A Discrete Naked Mole Algorithm to Solve Traveling Salesman Problem Based on Multiple Local Dynamic Searching Strategies

Zhen-Long Zhao, Jie-Sheng Wang, Sha-Sha Guo, Zhong-Feng Li *, Ji-Sheng Yu, Ji Sun

Abstract—**Naked mole rat (NMR) algorithm is a swarm intelligence optimization algorithm by imitating the pair breeding behavior of naked mole rats. A discrete NMR algorithm was proposed based on multiple local dynamic searching strategies to solve traveling salesman problem (TSP) and multiple traveling salesman problem (MTSP). The discrete NMR algorithm to solve TSP adopts sequential coding and individual update strategies (worker stage and reproduction stage). In the worker update mechanism of NMR algorithm, three local dynamic search operators (2-opt operator, 3-opt operator and double bridge operator) were added to make the algorithm far from the local optimum when obtaining optimal path. The data sets in the TSPLIB library were used to carry out simulation experiments. Basic discrete NMR (DNMR), DNMR-2opt, DNMR-3opt and DNMR-double Bridges were used to obtain the optimum of single TSP and MTSP with three different situations. The experimental results show that the improved discrete NMR algorithm can approach the theoretical optimal value in a reasonable time and has strong robustness in solving single TSP and three MTSP.**

Index Terms—**discrete naked mole rat algorithm, local dynamic search, traveling salesman problem, multiple traveling salesman problem, optimization performance**

I. Introduction

THE traveling salesman problem (TSP) is a classic combination optimization problem with NP-complete characteristic [1], whose purpose of TSP is intend to minimize traveling distance of the salesmen. Many practical engineering problems can be can be modeled as TSP, such as vehicle routing problem, robot path planning, logistics distribution [2-4]. On the other hand, multiple travelers traverse multiple cities, and find the shortest path to traverse all cities on the premise that each city is passed by a traveler once [5]. This kind of problems is defined as multiple traveling salesman problems (MTSP) [6-7]. According to the theory of computational complexity, TSP is a typical NP-complete (NP-C or NPC) problem. NP-complete problems should be the set of decisive problems that are most unlikely to be reduced to P (polynomial time determinable). So far, no polynomial time algorithm has been found to solve this class of problems. Therefore, many swarm intelligent optimization algorithms have been adopted to tackle with TSP and MTSP with better optimization performance in computational and time complexity. A hybrid algorithm combining ant colony algorithm (HACO) particle swarm optimization (PSO) algorithm was proposed based on deletion strategy to solve TSP with fast convergence velocity of local search [8]. On the other hand, a 3-opt heuristic operator was added to improve the local solution [9]. An improved cyclic crossover operator was proposed to minimize the total distance so as to seek the solution of TSP that the genetic algorithm (GA) could not provide an accurate optimal solution when solving TSP [10]. The imperial competition algorithm and local strategy operator was combined to solve TSP [11]. An adaptive pheromone initialization mechanism and a searching mechanism based on the guidance optimization on ACO algorithm was proposed to find the optimum of TSP [12]. A biometric heuristic method was proposed based on sequential crossing and pollen discarding behavior to solve the circular TSP [13].

NMR algorithm was inspired by the naked mole rats' reproductive behavior [14]. It has the following four characteristics. (1) Naked mole rats live in groups of 295 members. (2) A female naked mole rat king led the herd and divided them into breeders and workers. The optimal naked mole rats are selected as breeders used only for mating, while worker naked moles carry out other works. (3) Workers carry out the necessary workings, the best of which will be replaced by keepers. In short, better workers change to breeders and worse workers are replaced into to work pool. (4) The best breeder in breeding pool will have the change to mate with queen. But the aforementioned four regulations are idea and NMR algorithm is proposed. NMR algorithm includes three stages. Firstly, the NMR algorithm is initialized the worker stage is proceeded. Then the breeder stage is carried out. The

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Zhen-Long Zhao is a senior engineer of School of Electrical Engineering, Yingkou Institute of Technology, Yingkou, 115014, P. R. China (e-mail: 308056442@qq.com).

Jie-Sheng Wang is a professor of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, 114051, P. R. China. (e-mail: wang_jiesheng@126.com).

Sha-Sha Guo is a postgraduate student of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, 114051, P. R. China (e-mail: gss420924@163.com).

Zhong-Feng Li is an associate professor of School of Electrical Engineering, Yingkou Institute of Technology, Yingkou, 115014, P. R. China (Corresponding author, phone: 86-0412-2538355; fax: 86-0412-2538244; e-mail: afeng0601@163.com).

Ji-Sheng Yu is an associate professor of School of Electrical Engineering, Yingkou Institute of Technology, Yingkou, 115014, P. R. China (e-mail: 121388969@qq.com).

Ji Sun is a senior engineer of School of Electrical Engineering, Yingkou Institute of Technology, Yingkou, 115014, P. R. China (e-mail: 5122585@qq.com).

breeding stage is realized according to the probability of breeding stage. Thus, a discrete NMR algorithm by adopting multiple local dynamic searching strategies was proposed to deal with the single TSP. Simulation experiments are carried out in order to show the capability of the proposed discrete NMR method to solve TSP and three MTSP.

II. Mathematical Model of TSP

TSP is a representative combination problem and NP complete problem [15]. The classic TSP can be represented as follows. n cities are represented with number i , and the distance $d_{ij} \geq 0$ between city i and j is represented by real values, where $i, j \in \{1, n\}$. In addition to a given city, it is needed to traverse the remaining $n-1$ cities, each city is visited once so as to reach the given city to minimize the total distance traveled. Assuming that $x_1, x_2, \cdots, x_m$ is a feasible path, Eq.(1) is the total length of the path.

$$L = \sum_{i=1}^{m-1} d(x_i, x_{i+1}) + d(x_m, x_1) \quad (1)$$

where, $d(x_i, x_{i+1})$ is the distance between x_i and x_{i+1}, and L is the shortest distance of the path.

MTSP is an extension of TSP in which more than one salesmen are on hand to visit the city, but each city can only be visited once by one salesman. Given a set of n cities that a salesman will visit, TSP looks for the shortest possible trip for that salesman, which is to visit each city only once. MTSP can be briefly described as follows. Give an undirected graph $G=(V, A)$, which is an ordered pair $G=(V, A)$ consisting of a set of vertices V and a set of arcs A , where m represents the total number of salesmen. The goal is to divide V into m non-empty subsets $\{S_i\}_{i=1}^{m}$ and find the minimum cost path of each vertex through each subset S_i . The objective function of MTSP can be described is as follows [5].

$$Minimize \sum_{i=1}^{m} \left(x_{n^i,1}^{i} + \sum_{j=1}^{n^i-1} x_{j,j+1}^{i} \right) \quad (2)$$

where, the first part represents the paths through m salesmen, and the second part represents the cycle of all cities visited by the i -th salesman (the index of the first city visited by the i -th salesman is 1, and the index of the last city is n^i); $x_{j,j+1}^{i}$ represents the distance between the j -th city visited by the i -th salesman and $j+1$; $x_{n^i,1}^{i}$ represents the distance between the last city n^i visited by the i -th salesman and the first city; The value of n^i should not be less than the minimum number of cities specified by each salesman; $x_{j,j+1}^{i}$ is equal to $x_{j+1,j}^{i}$.

Multiple TSP can be divided into different conditions based on the difference of the departure and destination of the traveling salesmen. For the MTSP with different conditions, its objective function can be expressed by Eq. (6), that is to obtain the minimum value of the sum of all travelling salesman' paths. The effectiveness of the proposed improved algorithm in solving the following three MTSP is verified in the simulation experiments.

(1) MTSP with the starting point and the destination point (MTSP). Each travelling salesman has its own starting point and destination, in which each travelling salesman destination is its own starting point.

(2) MTSP with the same starting point and back to the same starting point (MTSP1). Each travelling salesman starts from the same starting point and returns to the starting point (starting point = end point).

(3) MTSP with the same starting point and the same destination point (MTSP2). Each travelling salesman has the same starting point and ending point (starting point ≠ ending point).

III. NMR Algorithm

A. Initialization

$$NMR_{i,j} = NMR_{\min,j} + U(0,1) \times (NMR_{\min,j} - NMR_{\max,j}) \quad (3)$$

where, $i \in [1, 2, \cdots n]$, $j \in [1, 2, \cdots D]$ and $NMR_{i,j}$ are the positions of the i -th individual on the j -th dimension; $NMR_{\min,j}$ and $NMR_{\max,j}$ represent the upper and lower boundary of the discussed problem; $U(0,1)$ is a uniformly distributed random number. Then, the fitness value of each individual is calculated based on the designed objective function. According to the fitness values, determine B breeders and W workers, then the global optimum d was obtained. Thus, the discrete NMR algorithm is repeatedly cycled or iterated through the searching process for workers and the multiplication stage.

B. Workers Stage

At this stage, the naked mole rat workers try to increase their physical fitness in order to make them have the opportunity to change into breeders and eventually have the chance to mate with the naked mole rat king. Thus, NMR workers' new solutions are generated according to their own experience and local information. Then evaluate the fitness values of the generated NMR individuals. If the new individual has better fitness, discard the old individual and remember the new individual. If not, the old individual will be retained. When all the workers completed the search, their final fitness was remembered. To obtain new solutions from old ones, NMR uses the following strategy.

$$w_i^{t+1} = w_i^t + \lambda(w_j^t - w_k^t) \quad (4)$$

where, w_i^t is the i -th worker in the t iteration; w_i^{t+1} is the new individual or worker; λ is the mating factor; w_j^t and w_k^t are two solutions randomly selected from the pool of workers; The value of λ is uniformly distributed in [0,1].

C. Reproductive Stage

The propagator NMR will also self renew so that choose to mate as a propagator. The NMR breeder updating strategy is based on the propagation probability (bp) relative to the holistic best d . The bp random number is in the range of [0,1]. Some breeders may not be able to update their health status and therefore may be returned to the worker category. Use the following rules to update the position of the breeder.

$$b_i^{t+1} = (1-\lambda)b_i^t + \lambda(d - b_i^t) \quad (5)$$

where, b_i^t is the i-th breeder in the t-th iteration; λ is the breeding frequency factor to control the breeder, which is used to determine the new breeder b_i^{t+1} in the next iteration, and the initial value of bp is set to 0.5.

As a matter of convenience, assume that there is only one naked mole king, and the best naked mole rat among breeders mates with the naked mole king, that is to say that only the male and female with the best reproductive ability are found to mate. The working principle of this algorithm is to distinguish or identify multipliers and workers in the NMR cell. After preliminary evaluation, the best breeders and workers are selected. The health status of worker naked mole rats has been updated, thereby improving their health and giving them an opportunity to become breeders. The other side of the shield, breeders also update their fitness based on the reproduction probability, so they are still breeders, and infertile breeders will be classified as workers.

IV. Discrete Naked Mole Rat Algorithm to Solve Traveling Salesman Problem

A. Coding Method and Fitness Function

TSP is often represented by natural numbers. So in this article, the sequential encoding method is adopted. If the number of cities is m, then a full array of cities ranging from 1 to m is used as a traveling salesman's path. For example, the number of cities is 5, the randomly generated array is [1, 3, 5, 2, 4], the traveling salesman's path is 1-3-5-2-4, and Eq. (1) was adopted to calculate its fitness function value. That is to say the smaller L, the better the fitness value.

B. Discrete NMR Algorithm to Solve Traveling Salesman Problem

The original naked mole rat algorithm was designed to optimize the continuous problems. Therefore, so as to seek the optimum of TSP, a discrete naked mole rat algorithm was introduced. In the discrete naked mole rat algorithm, the update strategy of its worker phase and reproduction phase are mainly changed. For the workers stage, the executions on path pieces (flip, switch, and move operations) are carried out to update individuals. Suppose $s=4$ and $e=6$, where s and e represent the fragments of the initial position and ending position, then S_1 is selected, and flip, switch and move on S_1 to carry out specific update process, which is shown in Fig. 1. Secondly, for the propagation stage and $U(0,1)>bp$, two points are randomly selected in the path for exchange. Otherwise, a full array randomly generated by a city ranging from 1 to m will be generated again as a route for travelling agents so as to avoid getting stuck at the local optimal. So as to further improve its performance in advance, three local search operators (2-opt operator, 3-opt operator and double bridge) are respectively added to the worker NMR updating mechanism.

C. Local Dynamic Searching Strategies (2-opt Operator, 3-opt Operator and Double-bridge Operator)

The 2-opt operator was a typical local searching strategy to find the optimum of TSP [16]. The 2-opt operator randomly deletes two edges in generated path, then reconnects them with newly produced paths. Only when the reconnected edge is shorter than the old edge, the reconnected edge is effective. This situation will continue until further improvements are possible. 3-opt operator works in a similar way, but it removes three edges instead of two [17]. By randomly selecting three edges in the path S, three path fragments S_1, S_2 and S_3 are obtained. So there are two kinds of re-connection, which is shown in Fig. 2. Assuming that the edges <c, d>, <e, f> and <a, b> are removed, then the edges <a, d>, <e, b> and <c, f> are reconnected or the edges <a, c> ,<b, e> and <d, f> reconnect. For the former, S_2 and S_3 are opposite; for the latter, in fact, S_2 and S_3 are reversible. The reconnected path must be better than the original path before the original path can be replaced.

Double bridge is a special 4-opt local search algorithm [18], and the example of double bridge movement is shown in Fig. 3 [19]. It can change the four sides of the loop to the other four sides, and can disturb the original loop, so as to avoid the search process falling into local optimal. By randomly selecting five edges in the path S, it can obtain five path fragments S_1, S_2, S_3, S_4 and S_5, where S_1 represents the reverse order of S_1.

D. Algorithm Procedure

The flowchart of discrete naked mole rat algorithm to solve TSP can be described as follows.

Step 1: Set up the initialization parameters. Initialize the population size n, the reproduction NMR number $B=n/5$, the worker NMR number $W=n-B$, and the reproduction probability bp.

Step 2: Initialize the population with positive integers. A random array of cities ranging from 1 to m is used as an NMR individual, where m is the number of cities. Other NMR individuals also reproduce with the same way.

Step 3: Obtain fitness function value for each solution.

Step 4: Use the following rules to update the reproduction NMR. Take any two points on the path, perform flip, switch and move operations in the segment to obtain a new path.

Step 5: Use the following rules to update the worker NMR. If $U(0,1)>bp$, randomly select two points in the path to exchange, otherwise follow Step 2 to reproduce.

The pseudo-code of the discrete NMR algorithm for solving TSP is described as follows.

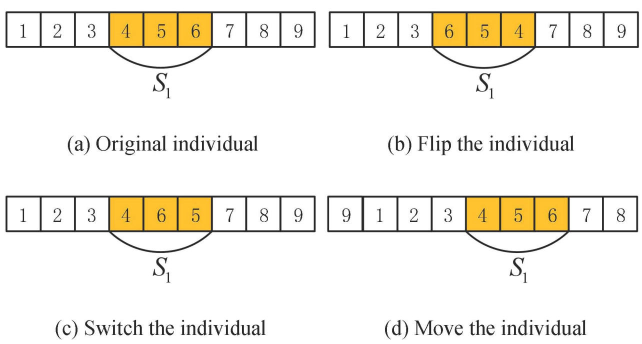


Fig. 1 Flip, switch, and move operations.

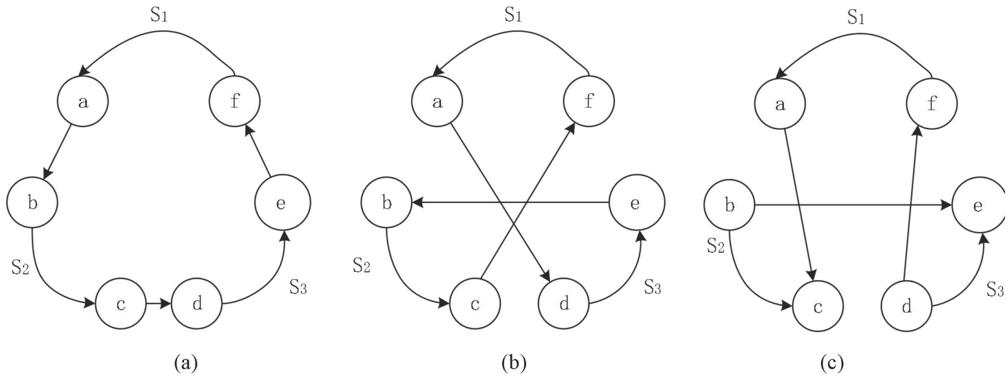


Fig. 2 Schematic diagram of 3-opt operator.

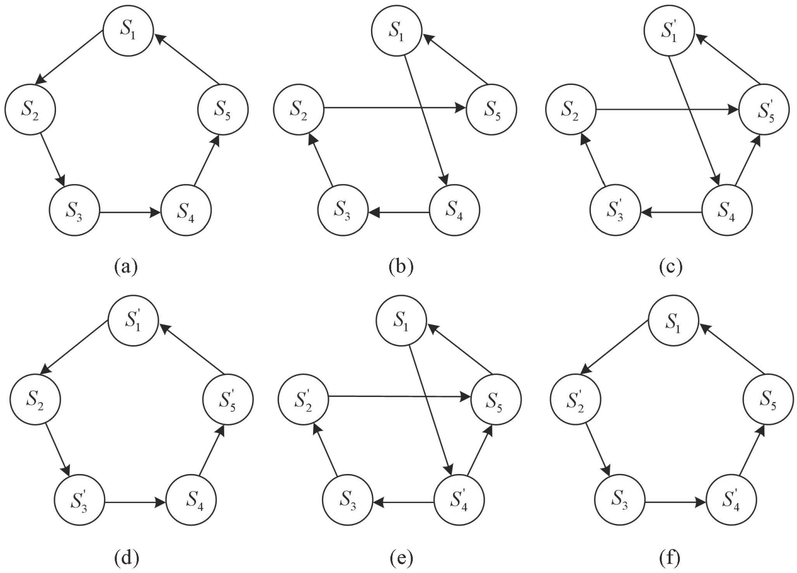


Fig. 3 Principle of double-bridge operator.

Begin:
  Inputs: Number of naked mole rats: n
    Number of cities to visit: N
    Breeders B : $n/5$
    Workers W : $n-B$
    Define breeding probability: bp
    Maximum number of iterations: MaxTter
  Use a randomly generated array ranging from 1 to N as NMR;
  $X(i,:) = randperm(N)$;
  Calculate the distance of the path formed by each individual, which is the fitness value $f(X(i,:))$, and sort them from the largest to the smallest;
  The first $n/5$ of the fitness value is defined as the breeders, followed by the $n-B$ as the workers;
  Output: The overall best MinDist;
  Do Until iter < MaxTter
    For $i=1:B$
      Take any two points of the path to perform the operation of flipping, switching and shifting inside the fragment to get a new path;
     Save the new path;
   End
   For $i=B+1:n$
     If $U(0,1) > bp$
      Two points in the path are randomly selected for exchange or 2-opt, 3-opt, and double Bridges;
    End for
    $X(i,:) = randperm(N)$
    Save the new path
  End
  Calculate the new fitness value
  Turnover MinDist
  Turnover iter
  End until
  Output the optimum (MinDist)
End

V. Experiments Results and Analysis

The cases in simulation experiments are all from the TSPLIB library, and the optimal values provided in the TSPLIB library are rounded to an integer. The basic discrete NMR algorithm (NMR), 2opt-based NMR algorithm (NMR-2opt), 3-opt-based NMR algorithm (NMR-3opt), and double bridge-based NMR algorithm (NMR-double bridges) are adopted in the simulation experiments to solve traveling salesman problems. Each test case in the simulation experiment is independently run 10 times. In order to compare with other algorithms, the optimization result retains two decimal numbers.

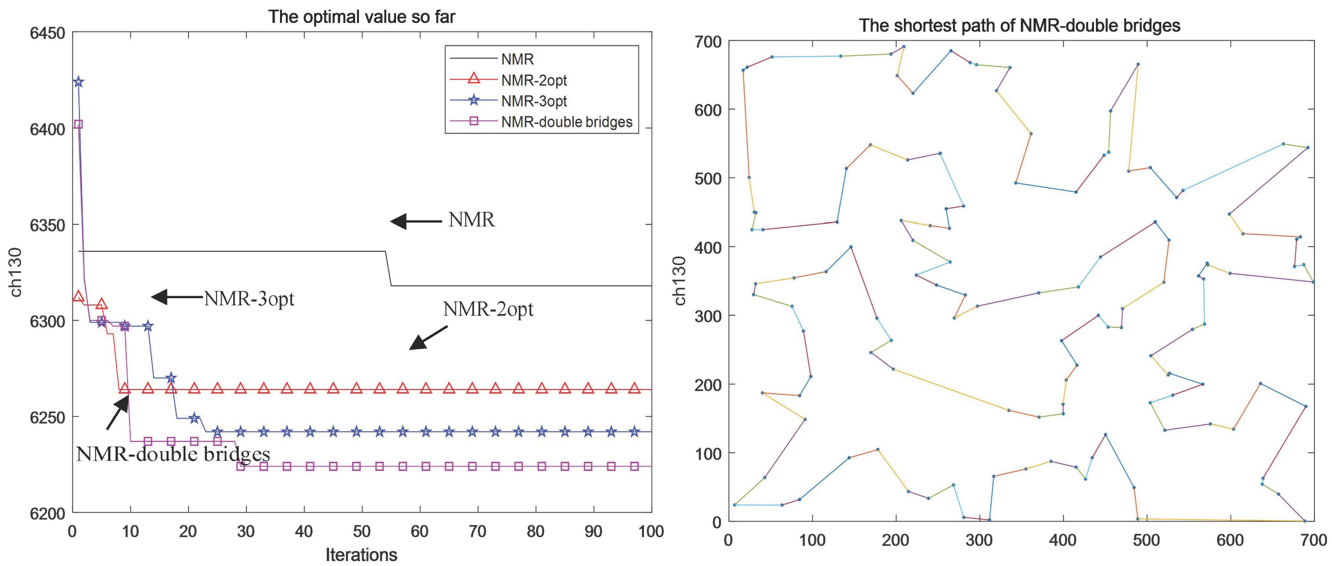
A. Discrete NMR Algorithm to Solve Single TSP

The 14 test sets in the TSPLIB library are selected for the simulation experiments on the discrete naked mole rat algorithm to solve typical TSP. The obtained experimental data and convergence curves are shown in Table 1 and Fig. 4. In addition, so as to further show the effectiveness of the algorithm, the compared results with other swarm intelligent optimization algorithms are listed in Table 2, such as a neuron-immune network [20], a self-organizing neural network based on immune system [21], an improved ant colony optimization [22], an ant colony optimization algorithm with 2-opt strategy [3], a hybrid ACO and delete-cross method [8], a discrete spider monkey optimization [23] and a swap sequence based artificial bee colony algorithm [24]. According to the simulation results listed in Table 1, the proposed discrete NMR method by adding the local searching operators has different degrees of improvement compared with the original NMR algorithm. For example, the deviation of att48 and berlin52 is 0, indicating that NMR-3opt and NMR-double bridges can obtain the best results and have good robustness. At the same time, it can be seen that in most cases, the optimal values are ranked from good to bad as NMR-double Bridges, NMR-3opt, NMR-2opt, NMR, that is to say, the local search ability can be enhanced to achieve better optimization effect.

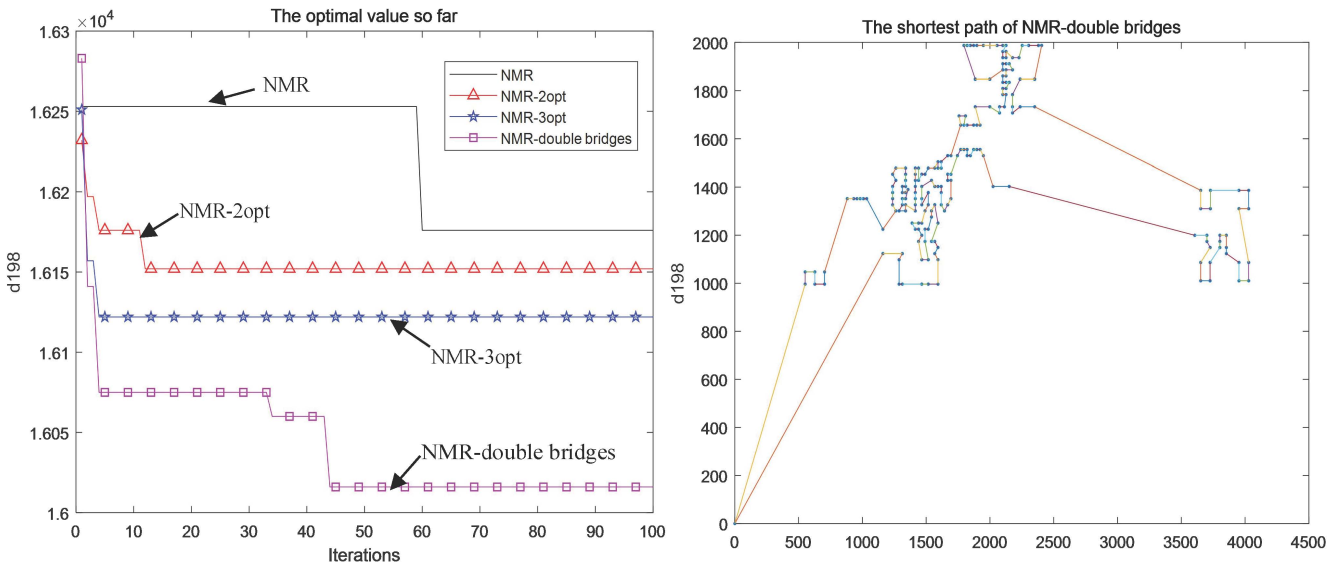
TABLE 1. OPTIMIZATION RESULTS OF SINGLE TRAVELING SALESMAN PROBLEMS

	Algorithm	Best	Worst	Ave	Std
att48	NMR	33564.00	33831.00	33655.00	91.29
	NMR-2opt	33522.00	33606.00	33568.40	32.85
	NMR-3opt	33522.00	33522.00	**33522.00**	0.00
	NMR-double bridges	33522.00	33522.00	**33522.00**	0.00
berlin52	NMR	7594.00	7881.00	7739.60	118.55
	NMR-2opt	7618.00	7724.00	7648.80	40.24
	NMR-3opt	7542.00	7542.00	**7542.00**	0.00
	NMR-double bridges	7542.00	7542.00	**7542.00**	0.00
ch130	NMR	6315.00	6469.00	6360.00	59.13
	NMR-2opt	6264.00	6326.00	6287.20	22.25
	NMR-3opt	6193.00	6242.00	6218.20	16.76
	NMR-double bridges	6141.00	6224.00	**6191.60**	28.10
ch150	NMR	6776.00	6988.00	6882.80	67.35
	NMR-2opt	6685.00	6786.00	6746.00	35.22
	NMR-3opt	6623.00	6680.00	6648.20	18.40
	NMR-double bridges	6585.00	6663.00	**6633.00**	31.81
d198	NMR	16124.00	16261.00	16176.40	46.02
	NMR-2opt	15991.00	16152.00	16075.00	57.74
	NMR-3opt	15959.00	16044.00	16013.00	29.56
	NMR-double bridges	15995.00	16030.00	**16011.40**	13.76
eil51	NMR	433.00	438.00	434.00	2.00
	NMR-2opt	429.00	433.00	431.40	1.62
	NMR-3opt	427.00	429.00	428.00	0.89
	NMR-double bridges	426.00	428.00	**427.00**	0.63
eil76	NMR	551.00	566.00	559.20	6.14
	NMR-2opt	540.00	555.00	549.00	5.18
	NMR-3opt	545.00	549.00	547.20	1.47
	NMR-double bridges	543.00	548.00	**546.00**	1.90
eil101	NMR	657.00	669.00	663.20	4.07
	NMR-2opt	645.00	658.00	653.60	4.63
	NMR-3opt	638.00	655.00	648.40	5.64
	NMR-double bridges	645.00	649.00	**647.00**	1.41
kroA100	NMR	21550.00	22056.00	21810.80	167.92
	NMR-2opt	21379.00	21696.00	21563.00	113.15
	NMR-3opt	21346.00	21388.00	21371.20	17.08
	NMR-double bridges	21282.00	21408.00	**21330.80**	48.63
kroA200	NMR	21677.00	30951.00	27136.40	4434.35

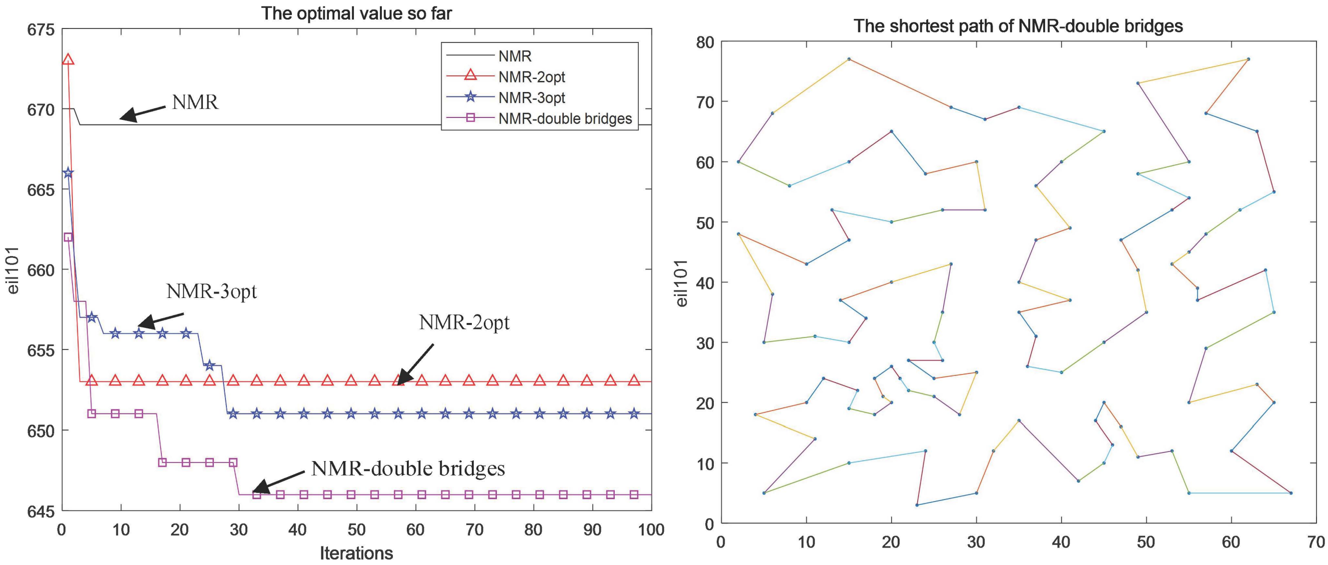
	NMR-2opt	21533.00	30710.00	27018.20	4399.03
	NMR-3opt	21292.00	30576.00	26750.80	4349.27
	NMR-double bridges	21346.00	30072.00	**26531.60**	4227.65
kroB200	NMR	31113.00	31496.00	31362.60	135.59
	NMR-2opt	30777.00	30962.00	30856.80	75.60
	NMR-3opt	30332.00	30766.00	30481.60	158.56
	NMR-double bridges	30200.00	30503.00	**30336.80**	104.59
pr76	NMR	108761.00	110784.00	109921.20	683.19
	NMR-2opt	108351.00	109480.00	108778.40	479.67
	NMR-3opt	108308.00	108593.00	108468.60	107.92
	NMR-double bridges	108202.00	108308.00	**108286.00**	42.03
pr107	NMR	44949.00	45670.00	45254.40	198.01
	NMR-2opt	44653.00	45295.00	45046.70	196.31
	NMR-3opt	44515.00	44852.00	44723.30	95.16
	NMR-double bridges	44402.00	44708.00	**44527.50**	97.06
pr226	NMR	80968.00	81982.00	81558.20	298.55
	NMR-2opt	80604.00	81406.00	81010.30	205.32
	NMR-3opt	80491.00	81123.00	80766.40	162.66
	NMR-double bridges	80405.00	80575.00	**80509.70**	56.68



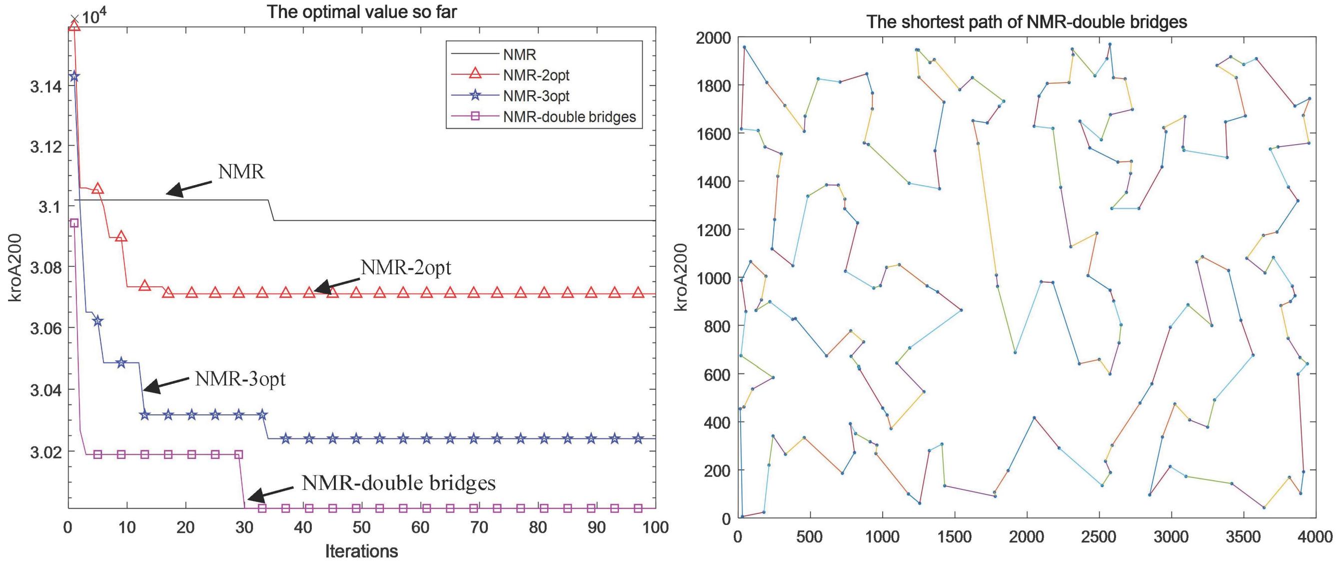
(a) ch130



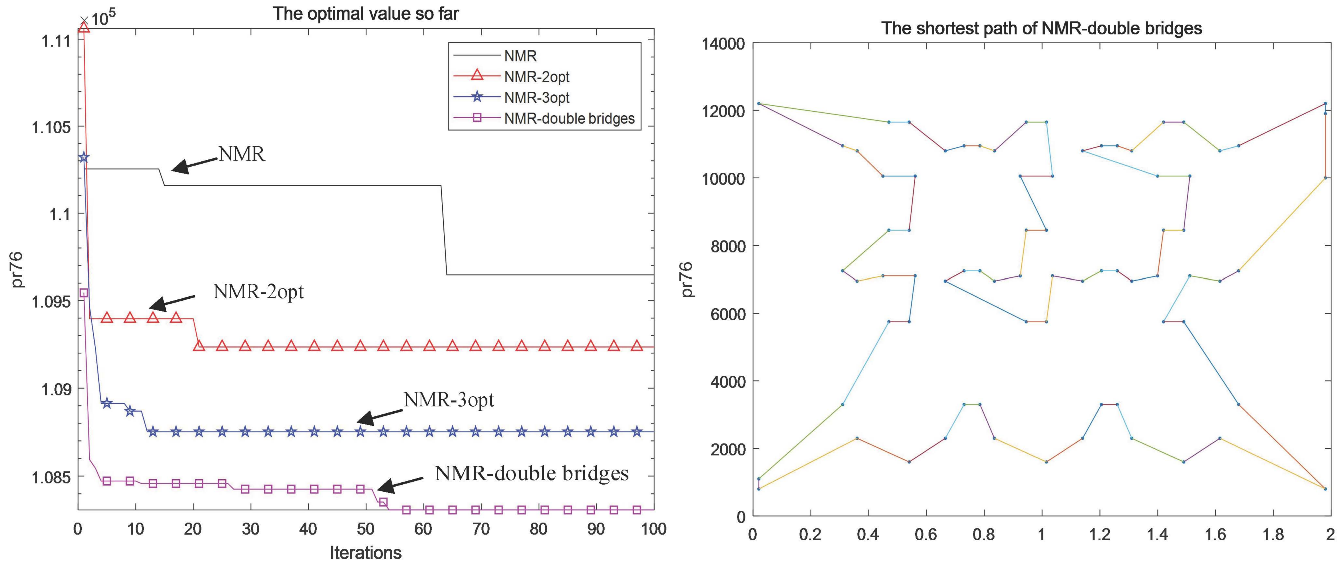
(b) d198



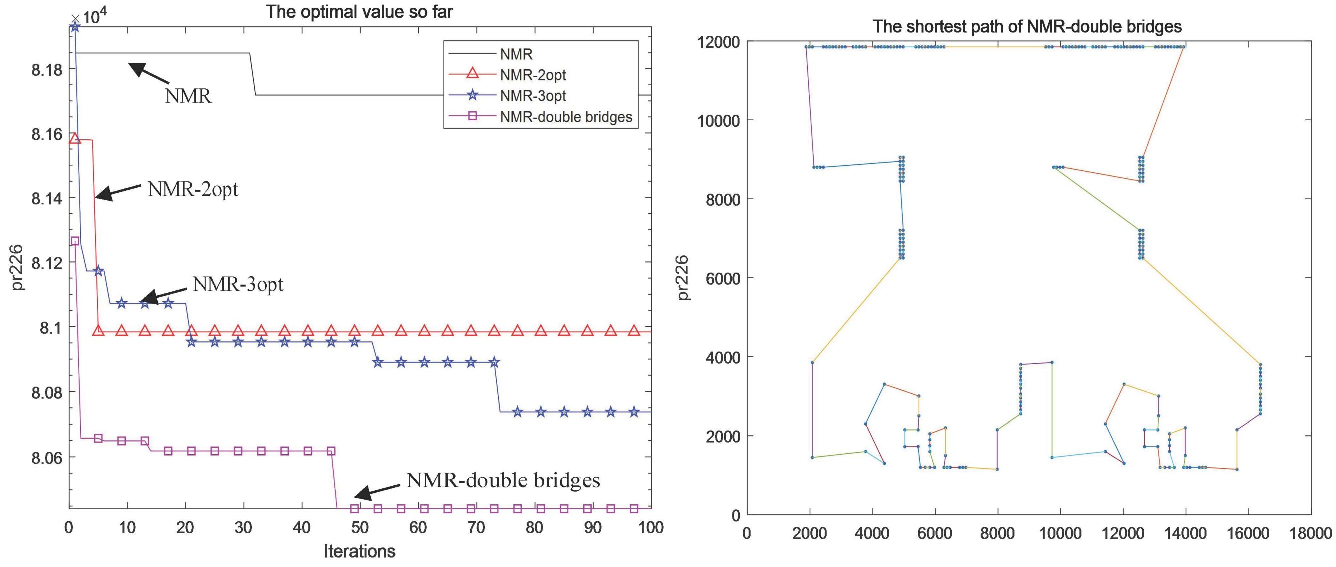
(c) eil101



(d) kroA200



(e) pr76



(f) pr226

Fig. 4 Paths and convergence curves of single traveling salesman problems.

TABLE 2. COMPARISON WITH OTHER OPTIMIZATION ALGORITHMS ON SINGLE TRAVELING SALESMAN PROBLEMS

Algorithm		berlin52	ch150	eil51	eil76	eil101
NMR	Ave	7739.60	6882.80	434.00	559.20	663.20
	Std	118.55	67.35	2.00	6.14	4.07
NMR-2opt	Ave	7648.80	6746.00	431.40	549.00	653.60
	Std	40.24	35.22	1.62	5.18	4.63
NMR-3opt	Ave	**7542.00**	6648.20	428.00	547.20	648.40
	Std	0.00	18.40	0.89	1.47	5.64
NMR-double bridges	Ave	**7542.00**	**6633.00**	**427.00**	546.00	647.00
	Std	0.00	31.81	0.63	1.90	1.41
RABNET−TSP [20]	Ave	8073.97	6753.20	438.70	556.10	654.83
	Std	270.14	83.01	3.52	8.03	6.57
Modified RABNET-TSP [21]	Ave	7932.50	6738.37	437.47	556.33	648.63
	Std	277.25	76.14	4.20	5.30	3.85
IVRS + 2opt [22]	Ave	7547.23	—	431.10	—	648.67
	Std	—	—	—	—	—
ACO + 2opt [3]	Ave	7556.58		439.25	—	672.37
	Std		—	—	—	—
HACO [8]	Ave	7560.54	—	431.20	—	—
	Std	67.48	—	2.00	—	—
DSMO [23]	Ave	7633.6	—	436.96	—	662.63
	Std	85.4	—	4.73	—	7.13
ABC+ Swap Sequence [24]	Ave	7543.00	—	427.01	**538.15**	**630.59**
	Std	0.00	—	0.46	0.60	2.37

Fig. 4 reflects the path diagram, convergence speed and accuracy of the algorithm when solving different problems. It can be seen from Fig. 4 that after adding the local searching operators, the convergence rate has decreased, but it is within an acceptable range. It can be seen form Table 2, NMR-double bridges get better results than other algorithms in most cases. For EIL51, Berlin52 and CH150 test sets, the optimization performance of NMR-double bridges algorithm is better than other algorithms. In addition, according to std, NMR-double bridges algorithm has the highest stability and better robustness.

B. *Discrete NMR Algorithm to Solve Multiple TSP*

The simulation experiments are carried out on the MTSP with three different situations, including multiple starting points and multiple terminal points (MTSP), MTSP with the same starting point and returning to the starting point (MTSP1), and MTSP with the same starting point and returning to the same destination (MTSP2). In the simulation experiment, because part of the data requires the traveling salesmen to go to a large number of cities or is affected by the distribution of cities, the paths between traveling salesmen inevitably overlap and cannot reflect the specific paths of traveling salesmen well. Therefore, for some data sets, only the shortest distance convergence curves of the path taken by the traveling salesman are displayed.

(1) Multiple TSP with Multiple Starting Points and Multiple Destinations (MTSP)

Ten test sets in the TSPLIB library are selected to perform the simulation experiments by adopting the discrete naked mole rat algorithm to solve MTSP. The results are shown in Table 3. Due to the influence of the number or distribution of cities, the test results of the gil262 and kroA200 datasets, we only provide the convergence curves. It can be seen from Table 3 that NMR-2opt, NMR-3opt and NMR-double bridges all have different degrees of improvement in the optimization accuracy, especially NMR-double bridges show the best effect in most cases. In particular, for att48, gil262, bier127 and pr226, ave data shows that NMR-double bridges have better optimization effects than other algorithms. At the same time, it can be known from std data that its std value is smaller than other algorithms, proving that it has better stability. For att48, eil101, gil262, kroA200, pr107, pr226 and rat99, the NMR-double bridges algorithm can be further optimized in the later iterations, so it has strong global optimization capabilities and is not easy to fall into local optimal. Although it has affected its convergence speed, it is within an acceptable range.

(2) MTSP with Same Starting Point and Back to Starting Point (MTSP1)

Ten test sets in the TSPLIB library are selected to perform the simulation experiments by using the discrete NMR algorithm to solve MTSP1. The results are shown in Table 4. Due to the influence of the number or distribution of cities, the test results of the gil262, pcb442, and pr136 data sets, we only provide the convergence curves. From the ave data in Table 4, it can be seen that for different test data, NMR-double bridges have different degrees of improvement compared with other algorithms. In particular, the test results of d198 and gil262 show that NMR-double bridges are better than the original NMR, about an improvement of 4.3% and 3.8%. It can be seen from the std data in Table 3 that in most cases, the variance of NMR-double bridges is smaller than other algorithms, indicating that the improved algorithm has good global convergence capabilities. This is because the algorithm enhances the local searching ability and can better prevent the algorithm from falling into the local optimal and premature.

(3) MTSP with Same Starting Point and Back to Same Destination (MTSP2)

The 14 test sets in the TSPLIB library are selected to perform the simulation experiments on the discrete NMR algorithm to solve MTSP2. The results are listed in Table 5. In Table 5, the ave column data shows the average value of four algorithms solved by running 10 tests, and the bold data represents the optimization result with the best effect in the experiments. It can be seen from the data in this column that NMR-double bridges, NMR-3opt, and NMR-2opt have better optimization effects than the original NMR algorithm. In particular, NMR-double bridges showed the best effect, as can be seen in the test results of att48, bier127, ch150, d198, eil51 and pr76, the improvement effect is the most obvious. The algorithm always searches near the optimal solution, which ensures that the convergence speed and the retention of high-quality populations are accelerated in the later stage.

TABLE 3. OPTIMIZATION RESULTS OF MTSP

	Algorithm	Best	Worst	Ave	Std
att48	NMR	37921.00	41649.00	39874.00	1499.21
	NMR-2opt	35673.00	37687.00	36744.00	1871.19
	NMR-3opt	33897.00	38968.00	36141.00	1670.01
	NMR-double bridges	35254.00	38260.00	**35994.60**	1143.87
bier127	NMR	161480.00	161480.00	170929.80	6064.13
	NMR-2opt	143452.00	143452.00	159005.60	9428.50
	NMR-3opt	147888.00	147888.00	158534.80	7065.91
	NMR-double bridges	150046.00	150046.00	**157223.00**	5090.00
eil51	NMR	497.00	536.00	510.80	15.26
	NMR-2opt	486.00	536.00	498.20	19.05
	NMR-3opt	470.00	488.00	**479.40**	5.71
	NMR-double bridges	465.00	495.00	480.40	9.71
eil101	NMR	832.00	883.00	853.00	17.81
	NMR-2opt	774.00	835.00	811.60	20.66
	NMR-3opt	775.00	824.00	801.00	19.46
	NMR-double bridges	723.00	841.00	**792.00**	39.96
gil262	NMR	5298.00	5446.00	5353.20	152.64

	NMR-2opt	4848.00	5265.00	5096.00	148.13
	NMR-3opt	5008.00	5398.00	5184.00	134.47
	NMR-double bridges	4867.00	5206.00	**5045.60**	125.32
kroA200	NMR	57821.00	63826.00	61565.80	2282.53
	NMR-2opt	53359.00	60592.00	56522.60	2447.38
	NMR-3opt	54407.00	56729.00	55934.40	837.74
	NMR-double bridges	51195.00	59691.00	**55679.60**	3333.03
pr76	NMR	132180.00	142760.00	136225.40	3865.62
	NMR-2opt	125001.00	133655.00	129701.00	2830.33
	NMR-3opt	123213.00	134077.00	**129424.80**	4223.49
	NMR-double bridges	126710.00	132933.00	129611.80	2371.82
pr107	NMR	37247.00	49341.00	39843.60	4752.73
	NMR-2opt	36890.00	38719.00	37577.60	634.33
	NMR-3opt	35440.00	36448.00	**35969.60**	421.55
	NMR-double bridges	36267.00	38719.00	37074.40	865.69
pr226	NMR	101317.00	109644.00	104526.60	2829.94
	NMR-2opt	90722.00	103773.00	96333.80	4677.49
	NMR-3opt	89014.00	100310.00	95084.80	3873.97
	NMR-double bridges	90722.00	96297.00	**93279.60**	2202.21
rat99	NMR	1407.00	1466.00	1443.25	24.30
	NMR-2opt	1333.00	1400.00	1364.50	27.37
	NMR-3opt	1385.00	1421.00	1400.25	14.75
	NMR-double bridges	1264.00	1435.00	**1349.75**	69.81

TABLE 4. OPTIMIZATION RESULTS OF MTSP1

	Algorithm	Best	Worst	Ave	Std
att48	NMR	46087.00	53027.00	48951.00	2381.68
	NMR-2opt	44531.00	47810.00	45870.20	1283.91
	NMR-3opt	43442.00	49971.00	45634.00	2274.68
	NMR-double bridges	43795.00	47643.00	**45523.20**	1276.86
d198	NMR	31249.00	32476.00	31688.67	537.79
	NMR-2opt	29379.00	32217.00	30689.83	998.34
	NMR-3opt	29274.00	31193.00	30502.50	631.64
	NMR-double bridges	29380.00	31240.00	**30381.67**	682.13
eil51	NMR	535.00	571.00	553.20	11.46
	NMR-2opt	524.00	569.00	537.40	16.50
	NMR-3opt	528.00	546.00	536.40	6.02
	NMR-double bridges	516.00	548.00	**531.20**	10.65
eil101	NMR	855.00	871.00	863.40	25.61
	NMR-2opt	785.00	876.00	820.80	32.76
	NMR-3opt	796.00	853.00	819.00	22.91
	NMR-double bridges	790.00	852.00	**818.80**	22.60
gil262	NMR	5219.00	5623.00	5377.50	145.06
	NMR-2opt	5012.00	5356.00	5186.67	136.56
	NMR-3opt	4848.00	5569.00	5198.17	235.14
	NMR-double bridges	5004.00	5373.00	**5179.83**	120.23
pcb442	NMR	109868.00	124680.00	117858.60	5280.39
	NMR-2opt	106985.00	116717.00	112921.40	3922.34
	NMR-3opt	105581.00	114462.00	110390.00	2843.67
	NMR-double bridges	107627.00	113958.00	**110356.60**	2133.27
pr76	NMR	164063.00	176168.00	170114.80	4676.60

	NMR-2opt	152317.00	166704.00	159292.40	4648.85
	NMR-3opt	151709.00	171566.00	159275.40	7340.21
	NMR-double bridges	149273.00	168732.00	**159024.80**	4618.74
pr107	NMR	59232.00	62065.00	60286.80	1482.46
	NMR-2opt	56238.00	60880.00	58802.80	1743.64
	NMR-3opt	51670.00	60116.00	57202.40	3116.47
	NMR-double bridges	54778.00	58819.00	**56812.20**	1327.66
pr136	NMR	145852.00	152362.00	148580.20	2154.34
	NMR-2opt	136974.00	148996.00	142188.40	4817.82
	NMR-3opt	138582.00	146689.00	143289.60	2978.97
	NMR-double bridges	137562.00	145432.00	**141964.00**	2844.29
rat99	NMR	1680.00	1867.00	1783.80	75.22
	NMR-2opt	1615.00	1801.00	1689.00	61.29
	NMR-3opt	1609.00	1859.00	1694.40	88.91
	NMR-double bridges	1639.00	1745.00	**1682.60**	38.62

TABLE 5. OPTIMIZATION RESULTS OF MTSP2

	Algorithm	Best	Worst	Ave	Std
att48	NMR	49885.00	54803.00	52009.00	2048.20
	NMR-2opt	45812.00	51465.00	48922.20	2116.96
	NMR-3opt	46238.00	50587.00	48498.80	1441.18
	NMR-double bridges	44427.00	49365.00	**47685.80**	1278.78
bier127	NMR	177707.00	190656.00	182874.67	3670.89
	NMR-2opt	168250.00	182316.00	176842.78	5558.82
	NMR-3opt	161275.00	180202.00	170087.89	8053.89
	NMR-double bridges	162853.00	177527.00	**168929.67**	4823.28
ch130	NMR	9769.00	10352.00	9979.78	199.03
	NMR-2opt	8702.00	10061.00	9424.44	418.09
	NMR-3opt	8747.00	9725.00	9269.78	341.52
	NMR-double bridges	8835.00	9748.00	**9267.44**	305.92
ch150	NMR	11446.00	12248.00	11951.50	274.41
	NMR-2opt	10812.00	11716.00	11177.60	315.17
	NMR-3opt	10418.00	12094.00	11225.20	408.18
	NMR-double bridges	10623.00	11910.00	**11144.50**	195.11
d198	NMR	31500.00	31850.00	31721.20	99.07
	NMR-2opt	30862.00	31384.00	31111.40	205.35
	NMR-3opt	30841.00	31662.00	31246.40	217.93
	NMR-double bridges	30648.00	31193.00	**30969.20**	187.75
eil76	NMR	747.00	827.00	773.40	29.92
	NMR-2opt	706.00	742.00	726.40	13.53
	NMR-3opt	706.00	755.00	724.80	17.41
	NMR-double bridges	693.00	752.00	**713.40**	21.33
eil51	NMR	576.00	616.00	588.80	14.99
	NMR-2opt	519.00	574.00	544.60	19.06
	NMR-3opt	535.00	584.00	555.40	17.00
	NMR-double bridges	517.00	541.00	**528.20**	10.63
eil101	NMR	884.00	922.00	902.80	14.08
	NMR-2opt	840.00	879.00	858.60	14.35
	NMR-3opt	827.00	889.00	855.60	26.74
	NMR-double bridges	805.00	892.00	**844.60**	29.60
gil262	NMR	5367.00	5551.00	5458.20	64.02

	NMR-2opt	5094.00	5442.00	5266.00	128.10
	NMR-3opt	5116.00	5437.00	5279.40	120.21
	NMR-double bridges	4891.00	5340.00	**5209.00**	176.09
kroA200	NMR	61361.00	64515.00	62824.80	1008.17
	NMR-2opt	57854.00	64432.00	60510.60	2308.73
	NMR-3opt	58059.00	61629.00	59719.80	1212.80
	NMR-double bridges	58307.00	61220.00	**59700.00**	994.60
pr76	NMR	168074.00	190497.00	178658.80	8513.72
	NMR-2opt	160196.00	184772.00	169425.00	8515.08
	NMR-3opt	158196.00	178085.00	169309.60	6803.24
	NMR-double bridges	161418.00	170943.00	**166157.00**	3907.98
pr107	NMR	78067.00	82048.00	79828.00	1324.13
	NMR-2opt	78271.00	79643.00	78756.20	519.64
	NMR-3opt	77806.00	79643.00	78498.40	654.91
	NMR-double bridges	77321.00	79856.00	**78412.20**	875.52
pr226	NMR	142143.00	146907.00	144733.20	1835.72
	NMR-2opt	137708.00	145686.00	140478.00	2722.98
	NMR-3opt	136018.00	142476.00	139583.00	2466.59
	NMR-double bridges	135109.00	144932.00	**139040.00**	3861.87
rat99	NMR	2016.00	2089.00	2064.00	25.37
	NMR-2opt	2013.00	2041.00	2028.40	11.16
	NMR-3opt	1971.00	2062.00	2022.20	32.47
	NMR-double bridges	1965.00	2059.00	**2015.40**	30.10

VI. Conclusion

A discrete MMR algorithm was proposed based on multiple local dynamic searching strategies to solve the single traveling salesman problems and multiple traveling salesman problems. In order to overcome the shortcoming that the algorithm is easy to fall into the local optimum, the 2-opt, 3-opt and double-bridge local search operators are added to the original discrete NMR algorithm. The proposed algorithm is tested by using benchmark test problems from TSPLIB database. Through the simulation experiments results of TSP and MTSP with three different situations, it can be seen that the improved NMR algorithm can obtain high-quality optimization results in a short time, and it has strong robustness. And NMR-double bridges have stronger local searching ability than other algorithms, so it shows better optimization accuracy.

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