases} \quad (2)$$

We have that

$$p_0 = \left[\sum_{n=0}^{c-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n + \frac{1}{c!} \left(\frac{\lambda}{\mu} \right)^c \cdot \frac{1}{1-\rho} \right]^{-1}, \quad (3)$$

$$p_n = \begin{cases} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n p_0, n = 1, \ldots, c-1, \\ \frac{1}{c!} \left(\frac{\lambda}{\mu} \right)^c \left(\frac{\lambda}{c\mu} \right)^{n-c} p_0, n = c, \cdots. \end{cases} \quad (4)$$

At this point, the steady-state average waiting queue length of the system

$$E(L) = \frac{1}{c!} \left(\frac{\lambda}{\mu} \right)^c \frac{\rho}{(1-\rho)^2} p_0 \quad (5)$$

Steady-state average queue length

$$E(Q) = E(L) + \frac{\lambda}{\mu} = \frac{1}{c!} \left(\frac{\lambda}{\mu} \right)^c \frac{\rho}{(1-\rho)^2} p_0 + \frac{\lambda}{\mu} \quad (6)$$

Steady-state average waiting time

$$E(W) = \frac{E(L)}{\lambda} = \frac{1}{c!} \left(\frac{\lambda}{\mu} \right)^c \frac{\rho}{(1-\rho)^2 \lambda} p_0 \quad (7)$$

B. Key indicators of the model

(1) Queueing mode 1:

According to equation (6), the steady-state average queue length of the first type of customers in queueing mode 1 can be obtained

$$E(Q_{11}) = \frac{\lambda_{11}}{\mu_{11} - \lambda_{11}}, \tag{8}$$

According to equation (7), the steady-state average waiting time of the first type of customers in queueing mode 1 can be obtained

$$E(W_{11}) = \frac{\lambda_{11}}{\mu_{11}(\mu_{11} - \lambda_{11})} = \frac{\lambda_{11}}{{\mu_{11}}^2 - \mu_{11}\lambda_{11}}. \tag{9}$$

According to equation (6), the steady-state average queue length of the second type of customers in queueing mode 1 can be obtained

$$E(Q_{12}) = \frac{\lambda_{12}}{\mu_{12} - \lambda_{12}}, \tag{10}$$

According to equation (7), the steady-state average waiting time of the second type of customers in queueing mode 1 can be obtained

$$E(W_{12}) = \frac{\lambda_{12}}{\mu_{12}(\mu_{12} - \lambda_{12})} = \frac{\lambda_{12}}{{\mu_{12}}^2 - \mu_{12}\lambda_{12}}. \tag{11}$$

(2) Queueing mode 2:

Queueing mode 2 is a system with two service desks and a single queue. According to equation (6), the steady-state average queue length of system in queueing mode 2 can be obtained

$$E(Q_2) = \frac{4\lambda_2\mu_2}{4{\mu_2}^2 - {\lambda_2}^2}. \tag{12}$$

The steady-state average waiting time of queueing mode 2 can be obtained from equation (7)

$$E(W_2) = \frac{{\lambda_2}^2}{4{\mu_2}^3 - \mu_2{\lambda_2}^2}. \tag{13}$$

IV. Comparison of two queueing methods

A. Customer classification and arrival rate and service rate analysis

The principle of customer classification is a key problem in the implementation of customer classification service. D-ifferent customer classification methods will lead to different types of customer input rate and service rate, which has a direct impact on the operation of the service system. The classification principle of customers can be considered in a variety of ways for the supermarket cashier service system, such as: according to the types of goods customers can buy, including food, clothing, and practical categories, etc., payment methods include wechat, alipay, membership card and cash payment. Obviously, different types of customers have different forms of service, and their arrival rate and service rate are also different. For other service systems, such as government administrative service, medical care, and financial system, etc., each has its unique characteristics of differentiated service objects.

This section is based on the supermarket cash register service system. Considering the significance and operability of the actual impact, the classification analysis is only carried out based on the different number of products purchased by customers. The number of items purchased by customers in the supermarket is random, and different customers are independent of each other, so its probability distribution should be analyzed by sampling investigation. Table 1 shows the sampling survey results of the number of items purchased by customers in a supermarket:

TABLE I: Frequency of goods purchased by customers

Pieces	$0\sim5$	$6\sim10$	$11\sim15$	$16\sim20$	More than 20
Frequency	13	101	74	11	1

Table 1 shows the statistical frequency of the number of items purchased by 200 customers in a supermarket within a day. By constructing the histogram, it can be predicted that the number of items purchased by customers approximately follows the Poisson distribution. Pearson χ^2 fitting method [11] is used to test. Since the parameter λ of Poisson distribution is unknown, the maximum likelihood estimator $\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X} = 9.775$ of λ is obtained through maximum likelihood estimation method of sample data. Now all the values of X are divided into five groups, mark $A_1 = [0,5], A_2 = [6,10], A_3 = [11,15], A_4 = [16,20], A_5 = [21,\infty]$. Have $\hat{p}_i = \hat{P}(A_i) = \sum_{l=a_{i-1}}^{a_i} \frac{\hat{\lambda}^l}{l!}e^{-\hat{\lambda}}, i = 1,2,\ldots,5$, where a_i and a_{i-1} are the upper and lower limits of group A_i, theoretical frequency $\hat{f}_i = n\hat{p}_i$, after combining the combinations with theoretical frequency less than 5, the grouping number is 4. See Table 2 for details.

TABLE II: Inspection of distribution of goods purchased by customers

A_i	f_i	$\widehat{p_i}$	$n\widehat{p_i}$	$\frac{{f_i}^2}{n\widehat{p_i}}$
A_1	13	0.0761	15.2184	11.105
A_2	101	0.5351	107.015	95.3233
A_3	74	0.3475	69.4946	78.7975
A_4	11 } 12	0.0401	8.0297 } 8.2723	17.4075
A_5	1	0.0012	0.2426	
$\sum$	200			202.6333

Statistics for

$$\chi^2 = \sum_{i=1}^{k} \frac{{f_i}^2}{n\hat{p}_i} - n,$$

According to the calculation in Table 2, we have that $\chi^2 = 2.6333$. In calculating the theoretical probability, estimated a parameter λ, therefore, the number of estimated parameters $r = 1$, the degree of freedom of the statistic χ^2 distribution is $K - r - 1 = 2$, take the significance level as $\alpha = 0.05$, look-up table to $\chi^2_{0.05}(2) = 5.991$, therefore, we have that $\chi^2 < \chi^2_{0.05}(k-r-1)$. That is, the number of goods purchased by customers follows the Poisson distribution is significant.

Assuming that the number of goods purchased by customers follows the Poisson distribution with parameter $\lambda_3 > 0$, the probability that a customer will buy less than or equal to m items is

$$p(k \le m) = \sum_{k=0}^{m} \frac{{\lambda_3}^k}{k!} e^{-\lambda_3} = \alpha \quad (0 < \alpha < 1), \tag{14}$$

The probability that a customer buys more than m items is

$$p(k > m) = 1 - \sum_{k=1}^{m} \frac{{\lambda_3}^k}{k!} e^{-\lambda_3} = 1 - \alpha. \quad (15)$$

Assuming that the arrival of customers is not affected by queueing mode, according to the nature of Poisson distribution and the law of total probability, the arrival rate λ_{11} and λ_{12} of the two types of customers in queueing mode 1 and the arrival rate λ_2 of queueing mode 2, namely the total arrival rate of the system, we have that $\lambda_2 = \lambda_{11} + \lambda_{12}$, and

$$\lambda_{11} = \alpha\lambda_2, \quad \lambda_{12} = (1-\alpha)\lambda_2. \quad (16)$$

Considering the relationship between customer service time and the number of products purchased by customers, assuming other factors are the same, the service time of any customer should be directly proportional to the number of products purchased. The average service time of customers who buy less than or equal to m items in queueing mode 1 should be less than that in queueing mode 2, while the average service time of customers who buy more than m items in queueing mode 1 should be more than that in queueing mode 2, that is, there should be a statistically significant quantity relationship $\mu_{11} > \mu_2$, $\mu_{12} < \mu_2$. As for the customers in the supermarket, it can be seen from the relationship between m and α that α is directly related to the service rate of the two types of customers. But in practice, the determination of the service rate of different types of customers in a specific service system needs the support of statistical data. For the supermarket cashier service system in this paper, the following relationship is constructed according to the influence of classification probability α on service rate:

$$\begin{aligned} \mu_{11} &= \frac{1+k_1}{\alpha+k_1}\mu_2 = p\mu_2, \\ \mu_{12} &= \frac{(1-\alpha)+k_2}{1+k_2}\mu_2 = q\mu_2, \end{aligned} \quad (17)$$

The values of parameters k_1 and k_2 meet the requirements of $p > 1, 0 < q < 1$, which must be determined by sampling statistics in practical application.(That is, we get the values of $\mu_{11}, \mu_{12}, \alpha$, and μ_2 through sampling statistics, and then we get the values of k_1 and k_2 from the above formula.)

B. System indicators of two queueing methods

According to the above discussion results, the average queue length and average waiting time of customers in two different queues of queueing mode 1 are as follows:

$$E(Q_{11}) = \frac{\lambda_{11}}{\mu_{11} - \lambda_{11}} = \frac{\alpha\lambda_2}{p\mu_2 - \alpha\lambda_2}, \quad (18)$$

$$E(Q_{12}) = \frac{\lambda_{12}}{\mu_{12} - \lambda_{12}} = \frac{(1-\alpha)\lambda_2}{q\mu_2 - (1-\alpha)\lambda_2}, \quad (19)$$

$$E(W_{11}) = \frac{\lambda_{11}}{{\mu_{11}}^2 - \mu_{11}\lambda_{11}} = \frac{\alpha\lambda_2}{p^2{\mu_2}^2 - \alpha p\lambda_2\mu_2}, \quad (20)$$

$$E(W_{12}) = \frac{\lambda_{12}}{{\mu_{12}}^2 - \mu_{12}\lambda_{12}} = \frac{(1-\alpha)\lambda_2}{q^2{\mu_2}^2 - (1-\alpha)q\lambda_2\mu_2}. \quad (21)$$

Then, according to equation (18) and equation (19), the average queue length of queueing mode 1 is

$$\begin{aligned} E(Q_1) &= E(Q_{11}) + E(Q_{12}) \\ &= \frac{\alpha\lambda_2}{p\mu_2 - \alpha\lambda_2} + \frac{(1-\alpha)\lambda_2}{q\mu_2 - (1-\alpha)\lambda_2}, \end{aligned} \quad (22)$$

Then, according to equation (20) and equation (21), the average waiting time of queueing mode 1 is

$$\begin{aligned} E(W_1) &= \alpha E(W_{11}) + (1-\alpha)E(W_{12}) \\ &= \frac{\alpha^2\lambda_2}{p^2{\mu_2}^2 - \alpha p\lambda_2\mu_2} + \frac{(1-\alpha)^2\lambda_2}{q^2{\mu_2}^2 - (1-\alpha)q\lambda_2\mu_2}. \end{aligned} \quad (23)$$

The average queue length and average waiting time of queueing mode 2 are respectively

$$E(Q_2) = \frac{4\lambda_2\mu_2}{4{\mu_2}^2 - {\lambda_2}^2}, \quad E(W_2) = \frac{{\lambda_2}^2}{4{\mu_2}^3 - \mu_2{\lambda_2}^2}. \quad (24)$$

C. A special case of queueing mode 1

When $\alpha = 0$, each system indicator of queueing mode 1 is

$$\begin{aligned} E(Q_{11}) = 0, &\quad E(Q_{12}) = \frac{\lambda_2}{\mu_2 - \lambda_2}, \\ E(W_{11}) = 0, &\quad E(W_{12)} = \frac{\lambda_2}{{\mu_2}^2 - \lambda_2\mu_2}, \\ E(Q_1) &= E(Q_{11}) + E(Q_{12}) = \frac{\lambda_2}{\mu_2 - \lambda_2}, \\ E(W_1) &= \alpha E(W_{11}) + (1-\alpha)E(W_{12}) = \frac{\lambda_2}{{\mu_2}^2 - \lambda_2\mu_2}. \end{aligned}$$

At this point, queueing mode 1 equals that all customers are classified into the second category, and only the second service desk works, and this service desk serves all customer-s.

When $\alpha = 1$, each system indicator of queueing mode 1 is

$$\begin{aligned} E(Q_{11}) = \frac{\lambda_2}{\mu_2 - \lambda_2}, &\quad E(Q_{12}) = 0, \\ E(W_{11}) = \frac{\lambda_2}{{\mu_2}^2 - \lambda_2\mu_2}, &\quad E(W_{12)} = 0, \\ E(Q_1) &= E(Q_{11}) + E(Q_{12}) = \frac{\lambda_2}{\mu_2 - \lambda_2}, \\ E(W_1) &= \alpha E(W_{11}) + (1-\alpha)E(W_{12}) = \frac{\lambda_2}{{\mu_2}^2 - \lambda_2\mu_2}. \end{aligned}$$

At this point, queueing mode 1 equals that all customers are classified into the first category, only the first service desk works, and this service desk serves all customers. These two special cases are extreme states of service mode 1 and will not be implemented in practice.

V. Numerical analysis

This section carries out numerical analysis on the system indicators of the two queueing modes.

1) Assuming $\alpha = \frac{4}{5}, \lambda_2 = 6$, in practical problems, k_1 and k_2 value associated with the value of $\mu_{11}, \mu_{12}, \alpha$, and μ_2, specific system needs to be confirmed statistical inference. In this case, inferred from sample data, we take $k_1 = -\frac{3}{5}$ and $k_2 = \frac{11}{5}$.

By equilibrium condition

$$\begin{cases} \lambda_{11} < \mu_{11}, \\ \lambda_{12} < \mu_{12}, \\ \lambda_2 < 2\mu_2, \end{cases} \Rightarrow \begin{cases} \alpha\lambda_2 < p\mu_2, \\ (1-\alpha)\lambda_2 < q\mu_2, \\ \lambda_2 < 2\mu_2, \end{cases}$$

plug in the data and we get $\mu_2 > 3$.

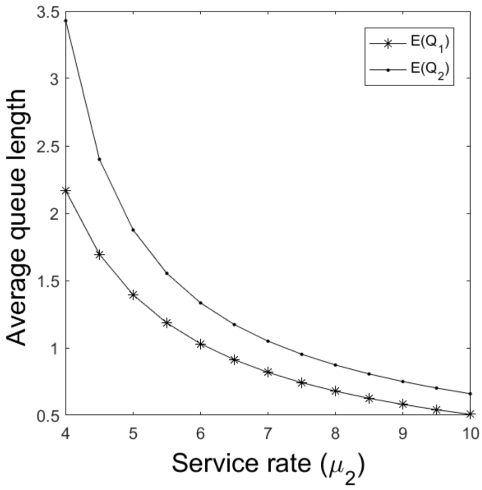

Fig. 1: Average queue length as a function of μ_2 ($\alpha = \frac{4}{5}, \lambda_2 = 6$, $k_1 = -\frac{3}{5}$ and $k_2 = \frac{11}{5}$)

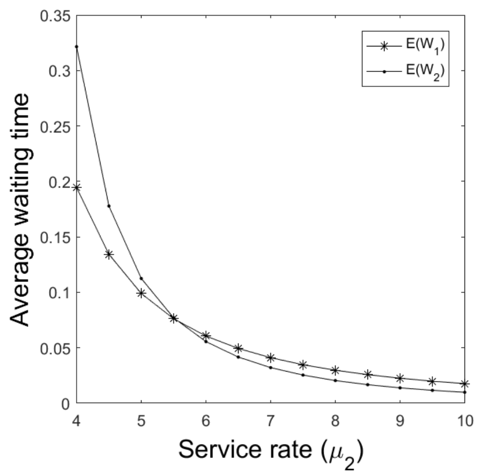

Fig. 2: Average waiting time as a function of μ_2 ($\alpha = \frac{4}{5}, \lambda_2 = 6$, $k_1 = -\frac{3}{5}$ and $k_2 = \frac{11}{5}$)

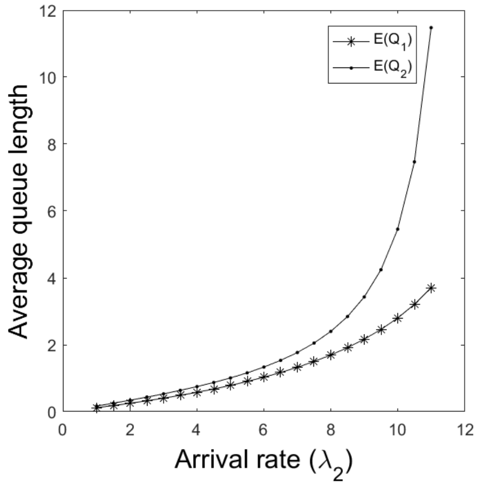

Fig. 3: Average queue length as a function of λ_2 ($\alpha = \frac{4}{5}, \mu_2 = 6$, $k_1 = -\frac{3}{5}$ and $k_2 = \frac{11}{5}$)

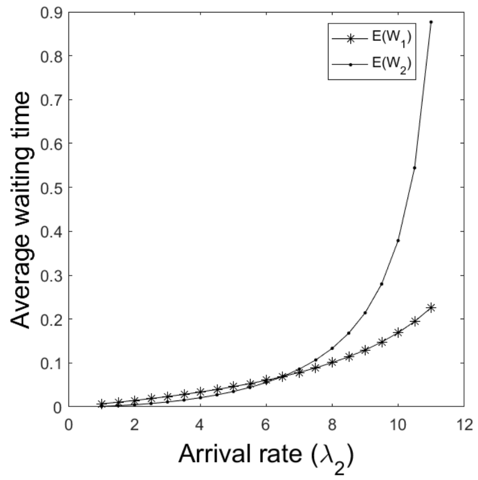

Fig. 4: Average waiting time as a function of λ_2 ($\alpha = \frac{4}{5}, \mu_2 = 6$, $k_1 = -\frac{3}{5}$ and $k_2 = \frac{11}{5}$)

According to equation (22) and equation (24), get

$$E(Q_1) = \frac{100\mu_2 - 192}{25{\mu_2}^2 - 100\mu_2 + 96}, \tag{25}$$

$$E(Q_2) = \frac{4\lambda_2\mu_2}{4{\mu_2}^2 - {\lambda_2}^2} = \frac{6\mu_2}{{\mu_2}^2 - 9}. \tag{26}$$

According to equation (23) and equation (24), get

$$E(W_1) = \frac{96}{225{\mu_2}^2 - 360\mu_2} + \frac{24}{25{\mu_2}^2 - 60\mu_2}, \tag{27}$$

$$E(W_2) = \frac{{\lambda_2}^2}{4{\mu_2}^3 - \mu_2{\lambda_2}^2} = \frac{9}{{\mu_2}^3 - 9\mu_2}. \tag{28}$$

According to the static equilibrium conditions, we take $4 \le \mu_2 \le 10$. Fig. 1 shows that the $E(Q_1)$ and $E(Q_2)$ with μ_2 variation characteristics. With the increase of μ_2, $E(Q_1)$ and $E(Q_2)$ have showed a trend of decline and $E(Q_1)$ less than $E(Q_2)$. Fig. 2 shows the $E(W_1)$ and $E(W_2)$ with μ_2 variation characteristics. With the increase of μ_2, $E(W_1)$ and $E(W_2)$ also have showed a trend of decline, but $E(W_2)$ decline faster, $E(W_1)$ and $E(W_2)$ intersect at $\mu_2 = 5.55$, which showed that the two ways can the average queue length and customer average waiting time no strict corresponding relation. This phenomenon is due to service the customer classification, because the service mode 1 of the second category of customers can be less number of customers, but in the case of low service rate will be a long waiting time. This example when $4 \le \mu_2 < 5.55$, $E(Q_1)$ less than $E(Q_2), E(W_1)$ less than $E(W_2)$. In this case, the effect of queueing mode 1 is better, the service efficiency of supermarket cashier system is faster, and customer satisfaction is higher.

2) Assuming $\alpha = \frac{4}{5}, \mu_2 = 6$, we take $k_1 = -\frac{3}{5}$ and $k_2 = \frac{11}{5}$.

By equilibrium condition

$$\begin{cases} \lambda_{11} < \mu_{11} \\ \lambda_{12} < \mu_{12} \\ \lambda_2 < 2\mu_2 \end{cases} \Rightarrow \begin{cases} \alpha\lambda_2 < p\mu_2 \\ (1-\alpha)\lambda_2 < q\mu_2 \\ \lambda_2 < 2\mu_2 \end{cases},$$

plug in the data and we get $0 < \lambda_2 < 12$.

According to equation (22) and equation (24), get

$$E(Q_1) = \frac{75\lambda_2 - 2{\lambda_2}^2}{2{\lambda_2}^2 - 75\lambda_2 + 675}, \tag{29}$$

$$E(Q_2) = \frac{4\lambda_2\mu_2}{4{\mu_2}^2 - {\lambda_2}^2} = \frac{24\lambda_2}{144 - {\lambda_2}^2}. \tag{30}$$

According to equation (23) and equation (24), get

$$E(W_1) = \frac{\lambda_2}{225 - 15\lambda_2} + \frac{4\lambda_2}{2025 - 90\lambda_2}, \quad (31)$$

$$E(W_2) = \frac{{\lambda_2}^2}{4{\mu_2}^3 - \mu{\lambda_2}^2} = \frac{{\lambda_2}^2}{864 - 6{\lambda_2}^2}. \quad (32)$$

According to the static equilibrium conditions, we take $1 \leq \lambda_2 \leq 11$, Fig. 3 shows the $E(Q_1)$ and $E(Q_2)$ with λ_2 variation characteristics. With the increase of λ_2, $E(Q_1)$ and $E(Q_2)$ have showed a trend of increase and $E(Q_1)$ less than $E(Q_2)$. Fig. 4 shows the $E(W_1)$ and $E(W_2)$ with λ_2 variation characteristics. With the increase of λ_2, $E(W_1)$ and $E(W_2)$ also have showed a trend of increase, but $E(W_2)$ increase faster, $E(W_1)$ and $E(W_2)$ intersect at $\lambda_2 = 6.49$, which showed that the two ways can the average queue length and customer average waiting time no strict corresponding relation. This phenomenon is due to service the customer classification, because the service mode 1 of the second category of customers can be less number of customers, but in the case of low service rate will be a long waiting time. When $6.49 < \lambda_2 \leq 11$, $E(Q_1)$ less than $E(Q_2), E(W_1)$ less than $E(W_2)$. In this case, the effect of queueing mode 1 is better, the service efficiency of supermarket cashier system is faster, and customer satisfaction is higher.

3) Assuming $\alpha = \frac{2}{5}, \lambda_2 = 6$, we take $k_1 = \frac{1}{5}$ and $k_2 = \frac{3}{5}$. By equilibrium condition

$$\begin{cases} \lambda_{11} < \mu_{11} \\ \lambda_{12} < \mu_{12} \\ \lambda_2 < 2\mu_2 \end{cases} \Rightarrow \begin{cases} \alpha\lambda_2 < p\mu_2 \\ (1-\alpha)\lambda_2 < q\mu_2 \\ \lambda_2 < 2\mu_2 \end{cases},$$

plug in the data and we get $\mu_2 > \frac{24}{5}$.

According to equation (22) and equation (24), get

$$E(Q_1) = \frac{150\mu_2 - 288}{25{\lambda_2}^2 - 150\lambda_2 + 144}, \quad (33)$$

$$E(Q_2) = \frac{4\lambda_2\mu_2}{4{\mu_2}^2 - {\lambda_2}^2} = \frac{6\mu_2}{{\mu_2}^2 - 9}. \quad (34)$$

According to equation (23) and equation (24), get

$$E(W_1) = \frac{6}{25{\mu_2}^2 - 30\lambda_2} + \frac{480}{125{\mu_2}^2 - 24\lambda_2}, \quad (35)$$

$$E(W_2) = \frac{{\lambda_2}^2}{4{\mu_2}^3 - \mu_2{\lambda_2}^2} = \frac{9}{{\mu_2}^3 - 9\mu_2}. \quad (36)$$

According to the static equilibrium conditions, we take $5 \leq \mu_2 \leq 10$, Fig. 5 shows the $E(Q_1)$ and $E(Q_2)$ with μ_2 variation characteristics. With the increase of μ_2, $E(Q_1)$ and $E(Q_2)$ have showed a trend of decline and $E(Q_1)$ greater than $E(Q_2)$. Fig. 6 shows the $E(W_1)$ and $E(W_2)$ with μ_2 variation characteristics. With the increase of μ_2, $E(W_1)$ and $E(W_2)$ also have showed a trend of decline and $E(W_1)$ greater than $E(W_2)$. when $5 \leq \mu_2 \leq 10$, $E(Q_1)$ greater than $E(Q_2), E(W_1)$ greater than $E(W_2)$. In this case, the effect of queueing mode 2 is better, the service efficiency of supermarket cashier system is faster, and customer satisfaction is higher.

4) Assuming $\alpha = \frac{2}{5}, \mu_2 = 6$, this example take $k_1 = \frac{1}{5}$ and $k_2 = \frac{3}{5}$.

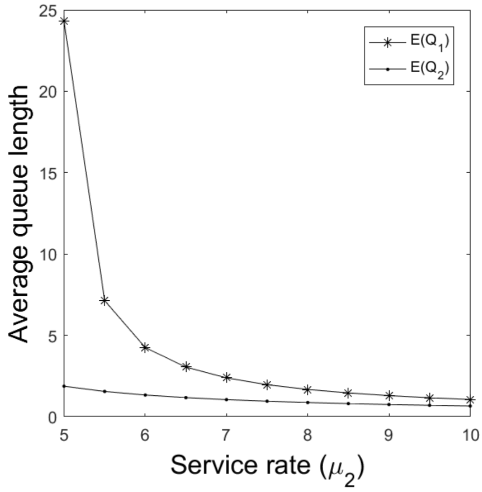

Fig. 5: Average queue length as a function of μ_2 ($\alpha = \frac{2}{5}, \lambda_2 = 6$, $k_1 = \frac{1}{5}$ and $k_2 = \frac{3}{5}$)

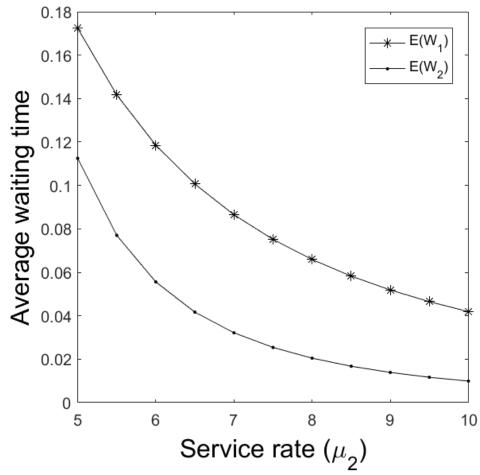

Fig. 6: Average waiting time as a function of μ_2 ($\alpha = \frac{2}{5}, \lambda_2 = 6$, $k_1 = \frac{1}{5}$ and $k_2 = \frac{3}{5}$)

By equilibrium condition

$$\begin{cases} \lambda_{11} < \mu_{11} \\ \lambda_{12} < \mu_{12} \\ \lambda_2 < 2\mu_2 \end{cases} \Rightarrow \begin{cases} \alpha\lambda_2 < p\mu_2 \\ (1-\alpha)\lambda_2 < q\mu_2 \\ \lambda_2 < 2\mu_2 \end{cases},$$

plug in the data and we get $0 < \lambda_2 < \frac{15}{2}$.

According to equation (22) and equation (24), get

$$E(Q_1) = \frac{75\lambda_2 - 4{\lambda_2}^2}{2{\lambda_2}^2 - 75\lambda_2 + 450}, \quad (37)$$

$$E(Q_2) = \frac{4\lambda_2\mu_2}{4{\mu_2}^2 - {\lambda_2}^2} = \frac{24\lambda_2}{144 - {\lambda_2}^2}. \quad (38)$$

According to equation (23) and equation (24), get

$$E(W_1) = \frac{\lambda_2}{900 - 30\lambda_2} + \frac{4\lambda_2}{225 - 30\lambda_2}, \quad (39)$$

$$E(W_2) = \frac{{\lambda_2}^2}{4{\mu_2}^3 - \mu_2{\lambda_2}^2} = \frac{{\lambda_2}^2}{864 - 6{\lambda_2}^2}. \quad (40)$$

According to the static equilibrium conditions, we take $1 \leq \mu_2 \leq 7$, Fig. 7 shows the $E(Q_1)$ and $E(Q_2)$ with λ_2 variation characteristics. With the increase of λ_2, $E(Q_1)$ and $E(Q_2)$ have showed a trend of increase and $E(Q_1)$

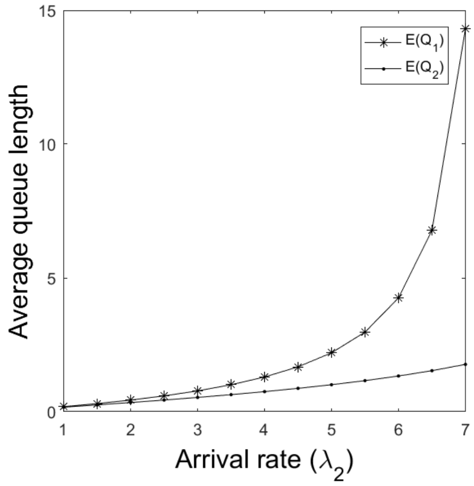

Fig. 7: Average queue length as a function of λ_2 ($\alpha = \frac{2}{5}, \mu_2 = 6$, $k_1 = \frac{1}{5}$ and $k_2 = \frac{3}{5}$)

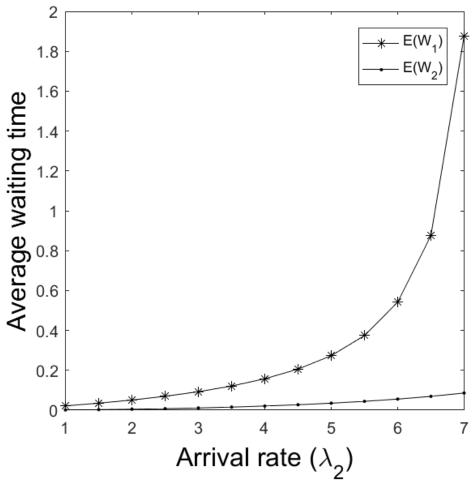

Fig. 8: Average waiting time as a function of λ_2 ($\alpha = \frac{2}{5}, \mu_2 = 6$, $k_1 = \frac{1}{5}$ and $k_2 = \frac{3}{5}$)

greater than $E(Q_2)$. Fig. 8 shows the $E(W_1)$ and $E(W_2)$ with λ_2 variation characteristics. With the increase of λ_2, $E(W_1)$ and $E(W_2)$ also have showed a trend of increase and $E(W_1)$ greater than $E(W_2)$. when $1 \leq \lambda_2 \leq 7$, $E(Q_1)$ greater than $E(Q_2), E(W_1)$ greater than $E(W_2)$. In this case, the effect of queueing mode 2 is better, the service efficiency of supermarket cashier system is faster, and customer satisfaction is higher.

5)According to 4.3, When $\alpha = 0$ or $\alpha = 1$, $E(Q_1)$ is the same, and $E(W_1)$ is the same, respectively

$$E(Q_1) = \frac{\lambda_2}{\mu_2 - \lambda_2}, E(W_1) = \frac{\lambda_2}{{\mu_2}^2 - \lambda_2\mu_2} \tag{41}$$

In queuing mode 1, only one service desk is working and serves all customers. When $\alpha = 0$, there is $p = \frac{1+k_1}{k_1} = 0, q = 1$, the $k_1 = -1$, k_2 takes any value(we take k_2=1).

Assuming $\lambda_2 = 6$.

By equilibrium condition

$$\begin{cases} \lambda_{11} < \mu_{11} \\ \lambda_{12} < \mu_{12} \\ \lambda_2 < 2\mu_2 \end{cases} \Rightarrow \begin{cases} \alpha\lambda_2 < p\mu_2 \\ (1-\alpha)\lambda_2 < q\mu_2 \\ \lambda_2 < 2\mu_2 \end{cases},$$

plug in the data and we get $\mu_2 > 6$.

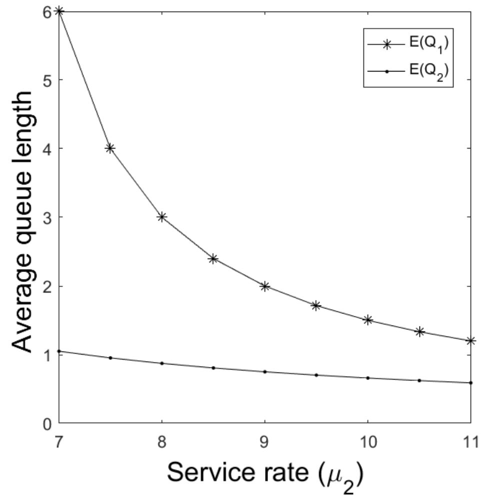

Fig. 9: Average queue length as a function of μ_2 ($\alpha = 0, \lambda_2 = 6$, $k_1 = -1$ and $k_2 = 1$)

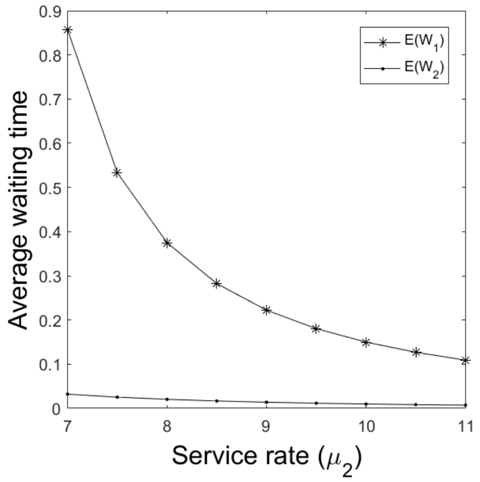

Fig. 10: Average waiting time as a function of μ_2 ($\alpha = 0, \lambda_2 = 6$, $k_1 = -1$ and $k_2 = 1$)

According to equation (22) and equation (24), get

$$E(Q_1) = \frac{6}{\mu_2 - 6}, E(Q_2) = \frac{4\lambda_2\mu_2}{4{\mu_2}^2 - {\lambda_2}^2} = \frac{6\mu_2}{{\mu_2}^2 - 9}. \tag{42}$$

According to equation (23) and equation (24), get

$$E(W_1) = \frac{6}{{\mu_2}^2 - 6\mu_2}, E(W_2) = \frac{9}{{\mu_2}^3 - 9\mu_2}. \tag{43}$$

According to the static equilibrium conditions, we take $7 \leq \mu_2 \leq 11$, Fig. 9 shows the $E(Q_1)$ and $E(Q_2)$ with μ_2 variation characteristics. With the increase of μ_2, $E(Q_1)$ and $E(Q_2)$ have showed a trend of decline and $E(Q_1)$ greater than $E(Q_2)$. Fig. 10 shows the $E(W_1)$ and $E(W_2)$ with μ_2 variation characteristics. With the increase of μ_2, $E(W_1)$ and $E(W_2)$ also have showed a trend of decline and $E(W_1)$ greater than $E(W_2)$. when $7 \leq \mu_2 \leq 11$, $E(Q_1)$ greater than $E(Q_2), E(W_1)$ greater than $E(W_2)$. In this case, the effect of queueing mode 2 is better, the service efficiency of supermarket cashier system is faster, and customer satisfaction is higher.

Assuming $\mu_2 = 6$

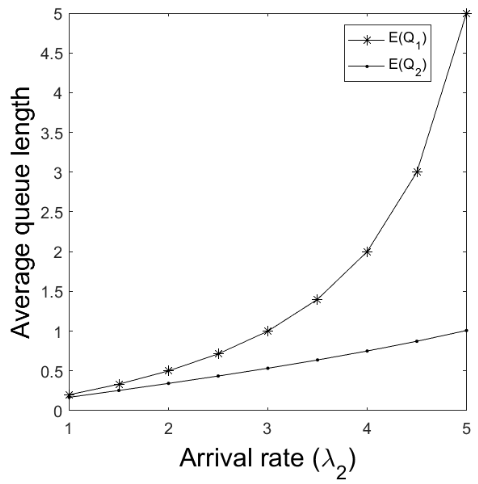

Fig. 11: Average queue length as a function of λ_2 ($\alpha = 0, \mu_2 = 6,\ k_1 = -1$ and $k_2 = 1$)

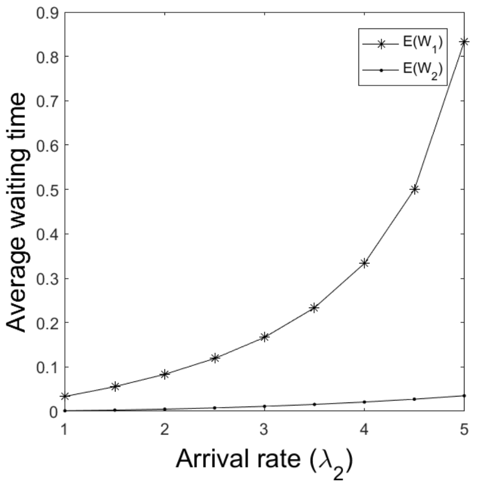

Fig. 12: Average waiting time as a function of λ_2 ($\alpha = 0, \mu_2 = 6,\ k_1 = -1$ and $k_2 = 1$)

By equilibrium condition

$$\begin{cases} \lambda_{11} < \mu_{11} \\ \lambda_{12} < \mu_{12} \\ \lambda_2 < 2\mu_2 \end{cases} \Rightarrow \begin{cases} \alpha\lambda_2 < p\mu_2 \\ (1-\alpha)\lambda_2 < q\mu_2 \\ \lambda_2 < 2\mu_2 \end{cases},$$

plug in the data and we get $0 < \lambda_2 < 6$.

According to equation (22) and equation (24), get

$$E(Q_1) = \frac{\lambda_2}{6-\lambda_2}, E(Q_2) = \frac{4\lambda_2\mu_2}{4{\mu_2}^2 - {\lambda_2}^2} = \frac{24\lambda_2}{144 - {\lambda_2}^2}. \tag{44}$$

According to equation (23) and equation (24), get

$$E(W_1) = \frac{\lambda_2}{36 - 6\lambda_2}, E(W_2) = \frac{{\lambda_2}^2}{864 - 6{\lambda_2}^2}. \tag{45}$$

According to the static equilibrium conditions, we take $1 \leq \mu_2 \leq 5$, Fig. 11 shows the $E(Q_1)$ and $E(Q_2)$ with λ_2 variation characteristics. With the increase of λ_2, $E(Q_1)$ and $E(Q_2)$ have showed a trend of increase and $E(Q_1)$ greater than $E(Q_2)$. Fig. 12 shows the $E(W_1)$ and $E(W_2)$ with λ_2 variation characteristics. With the increase of λ_2, $E(W_1)$ and $E(W_2)$ also have showed a trend of increase and $E(W_1)$ greater than $E(W_2)$. when $1 \leq \lambda_2 \leq 5$, $E(Q_1)$ greater than $E(Q_2), E(W_1)$ greater than $E(W_2)$. In this case, the effect of queueing mode 2 is better, the service efficiency of supermarket cashier system is faster, and customer satisfaction is higher.

When $\alpha = 1$, there is $p = 1, q = \frac{k_2}{1+k_2} = 0$, the k_1 takes any value, $k_2 = 0$. Respectively take $\lambda_2 = 6$ and $\mu_2 = 6$ is the same as $\alpha = 0$.

It can be seen from the numerical experiment that for the queueing system of two service desks, the quality of the service mode is not absolute. In the classification service mode, the classification standard also has a significant impact on the operation index of the system. Therefore, for a specific service system in practical application, the selection of service mode and the determination of customer classification principles and standards must be based on the reality, according to the actual operation of the system sample data to determine the best service mode.

VI. Conclusion

This paper makes a comparative analysis of the different service modes of the queueing system of the two service desks, and the analysis shows that there are significant differences in the operating indicators of the system under different service modes. The service system of two service desks is common in practice. The analysis of this paper provides a method of calculation comparison and decision analysis for the operation design and optimization of similar systems in practice. This paper only gives a comparative analysis method in theory, for the actual operation process, how to make customers know and abide by the classification rules is a practical problem to be considered. To solve this practical problem, prompt signs should be set up at the service desk and customer waiting area, purchasing guides can be set up in the early stage of implementation, and rewards and penalties should be carried out for complying with the rules.

In this paper, the supermarket cashier system of two service counters is taken as the research object, and the number of goods purchased is taken as the classification standard to classify customers. Different types of customers must be served at their own service counters. It can be expected that various service systems in practice are complex and diverse. In order to better realize the optimization of actual operation, more research and analysis in this direction are needed. On the basis of the research in this paper, it can be further considered that when a service desk is idle, it can provide temporary auxiliary services for another kind of customers waiting, and the customers of this service desk have a non-preemptive priority service strategy after arrival, which has good operability and effectiveness in practice. In addition, the two queues in service mode 1 in this paper, the optimal comparative analysis of their operation indicators, is also the further work of this paper. For the service system with three or more service desks, it is an important direction to classify customers and consider different classification principles and standards.

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