A Combination of A Shoreline Evolution Model and A Wave Crest Model on T-Head Groin Structures With the Breaking Wave Effect

Pidok Unyapoti, Nopparat Pochai

***Abstract*—Beach erosion is a naturally occurring phenomenon that occurs when the transfer of material away from the beach is not balanced by the deposit of new material on the shoreline. Beach erosion have always existed and have influenced the shoreline shape. This is a problem that contributes to the loss of shorelines. Structures invented to prevent beach erosion, such as seawalls, groins, and breakwaters. To avoid coastal erosion and sedimentation, a groin and a sea wall were constructed. Shoreline evolution analysis is being used to research the future topography of the beach. Beach erosion and beach deposition research requires a qualitative analysis of the model shoreline behavior with respect to the driving process. In this research, we focus on the effects of the T-head groin structure on shoreline evolution. The average wave crest impact is analyzed for eight sizes of T-head groin construction. An initial condition setting technique and boundary conditions techniques, as well as the structural impacts of the T-head groin, are discussed. Each year, the shoreline evolution is approximated using the traditional forward time centered space techniques and the unconditionally stable Saulyev finite differential techniques. The calculated impacts of shoreline evolution for eight different T-head groin sizes were consistent with the wave crest impact model.**

***Index Terms*— shoreline evolution, T- head groin system, explicit finite method, wave crest impact, mathematical model**

I. Introduction

Beach erosion is a naturally occurring phenomenon by which local sea-level rise, strong wave action, and coastal flooding wear down or carry away rocks, soils, and sands along the beach. Beach erosion and accretion have always existed and have influenced the current shoreline's shape. In many countries, beach erosion is responsible for coastal property loss, including damage to structures and loss of land. This is a problem that contributes to the loss of shorelines.

Nowadays, various construction structures are being invented to prevent beach erosion, such as seawalls, groins, breakwaters, etc. In this paper, we focused on the groin structure in the shape of the T-head groin. They demonstrated a novel approach in [1] that combines citizen science with low-cost unmanned aerial vehicles to generate survey-grade morphological data that can be used to model sediment dynamics at event to annual scales. The high-energy wavedominated coast of Victoria in south-eastern Australia serves as a field laboratory for testing the reliability of our protocol and developing a set of indices for studying multi-scale erosional dynamics. In [2], they presented sediment transport and erosion-deposition patterns near a detached, low-crested breakwater protecting Carey Island's cohesive shore in Malaysia. Their study found that the conductivity of the breakwater structure is essential to reducing erosion issues on Carey Island's cohesive coasts and to the effectiveness of mangrove rehabilitation initiatives in the area. In [3], they predicted the most likely total water level scenarios that result in overtopping at Santos Bay beaches and examined overtopping events in 2016. The prediction shows that the wider and flatter profiles in the western portion of Santos and Itararé provide greater protection from storm events, while the steeper eastern stretch of Santos Beach is more vulnerable to overtopping events. Their research focuses on beaches in Santos and So Vicente (So Paulo, Brazil). A seawall surrounds the entire 7 kilometer stretch of shoreline. In [4], they presented a study on the effect of groin application on shoreline erosion. Bathymetry and topography data from the north beach of Balongan, West Java, were used in the procedure. GENESIS software was used to model the coastline change caused by groin installation. They concluded that in the research area of west Java's north beach, an I-groin with a length of 70 meters and a T-head groin with a length of 60 meters efficiently overcomes erosion and advances the shoreline by 6,3 meters.

Many authors have developed one-line theory, and several contributors to the analytical solution of the shoreline evolution include [5], [6], [7], [8], [9], [10], and [11]. Analytical solutions cannot be expected to provide quantitatively precise solutions to situations with complicated boundary conditions and wave inputs. In the case of complicated boundary conditions and wave inputs, a numerical model of shoreline evolution would be more fitting than analytical solutions. A numerical model of shoreline evolution would be more fitting in the actual case.

In [12], [13], they have examined and presented two numerical schemes of shoreline evolution for simplified

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configuration beach. In [14], [15], [16], [17], [18], they have used the conditionally stable explicit finite difference methods to approximate their model solutions. In [19], [20], [21], they have used the numerical methods to approximate their model solution.

In [22], they proposed the Equilibrium energy function (EEF) analytical method and the shoreline evolution model. Testing of the proposed model at Nova Icaria reveals the same capabilities with only one measurement parameter as state-of-the-art models with more than 4 free parameters. In [23], a basic coastline profile model behavioral template was proposed to be calibrated and tested against a 6-year coastline location time series derived from a shoreline imaging system on the Gold Coast, Australia. Monitoring the model on unknown data shows that it can reproduce the dominant different seasons coastline transition observed at this site and up to 77% of the degraded coastline variability.

In this paper, we introduce a one-dimensional shoreline evolution model, wave crest impact model to obtain the breaking wave crest, the initial condition, and boundary conditions setting when T-head groins structure is added, eight lengths of considered T-head groins. The model solution will be approximated using finite difference techniques. We are focused on predicting the efficiency of T-head groin structure on shoreline evolution.

II. Governing Equation

A. Shoreline evolution model

In a one-dimensional shoreline evolution model, all of the bottom outlines should become parallel while the beach form remains constant and moves toward the land and the sea. Consequently, as the beach reduces and increases, so should be doing the design and volume of the beach level. Sand is moved along the shore on a profile between two clearly specified limit heights, which is the model's core idea. Where there is a difference between the rate of longshore sand transfer on the side of the segment and the associated sand condition, the adjustment in volume is affected. The laws of mass conservation must regularly be modified for the system [24]:

$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2}, \qquad (1)$$

for all $(x,t)\in(L,T)$, where $D=\dfrac{2Q_0}{D_B+D_C}$.

where x is the alongshore coordinate (m), y is the shoreline positions (m) and perpendicular to the x-axis, t is time (day), Q_0 is the long-shore sand transport rate amplitude (m^3/day), D_B is the average berm height (m) and D_C is the average closure depth (m).

B. Shoreline evolution parameters

The physical parameters of the shoreline evolution model are illustrated in Fig. 1-2. which are listed below.

α_0 is the angle between breaking wave crests impact angle and x-axis.

Q_0 is the long-shore sand transport rate amplitude.

D_B is the average berm height.

D_C is the average closure depth.

L is the length of alongshore.

T is the Time of simulation.

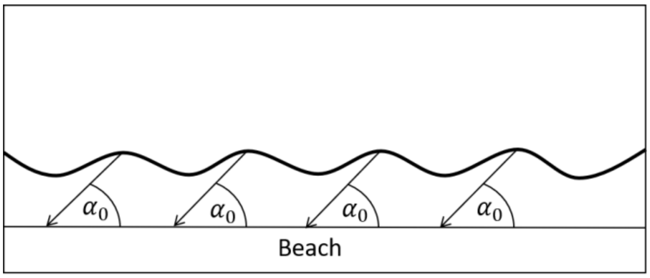

Fig. 1 Breaking wave crests impact angle when the beach is parallel to the x-axis

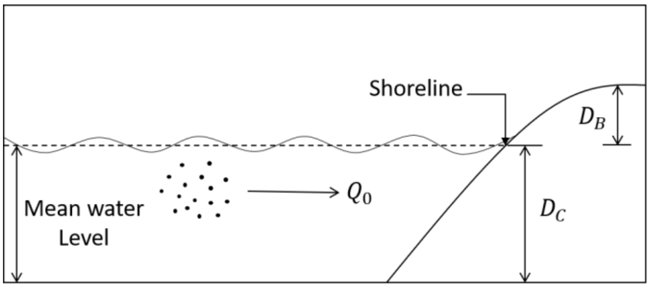

Fig. 2. Shoreline evolution parameters

C. The initial and boundary conditions for the shoreline evolution model

We assumed the initial beach to be parallel to the x-axis.

Assuming that, the angle between breaking wave crests impact angle and the shoreline is α_0. It follows that the sand transport rate along shoreline is consistent. The T-head groin is added on both side at $x=0$ and $x=L$ are illustrated in Fig. 3. Under this assumption, the initial condition becomes

$$y(x,\mathrm{t})=0, \quad \text{at} \quad t=0, \qquad (2)$$

boundary conditions are also assumed by,

$$\frac{\partial y(x,t)}{\partial x}=-\tan(\alpha_0) \quad \text{at} \quad x=0, \qquad (3)$$

and

$$\frac{\partial y(x,t)}{\partial x}=-\tan(-\alpha_0) \quad \text{at} \quad x=L, \qquad (4)$$

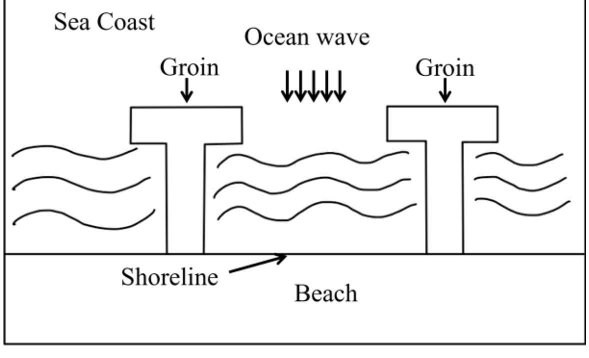

Fig. 3. Initial shoreline with configuration T-head groins.

D. Wave crest impact model

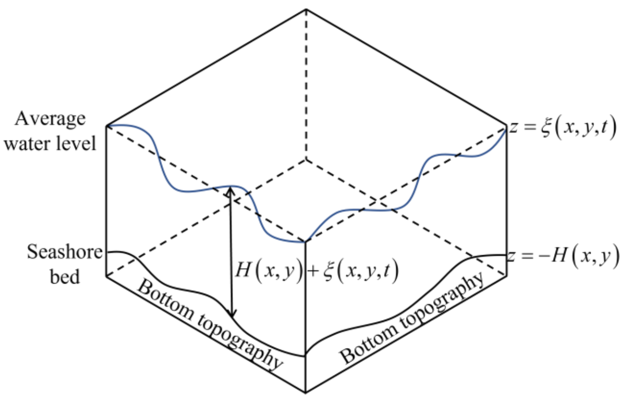

Fig. 4. Water elevation and bottom topography.

To achieve the wave crest impact in the shoreline evolution model, the hydrodynamic model is introduced [25].

A system of shallow water equations that takes into account momentum and mass conservation can be used to determine the two-dimensionally unstable water flows into and out of the coastline. The equations for this method should be derived from the depth-averaged Navier-Stokes equations in the vertical direction, omitting out the variables for the effects of friction, surface wind, Coriolis factor, and shear stress as well as the momentum diffusion caused by vibration. The equation of continuity is then expressed as follows:

$$\frac{\partial h}{\partial t}+\frac{\partial(uh)}{\partial x}+\frac{\partial(vh)}{\partial y}=0, \tag{5}$$

and the momentum equations are expressed as below:

$$\frac{\partial(uh)}{\partial t}+\frac{\partial\left(u^2h+\frac{1}{2}gh^2\right)}{\partial x}+\frac{\partial(uvh)}{\partial y}=0, \tag{6}$$

$$\frac{\partial(uh)}{\partial t}+\frac{\partial(uvh)}{\partial x}+\frac{\partial\left(v^2h+\frac{1}{2}gh^2\right)}{\partial y}=0, \tag{7}$$

where

$h(x,y,t)$ is the estimated depth from the average water surface to the seashore bed (m) $h=H+\xi$,

$\xi(x,y,t)$ is the elevation of the water surface above the average seashore water level (m),

$H(x,y)$ is the seashore's interpolated bottom topography function (m),

$u(x,y,t)$ is the velocity in the direction of x (m/s),

$v(x,y,t)$ is the velocity in the direction of y (m/s),

g is a gravity constant $(9.8\, m/s^2)$.

Such time (t), and two space coordinates, x and y are the independent variables. Likewise, the conserved quantities are mass, which is proportional to h, and momentum, which is proportional to (uh) and (vh). As taken with respect to the same term, the partial derivatives are grouped into vectors $(\partial x,\partial y,\partial t)$ and then rewritten as a partial differential hyperbolic equation as follows:

$$U=\begin{pmatrix} h \\ uh \\ vh \end{pmatrix}, F(U)=\begin{pmatrix} uh \\ u^2h+\frac{1}{2}gh^2 \\ uvh \end{pmatrix}, \tag{8}$$

$$G(U)=\begin{pmatrix} vh \\ uvh \\ v^2h+\frac{1}{2}gh^2 \end{pmatrix}, \tag{9}$$

the hyperbolic PDE:

$$\frac{\partial U}{\partial t}+\frac{\partial F(U)}{\partial x}+\frac{\partial G(U)}{\partial y}=0. \tag{10}$$

E. The initial and boundary condition for wave crest impact model

The initial condition of the shoreline was as follows: the x and y velocity components, as well as the water elevation, were all zero: $u=0, v=0$ and $\xi=0$.

Assume that the T-head groin is not a perfect water barrier because of its rock composition, which has large gaps. Under this assumption, the boundary condition was as follows: $(i)\ u=0,\frac{\partial v}{\partial y}=0,\xi=f(x,y,t)$ for wave coming, $(ii)\ \frac{\partial u}{\partial x}=0, \mathrm{v}=0,\frac{\partial \xi}{\partial x}=0$ for left and right boundary, $(iii)\ u=0,\frac{\partial v}{\partial y}=0,\frac{\partial \xi}{\partial y}=0$ for along the beach, $(iv)\ u=0,\frac{\partial v}{\partial y}=0,\frac{\partial \xi}{\partial y}=0$ for top T-head groin structure, $(v)\ u=0,\frac{\partial v}{\partial y}=0,\frac{\partial \xi}{\partial y}=0$ for bottom T-head groin structure, and $(vi)\ \frac{\partial u}{\partial x}=0, \mathrm{v}=0,\frac{\partial \xi}{\partial x}=0$ for left and right T-head groin structure. The boundary conditions are illustrated in Fig. 5-7.

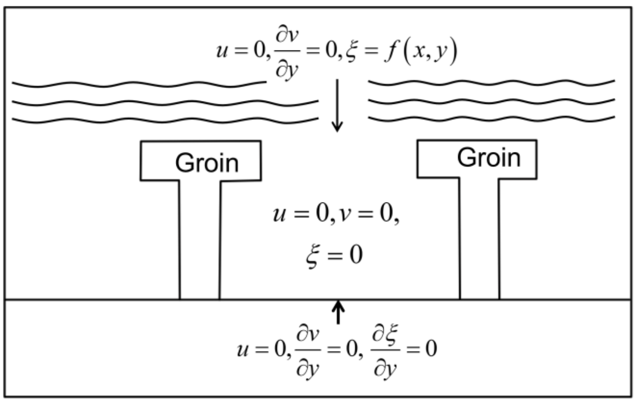

Fig. 5. Initial and boundary condition.

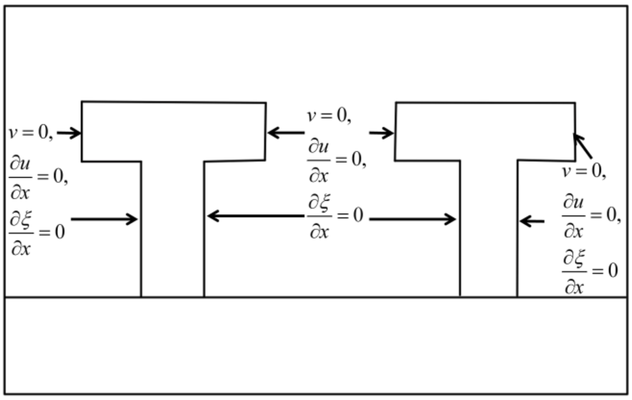

Fig. 6. Initial and boundary condition for T-head groin structure (1).

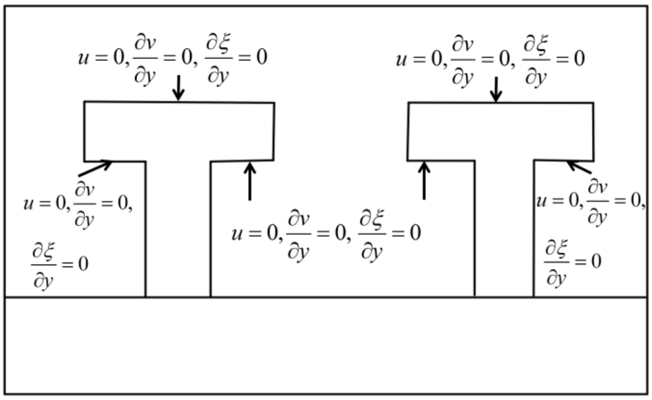

Fig. 7. Initial and boundary condition for T-head groin structure (2).

III. NUMERICAL TECHNIQUES

A. Grid Spacing

We are discretizing (1) by splitting the interval $[0,L]$ into I subintervals such as $I\Delta x = L$ and the interval $[0,T]$ into N subintervals such as $N\Delta t = T$. Then we approximate $y(x_i,t_n)$ by y_i^n, at the points $x_i = i\Delta x$ and $t_n = n\Delta t$, where $0 \le i \le I$ and $0 \le n \le N$ are positive integers of I and N.

B. Traditional forward time centered space techniques

The forward time centered space techniques will also be used. We can obtain that the finite difference approximation is [26],

$$y \cong y_i^n, \tag{11}$$

$$\frac{\partial y}{\partial t} \cong \frac{y_i^{n+1} - y_i^n}{\Delta t}, \tag{12}$$

$$\frac{\partial y}{\partial x} \cong \frac{y_{i+1}^n - y_{i-1}^n}{2\Delta x}, \tag{13}$$

$$\frac{\partial^2 y}{\partial x^2} \cong \frac{y_{i+1}^n - 2y_i^n + y_{i-1}^n}{(\Delta x)^2}, \tag{14}$$

where $A = \frac{D\Delta t}{(\Delta x)^2}$.

Substituting (11) – (14), in (1), we are obtaining,

$$\frac{y_i^{n+1} - y_i^n}{\Delta t} = D\left(\frac{y_{i+1}^n - 2y_i^n + y_{i-1}^n}{(\Delta x)^2}\right), \tag{15}$$

for $1 \le i \le I-1$ and $0 \le n \le N-1$. (15), can be written in an explicit form of finite difference as follows,

$$y_i^{n+1} = Ay_{i+1}^n + (1-2A)y_i^n + Ay_{i-1}^n, \tag{16}$$

for $1 \le i \le I-1$ and $0 \le n \le N-1$.

C. An unconditionally Saulyev finite difference techniques

The Saulyev finite difference techniques will also be used. We can obtain that the finite difference approximation is

$$y \cong y_i^n, \tag{17}$$

$$\frac{\partial y}{\partial t} \cong \frac{y_i^{n+1} - y_i^n}{\Delta t}, \tag{18}$$

$$\frac{\partial^2 y}{\partial x^2} \cong \frac{y_{i+1}^n - y_i^n - y_i^{n+1} + y_{i-1}^{n+1}}{(\Delta x)^2}, \tag{19}$$

where $A = \frac{D\Delta t}{(\Delta x)^2}$.

Substituting (17) – (19), in (1), we are obtaining,

$$\frac{y_i^{n+1} - y_i^n}{\Delta t} = D\left(\frac{y_{i+1}^n - y_i^n - y_i^{n+1} + y_{i-1}^{n+1}}{(\Delta x)^2}\right), \tag{20}$$

for $1 \le i \le I-1$ and $0 \le n \le N-1$. (20), can be written in an explicit form of finite difference as follows,

$$y_i^{n+1} = \frac{1}{(1+A)}\left(Ay_{i+1}^n + (1-A)y_i^n + Ay_{i-1}^{n+1}\right), \tag{21}$$

for $1 \le i \le I-1$ and $0 \le n \le N-1$.

D. Numerical method for the wave crest impact model

The finite difference technique is

$$U_{i,j}^{n+1} = U_{i,j}^n - \frac{\Delta t}{\Delta x}\left(F_{i+\frac{1}{2},j}^{n+\frac{1}{2}} - F_{i-\frac{1}{2},j}^{n+\frac{1}{2}}\right) - \frac{\Delta t}{\Delta y}\left(G_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} - G_{i,j-\frac{1}{2}}^{n+\frac{1}{2}}\right). \tag{22}$$

E. The averaged wave crest impact

We can determine that the wave crest impact is

$$\alpha(x_i,y_j,t) = \tan^{-1}\left(\frac{v(x_i,y_j,t)}{u(x_i,y_j,t)}\right), \tag{23}$$

We assume that the averaged wave crest impact is assumed by

$$\alpha_0(t) = \frac{\sum_{i=1}^{N_P}\alpha(x_i,0,t)}{N_P}, \tag{24}$$

where N_P is several sample points along the shoreline for wave crest impact.

F. The application of finite difference techniques to the left and right boundary conditions

The forward time centered space techniques will also be used. We can obtain that the finite difference approximation is,

$$y \cong y_i^n, \tag{25}$$

$$\frac{\partial y}{\partial t} \cong \frac{y_i^{n+1} - y_i^n}{\Delta t}, \tag{26}$$

$$\frac{\partial y}{\partial x} \cong \frac{y_{i+1}^n - y_{i-1}^n}{2\Delta x}, \tag{27}$$

where $A = \frac{D\Delta t}{(\Delta x)^2}$.

Substituting (25) - (27), in (1), we are obtaining,

$$\frac{y_i^{n+1} - y_i^n}{\Delta t} = D\left(\frac{y_{i+1}^n - 2y_i^n + y_{i-1}^n}{(\Delta x)^2}\right), \tag{28}$$

We approximated the substitution of the uncertain value of the left and right boundaries by using the center difference with the specified left and right boundary conditions.

For the left boundary i = 0, we are obtaining,

$$y_{-1}^n = y_1^n - 2(\Delta x)(-\tan(\alpha_0)), \tag{29}$$

substituting (29), in (28), we are obtaining,

$$y_i^{n+1} = (1-2A)y_i^n + 2Ay_{i+1}^n - 2A(\Delta x)(-\tan(\alpha_0)), \tag{30}$$

For the right boundary i = I, we are obtaining,

$$y_{I+1}^n = y_{M-1}^n + 2(\Delta x)(-\tan(-\alpha_0)), \tag{31}$$

substituting (31), in (28), we are obtaining,

$$y_i^{n+1} = 2Ay_{i-1}^n + (1-2A)y_i^n + 2A(\Delta x)(-\tan(-\alpha_0)), \tag{32}$$

(30), and (32), could be used to approximate the values y_i^n of the solution domain grid points.

IV. Groin Setting Techniques

We will consider eight lengths of considered T-head groin is 16, 18, 20, 22, 24, 26, 28, and 30 m. The consideration alongshore is illustrated in Fig 8.

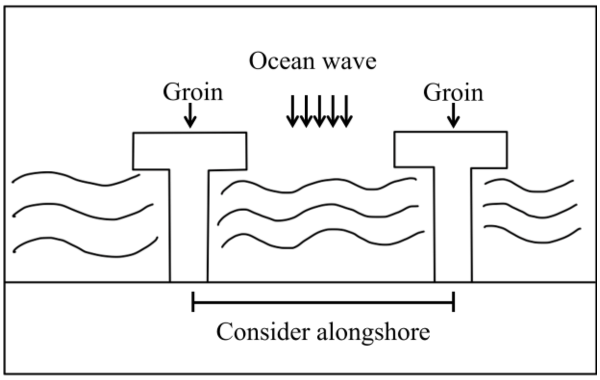

Fig. 8. Consider alongshore.

For eight lengths of the T-head groin that are being taken into consideration, the approximate wave crest impact model solution will be approximated using finite difference methods (22) are illustrated in Fig.9-16.

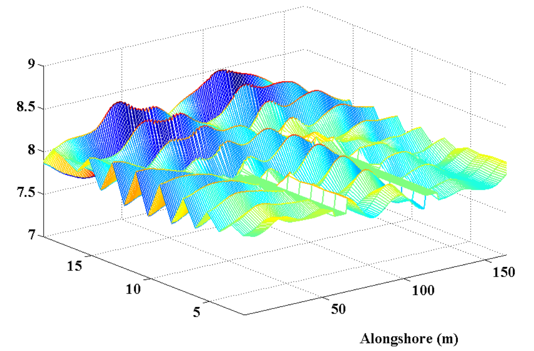

Fig. 9. Wave crest impact in 9 years when T-head groin 16 m.

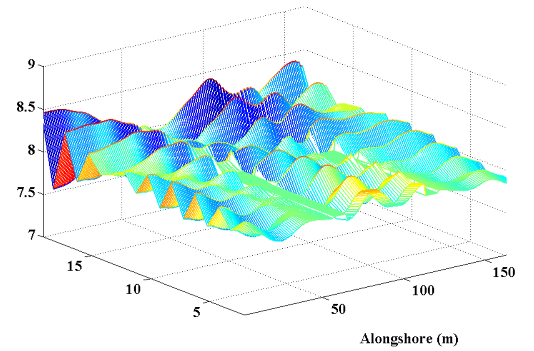

Fig. 10. Wave crest impact in 11 years when T-head groin 18 m.

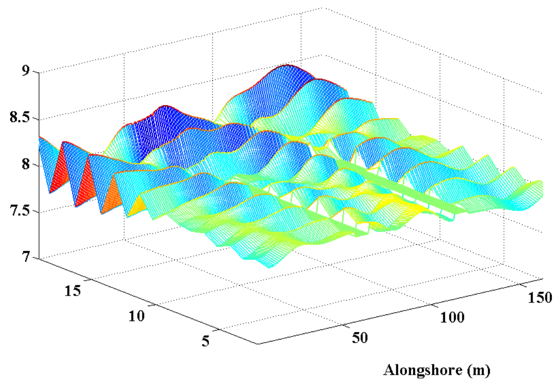

Fig. 11. Wave crest impact in 15 years when T-head groin 20 m.

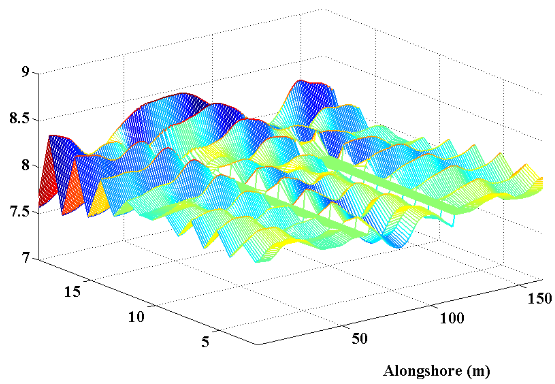

Fig. 12. Wave crest impact in 13 years when T-head groin 22 m.

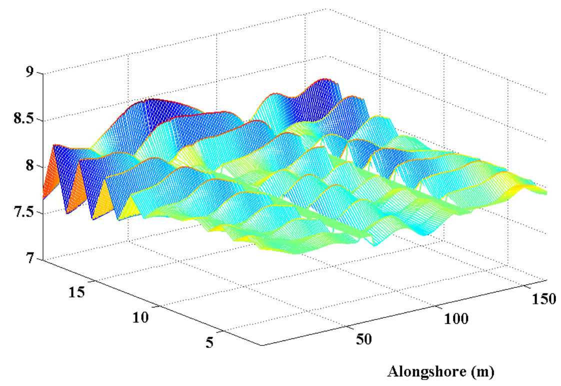

Fig. 13. Wave crest impact in 20 years when T-head groin 24 m.

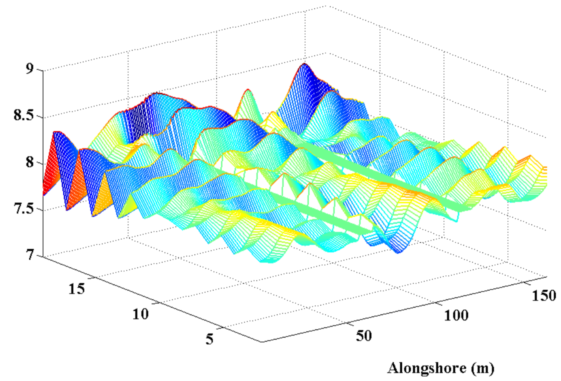

Fig. 14. Wave crest impact in 20 years when T-head groin 26 m.

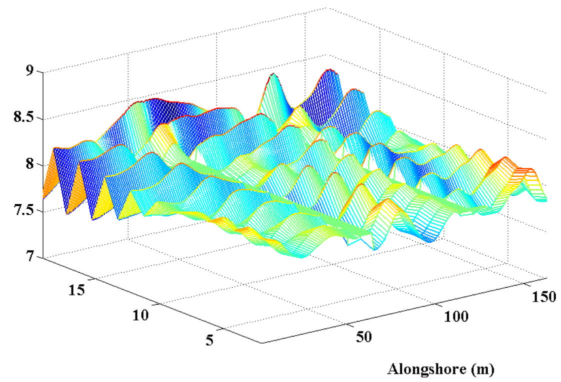

Fig. 15. Wave crest impact in 20 years when T-head groin 28 m.

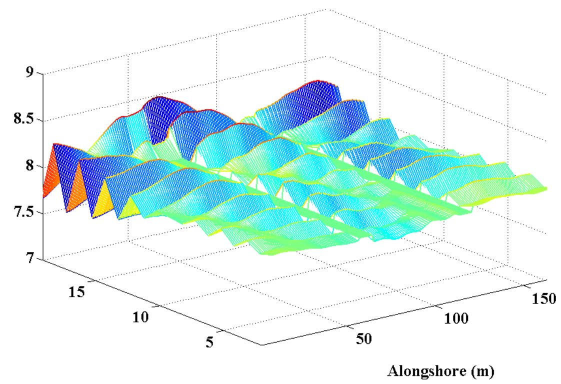

Fig. 16. Wave crest impact in 20 years when T-head groin 30 m.

Table 1-8 shows the averaged wave crest impact (α_0) obtained by (24) for eight lengths of the considered T-head groin.

TABLE I
THE AVERAGED WAVE CREST IMPACT 9 YEARS WHEN T-HEAD GROIN SIZE 16 M

Time (Years)	Minute					
	0-15	15-30	30-45	45-60	60-75	75-90
1	0.2635	0.2054	0.2100	0.2145	0.2189	0.2232
5	0.2645	0.2624	0.2604	0.2584	0.2564	0.2543
9	0.0264	0.0223	0.0181	0.0138	0.0093	0.0046
Time (Years)	…	1365-1380	1380-1395	1395-1410	1410-1425	1425-1440
1	…	-0.6192	-0.5621	-0.5047	-0.5098	-0.3890
5	…	0.0661	0.0633	0.0604	0.0575	0.0545
9	…	0.3304	0.3267	0.3859	0.3823	0.3786

TABLE II
THE AVERAGED WAVE CREST IMPACT 11 YEARS WHEN T-HEAD GROIN SIZE 18 M

Time (Years)	Minute					
	0-15	15-30	30-45	45-60	60-75	75-90
1	-0.0830	-0.0677	-0.0526	-0.1004	-0.0856	-0.0708
5	0.1572	0.1530	0.1488	0.1446	0.1404	0.1361
10	-0.1413	-0.1425	-0.2584	-0.0550	0.0099	0.1341
11	-0.0119	-0.0652	-0.0557	-0.0462	-0.0994	-0.0896
Time (Years)	…	1365-1380	1380-1395	1395-1410	1410-1425	1425-1440
1	…	-0.4803	-0.4885	-0.4339	-0.3791	-0.3242
5	…	0.0504	0.0493	0.0483	0.0472	0.0461
10	…	0.3557	0.3561	0.3564	0.3568	0.3572
11	…	-0.4077	-0.4091	-0.4107	-0.4123	-0.4769

TABLE III
THE AVERAGED WAVE CREST IMPACT 15 YEARS WHEN T-HEAD GROIN SIZE 20 M

Time (Years)	Minute					
	0-15	15-30	30-45	45-60	60-75	75-90
1	0.3146	0.3102	0.3052	0.2990	0.2906	0.3429
5	0.3369	0.3351	0.3333	0.3311	0.3284	0.3255
10	0.1009	0.1625	0.1610	0.1594	0.1577	0.1559
15	-0.4382	-0.3815	-0.3875	-0.3306	-0.2735	-0.2165
Time (Years)	…	1365-1380	1380-1395	1395-1410	1410-1425	1425-1440
1	…	-0.2437	-0.1238	-0.1295	-0.0096	-0.0154
5	…	0.1439	0.1422	0.1405	0.1388	0.1370
10	…	0.0145	0.0125	0.0104	0.0084	0.0062
15	…	0.2597	0.2581	0.2566	0.2551	0.2536

TABLE IV
THE AVERAGED WAVE CREST IMPACT 13 YEARS WHEN T-HEAD GROIN 22 M

Time (Years)	Minute					
	0-15	15-30	30-45	45-60	60-75	75-90
1	-0.2305	-0.1621	-0.1569	-0.1521	-0.1476	-0.1436
5	0.1447	0.1365	0.1287	0.1844	0.1782	0.1736
10	-0.0869	0.0900	0.1422	0.2579	0.3114	0.3654
13	0.1755	0.1749	0.1741	0.1733	0.1723	0.2340
Time (Years)	…	1365-1380	1380-1395	1395-1410	1410-1425	1425-1440
1	…	-0.3895	-0.3405	-0.3536	-0.3033	-0.2524
5	…	0.0922	0.0925	0.0928	0.0931	0.0934
10	…	0.3630	0.3630	0.3631	0.3631	0.3632
13	…	0.4394	0.4358	0.4324	0.4291	0.4260

TABLE V
THE AVERAGED WAVE CREST IMPACT 20 YEARS WHEN T-HEAD GROIN SIZE 24 M

Time (Years)	Minute					
	0-15	15-30	30-45	45-60	60-75	75-90
1	0.2364	0.2312	0.2260	0.2837	0.2785	0.2734
5	0.6124	0.6008	0.5904	0.5802	0.5703	0.5607
10	-0.3096	-0.3150	-0.2575	-0.2628	-0.2680	-0.1474
15	0.3091	0.2975	0.3488	0.4002	0.3888	0.4403
20	-0.3199	-0.3810	-0.3794	-0.3778	-0.3763	-0.4377

Time (Years)	Minute					
	…	1365-1380	1380-1395	1395-1410	1410-1425	1425-1440
1	…	0.0385	0.0926	0.1468	0.1381	0.1293
5	…	0.2496	0.2483	0.2470	0.2458	0.2445
10	…	-0.3180	-0.3202	-0.3224	-0.3246	-0.3268
15	…	0.3869	0.3849	0.3829	0.3809	0.3790
20	…	-0.2951	-0.2964	-0.2977	-0.2991	-0.3004

TABLE VI
THE AVERAGED WAVE CREST IMPACT 20 YEARS WHEN T-HEAD GROIN SIZE 26 M

Time (Years)	Minute					
	0-15	15-30	30-45	45-60	60-75	75-90
1	-0.2234	-0.1599	-0.2219	-0.2211	-0.2201	-0.2190
5	-0.1300	-0.1251	-0.1206	-0.1163	-0.1122	-0.0454
10	-0.2985	-0.2936	-0.2888	-0.2842	-0.2797	-0.2753
15	0.6864	0.7542	0.7602	0.7676	0.7130	0.5957
20	0.3344	0.3335	0.3324	0.3311	0.3926	0.3912

Time (Years)	Minute					
	…	1365-1380	1380-1395	1395-1410	1410-1425	1425-1440
1	…	0.3617	0.2390	0.2425	0.1210	0.1255
5	…	0.1702	0.1719	0.1736	0.1754	0.1771
10	…	-0.0298	-0.0289	-0.0280	-0.0272	-0.0263
15	…	0.1829	0.1846	0.1862	0.1879	0.1896
20	…	-0.1046	-0.1028	-0.1010	-0.0993	-0.0977

TABLE VII
THE AVERAGED WAVE CREST IMPACT 20 YEARS WHEN T-HEAD GROIN SIZE 28 M

Time (Years)	Minute					
	0-15	15-30	30-45	45-60	60-75	75-90
1	0.2891	0.3499	0.3478	0.3457	0.3436	0.3415
5	0.3573	0.4161	0.4121	0.4080	0.4040	0.4000
10	-0.4099	-0.4122	-0.4145	-0.3541	-0.3567	-0.2965
15	0.2809	0.2764	0.2720	0.3304	0.3260	0.3216
20	-0.5731	-0.5733	-0.5735	-0.5738	-0.5742	-0.6375

Time (Years)	Minute					
	…	1365-1380	1380-1395	1395-1410	1410-1425	1425-1440
1	…	-0.0372	-0.0400	-0.0428	0.0173	0.0146
5	…	0.2837	0.2812	0.2788	0.2764	0.2740
10	…	-0.2584	-0.2628	-0.2672	-0.2716	-0.2760
15	…	0.4681	0.4643	0.4606	0.4570	0.4534
20	…	-0.1795	-0.1837	-0.1879	-0.1921	-0.1963

TABLE VIII
THE AVERAGED WAVE CREST IMPACT 20 YEARS WHEN T-HEAD GROIN SIZE 30 M

Time (Years)	Minute					
	0-15	15-30	30-45	45-60	60-75	75-90
1	0.0787	0.0682	0.1205	0.1100	0.0995	0.1519
5	0.4851	0.4890	0.4296	0.4322	0.4338	0.4341
10	-0.3801	-0.3713	-0.3625	-0.4168	-0.4086	-0.4013
15	0.4475	0.5016	0.4929	0.4842	0.5385	0.5928
20	-0.1901	-0.1795	-0.1690	-0.2213	-0.2110	-0.2007

Time (Years)	Minute					
	…	1365-1380	1380-1395	1395-1410	1410-1425	1425-1440
1	…	0.5101	0.4583	0.4694	0.4804	0.4283
5	…	0.4199	0.4210	0.4222	0.4235	0.4248
10	…	-0.4248	-0.4256	-0.4266	-0.4276	-0.4286
15	…	0.4115	0.4115	0.4114	0.4115	0.4116
20	…	-0.4275	-0.4266	-0.4258	-0.4251	-0.4244

V. NUMERICAL EXPERIMENT

In this section, the numerical results of the various beach scenarios of T-head groin structure are considered, and the solution to the idealized problem is introduced. Assuming, during the experiments, that the length of the shoreline (L) under consideration is 100 m and the averaged wave crest impact (α_0) for eight T-head groin sizes. Table 1-8 shows the averaged wave crest impact of eight T-head groin structure sizes. Table 9 shows the long-shore transport rate (D) [27]. The simulation setting is illustrated in Fig. 17.

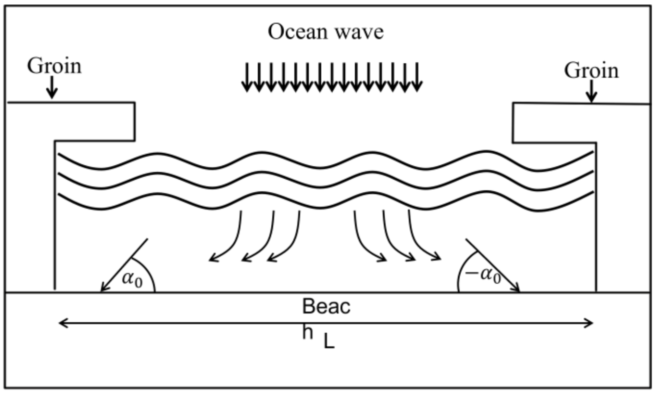

Fig. 17. Initial shoreline.

TABLE IX
THE LONG-SHORE TRANSPORT RATE

Month	$D\ (m/day)$
Jan	79.4659
Feb	62.1307
Mar	5.7869
Apr	61.4403
May	5.6420
Jun	5.4716
Jul	73.0227
Aug	83.071
Sep	121.7301
Oct	372.017
Nov	96.5710
Dec	101.1233

We are going to employ the traditional forward time centered space techniques (FTCS) (16), and the Saulyev finite difference techniques (21), to approximate the shoreline evolution model solution are illustrated in Fig. 18-25. Table 10-25 shows the calculated results.

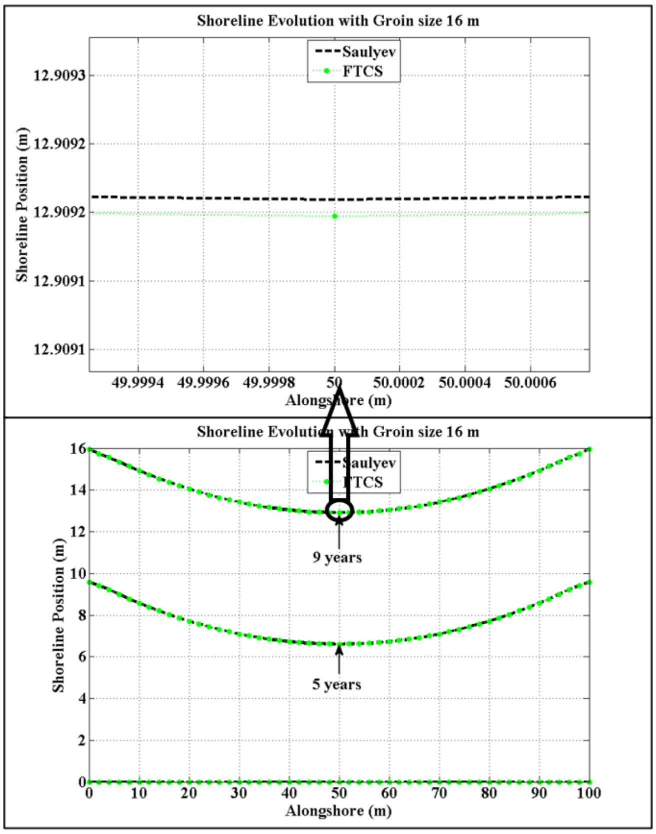

Fig. 18. Shoreline evolution in 9 years when T-head Groin size 16 m.

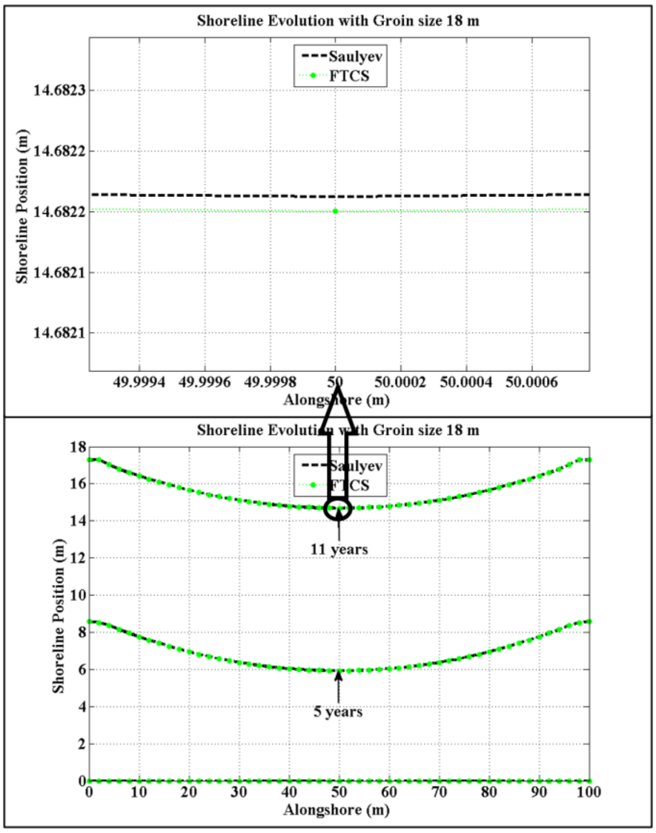

Fig. 19. Shoreline evolution in 11 years when T-head Groin size 18 m.

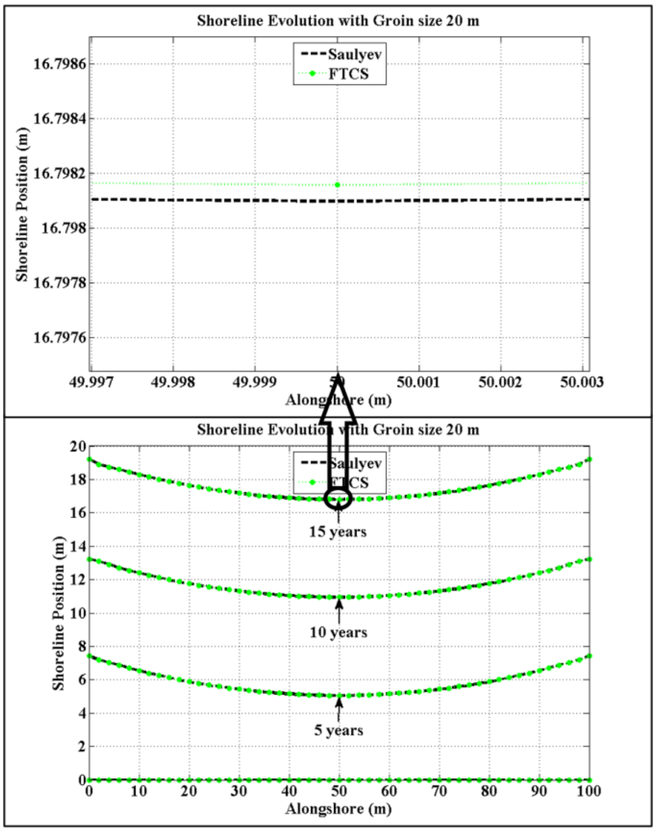

Fig. 20. Shoreline evolution in 15 years when T-head Groin size 20 m.

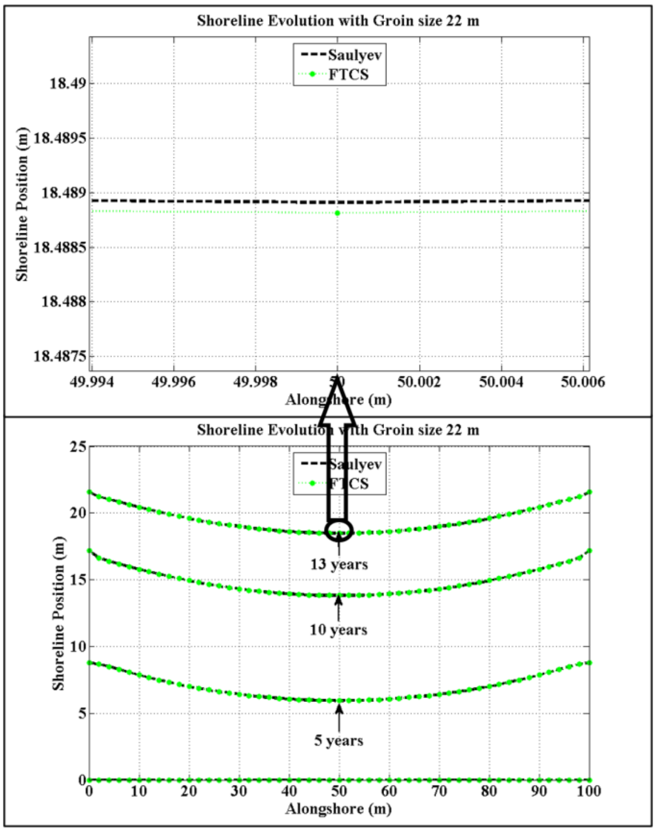

Fig. 21. Shoreline evolution in 13 years when T-head Groin size 22 m.

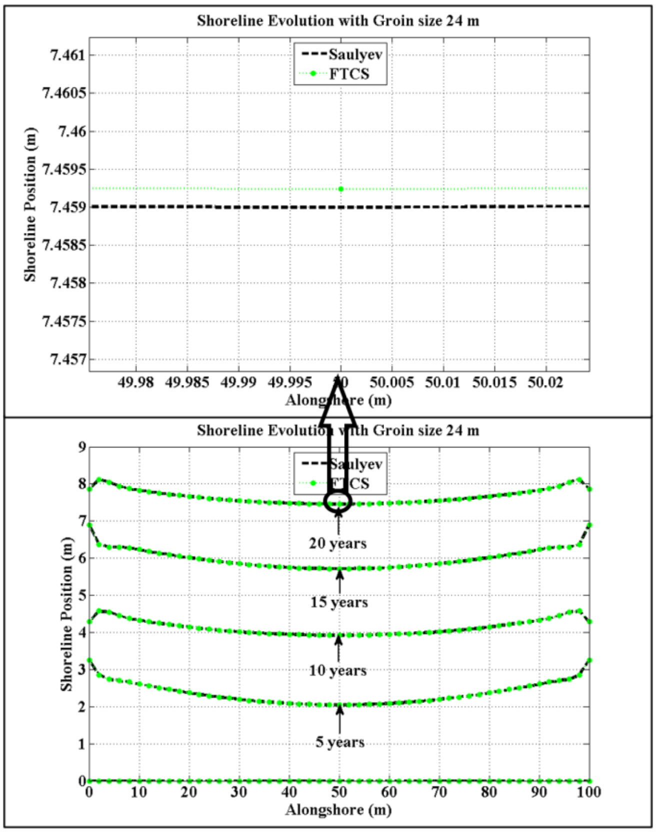

Fig. 22. Shoreline evolution in 20 years when T-head Groin size 24 m.

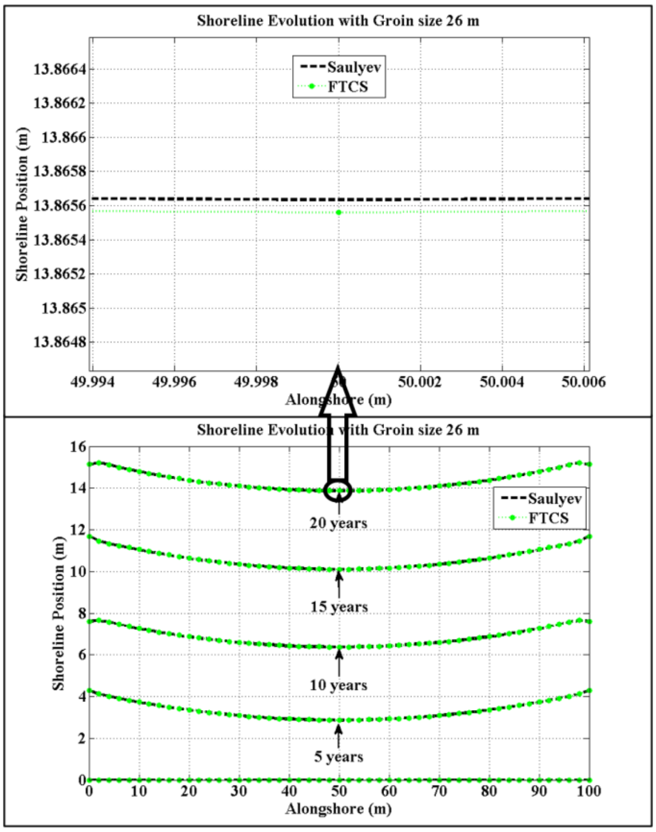

Fig. 23. Shoreline evolution in 20 years when T-head Groin size 26 m.

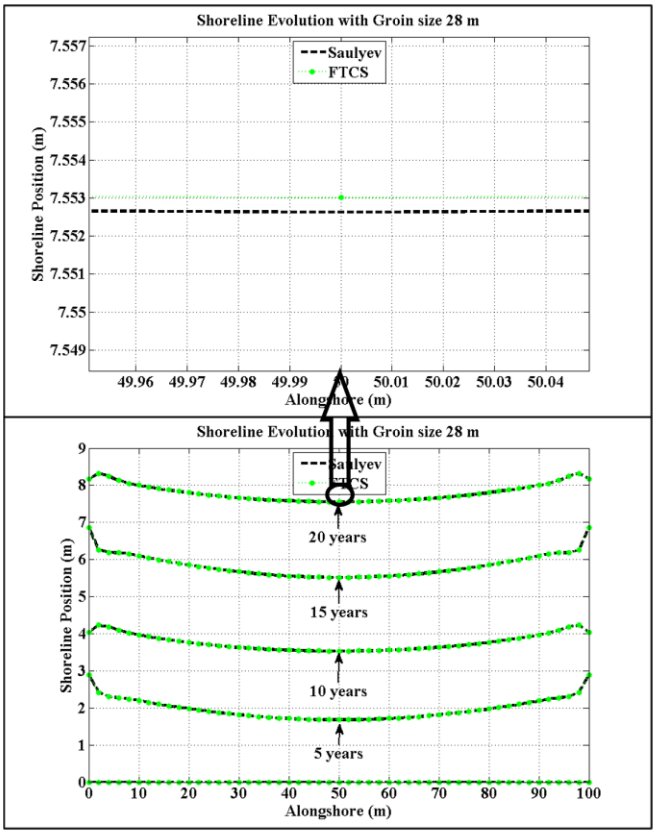

Fig. 24. Shoreline evolution in 20 years when T-head Groin size 28 m.

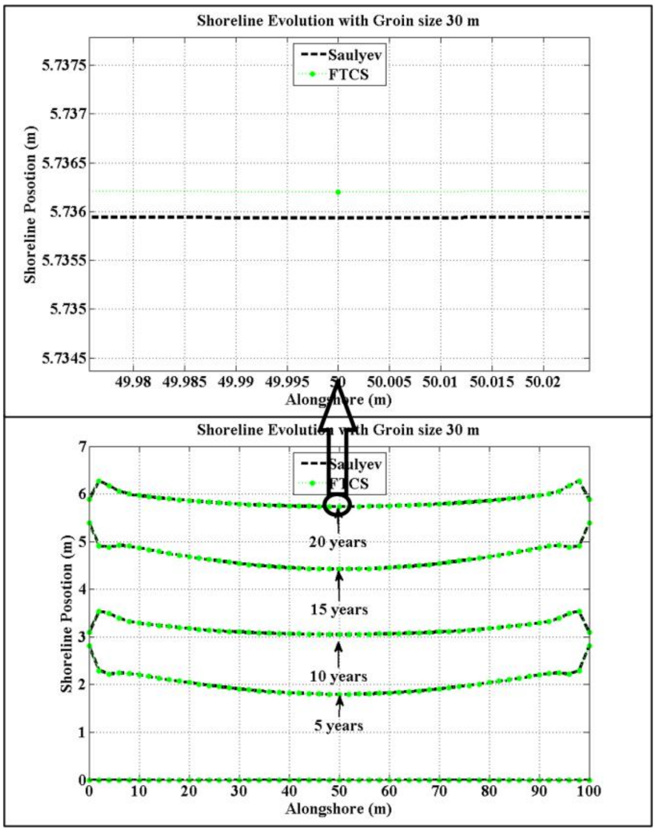

Fig. 25. Shoreline evolution in 20 years when T-head Groin size 30 m.

TABLE X
APPROXIMATED SHORELINE EVOLUTION ALONG 9 YEARS USING THE TRADITIONAL FORWARD TIME CENTERED SPACE TECHNIQUES WHEN T-HEAD GROIN SIZE 16 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	2.8710	1.4100	0.6302	0.6302	1.4100	2.8710
5	9.5859	7.7058	6.7258	6.7258	7.7058	9.5859
9	15.9610	14.0431	13.0345	13.0345	14.0431	15.9610

TABLE XI
APPROXIMATED SHORELINE EVOLUTION ALONG 9 YEARS USING THE SAULYEV FINITE DIFFERENCE TECHNIQUES WHEN T-HEAD GROIN SIZE 16 M

Time (Years)	Distance (m)					
	0	20	40	60	80	100
1	2.8718	1.4107	0.6305	0.6300	1.4094	2.8701
5	9.5859	7.7059	6.7259	6.7260	7.7059	9.5860
9	15.9615	14.0435	13.0346	13.0345	14.0429	15.9609

TABLE XII
APPROXIMATED SHORELINE EVOLUTION ALONG 11 YEARS USING THE TRADITIONAL FORWARD TIME CENTERED SPACE TECHNIQUES WHEN T-HEAD GROIN SIZE 18 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	2.4433	1.2654	0.5617	0.5617	1.2654	2.4433
5	8.5988	6.9462	6.0354	6.0354	6.9462	8.5988
10	16.3731	14.2624	13.3619	13.3619	14.2624	16.3731
11	17.2722	15.6527	14.7912	14.7912	15.6527	17.2722

TABLE XIII
APPROXIMATED SHORELINE EVOLUTION ALONG 11 YEARS USING THE SAULYEV FINITE DIFFERENCE TECHNIQUES WHEN T-HEAD GROIN SIZE 18 M

Time (Years)	Distance (m)					
	0	20	40	60	80	100
1	2.4436	1.2659	0.5619	0.5615	1.2649	2.4427
5	8.5988	6.9463	6.0353	6.0355	6.9464	8.5989
10	16.3738	14.2626	13.3621	13.3618	14.2622	16.3731
11	17.2720	15.6525	14.7911	14.7912	15.653	17.2724

TABLE XIV
APPROXIMATED SHORELINE EVOLUTION ALONG 15 YEARS USING THE TRADITIONAL FORWARD TIME CENTERED SPACE TECHNIQUES WHEN T-HEAD GROIN SIZE 20 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	2.2494	1.1606	0.5209	0.5209	1.1606	2.2494
5	7.4274	5.8894	5.1479	5.1479	5.8894	7.4274
10	13.2267	11.7631	11.0379	11.0379	11.7631	13.2267
15	19.2122	17.6409	16.8916	16.8916	17.6409	19.2122

TABLE XV
APPROXIMATED SHORELINE EVOLUTION ALONG 15 YEARS USING THE SAULYEV FINITE DIFFERENCE TECHNIQUES WHEN T-HEAD GROIN SIZE 20 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	2.2499	1.1610	0.5211	0.5207	1.1601	2.2488
5	7.4275	5.8894	5.1479	5.1479	5.8894	7.4273
10	13.2269	11.7631	11.0378	11.0379	11.7632	13.2268
15	19.2123	17.641	16.8916	16.8915	17.6408	19.2122

TABLE XVI
APPROXIMATED SHORELINE EVOLUTION ALONG 13 YEARS USING THE TRADITIONAL FORWARD TIME CENTERED SPACE TECHNIQUES WHEN T-HEAD GROIN SIZE 22 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	2.4577	1.1767	0.5171	0.5171	1.1767	2.4577
5	8.7941	7.0079	6.063	6.063	7.0079	8.7941
10	17.1728	14.9308	13.9419	13.9419	14.9308	17.1728
13	21.5783	19.5870	18.6116	18.6116	19.5870	21.5783

TABLE XVII
APPROXIMATED SHORELINE EVOLUTION ALONG 13 YEARS USING THE SAULYEV FINITE DIFFERENCE TECHNIQUES WHEN T-HEAD GROIN SIZE 22 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	2.4578	1.1770	0.5173	0.5169	1.1763	2.4574
5	8.7940	7.0079	6.0628	6.0629	7.0079	8.7940
10	17.1735	14.9311	13.942	13.9417	14.9306	17.1728
13	21.5774	19.5868	18.6117	18.6116	19.5870	21.5786

TABLE XVIII
APPROXIMATED SHORELINE EVOLUTION ALONG 20 YEARS USING THE TRADITIONAL FORWARD TIME CENTERED SPACE TECHNIQUES WHEN T-HEAD GROIN SIZE 24 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	1.0972	0.5544	0.2391	0.2391	0.5544	1.0972
5	3.2598	2.3726	2.0848	2.0848	2.3729	3.2598
10	4.285	4.1421	3.9454	3.9454	4.1421	4.285
15	6.8976	6.014	5.7506	5.7506	6.014	6.8976
20	7.8626	7.6623	7.4815	7.4815	7.6623	7.8626

TABLE XIX
APPROXIMATED SHORELINE EVOLUTION ALONG 20 YEARS USING THE SAULYEV FINITE DIFFERENCE TECHNIQUES WHEN T-HEAD GROIN SIZE 24 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	1.0975	0.5547	0.2392	0.2389	0.554	1.0968
5	3.2607	2.3731	2.0852	2.0848	2.3724	3.2596
10	4.2846	4.1417	3.945	3.9455	4.1424	4.2851
15	6.8987	6.0147	5.751	5.7506	6.0137	6.8975
20	7.8624	7.6618	7.4811	7.4815	7.6626	7.8627

TABLE XX
APPROXIMATED SHORELINE EVOLUTION ALONG 20 YEARS USING THE TRADITIONAL FORWARD TIME CENTERED SPACE TECHNIQUES WHEN T-HEAD GROIN SIZE 26 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	1.4976	0.5985	0.2596	0.2596	0.5985	1.4976
5	4.3015	3.3562	2.9214	2.9214	3.3562	4.3015
10	7.6055	6.8702	6.4288	6.4288	6.8702	7.6055
15	11.6917	10.6384	10.1618	10.1618	10.6384	11.6917
20	15.1301	14.3789	13.922	13.922	14.3789	15.1301

TABLE XXI
APPROXIMATED SHORELINE EVOLUTION ALONG 20 YEARS USING THE SAULYEV FINITE DIFFERENCE TECHNIQUES WHEN T-HEAD GROIN SIZE 26 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	1.4979	0.5988	0.2598	0.2595	0.5983	1.4974
5	4.3016	3.3563	2.9215	2.9213	3.3561	4.3014
10	7.6051	6.8701	6.4288	6.4289	6.8703	7.6056
15	11.6919	10.6386	10.1621	10.1619	10.6382	11.6916
20	15.1297	14.3787	13.922	13.9222	14.3791	15.1303

TABLE XXII
APPROXIMATED SHORELINE EVOLUTION ALONG 20 YEARS USING THE TRADITIONAL FORWARD TIME CENTERED SPACE TECHNIQUES WHEN T-HEAD GROIN SIZE 28 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	0.7186	0.3100	0.0987	0.0987	0.3100	0.7186
5	2.8822	1.9768	1.7127	1.7127	1.9768	2.8822
10	4.0363	3.7656	3.5555	3.5555	3.7656	4.0363
15	6.8627	5.8516	5.5533	5.5533	5.8516	6.8627
20	8.1678	7.800	7.5806	7.5806	7.800	8.1678

TABLE XXIII
APPROXIMATED SHORELINE EVOLUTION ALONG 20 YEARS USING THE SAULYEV FINITE DIFFERENCE TECHNIQUES WHEN T-HEAD GROIN SIZE 28 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	0.7189	0.3103	0.0989	0.0986	0.3095	0.718
5	2.8834	1.9772	1.7129	1.7125	1.9764	2.8819
10	4.0361	3.7650	3.5549	3.5555	3.7658	4.0364
15	6.8640	5.8523	5.5536	5.5532	5.8513	6.8625
20	8.1674	7.7994	7.5800	7.5805	7.8002	8.1679

TABLE XXIV
APPROXIMATED SHORELINE EVOLUTION ALONG 20 YEARS USING THE TRADITIONAL FORWARD TIME CENTERED SPACE TECHNIQUES WHEN T-HEAD GROIN SIZE 30 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	1.2549	0.4986	0.2042	0.2042	0.4986	1.2549
5	2.8227	2.0405	1.8249	1.8249	2.0405	2.8227
10	3.0897	3.1776	3.0660	3.0660	3.1776	3.0897
15	5.4009	4.6835	4.4527	4.4527	4.6835	5.4009
20	5.8824	5.8600	5.7502	5.7502	5.8600	5.8824

TABLE XXV
APPROXIMATED SHORELINE EVOLUTION ALONG 20 YEARS USING THE SAULYEV FINITE DIFFERENCE TECHNIQUES WHEN T-HEAD GROIN SIZE 30 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	1.2554	0.4992	0.2044	0.2041	0.4982	1.2546
5	2.8240	2.0415	1.8256	1.8249	2.0401	2.8224
10	3.0883	3.1765	3.0652	3.0660	3.1781	3.0900
15	5.4021	4.6844	4.4533	4.4527	4.6830	5.4006
20	5.8817	5.8593	5.7497	5.7502	5.8604	5.8824

VI. DISCUSSION

In this paper, we considered the averaged wave crest impact (α_0) as obtained by (29) for eight T-head groin sizes as seen in Table 1-8. The long-shore transport rate (D) for each month as seen in Table 9.

We used numerical techniques, the traditional forward time centered space techniques (FTCS), and the Saulyev finite difference techniques to approximate the shoreline evolution for eight T-head groin sizes.

The approximated shoreline evolution for T-head groin size 16 m with a time duration of eight years is seen in Table 10, 11, and Fig 18. As a result of shoreline evolution, the longest distance is 15.9615 meters, and the shortest distance is 12.9092 meters.

The approximated shoreline evolution for T-head groin size 18 m with a time duration of 11 years is seen in Table 12, 13, and Fig 19. As a result of shoreline evolution, the longest distance is 14.6822 meters, and the shortest distance is 17.2724 meters.

The approximated shoreline evolution for T-head groin size 20 m with a time duration of 15 years is seen in Table 14, 15, and Fig 20. As a result of shoreline evolution, the longest distance is 19.2123 meters, and the shortest distance is 16.7981 meters.

The approximated shoreline evolution for T-head groin size 22 m with a time duration of 13 years is seen in Table 16, 17, and Fig 21. As a result of shoreline evolution, the longest distance is 21.5786 meters, and the shortest distance is 18.4889 meters.

The approximated shoreline evolution for T-head groin size 24 m with a time duration of 20 years is seen in Table 18, 19, and Fig 22. As a result of shoreline evolution, the longest distance is 7.8627 meters, and the shortest distance is 7.4590 meters.

The approximated shoreline evolution for T-head groin size 26 m with a time duration of 20 years is seen in Table 20, 21, and Fig 23. As a result of shoreline evolution, the longest distance is 15.1303 meters, and the shortest distance is 13.8656 meters.

The approximated shoreline evolution for T-head groin size 28 m with a time duration of 20 years is seen in Table 22, 23 and Fig 24. As a result of shoreline evolution, the longest distance is 8.1679 meters, and the shortest distance is 7.5526 meters.

The approximated shoreline evolution for T-head groin size 30 m with a time duration of 9 years is seen in Table 24, 25, and Fig 25. As a result of shoreline evolution, the longest distance is 5.8824 meters, and the shortest distance is 5.7362 meters.

Approximate shoreline evolutions of all numerical approaches in eight sizes of the considered T-head groin are compatible.

The approximate shoreline evolution of T-head groin sizes of 16, 18, 20, and 22 m is used over time durations of 9, 11, 15, and 13 years, respectively, making the approximate shoreline comparable in size to the T-head groin. Other approximate T-head groin sizes are used over a time duration of 20 years. The approximate shoreline is still in the T-head groin area. The approximate shoreline tends to decrease with T-head groin sizes of 26, 28, and 30 m.

VII. CONCLUSION

In this paper, we introduce a shoreline evolution model was created in this research to adjust for the T-head groin structure. The nonuniform breaking wave crest impact is estimated using the wave crest impact model. The average wave crest impact for eight sizes of T-head groin structures is considered. The shoreline evolution in areas where T-head groins are installed on both sides. The initial condition setting approach and boundary conditions techniques, as well as the structural impacts of the T-head groin, are discussed. The traditional forward time centered space techniques (FTCS) and the unconditionally stable Saulyev finite differential techniques are used to approximate shoreline evolution each year. The estimated impacts of shoreline evolution were consistent with the wave crest impact model for eight different T-head groin sizes. As a result, the size of T-head groin influences the approximated shoreline evolution. The time duration of the approximate shoreline comparable in size to the T-head groin increases as the size of the T-head groin increases. But the size of the T-head groin is too large, the approximate shoreline evolution rate is lower, and the approximate shoreline evolution has resulted in a smaller shoreline area.

REFERENCES

[1] 1 C. Netzold and I. Mohamed, "Seawalls as a response to coastal erosion and flooding: a case study from Grande Comore, Comoros (West Indian Ocean)," Regional Environmental Change 17, pp. 1077–1087, 2017.

[2] 2 A. Fitri. R. Hashim, S. Abolfathi, and Khairul Nizam Abdul Maulud, “Dynamics of Sediment Transport and Erosion-Deposition Patterns in the Locality of a Detached Low-Crested Breakwater on a Cohesive Coast,” Water, Vol. 11, 28 pages, 2019.
[3] 3 Le Xuan Roan, “Some results of comparison between numerical and analytic solutions of the one-line model for shoreline change,” Vietnam Journal of Mechanics, VAST, Vol. 28, pp. 94-102, 2006.
[4] 4 Alia Khurram, and David W Kammler, "Numerical Generation of Images for the Gibbs Phenomenon Near a Corner in the Plane," IAENG International Journal of Applied Mathematics, vol. 44, no. 1, pp. 15-22, 2014.
[5] 5 W. T. Bakker, “The dynamics of coast with a groin system,” Proceeding of 11th Coastal Engineering Conference 1969, pp. 492–517.
[6] 6 W. T. Bakker, and T. Edelman, “The coastline of river deltas,” Proceeding of 9th Coastal Engineering Conference 1965, pp. 199–218.
[7] 7 W. Grijimr, “Theoretical form of shoreline,” Proceeding of 7th Coastal Engineering Conference 1961, pp. 197–202.
[8] 8 W. Grijimr, “Theoretical form of shoreline,” Proceeding of 9th Coastal Engineering Conference 1965, pp. 219–235.
[9] 9 B. Le Mahute, and M. Soldate, “Mathematical Modeling of Shoreline Evolution,” US Army Corps of Engineer Waterways Experiment Station, CERC, 1977.
[10] 10 H. Hanson, M. Larson, and N. C. Kraus, “Analytical Solution of the One-line Model for Shoreline Changel,” US Army Corps of Engineer Waterways Experiment Station, CERC, 1987.
[11] 11 T. Waltonand, and T. Chiu, “A review of analytical technique to solve the sand transport equation and some simplified solution,” Coastal Structure, pp. 809–837, 1979.
[12] 12 M. Mamat Subiyanto, M. F. Ahmad, and M. L. Husain, “Comparison of numerical method for forward and backward time centered space for long – term simulation of shoreline evolution,” Applied Mathematical Sciences, Vol. 7, pp. 5165–5173, 2013.
[13] 13 Pidok Unyapoti and Nopparat Pochai, “A One-Dimensional Mathematical Model of Long-Term Shoreline Evolution with Groin System using an Unconditionally Stable Explicit Finite Difference Method,” International Journal of Simulation: Systems, Science and Technology, vol. 21, no.3, pp. 2.1-2.6, 2020.
[14] 14 N. Pochai, “Unconditional stable numerical techniques for a water-quality model in a non-uniform flow stream,” Advances in Difference Equations, vol. 2017, Article ID 286, 13 pages, 2017.
[15] 15 P. Samalerk, and N. Pochai, “Numerical Simulation of a One-Dimensional Water-Quality Model in a Stream Using a Saulyev Technique with Quadratic Interpolated Initial-Boundary Conditions,” Abstract and Applied Analysis, vol. 2018, Article ID 1926519, 7 pages, 2018.
[16] 16 Shixiang Zhou, Fanwei Meng, Qinghua Feng, and Li Dong, "A Spatial Sixth Order Finite Difference Scheme for Time Fractional Sub-diffusion Equation with Variable Coefficient," IAENG International Journal of Applied Mathematics, vol. 47, no. 2, pp. 175-181, 2017.
[17] 17 Kewalee Suebyat, and Nopparat Pochai, "A Numerical Simulation of a Three-dimensional Air Quality Model in an Area Under a Bangkok Sky Train Platform Using an Explicit Finite Difference Scheme," IAENG International Journal of Applied Mathematics, vol. 47, no. 4, pp. 471-476, 2017.
[18] 18 Tata Sutardi, Linwei Wang, Manosh C. Paul, and Nader Karimi, "Numerical Simulation Approaches for Modelling a Single Coal Particle Combustion and Gasification," Engineering Letters, vol. 26, no. 2, pp. 257-266, 2018.
[19] 19 Ben Wongsaijai, Kanyuta Poochinapan, and Thongchai Disyadej, "A Compact Finite Difference Method for Solving the General Rosenau-RLW Equation," IAENG International Journal of Applied Mathematics, vol. 44, no. 4, pp. 192-199, 2014.
[20] 20 Linna Li, Zhirou Wei, and Qiongdan Huang, "A Numerical Method for Solving Fractional Variational Problems by the Operational Matrix Based on Chelyshkov Polynomials," Engineering Letters, vol. 28, no. 2, pp. 486-491, 2020.
[21] 21 Pornpon Othata and Nopparat Pochai, "A Mathematical Model of Salinity Control in a River with an Effect of Internal Waves using Two Explicit Finite Difference Methods," Engineering Letters, vol 29, no 2, pp.689-696, 2021.
[22] 22 M. S. Jara, M. González, and R. Medina, “Shoreline evolution model from a dynamic equilibrium beach profile,” Coastal Engineering, vol. 99, pp. 1-14, 2015.
[23] 23 L. P. Stein, and E. Siegle, “Overtopping events on seawall-backed beaches: Santos Bay, SP, Brazil,” Regional Studies in Marine Science, Vol. 40, 101492, 2020.
[24] 24 Dalrino, R. Herdianto and D. B. Silitonga, “Study of groin structures effectiveness for against abrasion in Padang Beach,” IOP Conference Series: Earth and Environmental Science, vol. 708, pp. 012035, 2021.
[25] 25 Witsarut Kraychang, and Nopparat Pochai, "A Simple Numerical Model for Water Quality Assessment with Constant Absorption around Nok Phrao Island of Trang River," Engineering Letters, vol. 28, no. 3, pp. 912-922, 2020.
[26] 26 Witsarut Kraychang and Nopparat Pochai, “Numerical Treatment to a Water-Quality Measurement Model in an Opened-Closed Reservoir,” Thai Journal of Mathematics, vol.13, pp. 775-788, 2015.
[27] 27 Pidok Unyapoti and Nopparat Pochai, “A Shoreline Evolution Model with a Twin Groins Structure using Unconditionally Stable Explicit Finite Difference Techniques.” Engineering Letters, vol. 29, no. 1, pp. 288-296, 2021.

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