Event-Triggered Adaptive Prescribed Performance Control of Uncertain MIMO Nonlinear Systems with Actuator Failure

Mengzhou Tang, Nannan Zhao, Xinyu Ouyang and Feng Zhang

Abstract—**For a class of multi-input multi-output (MIMO) uncertain nonlinear systems, an event-triggered adaptive fuzzy fault-tolerant tracking control method with prescribed performance is proposed in the presence of unknown actuator faults and external disturbances. To reduce the communication burden, the event-triggered signal based on the relative threshold is introduced; and a novel error conversion function is designed to achieve the preset performance and track the system output. The fuzzy logic system is used to approximate all unknown nonlinearities of the closed loop system. By using backstepping method, the adaptive controller of the system is designed, and the stability of the system is analyzed. Finally, the simulation results verify the effectiveness of the proposed control method.**

Index Terms—**Event-triggered control(ETC), prescribed performance control(PPC), fault-tolerant control(FTC), fuzzy systems, MIMO unknown nonlinear systems**

I. Introduction

IN the actual automation sector, actuators or sensors frequently fail or malfunction [1], resulting in a deterioration in system performance or even process disruption and ensuing losses. In response to industry demands, fault-tolerant control (FTC) has emerged as a popular topic that piques the interest of many academics. In the past few decades, several FTC-related findings have progressed [2]–[8]. Separately, an fault-tolerant control was presented for single-input single-output (SISO) nonlinear systems [9] in the presence of prescribed performance [3], unknown direction [4], and finite time restriction [5] was researched. Fulfil the demands of increasingly complex industrial environments, the fault-tolerant technique was extended to MIMO nonlinear systems [6], [7] and Markov jump systems [8] in which we have taken both lock-in-place and loss of effectiveness mistakes into account.

Manuscript received May 5, 2022; revised December 31, 2022. This work is partly supported by the Scientific Research Foundation of the Liaoning Provincial Education Department of China (Grant Nos. LJKZ0293 and LJKZ0307).

Mengzhou Tang is a Postgraduate of the School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, 114051, P. R. China. (e-mail:445700237@qq. com)

Nannan Zhao is a Professor of the School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, 114051, P. R. China. (corresponding author, e-mail:723306003@qq. com)

Xinyu Ouyang is a Professor of the School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, 114051, P. R. China. (corresponding author, e-mail:13392862@qq. com)

Feng Zhang is a doctoral candidate of the School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, 114051, P. R. China. (e-mail:15941237106@163.com)

In addition, one of the most critical factors to consider when operating is dealing with the system's performance limitations. It is primarily because transient and steady-state tracking performances are typically needed in many real-world systems and theories. Consequently, certain projects [10]–[16] have utilized the recommended performance control measures to overcome performance constraints. To meet the performance restriction for nonlinear systems, the writers of [10] created a prescribed performance control first (PPC). In addition, Bechlioulis C P et al. in [11] suggested an improved performance-constrained control approach for affine MIMO nonlinear systems to eliminate the affine problem.Using dynamic surface [12], FLS [13], and Approximation-Free [14]; subsequently, they use the PPC approach to construct adaptive controllers for several kinds of unknown nonlinear systems. However, few results exist in the literature for the PPC with unknown nonlinear systems that account for actuator or sensor failure. [15] suggested one robust finite-time control strategy with specified performance for a class of high-order nonlinear systems. Then, to attain asymptotic stability for a category of unknown MIMO nonlinear systems with actuator faults, [16] proposes a learning-based fault-tolerant controller.

It is crucial to point out that the literature mentioned above ignores the control system's communication burden, which might also result in unneeded work overload. The development of event-triggered control technology was encouraged to bypass these issues, which effectively conserves energy resources while reducing the controllers' computational costs [17]–[24]. Based on three different tactics, Xing et al. established some innovative event-triggered design methodologies for a range of nonlinear systems and published their findings [17]. After that, an event-triggered output feedback control strategy was proposed to achieve effectiveness while assuring the presetted disturbance attenuation level. This level was specified by the $L_2 \sim L_\infty$ performance index in [18]. Also, in [19], [20], two event-triggered adaptive fuzzy tracking control techniques for stochastic nonlinear systems were investigated. Further, in [21]–[24], Sahoo et al. applied an event-triggered control technique to MIMO nonlinear systems. Recently, in [25], [26], important developments combining event-triggered and prescribed performance for classes of SISO nonlinear systems have been accomplished. By introducing the prescribed performance functions [25], it is feasible to specify the constraint requirements on the tracking error. Then these errors can converge to a small residual set, all while the maximum overshoot is smaller than a predetermined amount. Also, to overcome the full-state limitations for unsettled nonstrict-feedback nonlinear systems, the authors in [26] presented an observer-based adaptive fixed-time prescribed performance control.

Several event-triggering adaptive performance control problems for various MIMO nonlinear systems with unpredictable nonlinear properties are researched due to the discussion above. The following are the primary benefits and contributions of the suggested technique in comparison to some of the previous findings:

1) Introducing a relative event-triggered mechanism minimizes communication costs. Therefore a more precise input signal than previous results [4], [5], [7] can be applied to the controller.

2) A new error transform function inspired by the inverse hyperbolic tangent function was first proposed to implement PPC. In addition, the prescribed adaptive control strategy can be adapted to nonlinear system uncertainties and actuator defects. Consequently, the assumption of a nonlinear system can be relaxed, allowing the use of the proposed control strategy in various nonlinear systems.

II. System descriptions and preparatory knowledge

A. System descriptions

The considered nonlinear system takes the form shown below:

$$\begin{aligned}
\dot{x}_{i,1} =& f_{i,1}\left(\underline{x}_{i,1}\right)+g_{i,1}\left(\underline{x}_{i,1}\right) x_{i,2}+d_{i,1}(t) \\
& \vdots \\
\dot{x}_{i,n_i-1} =& f_{i,n_i-1}\left(\underline{x}_{i,n_i-1}\right)+g_{i,n_i-1}\left(\underline{x}_{i,n_i-1}\right) x_{i,n_i} \\
& +d_{i,n_i-1}(t) \\
\dot{x}_{i,n_i} =& f_{i,n_i}(x)+g_{i,n_i}(x) u_{i,f}+d_{i,n_i}(t) \\
y_i =& x_{i,1}
\end{aligned} \tag{1}$$

where $\underline{x}_{i,j}=\left[x_{i,1}, x_{i,2}, \cdots, x_{i,j}\right]^T \in \mathbb{R}^j, i=1,2,\cdots,m; x=\left[\underline{x}_{1,n_1}^T, \underline{x}_{2,n_2}^T, \cdots, \underline{x}_{m,n_m}^T\right]^T \in \mathbb{R}^{\sum_{i=1}^m n_i}$ indicate the system states; $y_{i,r} \in \mathbb{R}$ is the reference signals; $f_{i,j}(\cdot):\in \mathbb{R}^j \rightarrow \mathbb{R}$ and $g_{i,j}(\cdot):\in \mathbb{R}^j \rightarrow \mathbb{R}$ stand for the unknown but smooth nonlinear functions; the external disturbances $d_{i,j}(t) \in \mathbb{R}$ are continuous in t. The signs of the virtual control coefficient serve as the role of control direction of the jth system of Equation (1), and $u_{i,f} \in \mathbb{R}$ represent actuator failure, which is described as follows

$$u_{i,f}=\varrho_i(t) \nu_i(t)+\zeta_i(t) \tag{2}$$

where $\nu_i(t)$ indicates the actual input signal, $\varrho_i(t)$ and $\zeta_i(t)$ respectively represent the partial loss of effectiveness and the failure satisfying Assumption 1.

Following that, a few lemmas and assumptions will need to be introduced. And, in the discussion that follows, for the sake of simplifying writing, denote $f_{i,j}\left(\underline{x}_{i,j}\right)$ and $g_{i,j}\left(\underline{x}_{i,j}\right)$ as $f_{i,j}$ and $g_{i,j}$, respectively. Additionally, the time variable t is missing from most variables. For instance, denote $\eta_{i,j}=\eta_{i,j}(t), \rho_{i,j}=\rho_{i,j}(t), y_{i,r}=y_{i,r}(t), y_{i,r}^{(r_i)}=y_{i,r}^{(r_i)}(t)$ and so on.

Assumption 1: Some undetermined positive constants $\underline{\varrho}_i, \bar{\zeta}_i$ and $\bar{d}_i$, respectively, in such a way that

$$0<\underline{\varrho}_i \leq \varrho_i \leq 1,\left|\zeta_i\right| \leq \bar{\zeta}_i,\left|d_{i,j}\right|<\bar{d}_i \tag{3}$$

with $i=1,2,\cdots,m, j=1,2,\cdots,n_i$

Assumption 2: The signs of $g_{i,j},(i=1,2,\cdots,m, j=1,2,\cdots,n_i)$ are determined, and undetermined constants b_m and b_M satisfying

$$0<b_m \leq\left|g_{i,j}\right| \leq b_M<\infty \tag{4}$$

Remark 1: Assumption 2 implies undefined function $g_{i,j},(i=1,2,\cdots,m, j=1,2,\cdots,n_i)$ are either strictly positive or strictly negative. Generally, $g_{i,j}>0$.

Assumption 3: Target tracking trajectory $y_{i,r},(i=1,2,\cdots,m)$ and its derivatives up to order r_i are continuous and bounded.

B. Prescribed performance control

The error variables of the jth subsystem are as follows:

$$\begin{aligned}
& z_{i,1}=x_{i,1}-\alpha_{i,0}, \quad \alpha_{i,0}=y_{i,r} \\
& z_{i,j}=x_{i,j}-\alpha_{i,j-1}, \quad j=2,3,\ldots,n_i
\end{aligned} \tag{5}$$

where $\alpha_{i,j},(i=1,2,\cdots,m, j=0,1,\cdots,n_i)$ denotes virtual control laws and $y_{i,r}$ are the expected trajectory. Achieving PPC for each subsystem j, the prescribed performance function is defined as

$$\rho_{i,j}=\left(\rho_{i,j}^{(0)}-\rho_{i,j}^{(\infty)}\right) e^{-\kappa_{i,j}}+\rho_{i,j}^{(\infty)} \tag{6}$$

with $\rho_{i,j}^{(0)}>\rho_{i,j}^{(\infty)}>0$, $\kappa_{i,j}>0$ such that

$$\left|z_{i,j}(0)\right|<\left|\rho_{i,j}(0)\right| \tag{7}$$

To ensure $\left|z_{i,j}\right|<\left|\rho_{i,j}\right|, \forall t \geq 0$, let's define error transformation functions as follows:

$$\eta_{i,j}(t)=\frac{\ln \left(\frac{\rho_{i,j}+z_{i,j}}{\rho_{i,j}-z_{i,j}}\right)}{1-\frac{z_{i,j}^2}{\rho_{i,j}^2}} \tag{8}$$

Remark 2: In the above equation, $\eta_{i,j}$ stands for the error transformation function, which is inspired by this hyperbolic tangent function $\tanh^{-1}(x)=\frac{1}{2} \ln \left(\frac{1+x}{1-x}\right)$. From the properties of hyperbolic tangent function $\tanh^{-1}(x)$, it will not be $\pm\infty$ for any given x satisfying $|x|<1$. This implies that $\eta_{i,j}$ is bounded if inequality $\left|z_{i,j}\right|<\left|\rho_{i,j}\right|$ holds. Later stability analysis will benefit from the boundedness of $\eta_{i,j}$.

C. Event-Triggered control

The following is a definition of control based on a relative threshold:

$$\begin{aligned}
\omega_i= & -\left(1+\tau_{i,0}\right)\Bigg(\alpha_{i,n_i} \tanh (\frac{\eta_{i,n_i} \alpha_{i,n_i}}{\epsilon_i \rho_{i,n_i}}) \\
& +\mu_{i,1} \tanh \left(\frac{\eta_{i,n_i} \mu_{i,1}}{\epsilon_i \rho_{i,n_i}}\right)\Bigg)
\end{aligned} \tag{9}$$

$$\nu_i=\omega_i\left(t_d\right), \quad \forall t \in\left[t_d, t_{d+1}\right) \tag{10}$$

$$t_{d+1}=\inf \left\{t \in R \| e_i(t) \mid \geq \tau_{i,0}\left|\nu_i(t)\right|+\mu_{i,2}\right\} \tag{11}$$

where the signal α_{i,n_i} will be design later, $e_i=\omega_i-\nu_i$. ϵ_i, $\mu_{i,1}$, $\mu_{i,2}$ are positive parameters, as well as $\mu_{i,2}>0$, $0<\tau_{i,0}<1, \mu_{i,1}>\frac{\mu_{i,2}}{1-\tau_{i,0}}, d \in z^{+}$, are controller updating time, i.e., whenever condition (9) is satisfied, the control value $\nu_i(t_{d+1})$ generated at that moment is updated. For the interval $[td, td+1)$, the control signal is a constant, namely $\omega_i(t_d)$. In addition, for any given $\forall\epsilon_i > 0$ and $\Upsilon \in \mathbb{R}$, the hyperbolic tangent function should satisfy the following conditions:

$$0 \leq |\Upsilon| - \Upsilon \tanh\left(\frac{\Upsilon}{\epsilon_i}\right) \leq \epsilon_i \varphi \tag{12}$$

in the above equation, φ represents a constant that satisfies the equation $\varphi = \mathrm{e}^{-(\varphi+1)}$ ($\varphi \approx 0.2785$) .

D. Fuzzy Logic Systems(FLSs)

Because the system considered in this paper contains unknown function terms, there will be some difficulties in the design process of the controller. Therefore, to solve this difficulty, the fuzzy logic system is introduced, which can produce an estimated value to replace the uncertain function contained in the system. The fuzzy logic system consists of four parts, where the knowledge base is an if-then set of rules [13], as shown below:

$$\begin{aligned} R^l: & IF \quad x_1 \quad is \quad \Psi_1^l \quad and \ldots and \quad x_n \quad is \quad \Psi_n^l, \\ & THEN \quad y \quad is \quad \aleph^l, l=1,\ldots,N. \end{aligned} \tag{13}$$

where N denotes the number of rules, $x_i (i=1,\ldots,n)$ and y represent inputs and outputs, respectively. $\Psi_i^l (i=1,\ldots,n, l=1,\ldots,N)$ and $\aleph^l$ denote fuzzy sets, $\mu_{\Psi^l}(x_i)$ and $\mu_{\mathbb{N}^l}(y)$ denote Gaussian membership functions. Based on [13], a known fuzzy logic system is shown below

$$y(x) = \frac{\sum_{l=1}^N \bar{y}_l \prod_{i=1}^n \mu_{\Psi_i^l}(x_i)}{\sum_{l=1}^N \left(\prod_{i=1}^n \mu_{\Psi_i^l}(x_i)\right)} \tag{14}$$

where $\bar{y}_l = \max_{u \in R} \mu_{\aleph^l}(y), l=1,\ldots,N$.
Describe fundamental functions as follows:

$$\phi_l(x) = \frac{\prod_{i=1}^n \mu_{\Psi_i^l}(x_i)}{\sum_{l=1}^N \left(\prod_{i=1}^n \mu_{\Psi_i^l}(x_i)\right)} \tag{15}$$

Let $W = [\bar{y}_1,\ldots,\bar{y}_N]^T = [W_1,\ldots,W_N]^T, \psi(x) = [\psi_1(x),\ldots,\psi_N(x)]^T, x = [x_1,\ldots,x_n]^T$, then one has

$$y(x) = W^T \psi(x) \tag{16}$$

Lemma 1: [7]: The following FLS expression represents a continuous function $f(x)$ defined on a compact set $\wp$

$$\sup_{x \in \wp} \left| f(x) - W^T \psi(x) \right| \leq \delta \tag{17}$$

where δ is a positive constant.

Lemma 2: [15]: For $\forall(a,b) \in R^2$, we have:

$$ab \leq \frac{\beta^p}{p}|a|^p + \frac{1}{q\beta^q}|b|^q \tag{18}$$

where $\beta > 0, p > 1$, and $q > 1$ are positive constants, p and q fulfill $(p-1)(q-1) = 1$.

III. Event-triggered adaptive controller design

In this section, an event-triggered adaptive prescribed performance fuzzy fault-tolerant tracking controller is designed with the backstepping method. The $\alpha_{i,j}$ virtual control for the first n_i step looks like this:

$$\begin{aligned} \alpha_{i,j} = & -\eta_{i,j}\rho_{i,j}\left(k_{i,j} + \frac{1}{2} + \frac{1}{2C_{i,j}^2}\hat{\theta}_{i,0}\psi_{i,j}^T\psi_{i,j}\right) \\ \alpha_{i,n_i} = & -\frac{\eta_{i,n_i}\rho_{i,n_i}}{\underline{\varrho}_i}\left(k_{i,n_i} + \frac{1}{2}\right. \\ & \left. + \frac{1}{2C_{i,n_i}^2}\hat{\theta}_{i,0}\psi_{i,n_i}^T\psi_{i,n_i}\right) \end{aligned} \tag{19}$$

where $k_{i,j}, C_{i,j} (i=1,2,\cdots,m; j=1,2,\cdots,n_i)$ indicates positive design constants, $Z_{i,j} = [\underline{x}_{i,j}, \hat{\theta}_{i,0}, \bar{y}_{i,r}^{(j)}, \bar{\rho}_{i,j}^{(j)}]^T$ and $\hat{\theta}_{i,0}$ denotes the estimate value of the adaptive law $\theta_{i,0}$, which is defined as

$$\theta_{i,0} = \max\left\{\frac{\left\|W_{i,j}^*\right\|^2}{b_m}\right\} \tag{20}$$

where b_m is defined in (4) and $W_{i,j}^*$ is defined subsequently. The parameter adaptive law designed in the controller is as follows

$$\dot{\hat{\theta}}_{i,0} = \sum_{j=1}^n \frac{\lambda_{i,0}}{2C_{i,j}^2}\eta_{i,j}^2\psi_{i,j}^T(Z_{i,j})\psi_{i,j}(Z_{i,j}) - \gamma_{i,0}\hat{\theta}_{i,0} \tag{21}$$

where $\lambda_{i,0}$ and $\gamma_{i,0}$ are positive design parameters.

For emphasis, the variable $Z_{i,j}$ will be omitted from the corresponding function $\psi_{i,j}(Z_{i,j})$ in the following writing, and let $\psi_{i,j}(Z_{i,j}) = \psi_{i,j}$ and $\delta_{i,j}(Z_{i,j}) = \delta_{i,j}$.

Step 1 : Choose a positive definite Lyapunov function as follows:

$$V_{i,1}(t) = \frac{1}{4}\ln^2\left(\frac{\rho_{i,1}+z_{i,1}}{\rho_{i,1}-z_{i,1}}\right) + \frac{b_m}{2\lambda_{i,0}}\tilde{\theta}_{i,0}^2 \tag{22}$$

where $\tilde{\theta}_{i,0} = \theta_{i,0} - \hat{\theta}_{i,0}$ denote parameter errors. By combining (1), (5) with (8), we can obtain $V_{i,1}(t)$'s derivative as

$$\begin{aligned} \dot{V}_{i,1} = & \frac{\eta_{i,1}}{\rho_{i,1}}\left(\dot{z}_{i,1} - z_{i,1}\frac{\dot{\rho}_{i,1}}{\rho_{i,1}}\right) - \frac{b_m}{\lambda_{i,0}}\tilde{\theta}_{i,0}\dot{\hat{\theta}}_{i,0} \\ = & \frac{\eta_{i,1}}{\rho_{i,1}}\left(f_{i,1} + g_{i,1}z_{i,2} + d_{i,1} - \dot{y}_{i,r} - z_{i,1}\frac{\dot{\rho}_{i,1}}{\rho_{i,1}}\right) \\ & - \frac{b_m}{\lambda_{i,0}}\tilde{\theta}_{i,0}\dot{\hat{\theta}}_{i,0} \end{aligned} \tag{23}$$

Based on Lemma 2 and Assumption 1, one has $\frac{\eta_{i,1}d_{i,1}}{\rho_{i,1}} \leq \frac{\eta_{i,1}^2}{2\rho_{i,1}^2} + \frac{\bar{d}_i^2}{2}$. Apply this inequality to (23)

$$\begin{aligned} \dot{V}_{i,1} \leq & \frac{\eta_{i,1}}{\rho_{i,1}}\left(f_{i,1} + g_{i,1}x_{i,2} + \frac{\eta_{i,1}}{2\rho_{i,1}} - \dot{y}_{i,r}\right. \\ & \left. - z_{i,1}\frac{\dot{\rho}_{i,1}}{\rho_{i,1}}\right) + \frac{\bar{d}_i^2}{2} - \frac{b_m}{\lambda_{i,0}}\tilde{\theta}_{i,0}\dot{\hat{\theta}}_{i,0} \\ \leq & \frac{\eta_{i,1}}{\rho_{i,1}}g_{i,1}x_{i,2} + \eta_{i,1}F_{i,1} + \frac{\bar{d}_i}{2} - \frac{b_m}{\lambda_{i,0}}\tilde{\theta}_{i,0}\dot{\hat{\theta}}_{i,0} \end{aligned} \tag{24}$$

where $F_{i,1} = \frac{1}{\rho_{i,1}}\left(f_{i,1} + \frac{\eta_{i,1}}{2\rho_{i,1}} - \dot{y}_{i,r} - z_{i,1}\frac{\dot{\rho}_{i,1}}{\rho_{i,1}}\right)$. Since $F_{i,1}$ contains the unknown function $f_{i,1}(\bar{x}_{i,1})$, $F_{i,1}$ cannot directly construct virtual control signal $\alpha_{i,1}$. By utilizing a FLS $W_{i,1}^T\psi_{i,1}$ to approximate $F_{i,1}$, $F_{i,1}$ can be expressed as

$$F_{i,1} = W_{i,1}^{*T}\psi_{i,1} + \delta_{i,1}, |\delta_{i,1}| \le \bar{\varepsilon}_{i,1} \tag{25}$$

where $\delta_{i,1}$ are the approximation errors and $\bar{\varepsilon}_{i,1}$ are unknown positive constants. In line with Lemma 2, one has

$$\begin{aligned}\eta_{i,1}F_{i,1} \le& \frac{b_m}{2C_{i,1}^2}\eta_{i,1}^2\frac{\left\|W_{i,1}^{*T}\right\|^2}{b_m}\psi_{i,1}^T\psi_{i,1} \\ &+ \frac{C_{i,1}^2}{2} + \frac{\eta_{i,1}^2}{2} + \frac{\bar{\varepsilon}_{i,1}^2}{2} \\ \le& \frac{b_m}{2C_{i,1}^2}\eta_{i,1}^2\theta_{i,0}\psi_{i,1}^T\psi_{i,1} \\ &+ \frac{C_{i,1}^2}{2} + \frac{\eta_{i,1}^2}{2} + \frac{\bar{\varepsilon}_{i,1}^2}{2}\end{aligned} \tag{26}$$

Based on (19), we have

$$\begin{aligned}\frac{\eta_{i,1}}{\rho_{i,1}}g_{i,1}\alpha_{i,1} \le& -k_{i,1}b_m\eta_{i,1}^2 - \frac{b_m\eta_{i,1}^2}{2} \\ &- \frac{b_m}{2C_{i,1}^2}\hat{\theta}_{i,0}\eta_{i,1}^2\psi_{i,1}^T\psi_{i,1}\end{aligned} \tag{27}$$

Combining(26), (27) with (24) procedure, then

$$\begin{aligned}\dot{V}_{i,1} \le& -k_{i,1}b_m\eta_{i,1}^2 + \frac{\eta_{i,1}}{\rho_{i,1}}b_Mz_{i,2} \\ &+ \frac{b_m}{\lambda_{i,0}}\tilde{\theta}_{i,0}\left(\frac{\lambda}{2C_{i,1}^2}\eta_{i,1}^2\psi_{i,1}^T\psi_{i,1} - \dot{\hat{\theta}}_{i,0}\right) \\ &+ \Delta_{i,1}\end{aligned} \tag{28}$$

where $\Delta_{i,1} = \frac{C_{i,1}^2}{2} + \frac{\bar{d}_i^2}{2} + \frac{\bar{\varepsilon}_{i,1}^2}{2}$.

Step 2 : Similar to Step 1, we have:

$$V_{i,2} = V_{i,1} + \frac{1}{4}\ln^2\left(\frac{\rho_{i,2}+z_{i,2}}{\rho_{i,2}-z_{i,2}}\right) \tag{29}$$

Combining (1), (5), (8), and (29), we can calculate the derivative of $V_{i,2}(t)$ as

$$\begin{aligned}\dot{V}_{i,2} =& \dot{V}_{i,1} + \frac{\eta_{i,2}}{\rho_{i,2}}\left(\dot{z}_{i,2} - z_{i,2}\frac{\dot{\rho}_{i,2}}{\rho_{i,2}}\right) \\ =& \dot{V}_{i,1} + \frac{\eta_{i,2}}{\rho_{i,2}}\Big(f_{i,2} + g_{i,2}x_{i,3} + d_{i,2} \\ &- \dot{\alpha}_{i,1} - z_{i,2}\frac{\dot{\rho}_{i,2}}{\rho_{i,2}}\Big)\end{aligned} \tag{30}$$

where

$$\begin{aligned}\dot{\alpha}_{i,1} =& -\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\left(f_{i,1} + g_{i,1}x_{i,2} + d_{i,1}\right) \\ &- \sum_{k=0}^{1}\frac{\partial\alpha_{i,1}}{\partial\rho_{i,1}^{(k)}}\rho_{i,1}^{(k+1)} \\ &- \sum_{k=0}^{1}\frac{\partial\alpha_{i,1}}{\partial y_{i,r}^{(k)}}y_{i,r}^{(k+1)} - \frac{\partial\alpha_{i,1}}{\partial\hat{\theta}_{i,0}}\dot{\hat{\theta}}_{i,0}\end{aligned} \tag{31}$$

Using Lemma 2 and Assumption 1, it is not difficult to achieve that $\frac{\eta_{i,2}d_{i,2}}{\rho_{i,2}} \le \frac{\eta_{i,2}^2}{2\rho_{i,2}^2} + \frac{\bar{d}_i^2}{2}$. Similar to (24), one has

$$\begin{aligned}\dot{V}_{i,2} \le& -k_{i,1}b_m\eta_{i,1}^2 + \Delta_{i,1} \\ &+ \frac{b_m}{\lambda_{i,0}}\tilde{\theta}_{i,0}\Big(\frac{\lambda_{i,0}}{2C_{i,1}^2}\eta_{i,1}^2\psi_{i,1}^T\psi_{i,1} \\ &- \dot{\hat{\theta}}_{i,0}\Big) + \frac{\eta_{i,2}}{\rho_{i,2}}g_{i,j}x_{i,3} + \eta_{i,2}F_{i,2} + \frac{\bar{d}_i^2}{2}\end{aligned} \tag{32}$$

where $F_{i,2} = \frac{1}{\rho_{i,2}}\left(f_{i,2} + \frac{\eta_{i,2}}{2\rho_{i,2}} - \dot{\alpha}_{i,1} + \frac{\eta_{i,1}\rho_{i,2}}{\rho_{i,1}\eta_{i,2}}b_Mz_{i,2}\right) - z_{i,1}\frac{\dot{\rho}_{i,1}}{\rho_{i,1}^2}$. Similar to (25), one has

$$F_{i,2} = W_{i,2}^{*T}\psi_{i,2} + \delta_{i,2}, |\delta_{i,2}| \le \bar{\varepsilon}_{i,2} \tag{33}$$

Furthermore, based on Lemma 2, we have

$$\begin{aligned}\eta_{i,2}F_{i,2} \le& \frac{b_m}{2C_{i,2}^2}\eta_{i,2}^2\frac{\left\|W_{i,2}^{*T}\right\|^2}{b_m}\psi_{i,2}^T\psi_{i,2} + \frac{C_{i,2}^2}{2} \\ &+ \frac{\eta_{i,2}^2}{2} + \frac{\bar{\varepsilon}_{i,2}^2}{2} \\ \le& \frac{b_m}{2C_{i,2}^2}\eta_{i,2}^2\theta_{i,0}\psi_{i,2}^T\psi_{i,2} + \frac{C_{i,2}^2}{2} \\ &+ \frac{\eta_{i,2}^2}{2} + \frac{\bar{\varepsilon}_{i,2}^2}{2}\end{aligned} \tag{34}$$

According to (19) and Assumption 1, we have

$$\begin{aligned}\frac{\eta_{i,2}}{\rho_{i,2}}g_{i,2}\alpha_{i,2} \le& -k_{i,2}b_m\eta_{i,2}^2 - \frac{b_m\eta_{i,2}^2}{2} \\ &- \frac{b_m}{2C_{i,2}^2}\hat{\theta}_{i,0}\eta_{i,2}^2\psi_{i,2}^T\psi_{i,2}\end{aligned} \tag{35}$$

Combining(34), (35) with (32) procedure, then

$$\begin{aligned}\dot{V}_{i,2} \le& -\sum_{a=1}^{2}k_{i,a}b_m\eta_{i,a}^2 + \sum_{a=1}^{2}\Delta_{i,a} \\ &+ \frac{b_m}{\lambda_{i,0}}\tilde{\theta}_{i,0}\left(\sum_{a=1}^{2}\frac{\lambda}{2C_{i,a}^2}\eta_{i,a}^2\psi_{i,a}^T\psi_{i,a} - \dot{\hat{\theta}}_{i,0}\right) \\ &+ \frac{\eta_{i,2}}{\rho_{i,2}}b_Mz_{i,3}\end{aligned} \tag{36}$$

where $\Delta_{i,2} = \frac{C_{i,2}^2}{2} + \frac{\bar{d}_i^2}{2} + \frac{\bar{\varepsilon}_{i,2}^2}{2}$.

Step $j\,(3 \le j \le n_i - 1)$: According to $z_{i,j} = x_{i,j} - \alpha_{i,j-1}$, we define Lyapunov function as follows:

$$V_{i,j} = V_{i,j-1} + \frac{1}{4}\ln^2\left(\frac{\rho_{i,j}+z_{i,j}}{\rho_{i,j}-z_{i,j}}\right) \tag{37}$$

Combining (1), (5), (8) and (37), we can obtain $V_{i,2}(t)$'s derivative as

$$\begin{aligned}\dot{V}_{i,j} =& \dot{V}_{i,j-1} + \frac{\eta_{i,j}}{\rho_{i,j}}\left(\dot{z}_{i,j} - z_{i,j}\frac{\dot{\rho}_{i,j}}{\rho_{i,j}}\right) \\ =& \dot{V}_{i,j-1} + \frac{\eta_{i,j}}{\rho_{i,j}}\Big(f_{i,j} + g_{i,j}x_{i,j+1} + d_{i,j} \\ &- \dot{\alpha}_{i,j-1} - z_{i,j}\frac{\dot{\rho}_{i,j}}{\rho_{i,j}}\Big)\end{aligned} \tag{38}$$

where

$$\begin{aligned}\dot{\alpha}_{i,j-1} =& -\frac{\partial\alpha_{i,j-1}}{\partial x_{i,j-1}}\left(f_{i,j-1} + g_{i,j-1}x_{i,j+1} + d_{i,j}\right) \\ &- \sum_{k=0}^{1}\frac{\partial\alpha_{i,j-1}}{\partial\rho_{i,j-1}^{(k)}}\rho_{i,j-1}^{(k+1)} \\ &- \sum_{k=0}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial y_{i,r}^{(k)}}y_{i,r}^{(k+1)} - \frac{\partial\alpha_{i,j-1}}{\partial\hat{\theta}_{i,0}}\dot{\hat{\theta}}_{i,0}\end{aligned} \tag{39}$$

Using Lemma 2 and Assumption 1, we have $\frac{\eta_{i,j} d_{i,j}}{\rho_{i,j}} \leq \frac{\eta_{i,j}^2}{2\rho_{i,j}^2} + \frac{\bar{d}_i^2}{2}$. By substituting (36) with (38), one has

$$\begin{aligned} \dot{V}_{i,j} \leq & -\sum_{a=1}^{j-1} k_{i,a} b_m \eta_{i,a}^2 + \sum_{a=1}^{j-1} \Delta_{i,a} \\ & + \frac{b_m}{\lambda_{i,0}} \tilde{\theta}_{i,0} \Bigg(\sum_{a=1}^{j-1} \frac{\lambda_{i,0}}{2C_{i,a}^2} \eta_{i,a}^2 \psi_{i,a}^T \psi_{i,a} \\ & - \dot{\hat{\theta}}_{i,0} \Bigg) + \frac{\eta_{i,j}}{\rho_{i,j}} g_{i,j} x_{i,j+1} + \eta_{i,j} F_{i,j} + \frac{\bar{d}_i^2}{2} \end{aligned} \tag{40}$$

where $F_{i,j} = \frac{1}{\rho_{i,j}} \left(f_{i,j} + \frac{\eta_{i,j}}{2\rho_{i,j}} - \dot{\alpha}_{i,j-1} + \frac{\eta_{i,j-1}\rho_{i,j}}{\rho_{i,j-1}\eta_{i,j}} b_M z_{i,j} \right) - z_{i,j} \frac{\dot{\rho}_{i,j}}{\rho_{i,j}^2}$. Similar to (33), we get

$$F_{i,j} = W_{i,j}^{*T} \psi_{i,j} + \delta_{i,j}, |\delta_{i,j}| \leq \bar{\varepsilon}_{i,j} \tag{41}$$

Furthermore, on the basis of Lemma 2, it follows from (34) that

$$\begin{aligned} \eta_{i,j} F_{i,j} \leq & \frac{b_m}{2C_{i,j}^2} \eta_{i,j}^2 \frac{\left\| W_{i,j}^{*T} \right\|^2}{b_m} \psi_{i,j}^T \psi_{i,j} + \frac{C_{i,j}^2}{2} \\ & + \frac{\eta_{i,j}^2}{2} + \frac{\bar{\varepsilon}_{i,j}^2}{2} \\ \leq & \frac{b_m}{2C_{i,j}^2} \eta_{i,j}^2 \theta_{i,0} \psi_{i,j}^T \psi_{i,j} + \frac{C_{i,j}^2}{2} \\ & + \frac{\eta_{i,j}^2}{2} + \frac{\bar{\varepsilon}_{i,j}^2}{2} \end{aligned} \tag{42}$$

According to (19) and Assumption 1, one has

$$\begin{aligned} \frac{\eta_{i,j}}{\rho_{i,j}} g_{i,j} \alpha_{i,j} \leq & -k_{i,j} b_m \eta_{i,j}^2 - \frac{b_m \eta_{i,j}^2}{2} \\ & - \frac{b_m}{2C_{i,j}^2} \hat{\theta}_{i,0} \eta_{i,j}^2 \psi_{i,j}^T \psi_{i,j} \end{aligned} \tag{43}$$

Combining(43), (42) with (40) procedure, then

$$\begin{aligned} \dot{V}_{i,j} \leq & -\sum_{a=1}^{j} k_{i,a} b_m \eta_{i,a}^2 + \sum_{a=1}^{j} \Delta_{i,a} \\ & + \frac{b_m}{\lambda_{i,0}} \tilde{\theta}_{i,0} \left(\sum_{a=1}^{j} \frac{\lambda}{2C_{i,a}^2} \eta_{i,a}^2 \psi_{i,a}^T \psi_{i,a} - \dot{\hat{\theta}}_{i,0} \right) \\ & + \frac{\eta_{i,j}}{\rho_{i,j}} b_M z_{i,j+1} \end{aligned} \tag{44}$$

where $\Delta_{i,j} = \frac{C_{i,j}^2}{2} + \frac{\bar{d}_i^2}{2} + \frac{\bar{\varepsilon}_{i,j}^2}{2}$.

Step n_i : In the final step, we will design the actual control input signals ν_i. Now, the Lyapunov functions are chosen as

$$V_{i,n_i} = V_{i,n_i-1} + \frac{1}{4} \ln^2 \left(\frac{\rho_{i,n_i} + z_{i,n_i}}{\rho_{i,n_i} - z_{i,n_i}} \right) \tag{45}$$

Combining (1), (2), (5) and (8), one has

$$\begin{aligned} \dot{V}_{i,n_i} = & \dot{V}_{i,n_i-1} + \frac{\eta_{i,n_i}}{\rho_{i,n_i}} \left(\dot{z}_{i,n_i} - z_{i,n_i} \frac{\dot{\rho}_{i,n_i}}{\rho_{i,n_i}} \right) \\ \leq & \dot{V}_{i,n_i-1} + \frac{\eta_{i,n_i}}{\rho_{i,n_i}} \Bigg(f_{i,n_i} + g_{i,n_i} \varrho_i \nu_i \\ & + g_{i,n_i} \zeta_i + d_{i,n_i} - \dot{\alpha}_{i,n_i-1} \\ & - z_{i,n_i} \frac{\dot{\rho}_{i,n_i}}{\rho_{i,n_i}} \Bigg) \end{aligned} \tag{46}$$

Given Young's inequality, we can derive

$$\frac{\eta_{i,n_i} d_{i,n_i}}{\rho_{i,n_i}} \leq \frac{\eta_{i,n_i}^2}{2\rho_{i,n_i}^2} + \frac{\bar{d}_i^2}{2} \tag{47}$$

Then

$$\begin{aligned} \dot{V}_{i,n_i} \leq & \dot{V}_{i,n_i-1} + \frac{\eta_{i,n_i}}{\rho_{i,n_i}} \Bigg(f_{i,n_i} + g_{i,n_i} \varrho_i \nu_i \\ & + \frac{\eta_{i,n_i}}{2\rho_{i,n_i}} + g_{i,n_i} \zeta_i - \dot{\alpha}_{i,n_i-1} \\ & - z_{i,n_i} \frac{\dot{\rho}_{i,n_i}}{\rho_{i,n_i}} \Bigg) + \frac{\bar{d}_i^2}{2} \end{aligned} \tag{48}$$

During the time intervals $[t_k, t_{k+1})$, and from (10), we have $|\omega_i(t) - \nu_i(t)| \geq \tau_{i,0} |\nu_i(t)| + \mu_{i,2}$. Therefore, $\forall t \in [t_k, t_{k+1})$, there are two continuous time-varying parameters $\ell_{i,1}(t)$ and $\ell_{i,2}(t)$, with $|\ell_{i,1}(t)| \leq 1$ and $|\ell_{i,2}(t)| \leq 1$, such that $\nu_i(t) = \frac{\omega_i(t) - \mu_{i,2}\ell_{i,2}(t)}{1 + \tau_{i,0}\ell_{i,1}(t)}$. we have

$$\begin{aligned} \dot{V}_{i,n_i} \leq & -\sum_{a=1}^{n_i-1} k_{i,a} b_m \eta_{i,a}^2 + \sum_{a=1}^{n_i-1} \Delta_{i,a} \\ & + \frac{b_m}{\lambda_{i,0}} \tilde{\theta}_{i,0} \left(\sum_{a=1}^{n_i-1} \frac{\lambda}{2C_{i,a}^2} \eta_{i,a}^2 \psi_{i,a}^T \psi_{i,a} - \dot{\hat{\theta}}_{i,0} \right) \\ & + \eta_{i,n_i} F_{i,n_i} + \frac{\bar{d}_i^2}{2} \\ & + \frac{\eta_{i,n_i} g_{i,n_i} \varrho_i}{\rho_{i,n_i}} \frac{\omega_i - \mu_{i,2}\ell_{i,2}}{1 + \tau_{i,0}\ell_{i,1}} \end{aligned} \tag{49}$$

where $F_{i,n_i} = \frac{1}{\rho_{i,n_i}} \left(f_{i,n_i} + \frac{\eta_{i,n_i}}{2\rho_{i,n_i}} + \frac{\eta_{i,n_i-1}\rho_{i,n_i}}{\rho_{i,n_i-1}\eta_{i,n_i}} b_M z_{i,n_i} \right) + \frac{g_{i,n_i}\zeta_i}{\rho_{i,n_i}} - \frac{\dot{\alpha}_{i,n_i-1}}{\rho_{i,n_i}} - z_{i,n_i} \frac{\dot{\rho}_{i,n_i}}{\rho_{i,n_i}^2}$. Since $\frac{\eta_{i,n_i}\omega_i}{1 + \ell_{i,1}\tau_{i,0}} \leq \frac{\eta_{i,n_i}\omega_i}{1 + \tau_{i,0}}$ and $\left| \frac{\eta_{i,n_i}\ell_{i,2}(t)\mu_{i,2}}{1 + \ell_{i,1}\tau_{i,0}} \right| \leq \frac{\eta_{i,n_i}\mu_{i,2}}{1 - \tau_{i,0}}$, then

$$\begin{aligned} \dot{V}_{i,n_i} \leq & -\sum_{a=1}^{n_i-1} k_{i,a} b_m \eta_{i,a}^2 + \sum_{a=1}^{n_i-1} \Delta_{i,a} \\ & + \frac{b_m}{\lambda_{i,0}} \tilde{\theta}_{i,0} \left(\sum_{a=1}^{n_i-1} \frac{\lambda}{2C_{i,a}^2} \eta_{i,a}^2 \psi_{i,a}^T \psi_{i,a} - \dot{\hat{\theta}}_{i,0} \right) \\ & + \eta_{i,n_i} F_{i,n_i} + \frac{\bar{d}_i^2}{2} + \frac{g_{i,n_i} \underline{\varrho}_i}{\rho_{i,n_i}} \left| \frac{\eta_{i,n_i}\mu_{i,2}}{1 - \tau_{i,0}} \right| \\ & + \frac{g_{i,n_i} \underline{\varrho}_i}{\rho_{i,n_i}} \Bigg(-\eta_{i,n_i} \alpha_{i,n_i} \tanh\Big(\frac{\eta_{i,n_i}\alpha_{i,n_i}}{\epsilon_i \rho_{i,n_i}}\Big) \\ & - \eta_{i,n_i} \mu_{i,1} \tanh\Big(\frac{\eta_{i,n_i}\mu_{i,1}}{\epsilon_i \rho_{i,n_i}}\Big) \Bigg) \end{aligned} \tag{50}$$

By putting (12) into the preceding inequality, we have

$$\begin{aligned} \dot{V}_{i,n_i} \leq & -\sum_{a=1}^{n_i-1} k_{i,a} b_m \eta_{i,a}^2 + \sum_{a=1}^{n_i-1} \Delta_{i,a} \\ & + \frac{b_m}{\lambda_{i,0}} \tilde{\theta}_{i,0} \left(\sum_{a=1}^{n_i-1} \frac{\lambda}{2C_{i,a}^2} \eta_{i,a}^2 \psi_{i,a}^T \psi_{i,a} - \dot{\hat{\theta}}_{i,0} \right) \\ & + \eta_{i,n_i} F_{i,n_i} + \frac{g_{i,n_i} \underline{\varrho}_i}{\rho_{i,n_i}} \eta_{i,n_i} \alpha_{i,n_i} \\ & + \frac{g_{i,n_i} \underline{\varrho}_i}{\rho_{i,n_i}} \left(\left| \frac{\eta_{i,n_i}\mu_{i,2}}{1 - \tau_{i,0}} \right| - |\eta_{i,n_i}\mu_{i,1}| \right) \\ & + \frac{\bar{d}_i^2}{2} + 0.557 b_M \epsilon_i \end{aligned} \tag{51}$$

According to (19) and Assumption 1, we have

$$\begin{aligned}\frac{\eta_{i,n_i}\underline{\varrho}_i}{\rho_{i,n_i}}g_{i,n_i}\alpha_{i,n_i}\leq&-k_{i,n_i}b_m\eta_{i,n_i}^2-\frac{b_m\eta_{i,n_i}^2}{2}\\&-\frac{b_m}{2C_{i,n_i}^2}\hat{\theta}_{i,0}\eta_{i,n_i}^2\psi_{i,j}^T\psi_{i,j}\end{aligned}\tag{52}$$

Similar to (41), we have

$$F_{i,n_i}=W_{i,n_i}^{*T}\psi_{i,n_i}+\delta_{i,n_i},|\delta_{i,n_i}|\leq\bar{\varepsilon}_{i,n_i}\tag{53}$$

Furthermore, based on Lemma 2, we have

$$\begin{aligned}\eta_{i,n_i}F_{i,n_i}\leq&\frac{b_m}{2C_{i,n_i}^2}\eta_{i,n_i}^2\frac{\left\|W_{i,n_i}^{*T}\right\|^2}{b_m}\psi_{i,n_i}^T\psi_{i,n_i}+\frac{C_{i,n_i}^2}{2}\\&+\frac{\eta_{i,n_i}^2}{2}+\frac{\bar{\varepsilon}_{i,n_i}^2}{2}\\\leq&\frac{b_m}{2C_{i,n_i}^2}\eta_{i,n_i}^2\theta_{i,0}\psi_{i,n_i}^T\psi_{i,n_i}+\frac{C_{i,n_i}^2}{2}\\&+\frac{\eta_{i,n_i}^2}{2}+\frac{\bar{\varepsilon}_{i,n_i}^2}{2}\end{aligned}\tag{54}$$

Substituting (54), (52) into (51) and $|\frac{\eta_{i,n_i}\mu_{i,2}}{1-\tau_{i,0}}|<|\eta_{i,n_i}\mu_{i,1}|$, we can gain

$$\begin{aligned}\dot{V}_{i,n_i}\leq&-\sum_{a=1}^{n_i}k_{i,a}b_m\eta_{i,a}^2+\sum_{a=1}^{n_i}\Delta_{i,a}\\&+\frac{b_m}{\lambda_{i,0}}\tilde{\theta}_{i,0}\left(\sum_{a=1}^{n_i}\frac{\lambda}{2C_{i,a}^2}\eta_{i,a}^2\psi_{i,a}^T\psi_{i,a}-\dot{\hat{\theta}}_{i,0}\right)\\&+0.557b_M\epsilon_i\end{aligned}\tag{55}$$

Substituting the adaptive law (21) into (55), we have

$$\begin{aligned}\dot{V}_{i,n_i}\leq&\sum_{a=1}^{n_i}-k_{i,a}b_m\eta_{i,a}^2+\sum_{a=1}^{n_i}\Delta_{i,a}+0.557b_M\epsilon_i\\&+\frac{b_m}{\lambda_{i,0}}\gamma_{i,0}\hat{\theta}_{i,0}\tilde{\theta}_{i,0}\end{aligned}\tag{56}$$

with the inequality as follows

$$\tilde{\theta}_{i,0}\hat{\theta}_{i,0}\leq\frac{\theta_{i,0}^2}{2}-\frac{\tilde{\theta}_{i,0}^2}{2}\tag{57}$$

Then

$$\begin{aligned}\dot{V}_{i,n_i}\leq&\sum_{a=1}^{n_i}-k_{i,a}b_m\eta_{i,a}^2-\frac{b_m}{2\lambda_{i,0}}\gamma_{i,0}\tilde{\theta}_{i,0}^2\\&+\Lambda_{i,0}\end{aligned}\tag{58}$$

We define $\Lambda_{i,0}=\sum_{a=1}^{n_i}\Delta_{i,a}+0.557b_M\epsilon_i+\frac{b_m}{\lambda_{i,0}}\gamma_{i,0}\hat{\theta}_{i,0}^2$. Define $\Gamma_{i,0}=\min\{k_{i,j},b_m,\gamma_{i,0},(i=1,2,\cdots,m,j=1,2,\cdots,n_i)\}$, so (59) can be rewritten as

$$\dot{V}_{i,n_i}\leq-\Gamma_{i,0}V_{i,n_i}+\Lambda_{i,0}\tag{59}$$

IV. Stability analysis

Next, we will complete the stability analysis and proof of the event-triggered adaptive preset controller proposed in this paper.

Theorem 1: In the case of closed-loop systems with external disturbances and actuator failures, such as (1). Their controllers (19) and adaptive laws (21), under Assumptions 1-3, a fuzzy logic system can achieve a desired degree of accuracy by approaching the initial conditions and $F_{i,j}\left(x_{i,j}\right)$ defined in a compact set Ω_0. Then the following results are true:

1) These signals generated in a closed-loop system, such as system 1, are semi-globally bounded. In particular, the error transformation function $\rho_{i,1}$, the error signal $z_{i,1},(i=2,3,\ldots,m)$, and $\hat{\theta}_{i,0}$ all converge to a compact set Ω_z defined as

$$\begin{aligned}\Omega_z=\Bigg\{\eta_{i,j},z_{i,j},\hat{\theta}_{i,0}\mid E\left(\eta_{i,j}^2\right)\leq\frac{2\Gamma_{i,0}}{\Lambda_{i,0}},\\E\left(z_{i,j}^2\right)\leq\frac{2\Gamma_{i,0}}{\Lambda_{i,0}}\\E\left(\tilde{\theta}_{i,0}^2\right)\leq\frac{2\gamma_{i,0}\Psi_0}{b_m\Upsilon_0}\Bigg\}\end{aligned}\tag{60}$$

2) There is an instantaneous moment $t^*>0$ where the intervals $\{t_{d+1}-t_d\}$ are constrained by $t^*,\forall d\in\mathbf{Z}^+$.

Proof 1: 1) To facilitate the stability analysis, the Lyapunov function is chosen as $V=V_{i,n_i}$, then (59) can be rewritten as

$$\dot{V}\leq-\Gamma_{i,0}V+\Lambda_{i,0}\tag{61}$$

further, we can gain the following inequality

$$E(V(t))\leq\left(V(0)-\frac{\Lambda_{i,0}}{\Gamma_{i,0}}\right)e^{-\Gamma_{i,0}t}+\frac{\Lambda_{i,0}}{\Gamma_{i,0}}\tag{62}$$

According to the definition of $V(t)$ and (61), it denotes that $V(t)$, $\rho_{i,j},z_{i,j},\hat{\theta}_{i,0}$ and $x_{i,j}$ are bounded. Moreover, as $t\to\infty$, one has $e^{-\rho_{i,0}t}\to0$, then

$$E(V(t))\leq\frac{\Lambda_{i,0}}{\Gamma_{i,0}}\tag{63}$$

Consequently, the constraint function $\eta_{i,j}$, the error signal $z_{i,j},(i=1,2,\ldots,m,j=1,2,\ldots,n_i)$ and $\hat{\theta}_{i,0}$ eventually converge to a compact set Ω_z provided by (60), which means that all signals are evenly constrained.

2) To demonstrate that there exists a constant $t^*>0$ such that $t_{d+1}-t_d\geq t^*,\forall d\in\mathbb{Z}^+$, recall the definition of $e_i(t)$: $e_i(t)=\omega_i(t)-v_i(t),\forall t\in[t_d,t_{d+1})$. Then, we get

$$\frac{d}{dt}|e_i|=\frac{d}{dt}\left(e_i*e_i\right)^{\frac{1}{2}}=\operatorname{sign}\left(e_i\right)\dot{e}_i\leq|\dot{\omega}_i|\tag{64}$$

From (9), $\dot{\omega}_i$ must be continuous and bounded. Consequently, there must exist a constant $\bar{\omega}_i>0$ such that $|\dot{\omega}_i|<\bar{\omega}_i$. From $e_i\left(t_d\right)=0$ and $\lim_{t\to t_{d+1}}e_i=\tau_{i,0}|\nu_i\mid+\mu_{i,2}$, it yields that the bound of t^* must satisfy $\tau_{i,0}|\nu_i\mid+\mu_{i,2}/\bar{\omega}_i$, the Zeno-behavior is therefore avoided.

V. Simulation

In this part, a numerical example demonstrating the applicability of the devised control mechanism is presented. Specifically, the following describes a third-order uncertain MIMO system with external disturbance and actuator failure:

$$\begin{aligned}\dot{x}_{1,1}=&x_{1,1}sin(x_{1,1})+(2+x_{2,1}^2)x_{1,2}+sin(t)\\\dot{x}_{1,2}=&x_{1,1}cos(x_{1,2}x_{2,1})+(1+x_{1,2}^2+x_{1,1}^2\\&+x_{2,1}^2)u_{1,f}-0.4cos(t)\\y_1=&x_{1,1}\end{aligned}\tag{65}$$

$$
\begin{aligned}
\dot{x}_{2,1} =& cos(x_{2,1}) + (2 + x_{2,1}^2)x_{2,2} + sin(t) \\
\dot{x}_{2,2} =& sin(x_{1,1}^2 x_{2,2})x_{3,1} + (1 + x_{2,2}^2 + x_{1,1}^2)u_{2,f} \\
& - 0.2cos(t) \\
y_2 =& x_{2,1}
\end{aligned} \tag{66}
$$

$$
\begin{aligned}
\dot{x}_{3,1} =& x_{3,1} + (1 + sin(x_{3,1}^2))x_{3,2} + sin(t) \\
\dot{x}_{3,2} =& x_{3,1}x_{3,2}^2 x_{1,2}^2 + (1 + x_{3,2}^2 + x_{2,1}^2)u_{3,f} \\
& - cos(t) \\
y_3 =& x_{3,1}
\end{aligned} \tag{67}
$$

The design parameters for system (65) are selected as $\lambda_{1,0} = 1, \tau_{1,0} = 0.3, \epsilon_1 = 10; k_{1,1} = 100, k_{1,2} = 50, C_{1,1} = 50, C_{1,2} = 50, \mu_{1,1} = 1, \mu_{1,2} = 0.2$. The reference trajectory is $y_{1,r} = 0.2\sin(t)$. The design parameters for system (66) are selected as $\lambda_{2,0} = 1, \tau_{2,0} = 0.2, \epsilon_2 = 10; k_{2,1} = 70, k_{2,2} = 50, C_{2,1} = 50, C_{2,2} = 50, \mu_{2,1} = 1, \mu_{2,2} = 0.1$. The reference trajectory is taken as $y_{2,r} = -0.3\cos(t)$. The design parameters for system (67) are selected as $\lambda_{3,0} = 1, \tau_{3,0} = 0.3, \epsilon_3 = 10; k_{3,1} = 100, k_{3,2} = 50, C_{3,1} = 30, C_{3,2} = 30, \mu_{3,1} = 1, \mu_{3,2} = 0.05$. The reference trajectory is $y_{3,r} = 0.1\cos(t)$.

As previously, by the performance bounds of the state errors (6) and error transformation functions (8), the simulation parameters are selected as:

$$
\begin{aligned}
\rho_{1,1} &= (2 - 0.1)e^{-t} + 0.1 \\
\rho_{1,2} &= (4 - 0.3)e^{-0.3t} + 0.3
\end{aligned} \tag{68}
$$

$$
\begin{aligned}
\rho_{2,1} &= (2 - 0.1)e^{-t} + 0.1 \\
\rho_{2,2} &= (4 - 0.5)e^{-0.t} + 0.5
\end{aligned} \tag{69}
$$

$$
\begin{aligned}
\rho_{3,1} &= (3 - 0.3)e^{-t} + 0.5 \\
\rho_{3,2} &= (5 - 0.2)e^{-0.25t} + 0.3
\end{aligned} \tag{70}
$$

Meanwhile, the initial values are taken as $x(0) = [0.2; -0.8; 0; -0.3; 0.6; 0; 0.1; -0.7; 0]^T$, by the intermediate control signals and the control law (20), the positive constant vector is given as $\gamma_{i,0} = [1; 1; 1]^T$. We can obtain the simulation results in Figs. 1-6 by applying the static controller designed in Section 2. The same as predicted in Theorem 1, despite actuator failure and external disturbances. The state errors $z_{1,j}, j = 1, 2$ and corresponding performance bounds $\pm\rho_{1,j}, j = 1, 2$ are shown in Fig. 1 and 2, respectively. Fig. 3 shows the output signals $y_{i,1}$ of subsystem 1, which can track the target reference signal $y_{1,r}$ well. Fig.4 shows the event-triggered control effect, wherein ν_1 is the continuous control input signal and w_1 is the time-driven control signal. Fig. 5 shows the time intervals $t_{d+1} - t_d$, wherein the number of event triggering is 285. The adaptive parameters $\hat{\theta}_{i,0}, i = 1, 2, 3$ are shown in Fig. 6.

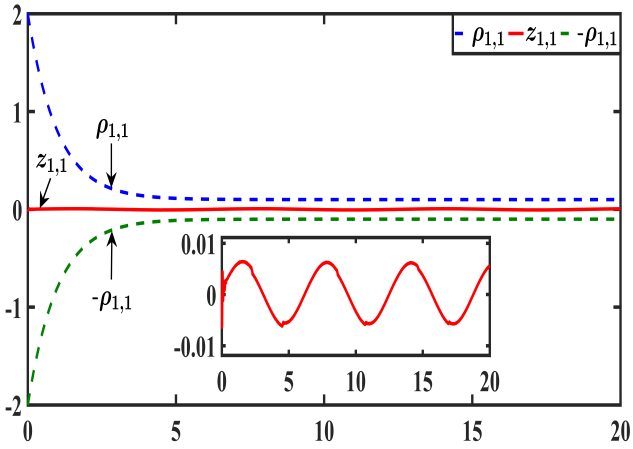

Fig. 1. Tracking error $z_{1,1}$

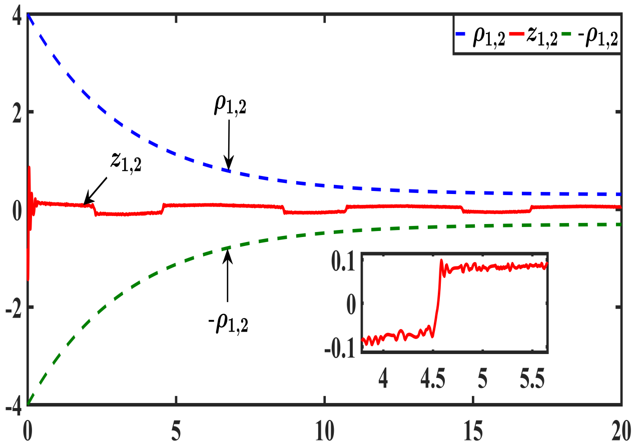

Fig. 2. Tracking error $z_{1,2}$

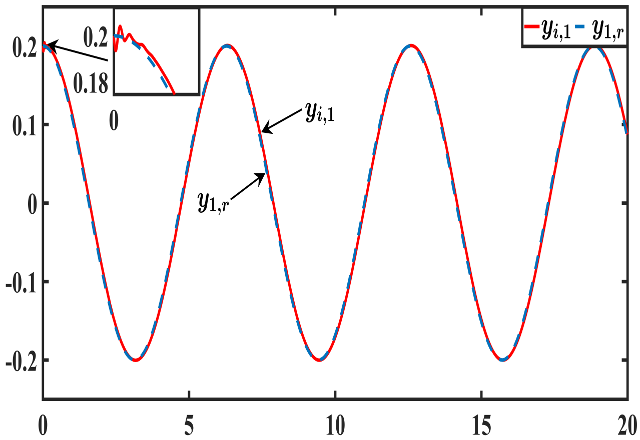

Fig. 3. Reference signal $y_{1,r}$ and system output $y_{i,1}$

VI. Conclusion

This paper proposes an event-triggered adaptive fuzzy controller with preset performance for a class of MIMO nonlinear systems with external disturbances and actuator faults. Unlike other related studies, this paper presents a new error transformation function, which can be used to implement performance constraints on output errors, and ensure that the dynamic and steady-state performance indicators of the nonlinear system meet the requirements. The controller is able to maintain the probability limit of all closed-loop system signals, and the tracking error converges to any small neighbourhood near the origin in the sense of quadric mean. Simulation results demonstrate the effectiveness and practicability of the proposed control strategy. Furthermore, it is noted that disturbances, such as unknown control di-

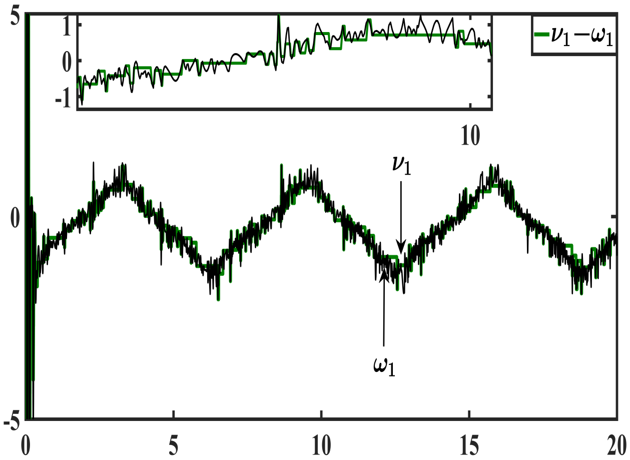

Fig. 4. Actual inputs ν_1 and event-trigger signals ω_1

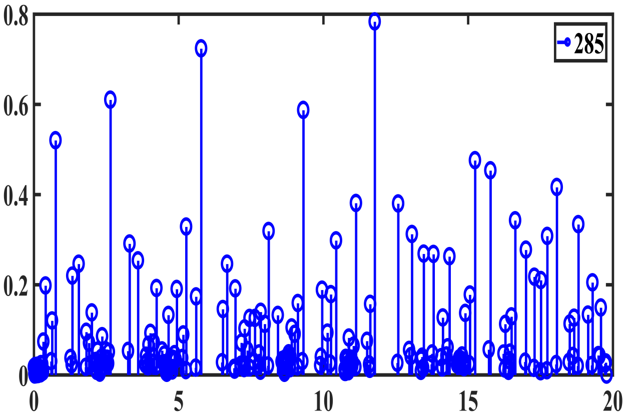

Fig. 5. Event trigger times

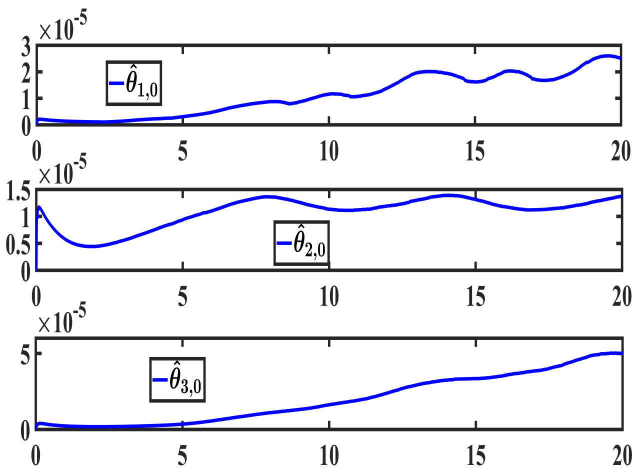

Fig. 6. Adaptive Law $\hat{\theta}_{i,0}$

rections, are prevalent in nonlinear systems. Therefore, it is an interesting future work to study the event-triggered PPC problem for MIMO nonlinear systems with unknown orientation constraints.

References

[1] X. Tian, Z. Yang, and Z. Yang, "Adaptive stabilization of fractional-order energy supply-demand system with dead-zone nonlinear inputs," *IAENG International Journal of Applied Mathematics*, vol. 49, no. 4, pp. 500–504, 2019.

[2] F. Gao, X. Zhu, J. Huang, and X. Wen, "Finite-time state feedback stabilization for a class of uncertain high-order nonholonomic feed-forward systems," *Engineering Letters*, vol. 27, no. 1, pp. 108–113, 2019.

[3] M. Chen, X. Liu, and H. Wang, "Adaptive robust fault-tolerant control for nonlinear systems with prescribed performance," *Nonlinear Dynamics*, vol. 81, no. 4, pp. 1727–1739, 2015.

[4] S. Yin, H. Gao, J. Qiu, and O. Kaynak, "Adaptive fault-tolerant control for nonlinear system with unknown control directions based on fuzzy approximation," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 8, pp. 1909–1918, 2017.

[5] S. Sui and C. P. Chen, "Finite-time fault-tolerant control for a nonlinear siso system with actuator faults," in *2018 International Conference on Security, Pattern Analysis, and Cybernetics (SPAC)*. IEEE, pp. 208–212, 2018.

[6] S. Zhou and Y. Song, "Prescribed performance neuroadaptive fault-tolerant compensation for mimo nonlinear systems under extreme actuator failures," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 9, pp. 5427–5436, 2019.

[7] Z. Ruan, Q. Yang, S. S. Ge, and Y. Sun, "Adaptive fuzzy fault tolerant control of uncertain mimo nonlinear systems with output constraints and unknown control directions," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 5, pp. 1224–1238, 2022.

[8] H. Yang, Y. Jiang, and S. Yin, "Adaptive fuzzy fault-tolerant control for markov jump systems with additive and multiplicative actuator faults," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 4, pp. 772–785, 2021.

[9] F. Osorio-Arteaga, J. J. Marulanda-Durango, and E. Giraldo, "Robust multivariable adaptive control of time-varying systems," *IAENG International Journal of Computer Science*, vol. 47, no. 4, pp. 605–612, 2020.

[10] Bechlioulis, Charalampos P and Rovithakis, George A, "Robust adaptive control of feedback linearizable mimo nonlinear systems with prescribed performance," *IEEE Transactions on Automatic Control*, vol. 53, no. 9, pp. 2090–2099, 2008.

[11] Bechlioulis, Charalampos P and Rovithakis, George A, "Prescribed performance adaptive control for multi-input multi-output affine in the control nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 5, pp. 1220–1226, 2010.

[12] Y. Li, S. Tong, L. Liu, and G. Feng, "Adaptive output-feedback control design with prescribed performance for switched nonlinear systems," *Automatica*, vol. 80, pp. 225–231, 2017.

[13] W. Shi and B. Li, "Adaptive fuzzy control for feedback linearizable mimo nonlinear systems with prescribed performance," *Fuzzy Sets and Systems*, vol. 344, pp. 70–89, 2018.

[14] I. S. Dimanidis, C. P. Bechlioulis, and G. A. Rovithakis, "Output feedback approximation-free prescribed performance tracking control for uncertain mimo nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 65, no. 12, pp. 5058–5069, 2020.

[15] W. Bai and H. Wang, "Robust adaptive fault-tolerant tracking control for a class of high-order nonlinear system with finite-time prescribed performance," *International Journal of Robust and Nonlinear Control*, vol. 30, no. 12, pp. 4708–4725, 2020.

[16] X. Wang, Q. Wang, and C. Sun, "Prescribed performance fault-tolerant control for uncertain nonlinear mimo system using actorcritic learning structure," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 33, no. 9, pp. 4479–4490, 2022.

[17] L. Xing, C. Wen, Z. Liu, H. Su, and J. Cai, "Event-triggered adaptive control for a class of uncertain nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 2071–2076, 2016.

[18] Y. Sun, D. Ding, S. Zhang, G. Wei, and H. Liu, "Non-fragile–control for discrete-time stochastic nonlinear systems under event-triggered protocols," *International Journal of General Systems*, vol. 47, no. 5, pp. 446–459, 2018.

[19] B. Li, J. Xia, H. Zhang, H. Shen, and Z. Wang, "Event-triggered adaptive fuzzy tracking control for stochastic nonlinear systems," *Journal of the Franklin Institute*, vol. 357, no. 14, pp. 9505–9522, 2020.

[20] T. Wang, M. Ma, J. Qiu, and H. Gao, "Event-triggered adaptive fuzzy tracking control for pure-feedback stochastic nonlinear systems with multiple constraints," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 6, pp. 1496–1506, 2020.

[21] A. Sahoo, H. Xu, and S. Jagannathan, "Neural network approximation-based event-triggered control of uncertain mimo nonlinear discrete

time systems," in *2014 American Control Conference*. IEEE, pp. 2017–2022, 2014.

[22] L.-B. Wu, J. H. Park, X.-P. Xie, and Y.-J. Liu, "Neural network adaptive tracking control of uncertain mimo nonlinear systems with output constraints and event-triggered inputs," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 32, no. 2, pp. 695–707, 2020.

[23] X. Huo, L. Ma, X. Zhao, and G. Zong, "Event-triggered adaptive fuzzy output feedback control of mimo switched nonlinear systems with average dwell time," *Applied Mathematics and Computation*, vol. 357, p. 11518–11544, 2020.

[24] T. Lei, W. Meng, K. Zhao, and L. Chen, "Adaptive asymptotic tracking control of constrained multi-input multi-output nonlinear systems via event-triggered strategy," *International Journal of Robust and Nonlinear Control*, vol. 31, no. 5, pp. 1479–1496, 2021.

[25] L. Wang and C. P. Chen, "Event-triggered-based adaptive output feedback control with prescribed performance for strict-feedback nonlinear systems," in *2019 IEEE International Conference on Systems, Man and Cybernetics (SMC)*. IEEE, pp. 2927–2932, 2019.

[26] W. Yang, Y. Pan, and H. Liang, "Event-triggered adaptive fixed-time nn control for constrained nonstrict-feedback nonlinear systems with prescribed performance," *Neurocomputing*, vol. 422, pp. 332–344, 2021.