Event-Trigger-Based Output Feedback Controller Design for a Class of Strict-Feedback Systems

Zhanbo Xu, Shaochuan Xu, and Haizhi Jiang

***Abstract*—In the article, to address the tracking control problem in nonlinear systems with immeasurable states, an event-trigger-based adaptive controller with fuzzy state observer is designed. Firstly, an adaptive control method based on backstepping is used for the rational design of control parameters. Then, a state observer is introduced to estimate the unknown states and a fuzzy-logic system (FLS) is used to approximate unknown nonlinear functions. The above control mode can make the controller more widely used. Furthermore, an event-trigger strategy with fixed threshold is adopted to save the network resources and improve the system performance. Finally, the novel tracking controller is developed through Lyapunov stability analysis to ensure all signals are bounded in the closed-loop system (CLS) and the tracking error can converge to a small number field ultimately. The effectiveness of the design approach is demonstrated by simulation.**

***Index Terms*—tracking control, event-trigger strategy, fuzzy state observer, adaptive controller**

I. Introduction

It is known that most systems in actual production are nonlinear systems. Thus the tracking control of nonlinear systems is crucial in the theoretic and practical research of the system control. It is noted that nonlinear systems are usually characterized by severe parameter uncertainty, unknown time-varying control coefficients, and not containing the precise information of their bounds. It is almost impossible to include known mathematical model structures and known parameters in actual nonlinear systems. In order to better achieve the preset goals of the system, the tracking error needs to be adjusted to ensure the output tracking the reference signal [1-3]. To accomplish this purpose, an adaptive control method based on backstepping [4-8] is presented for a class of nonlinear systems. The method can design parameters in the process of recurrence and finally obtain the control law of the system. With immeasurable states and unknown mathematical constructions contained in the nonlinear systems, the fuzzy state observers [9-13] are introduced to estimate the unknown states and approximate unknown nonlinear functions. The application of the observer can make the controller more widely used and improve the flexibility of the system control. In the control design of the nonlinear systems, it is inappropriate that the output is continuously fed back to the system, whether required or not. This will result in limited system resources. To address the above issues, the event-triggered strategies [14-18] with a fixed threshold are designed into the controller. The strategies can save network resources and improve system performance. It can be seen that there no article in the above simultaneously designs the controller with immeasurable states in the system and saves the network resources. Thus a novel event-trigger-based adaptive controller containing fuzzy logic observer is designed. The controller can estimate the immeasurable states and the unknown nonlinear functions. It can also improve the system performance by releasing the system resource space. Compared with the two-wheeled mobile manipulator controller designed in [19], the proposed controller is able to better adapt to systems containing immeasurable states. For the fuzzy observer based nonlinear controller designed in [20], the proposed controller have the advantage of reducing excess resource consumption by applying the event-triggered strategy. Eventually, it can be proven all signals are bounded in the CLS and the controller can achieve the goal of tracking control.

Manuscript received September 1, 2022; revised March 12, 2023.

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II. Problem formulations

Consider the following nonlinear system:

$$\begin{cases} \dot{x}_m = x_{m+1} + f_m(\overline{x}_m), \\ 1 \le m \le n-1, \\ \dot{x}_n = u(t) + f_n(\overline{x}_n), \\ y = x_1, \end{cases} \tag{1}$$

In the above system equation, $\overline{x}_i = [x_1, ..., x_i]^T \in R^i$ represents the system state vector, $i = 1, 2, ..., n$. $y \in R$ is the system output. $f_m(\cdot)$ $(m = 1, ..., n)$ represents the unknown nonlinear function and $f_m(0) = 0$. In addition, $u(t)$ represents the input of the considered system, and it has $u(t) = 0$ when $t \le 0$.

For system (1), the control objectives are described as follows: under the condition that the states $x_2, ..., x_n$ are immeasurable, an adaptive tracking controller with a fuzzy observer is designed to ensure that the output y can track the given signal y_r , and the error between two signals is convergent to a small field finally. All signals are bounded in the strict feed-back system. To accomplish the above objectives, the following assumption need to be established:

Assumption 1: The given signal y_r and its derivatives y_r^k are bounded.

III. Observer design

The advantages of the fuzzy state observer include that the control law introduced by recursion does not contain the unknown functions, and it also can estimate the immeasurable states in the system. It can make the design methods apply in more occasions. For error estimation, good observation performance can be guaranteed by fuzzy logic system approximation. Based on the problem of uncertainty in the system, fuzzy state observers with fuzzy logic systems are more widely used in practical nonlinear systems. The above fuzzy logic system is mainly composed of these four parts of fuzzifier, defuzzier, knowledge base, and fuzzy inference engine. The fuzzy logic system has the following inference rules.

R^l : If χ_1 is F_1^l, and χ_2 is F_2^l, and,..., χ_n is F_n^l, then Y is E^l where $l=1,...,N$. $x=[\chi_1,...,\chi_n]^T$ stands for the input of FLS and Y stands for the output of FLS. E^l and F_n^l represent fuzzy sets which correspond to fuzzy membership function and N represents the number of rules. By the analysis and application of product inference, center average defuzzifcation and Singleton function , we define FLS as:

$$Y(\chi)=\frac{\sum_{l=1}^{N}\bar{Y}_L\prod_{i=1}^{n}\mu F_i^l(\chi_i)}{\sum_{l=1}^{N}\left[\prod_{i=1}^{n}\mu F_i^l(\chi_i)\right]}, \tag{2}$$

where $\bar{Y}_l=\max_{y\in R}\mu E^l(y)$, $\mu E_i^l(\chi_i)$ and $\mu F_i^l(\chi_i)$ represent fuzzy membership functions.

We define the fuzzy basis functions as:

$$\varsigma_l=\frac{\prod_{i=1}^{n}\mu F_i^l(\chi_i)}{\sum_{l=1}^{N}\left[\prod_{i=1}^{n}\mu F_i^l(\chi_i)\right]}. \tag{3}$$

By applying (3), (2) can be transformed as:

$$Y(\chi)=\hat{\theta}^T\varsigma(\chi), \tag{4}$$

where $\varsigma(\chi)=\left[\varsigma_1(\chi),\varsigma_2(\chi),...,\varsigma_N(\chi)\right]^T$, and $\hat{\theta}=\left[\bar{Y}_1,...,\bar{Y}_N\right]^T$.

Lemma 1 [9]: If $f(x)$ is defined on a compact set Ω and it is a continuous function, then the FLS (4) can have the following form for any positive constant ε.

$$\sup_{\chi\in R}\left|f(\chi)-\hat{\theta}^T\varsigma(\chi)\right|\le\varepsilon. \tag{5}$$

From **Lemma 1**, We can estimate the nonlinear function $f_m(\bar{x}_m)(1\le m\le n)$ in the above system by using the FLS of the following form.

$$\hat{f}_m\left(\hat{\bar{x}}_m\mid\hat{\theta}_m\right)=\hat{\theta}_m^T\varsigma_m\left(\hat{\bar{x}}_m\right), \tag{6}$$

where $\hat{\bar{x}}_m=\left[\hat{x}_1,...,\hat{x}_m\right]^T$ is the estimation of $\bar{x}_m=\left[x_1,...,x_m\right]^T$.

Define the optimal parameter vector θ_m^* in θ_m to be of the following form.

$$\theta_m^*=\arg\min_{\theta_m\in\Omega_m}\left\{\sup_{\left(\hat{\bar{x}}_m,\bar{x}_m\right)\in U_m}\left|\hat{f}_m\left(\hat{\bar{x}}_m\mid\theta_m\right)-f_m\left(\bar{x}_m\right)\right|\right\}, \tag{7}$$

where Ω_m is the bounded set of θ_m and U_m is the bounded set of $\hat{\bar{x}}_m$ and $\bar{x}_m$. Furthermore, the minimal approximation error can be defined by

$$\varepsilon_m=f_m(\bar{x})-\hat{f}_i\left(\hat{\bar{x}}_m\mid\theta_m^*\right), \tag{8}$$

where $|\varepsilon_m|\le\varepsilon_m^*$ and $\varepsilon^*_m>0$ is a constant. Then, to estimate the immeasurable states in the system, the following structure of the fuzzy state observer is established.

$$\begin{cases}\dot{\hat{x}}_1=k_1e_1+\hat{f}_1(\hat{\bar{x}}_1)+\hat{x}_2,\\ \dot{\hat{x}}_2=k_2e_1+\hat{f}_2(\hat{\bar{x}}_2)+\hat{x}_3,\\ \vdots\\ \dot{\hat{x}}_n=k_ne_1+\hat{f}_n(\hat{\bar{x}}_n)+u.\end{cases} \tag{9}$$

Thus, (9) can be obtained as follows:

$$\begin{cases}\dot{\hat{X}}=A\hat{X}+Wy+\sum_{i=1}^{n}H_i\hat{f}_i\left(\hat{\bar{x}}_i\,|\theta_i\right)+Mu,\\ \hat{y}=D\hat{X},\end{cases} \tag{10}$$

where $A=\begin{bmatrix}-k_1 & \\ \vdots & I_{n-1}\\ -k_n & 0\ \cdots\ 0\end{bmatrix}$, $W=[k_1,k_2,...,k_n]^T$, $\hat{X}=\left[\hat{x}_1,...,\hat{x}_n\right]^T$, $M=\left[0,...,1\right]^T$, $H_i=\left[\underbrace{0,...,1}_{i},...,0\right]^T$, and $D=\left[1,...,0\right]$.

Select suitable parameters so that A is a strict Hurwitz matrix. Then, from [9], one has

$$A^TP+PA=-2Q \tag{11}$$

where P and Q are both positive definite symmetric matrices.

Similar to (10), the above system is expressed as:

$$\begin{cases}\dot{X}=AX+Wy+\sum_{i=1}^{n}H_if_i(\bar{x}_i)+Mu,\\ \hat{y}=D\hat{X},\end{cases} \tag{12}$$

The error vector e is described as:

$$e=X-\hat{X}, \tag{13}$$

where $e=\left[e_1,...,e_n\right]^T$.

Therefore, from $\tilde{\theta}_i=\theta_i^*-\hat{\theta}_i, i=1,...,n$, the derivative of e is

$$\dot{e}=\dot{X}-\dot{\hat{X}}=Ae+\varepsilon+\sum_{i=1}^{n}H_i\tilde{\theta}_i^T\varsigma_i\left(\hat{\bar{x}}_i\right), \tag{14}$$

where $\varepsilon=\left[\varepsilon_{1,}\cdots\varepsilon_n\right]^T$.

Choose the Lyapunov function as:

$$V_0=\frac{1}{2}e^TPe. \tag{15}$$

From (15), the derivative of V_0 yields:

$$\dot{V}_0=e^TP\dot{e}. \tag{16}$$

Substituting (14) into (16) gives

$$\begin{aligned}\dot{V}_0&=e^TP\left(Ae+\varepsilon+\sum_{i=1}^{n}H_i\tilde{\theta}_i^T\varsigma_i\left(\hat{\bar{x}}_i\right)\right)\\&=-e^TQe+e^TP\varepsilon+e^TP\sum_{i=1}^{n}H_i\tilde{\theta}_i^T\varsigma_i\left(\hat{\bar{x}}_i\right).\end{aligned}\tag{17}$$

Generally, $\varsigma_i\left(\hat{\bar{x}}_i\right)$ is chosen as Gaussian function. It gives $\varsigma_m\left(\hat{x}_m\right)^T\varsigma_m\left(\hat{x}_m\right)\leq 1$. Using Young's inequality, it gives

$$e^TP\sum_{i=1}^{n}H_i\tilde{\theta}_i^T\varsigma_i\left(\hat{\bar{x}}_i\right)\leqslant\frac{n}{2}\|e\|^2+\frac{1}{2}\|P\|^2\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i,\tag{18}$$

$$e^TP\varepsilon\leqslant\frac{1}{2}\|P\|^2\left\|\varepsilon^*\right\|^2+\frac{1}{2}\|e\|^2,\tag{19}$$

where $\varepsilon^*=\left[\varepsilon_1,\varepsilon_2,...,\varepsilon_n\right]^T$.

Substituting (18) and (19) into (17) produces

$$\dot{V}_0\leqslant-p_0\|e\|^2+\sum_{i=1}^{n}\frac{1}{2}\|P\|^2\tilde{\theta}_m^T\tilde{\theta}_m+q,\tag{20}$$

where $p_0=\min\left\{\lambda_{\min}(Q)-\frac{n}{2}-\frac{1}{2}\right\}>0$, $q=\frac{1}{2}\sum_{m=1}^{n}\|P\|^2\left\|\varepsilon^*\right\|^2$, and $\lambda_{\min}(Q)$ is the minimal eigenvalue of Q.

IV. Backstepping Controller Design

In this part, by using adaptive control based on backstepping, the controller of the nonlinear system can be designed in the last step finally.

Firstly, the coordinate transformation is expressed in the following form.

$$\begin{cases}z_1=x_1-y_r,\\ z_i=\hat{x}_i-\alpha_{i-1}-y_r^{(i-1)},\quad i=2,...,n.\end{cases}\tag{21}$$

where z_1 and z_i represent transformation errors, and $\alpha_1,\alpha_2,...,\alpha_{n-1}$ are the virtual control laws which appear at every step of the design.

Step 1:

According to (1) and (21), the derivative of z_1 is as follows:

$$\begin{aligned}\dot{z}_1&=\dot{x}_1-\dot{y}_r\\&=x_2+f_1\left(\bar{x}_1\right)-\dot{y}_r\\&=\hat{x}_2+e_2+\hat{f}_1\left(\hat{\bar{x}}_1\right)+\varepsilon_1-\dot{y}_r.\end{aligned}\tag{22}$$

Substituting $z_2=\hat{x}_2-\alpha_1-\dot{y}_r$ into (22) produces

$$\dot{z}_1=z_2+\alpha_1+e_2+\hat{f}_1(\hat{\bar{x}}_1)+\varepsilon_1.\tag{23}$$

For the first subsystem in (1), select the Lyapunov function as:

$$V_1=\frac{1}{2}z_1^2+\frac{1}{2\gamma_1}\tilde{\theta}_1^T\tilde{\theta}_1,\tag{24}$$

where $r_1>0$ is a positive parameter that is designed later.

Then, substitute $\dot{z}_1$ and $\tilde{\theta}_1=\theta_1^*-\hat{\theta}_1$ into the derivative of V_1, it yields

$$\begin{aligned}\dot{V}_1&=z_1\dot{z}_1-\frac{1}{\gamma_1}\tilde{\theta}_1^T\dot{\hat{\theta}}_1\\&=z_1(z_2+\alpha_1+e_2+\hat{\theta}_1^T\varsigma_1(\hat{\bar{x}}_1)+\varepsilon_1)\\&\quad-\frac{1}{\gamma_1}\tilde{\theta}_1^T\dot{\hat{\theta}}_1+z_1\tilde{\theta}_1^T\varsigma_1(\hat{\bar{x}}_1).\end{aligned}\tag{25}$$

By using Young's inequality, it gives

$$\begin{aligned}z_1e_2&\leq\frac{1}{2}z_1^2+\frac{1}{2}\|e\|^2,\\ z_1\varepsilon_1&\leq\frac{1}{2}z_1^2+\frac{1}{2}\left\|\varepsilon^*\right\|^2.\end{aligned}\tag{26}$$

From (25) and (26), the following result is obtained.

$$\begin{aligned}\dot{V}_1&\leq z_1z_2+z_1\left(\alpha_1+\hat{\theta}_1^T\varsigma_1(\hat{\bar{x}}_1)+z_1\right)\\&\quad+\frac{1}{2}\|e\|^2-\frac{1}{\gamma_1}\tilde{\theta}_1\left(\dot{\hat{\theta}}_1-z_1\gamma_1\varsigma_1\left(\hat{\bar{x}}_1\right)\right)+\frac{1}{2}\left\|\varepsilon^*\right\|^2.\end{aligned}\tag{27}$$

Therefore, α_1 and $\dot{\hat{\theta}}_1$ can be designed as:

$$\begin{cases}\alpha_1=-\hat{\theta}_1^T\varsigma_1(\hat{\bar{x}}_1)-z_1-c_1z_1,\\ \dot{\hat{\theta}}_1=z_1\gamma_1\varsigma_1(\hat{\bar{x}}_1)-w_1\hat{\theta}_1,\end{cases}\tag{28}$$

where $c_1>0$ and $w_1>0$ the the parameters being designed.

Substituting α_1 and $\dot{\hat{\theta}}_1$ into (27) gives

$$\dot{V}_1\leq z_1z_2-c_1z_1^2+\frac{1}{2}\|e\|^2+\frac{w_1}{\gamma_1}\tilde{\theta}_1^T\hat{\theta}_1+\frac{1}{2}\left\|\varepsilon^*\right\|^2.\tag{29}$$

From $\tilde{\theta}_1^T\hat{\theta}_1\leq\tilde{\theta}_1^T\left(\theta_1^*-\tilde{\theta}_1\right)\leq-\frac{1}{2}\tilde{\theta}_1^T\tilde{\theta}_1+\frac{1}{2}\|\theta_1^*\|^2$, it produces

$$\dot{V}_1\leq z_1z_2-c_1z_1^2+\frac{1}{2}\|e\|^2-\frac{w_1}{2\gamma_1}\tilde{\theta}_1^T\tilde{\theta}_1+\sigma_1+\frac{1}{2}\left\|\varepsilon^*\right\|^2,\tag{30}$$

where $\sigma_1=\frac{w_1}{2\gamma_1}\|\theta_1^*\|^2$.

Step i:

Similarly, from (9) and (21), the derivative of z_i is as follows:

$$\begin{aligned}\dot{z}_i&=\dot{\hat{x}}_i-\dot{\alpha}_{i-1}-y_r^{(i)}\\&=k_ie_1+\hat{f}_i\left(\hat{\bar{x}}_i\right)+z_{i+1}+\alpha_i-\frac{\partial\alpha_{i-1}}{\partial x_1}e_2-\frac{\partial\alpha_{i-1}}{\partial x_1}\varepsilon_1-D_{i-1},\end{aligned}\tag{31}$$

where

$$\begin{aligned}D_{i-1}&=\frac{\partial\alpha_{i-1}}{\partial x_1}(\hat{x}_2+\hat{f}_1(\hat{\bar{x}}_1))+\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j\\&\quad+\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j+\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(j-1)}}y_r^{(j)}.\end{aligned}\tag{32}$$

Select the Lyapunov function for the i-th subsystem as:

$$V_i=V_{i-1}+\frac{1}{2}z_i^2+\frac{1}{2\gamma_i}\tilde{\theta}_i^T\tilde{\theta}_i,\tag{33}$$

where $r_i>0$ is a positive parameter that is designed later.

Substitute $\dot{z}_i$ and $\tilde{\theta}_i=\theta_i^*-\hat{\theta}_i$ into the derivative of V_i, it produces

$$\begin{aligned}\dot{V}_i&=\dot{V}_{i-1}+z_i\dot{z}_i-\frac{1}{\gamma_i}\tilde{\theta}_i^T\dot{\hat{\theta}}_i\\&=\dot{V}_{i-1}+z_i\left(k_ie_1+\hat{f}_i\left(\hat{\bar{x}}_i\right)+z_{i+1}+\alpha_i-\frac{\partial\alpha_{i-1}}{\partial x_1}e_2-\frac{\partial\alpha_{i-1}}{\partial x_1}\varepsilon_1-D_{i-1}\right)-\frac{1}{\gamma_i}\tilde{\theta}_i^T\dot{\hat{\theta}}_i\\&=\dot{V}_{i-1}+z_i\left(k_ie_1+\hat{\theta}_i\varsigma_i\left(\hat{\bar{x}}_i\right)+z_{i+1}+\alpha_i-\frac{\partial\alpha_{i-1}}{\partial x_1}e_2-\frac{\partial\alpha_{i-1}}{\partial x_1}\varepsilon_1-D_{i-1}\right)\\&\quad+\frac{1}{\gamma_i}\tilde{\theta}_i^T\left(\gamma_iz_i\varsigma_i\left(\hat{\bar{x}}_i\right)-\dot{\hat{\theta}}_i\right)-z_i\tilde{\theta}_i\varsigma_i(\hat{\bar{x}}_i).\end{aligned}\tag{34}$$

According to Young's inequality, it gives:

$$-z_i\tilde{\theta}_i^T\varsigma_i(\hat{\bar{x}}_i) \le \frac{1}{2}z_i^2 + \frac{1}{2}\tilde{\theta}_i^T\tilde{\theta}_i, \quad (35)$$

$$-z_i\frac{\partial\alpha_{i-1}}{\partial x_1}e_2 \le \frac{1}{2}z_i^2\left(\frac{\partial\alpha_{i-1}}{\partial x_1}\right)^2 + \frac{1}{2}\|e\|^2, \quad (36)$$

$$-z_i\frac{\partial\alpha_{i-1}}{\partial x_1}\varepsilon_1 \le \frac{1}{2}z_i^2\left(\frac{\partial\alpha_{i-1}}{\partial x_1}\right)^2 + \frac{1}{2}\|\varepsilon^*\|^2. \quad (37)$$

Substitute (35), (36), and (37) into (34), the following result holds.

$$\dot{V}_i \le \dot{V}_{i-1} + z_i\left(k_ie_1 + z_i\hat{\theta}_i\varsigma_i(\hat{\bar{x}}_i) + z_{i+1} + \alpha_i + z_i\left(\frac{\partial\alpha_{i-1}}{\partial x_1}\right)^2 - D_{i-1} + \frac{1}{2}z_i\right)$$
$$+\frac{1}{2}\|e\|^2 + \frac{1}{\gamma_i}\tilde{\theta}_i^T\left(\gamma_iz_i\varsigma_i(\hat{\bar{x}}_i) - \dot{\hat{\theta}}_i\right) + \frac{1}{2}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2}\|\varepsilon^*\|^2. \quad (38)$$

Thus, α_i and $\dot{\theta}_i$ can be designed as:

$$\begin{cases}\alpha_i = -c_iz_i - k_ie_i - z_i\hat{\theta}_i\varsigma_i(\hat{\bar{x}}_i) - z_{i-1} + D_{i-1} - z_i\left(\frac{\partial\alpha_{i-1}}{\partial x_1}\right)^2 - \frac{1}{2}z_i, \\ \dot{\hat{\theta}} = z_i\gamma_i\varsigma_i(\hat{\bar{x}}_i) - w_i\hat{\theta}_i,\end{cases} \quad (39)$$

where $c_i > 0$ and $w_i > 0$ are the parameters being designed.

Substituting α_i and $\dot{\hat{\theta}}_i$ into (38) gives:

$$\dot{V}_i \le \dot{V}_{i-1} + z_iz_{i+1} + \frac{1}{2}\|e\|^2 - c_iz_i^2$$
$$+\frac{w_i}{\gamma_i}\tilde{\theta}_i^T\hat{\theta}_i + \frac{1}{2}\|\varepsilon^*\|^2 + \frac{1}{2}\tilde{\theta}_i^T\tilde{\theta}_i. \quad (40)$$

From $\tilde{\theta}_i^T\hat{\theta}_i \le \tilde{\theta}_i^T\left(\theta_i^* - \tilde{\theta}_i\right) \le -\tilde{\theta}_i^T\tilde{\theta}_i/2 + \|\theta_i^*\|^2/2$, it produces that

$$\dot{V}_i \le z_iz_{i+1} - \sum_{j=1}^{i}c_jz_j^2 + \frac{i}{2}\|e\|^2$$
$$+\sum_{j=1}^{i}\frac{w_i}{2\gamma_j}\tilde{\theta}_j^T\tilde{\theta}_j + \sigma_i + \frac{i}{2}\|\varepsilon^*\|^2 + \frac{1}{2}\sum_{j=2}^{i}\tilde{\theta}_j^T\tilde{\theta}_j, \quad (41)$$

where $\sigma_i = \sum_{j=1}^{i}\frac{w_j}{2\gamma_j}\|\theta_j^*\|^2$.

Step n:

Similarly, from (9) and (21), the derivative of z_i has the following form.

$$\dot{z}_n = \hat{x}_n - \dot{\alpha}_{n-1} - y_r^{(n)}$$
$$= k_ne_1 + \hat{f}_n(\hat{\bar{x}}_n) + u - \frac{\partial\alpha_{n-1}}{\partial x_1}e_2 - \frac{\partial\alpha_{n-1}}{\partial x_1}\varepsilon_1 - D_{n-1} - y_r^{(n)}, \quad (42)$$

where

$$D_{n-1} = \frac{\partial\alpha_{n-1}}{\partial x_1}(\hat{x}_2 + f_1(\hat{\bar{x}}_1)) + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j$$
$$+\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(j-1)}}y_r^{(j)}. \quad (43)$$

Then, α_n can be designed as follows:

$$\alpha_n = -c_nz_n - k_ne_1 - \hat{\theta}_n\varsigma_n(\hat{\bar{x}}_n) - z_{n-1}$$
$$+D_{n-1} - z_n\left(\frac{\partial\alpha_{n-1}}{\partial x_1}\right)^2 - \frac{1}{2}z_n. \quad (44)$$

To be able to better improve the performance of the system, an effective event triggered control strategy is applied. For this system, based on the the consideration of the system's application condition and the tracking error, fixed threshold scheme is used. Thus, the controller is designed as the following form.

$$h(t) = \alpha_n + y_r^{(n)} - \bar{b}\tanh\left(\frac{z_n\bar{b}}{\Omega}\right), \quad (45)$$

$$\dot{\hat{\theta}}_n = z_n\gamma_n\varsigma_n(\hat{\bar{x}}_n) - w_n\hat{\theta}_n. \quad (46)$$

The triggered event can be rendered as

$$u(t) = h(t_k), \quad \forall t \in [t_k, t_{k+1}), \quad (47)$$

$$t_{k+1} = \inf\{t \in R\,|\,|\omega(t) \ge b|\}, t_1 = 0, \quad (48)$$

where $\omega(t) = h(t) - u(t)$ denotes the input error, Ω, b, and $\bar{b} > b$ are design parameters which are positive. $t_k, k \in Z^+$ is the controller update time. Whenever $|\omega(t) \ge b|$, the time will be updated to t_{k+1}, then $u(t_{k+1})$ will be applied to the system. When $t \in [t_k, t_{k+1})$, the input signal is kept as a constant $h(t_k)$.

When t is within the interval of $[t_k, t_{k+1})$, combined with (48), $|h(t) - u(t)| \le b$ is obtained. Therefore, a continuous time-varying parameter $\xi(t)$ is existed and it satisfies $\xi(t_k) = 0$, $\xi(t_{k+1}) = \pm 1$ and $|\xi(t)| \le 1$, $\forall t \in [t_k, t_{k+1})$, such that $h(t) = u(t) + \xi(t)b$.

Select the Lyapunov function for the overall system as:

$$V = V_0 + V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2\gamma_n}\tilde{\theta}_n^T\theta_n. \quad (49)$$

Thus, the derivative of V is as follows:

$$\dot{V} = \dot{V}_0 + \dot{V}_{n-1} + z_n\dot{z}_n - \frac{1}{\gamma_n}\tilde{\theta}_n^T\tilde{\theta}_n$$
$$\le \dot{V}_0 + z_{n-1}z_n - \sum_{j=1}^{n-1}c_jz_j^2 + \frac{n-1}{2}\|e\|^2 + \frac{n-1}{2}\|\varepsilon^*\|^2$$
$$+\frac{1}{2}\sum_{j=2}^{n-1}\tilde{\theta}_j^T\tilde{\theta}_j + \sum_{j=1}^{n-1}\frac{w_j}{2\gamma_j}\tilde{\theta}_j^T\tilde{\theta}_j + \sigma_{n-1} \quad (50)$$
$$+z_n\left(k_ne_1 + \hat{f}_n(\hat{x}_n) + h(t) - \xi(t)b - \frac{\partial\alpha_{n-1}}{\partial x_1}e_2 - D_{n-1} - y_r^{(n)} - \frac{\partial\alpha_{n-1}}{\partial x_1}\varepsilon_1\right)$$
$$+\frac{1}{\gamma_n}\tilde{\theta}_n^T\left(\gamma_nz_n\varsigma_n(\hat{\bar{x}}_n) - \dot{\hat{\theta}}_n\right) - z_n\tilde{\theta}_n\varsigma_n(\hat{\bar{x}}_n).$$

According to Young's inequality, it produces

$$-z_n\tilde{\theta}_n^T\varsigma_n(\hat{\bar{x}}_n) \le \frac{1}{2}z_n^2 + \frac{1}{2}\tilde{\theta}_n^T\tilde{\theta}_n, \quad (51)$$

$$-z_n\frac{\partial\alpha_{n-1}}{\partial x_1}e_2 \le \frac{1}{2}z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_1}\right)^2 + \frac{1}{2}\|e\|^2, \quad (52)$$

$$-z_n\frac{\partial\alpha_{n-1}}{\partial x_1}\varepsilon_1 \le \frac{1}{2}z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_1}\right)^2 + \frac{1}{2}\|\varepsilon^*\|^2. \quad (53)$$

Substitute (51), (52), (53), $h(t)$, and $\dot{\hat{\theta}}_n$ into (50), $\dot{V}$ can be obtained.

$$\dot{V} \le \dot{V}_0 + z_n\left(-\bar{b}\tanh\left(\frac{z_n\bar{b}}{\varepsilon}\right) - \xi(t)b\right) - \sum_{j=1}^{n} c_j z_j^2 + \frac{n}{2}\|e\|^2 + \frac{n}{2}\|\varepsilon^*\|^2 + \frac{1}{2}\sum_{j=2}^{n}\tilde{\theta}_j^T\tilde{\theta}_j + \sum_{j=1}^{n}\frac{k_j}{2\gamma_j}\tilde{\theta}_j^T\tilde{\theta}_j + \sigma_n, \tag{54}$$

where

$$\sigma_n = \sum_{j=1}^{n}\frac{w_j}{2\gamma_j}\|\theta_j^*\|^2. \tag{55}$$

From [14], it gives that

$$0 \le |l| - b\tanh\left(\frac{l}{A}\right) \le 0.2785\Omega, \tag{56}$$

where $l \in R$ and $A > 0$.

Then, we have

$$\dot{V} \le \dot{V}_0 - \sum_{j=1}^{n} c_j z_j^2 + \frac{n}{2}\|e\|^2 + \frac{n}{2}\|\varepsilon^*\|^2 + \frac{1}{2}\sum_{j=2}^{n}\tilde{\theta}_j^T\tilde{\theta}_j - \sum_{j=1}^{n}\frac{k_j}{2\gamma_j}\tilde{\theta}_j^T\tilde{\theta}_j + \sigma_n + 0.2785\Omega. \tag{57}$$

From (20) and (57), one produces

$$\begin{aligned}\dot{V} \le & -\sum_{j=1}^{n} c_j z_j^2 + \frac{n}{2}\|e\|^2 + \frac{n}{2}\|\varepsilon^*\|^2 + \frac{1}{2}\sum_{j=2}^{n}\tilde{\theta}_j^T\tilde{\theta}_j - \sum_{j=1}^{n}\frac{k_j}{2\gamma_j}\tilde{\theta}_j^T\tilde{\theta}_j + \sigma_n + 0.2785\Omega \\ & - p_0\|e\|^2 + \sum_{j=1}^{n}\frac{1}{2}\|P\|^2\tilde{\theta}_j^T\tilde{\theta}_j + q \\ \le & -\sum_{j=1}^{n} c_j z_j^2 - p\|e\|^2 - \beta_1\tilde{\theta}_1^T\tilde{\theta}_1 - \sum_{j=2}^{n}\beta_j\tilde{\theta}_j^T\tilde{\theta}_j + c,\end{aligned} \tag{58}$$

where $p = p_0 - \frac{n}{2}$, $\beta_1 = \frac{w_1}{2\gamma_1} - \frac{1}{2}\|P\|^2$, $\beta_j = \frac{w_j}{2\gamma_j} - \frac{1}{2} - \frac{1}{2}\|P\|^2, j = 2,..,n,$ and $c = q + \frac{n}{2}\|\varepsilon^*\|^2 + 0.2785\Omega + \sigma_n$.

Choose the suitable parameters to make $p > 0$ and $\beta_i > 0$ and construct the following definition:

$$\alpha = \min\{2p/\lambda_{\min}(P), 2c_i, 2\beta_i\gamma_i\}, i = 1,\ldots,n. \tag{59}$$

Then, (58) can be rewritten as

$$\dot{V} \leqslant -\alpha V + c. \tag{60}$$

Thus, we can conclude from (60) that:

$$0 \leqslant V(t) \leqslant V(0)e^{-\alpha(t-t_0)} + \frac{c}{\alpha}, \tag{61}$$

where t_0 is the initial time. From (61), it shows that z_1^2 is bounded. Next we can demonstrate that there is a $t^* > 0$ satisfying $\forall k \in z^+, \{t_{k+1} - t_k\} \ge t^*$.

From $\omega(t) = h(t) - u(t)$, $\forall t \in [t_k, t_{k+1})$, we have

$$\frac{d}{dt}|\omega| = \frac{d}{dt}(\omega * \omega)^{\frac{1}{2}} = sign(\omega)\dot{\omega} \le |\dot{h}| \tag{62}$$

From (45), one has

$$\dot{h} = \dot{\alpha}_n + y_r^{n+1} - \frac{\bar{b}\dot{z}_n}{\cosh^2\left(\frac{z_n\bar{b}}{\Omega}\right)} \tag{63}$$

Based on the above, it is clear that $\dot{h}$ must be continuous. Since z and $y_r^{(k)}$ is bounded respectively, x is also bounded. As h and $\dot{h}$ is for the equation of x and θ, all the signals in CLS are bounded. It can also be proven that $\dot{h}$ is bounded. Therefore, a constant $\Delta > 0$ exists that satisfies $|\dot{h}| \le \Delta$. From $\omega(t_k) = 0$ and $\lim_{t \to t_{k+1}} \omega(t) = b$, it can be concluded that the lower bound of event triggered intervals t^* must satisfy $t^* \ge b/\Delta$.

V. Simulation

We can use simulation to check the feasibility of the designed control method and controller in this section. To validate our argument, we use a practical second-order nonlinear system for the simulation. The single-link system is given as:

$$\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = u + \frac{1}{M}\left(\frac{mmgl\sin(x_1)}{2}\right), \\ y = x_1\end{cases} \tag{64}$$

The parameters of the system are $mm = 0.05$, $l = 0.5$, $M = 1$, $g = 9.8$. The adaptive laws parameters are $r_1 = 0.4$, $r_2 = 0.4$, $w_1 = 60$, $w_2 = 60$. The observer parameters are $k_1 = 1.5$, $k_2 = 2.5$. The controller parameters are $c_1 = 0.5$, $c_2 = 0.5$. The event trigger parameters are $\bar{b} = 10$, $b = 0.3$, $\Omega = 8$. The initial condition are $\hat{x}_1(0) = 0$, $\hat{x}_2(0) = 0$, $x_1(0) = 1.5$, $x_2(0) = 0$, $\hat{\theta}_1(0) = 0$, $\hat{\theta}_2(0) = 0$. By choosing the suitable parameters, the output can track $y_r = \sin(t)$. For better simulation effect, the following fuzzy membership function is chosen.

$$\varsigma_{F_i^Z}(\hat{x}_i) = \exp\left(-\frac{(\hat{x}_i + 3 - Z)^2}{4}\right), \quad i = 1,2, \quad Z = 1,2,3,4,5. \tag{65}$$

By using the design approach in this paper, we show the simulation results in Fig. 1-6.

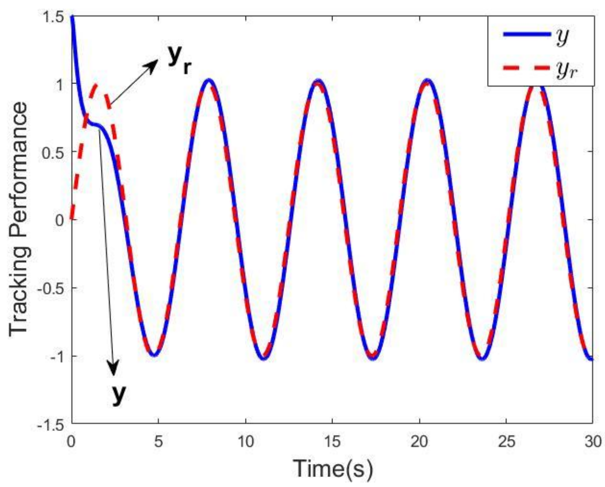

Fig. 1. Tracking effect of the controller

Fig. 1 presents the case of tracking between the output y and the reference signal. From the figure, it can be known that the control method can realize excellent tracking effect.

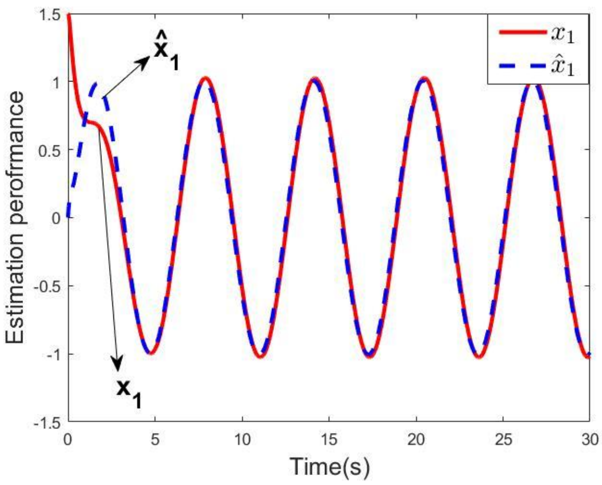

Fig. 2. Observation performance of the state x_1

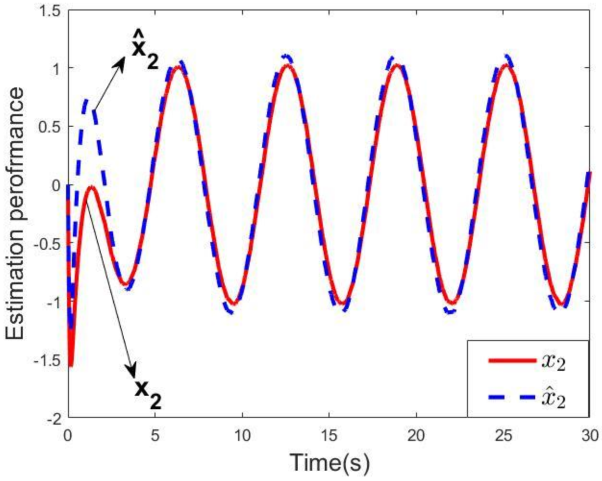

Fig. 3. Observation performance of the state x_2

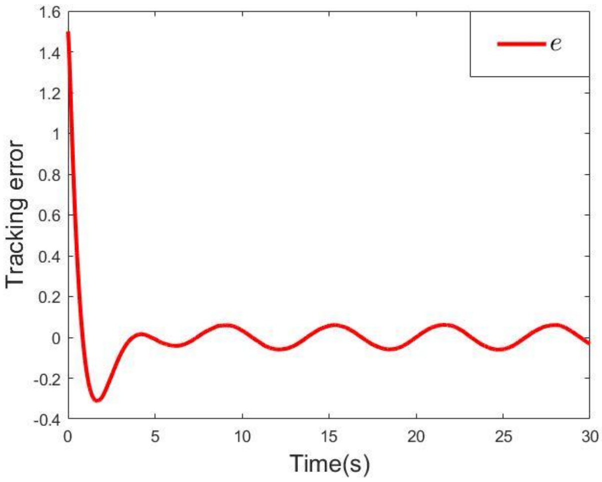

Fig. 4. Trend of tracking error

In Fig. 2 and Fig. 3, the fuzzy state observer can estimate the states x_1 and x_2 respectively when the states are unknown or immeasurable in the system. Fig. 4 illustrates the trend of the tracking error for the nonlinear system. As can be seen from the figure, the tracking error gradually approaches a very small region. We can conclude from the above points that the controller can accomplish the tracking objective and have good observation performance. Besides, the designed approach ensures state and observation signals in the CLS are consistent and bounded.

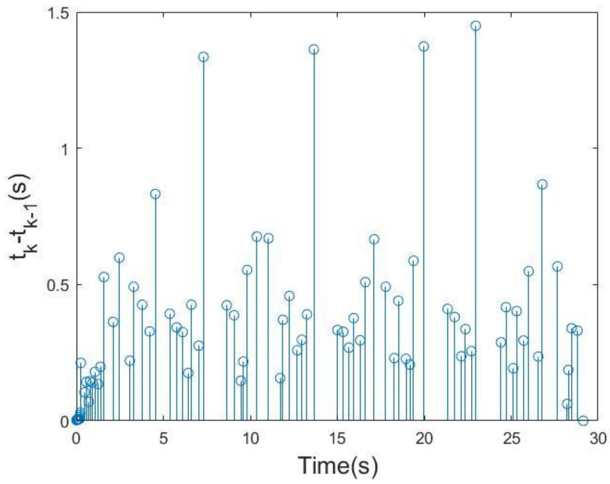

Fig. 5. The time intervals of triggered events

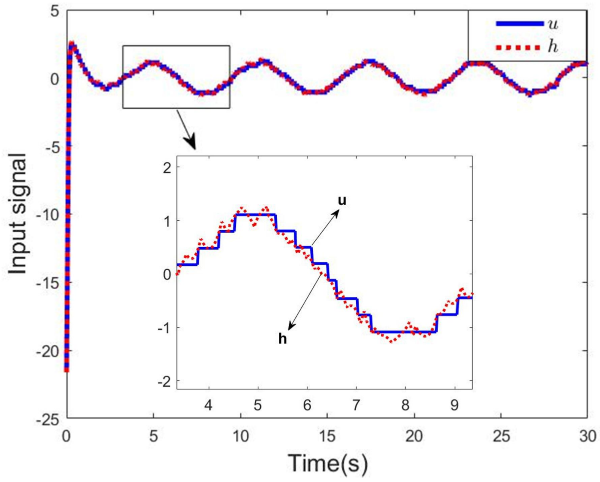

Fig. 6. Controller input u and h

To improve the performance of the system, an event triggered control method with a fixed threshold is used in the process of the design. Based on the above parameters, the number of event trigger is 119. The event triggered situation can be seen in Fig. 5. We can clearly see the time intervals of each event trigger. The curve of input u and h is shown in the following Fig. 6. It can be clearly shown the impact of the event triggering strategy on the input signal. We can see that the designed controller can achieve flexible tracking control of nonlinear systems while also effectively improving system performance.

VI. CONCLUSION

To address the tracking problem of nonlinear system with immeasurable states, an adaptive tracking control scheme based on a fuzzy state observer is designed and an event triggered strategy with a fixed threshold is used in the controller. In the process of the design, the adaptive control and fuzzy state observer are introduced to address the problems that the system states are immeasurable and the system contains parameters that need to be designed and unknown functions. In addition, control laws are drawn which do not contain the uncertainties of the state expressions by backstepping. Furthermore, the usage of the event-triggered method can solve the problem of limited network resources and improve the computational efficiency. The proposed controller can implement the object of tracking control and make sure that all the signals in the CLS are bounded. The tracking error eventually converges a small region. Finally, the effectiveness of the design method is proved by the simulation results.

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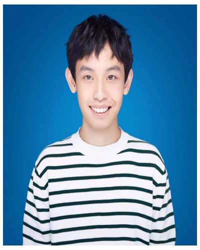

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