A New Types Interval Valued Fuzzy Ideals in Gamma-Semigroups

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***Abstract*—In this paper, we define the new types of interval valued fuzzy ideals, and interval valued fuzzy almost ideals in Γ-semigroups. We discuss the properties of new types interval valued fuzzy ideals. The relation between interval valued fuzzy almost ideals and almost ideals are proved. Finally, we examine the properties of minimal interval valued fuzzy ideals, and minimal interval valued fuzzy almost ideals in Γ-semigroups.**

***Index Terms*—Interval valued fuzzy set, interval valued fuzzy ideal, interval valued fuzzy almost ideal, minimal interval valued fuzzy ideal, minimal interval valued fuzzy almost ideal.**

I. Introduction

THE theory for dealing with uncertainty is the fuzzy set theory initiated by Zadeh in 1965 [1], which has been applied in many areas, such as medical science, robotics, computer science, information science, control engineering, measure theory, logic, set theory, topology, etc. Later in 1979, Kuroki [2] studied some properties of fuzzy subsemigroups of a semigroup and characterized a semigroup in terms of fuzzy subsets of semigroups. The theory of interval valued fuzzy sets was conceptualized by Zadeh in 1975 [3]. These concepts were applied in some fields like medical science [4], image processing [5], topsis decision model [6], decision making [7], etc. In 1994, Biswas [8] applied this concept was to algebra structure. In 2006, Narayanan and Manikantan [9] studied interval valued fuzzy subsemigroups and types of interval valued fuzzy ideals in semigroups. In 2014, Bashir and Sarwar [10] developed interval valued fuzzy on a semigroup, and introduced it to interval valued fuzzy on a Γ-semigroup. The theory of an ideal is properties important in studies thoery semigroups. Furthermore, many researchers studied ideal in Γ-semigroups. For example, Chinram [11] studied quasi-Γ-ideal in Γ-semigroup. P. Kummoon and T. Changhas [12] studied bi-bases of Γ-semigroups, and Iampan [13] studied bi-ideal in Γ-semigroups. In 2021 A. Simuen et al. [14] studied a novel of ideals and fuzzy ideals of Γ-semigroups. Recently, in 2022 T. Gaketem and P. Khamrot [15], [16] given concepts and investigated propertes of a novel of ideals on intuitionistic fuzzy ideals and cubic ideals of Γ-semigroups.

In this paper we extend new fuzzy ideal to interval valued fuzzy ideal of Γ-semigroups, and we investigate properties of new types interval valued fuzzy ideals on Γ-semigroups. Additionally, we prove the relation between interval valued fuzzy almost ideals and almost ideals in Γ-semigroups.

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II. Preliminaries

In this section, we review some fundamental concepts. Γ-semigroups, fuzzy sets, and interval valued fuzzy sets are presented.

A *sub-Γ-semigroup* of a Γ-semigroup $\mathfrak{P}$ is a non-empty set $\mathfrak{K}$ of $\mathfrak{P}$ such that $\mathfrak{K}\Gamma\mathfrak{K} \subseteq \mathfrak{K}$. A *left (right) ideal* of a Γ-semigroup $\mathfrak{P}$ is a non-empty set $\mathfrak{K}$ of $\mathfrak{P}$ such that $\mathfrak{P}\Gamma\mathfrak{K} \subseteq \mathfrak{K}$ ($\mathfrak{K}\Gamma\mathfrak{P} \subseteq \mathfrak{K}$). By an *ideal* of a Γ-semigroup $\mathfrak{P}$, we refwe to a non-empty set of $\mathfrak{P}$, which is both a left and a right ideal of $\mathfrak{P}$. A *quasi-ideal* of a Γ-semigroup $\mathfrak{P}$ is a non-empty set $\mathfrak{K}$ of $\mathfrak{P}$ such that $\mathfrak{K}\Gamma\mathfrak{P}\cap\mathfrak{P}\Gamma\mathfrak{K} \subseteq \mathfrak{K}$. A sub-$\Gamma$-semigroup $\mathfrak{K}$ of a Γ-semigroup $\mathfrak{P}$ is called a *bi-ideal* of $\mathfrak{P}$ if $\mathfrak{K}\Gamma\mathfrak{P}\Gamma\mathfrak{K} \subseteq \mathfrak{K}$.

Definition 2.1. *[14] Let $\mathfrak{P}$ be a Γ-semigroup, $\mathfrak{K}$ be a non-empty subset of $\mathfrak{P}$, for all $\mathfrak{p} \in \mathfrak{P}$ and $\alpha, \beta \in \Gamma$. Then $\mathfrak{K}$ is said to be*

(1) *a* left (right) almost ideal *of Γ-semigroup $\mathfrak{P}$ is a non-empty set $\mathfrak{K}$ such that $(\mathfrak{p}\Gamma\mathfrak{K}) \cap \mathfrak{K} \neq \emptyset$ $((\mathfrak{K}\Gamma\mathfrak{p}) \cap \mathfrak{K} \neq \emptyset)$.*
(2) *an* almost bi-ideal *of Γ-semigroup $\mathfrak{P}$ is a non-empty set $\mathfrak{K}$ such that $(\mathfrak{K}\Gamma\mathfrak{p}\Gamma\mathfrak{K}) \cap \mathfrak{K} \neq \emptyset$*
(3) *an* almost quasi-ideal *of Γ-semigroup $\mathfrak{P}$ is a non-empty set $\mathfrak{K}$ such that $(\mathfrak{p}\Gamma\mathfrak{K} \cap \mathfrak{K}\Gamma\mathfrak{p}) \cap K \neq \emptyset$.*
(4) *an* almost interior ideal *of a Γ-semigroup $\mathfrak{P}$ is a non-empty set $\mathfrak{K}$ such that $(\mathfrak{p}\Gamma\mathfrak{K}\Gamma\mathfrak{p}) \cap \mathfrak{K} \neq \emptyset$.*
(5) *a left α-ideal of a Γ-semigroup $\mathfrak{P}$ is a non-empty set $\mathfrak{K}$ such that $\mathfrak{P}\alpha\mathfrak{K} \subseteq \mathfrak{K}$. A right α-ideal of a Γ-semigroup $\mathfrak{P}$ is a non-empty set $\mathfrak{K}$ such that $\mathfrak{K}\beta\mathfrak{P} \subseteq \mathfrak{K}$.*
(6) *an* (α, β)-ideal *of a Γ-semigroup $\mathfrak{P}$ is a non-empty set $\mathfrak{K}$ such that it is both a left α-ideal and a right β-ideal of $\mathfrak{P}$.*

Definition 2.2. *[1] A* fuzzy set *ϖ of a non-empty set $\mathfrak{T}$ is a function $\varpi : \mathfrak{T} \to [0, 1]$.*

For any two fuzzy sets ϖ and ν of a non-empty set $\mathfrak{T}$, define $\geq, =, \wedge$ and $\vee$ as follows:

(1) $\varpi \geq \nu \Leftrightarrow \varpi(\mathfrak{k}) \geq \nu(\mathfrak{k})$ for all $\mathfrak{k} \in \mathfrak{T}$,
(2) $\varpi = \nu \Leftrightarrow \varpi \geq \nu$ and $\nu \geq \varpi$,
(3) $(\varpi \wedge \nu)(\mathfrak{k}) = \min\{\varpi(\mathfrak{k}), \nu(\mathfrak{k})\} = \varpi(\mathfrak{k}) \wedge \nu(\mathfrak{k})$ for all $\mathfrak{k} \in \mathfrak{T}$,
(4) $(\varpi \vee \nu)(\mathfrak{k}) = \max\{\varpi(\mathfrak{k}), \nu(\mathfrak{k})\} = \varpi(\mathfrak{k}) \vee \nu(\mathfrak{k})$ for all $\mathfrak{k} \in \mathfrak{T}$.
For the symbol $\varpi \leq \nu$, we mean $\nu \geq \varpi$.

For any element $\mathfrak{k}$ in a semigroup $\mathfrak{P}$, define the set $F_{\mathfrak{k}}$ by

$$F_{\mathfrak{k}} := \{(\mathfrak{y}, \mathfrak{z}) \in \mathfrak{P} \times \mathfrak{P} \mid \mathfrak{k} = \mathfrak{y}\mathfrak{z}\}.$$

For two fuzzy sets ϖ and ν on a semigroup $\mathfrak{P}$, define the product $\varpi \circ \nu$ as follows: for all $\mathfrak{k} \in \mathfrak{P}$,

$$(\varpi \circ \nu)(\mathfrak{k}) = \begin{cases} \bigvee_{(\mathfrak{y},\mathfrak{z}) \in F_{\mathfrak{k}}} \{\varpi(\mathfrak{y}) \wedge \nu(\mathfrak{z}) \mid (\mathfrak{y},\mathfrak{z}) \in F_{\mathfrak{k}}\} & \text{if} \quad F_{\mathfrak{k}} \neq \emptyset, \\ 0 & \text{if} \quad F_{\mathfrak{k}} = \emptyset. \end{cases}$$

The following definitions are types of fuzzy almost ideal on semigroups.

Definition 2.3. *[14] A fuzzy set ϖ of a semigroup $\mathfrak{P}$ is said to be*

(1) *a fuzzy almost left (right) ideal of $\mathfrak{P}$ if $\mathfrak{P} \circ \varpi \cap \varpi \neq \emptyset$ ($\varpi \circ \mathfrak{P} \cap \varpi \neq \emptyset$).*
(2) *a fuzzy almost ideal of $\mathfrak{P}$ if it is both a fuzzy almost left ideal and a fuzzy almost right ideal of $\mathfrak{P}$,*
(3) *a fuzzy almost bi-ideal of $\mathfrak{P}$ if $\varpi \circ \mathfrak{P} \circ \varpi \cap \varpi \neq \emptyset$.*
(4) *a fuzzy almost quasi-ideal of $\mathfrak{P}$ if $(\mathfrak{P} \circ \varpi \cap \varpi \circ \mathfrak{P}) \cap \varpi \neq \emptyset$.*

The following definitions are types of fuzzy subsemigroups on Γ-semigroups.

Definition 2.4. *[17] A fuzzy set ϖ of a Γ-semigroup $\mathfrak{P}$ is said to be*

(1) *a fuzzy subsemigroup of $\mathfrak{P}$ if $\varpi(\mathfrak{u}\gamma\mathfrak{v}) \geq \varpi(\mathfrak{u}) \wedge \varpi(\mathfrak{v})$ for all $\mathfrak{u}, \mathfrak{v} \in \mathfrak{P}$ and $\gamma \in \Gamma$,*
(2) *a fuzzy left (right) ideal of $\mathfrak{P}$ if $\varpi(\mathfrak{u}\gamma\mathfrak{v}) \geq \varpi(\mathfrak{v})$ ($\varpi(\mathfrak{u}\gamma\mathfrak{v}) \geq \varpi(\mathfrak{u})$) for all $\mathfrak{u}, \mathfrak{v} \in \mathfrak{P}$ and $\gamma \in \Gamma$,*
(3) *a fuzzy ideal of $\mathfrak{P}$ if it is both a fuzzy left ideal and a fuzzy right ideal of $\mathfrak{P}$,*
(4) *a fuzzy bi-ideal of $\mathfrak{P}$ if ϖ is a fuzzy subsemigroup of $\mathfrak{P}$ and $\varpi(\mathfrak{u}\gamma\mathfrak{v}\beta\mathfrak{w}) \geq \varpi(\mathfrak{u}) \wedge \varpi(\mathfrak{w})$ for all $\mathfrak{u}, \mathfrak{v}, \mathfrak{w} \in \mathfrak{P}$ and $\gamma, \beta \in \Gamma$.*

Now, we review the interval valued fuzzy set.

Let $\Omega[0,1]$ be the set of all closed subintervals of $[0,1]$, i.e.,

$$\Omega[0,1] = \{\overline{p} = [p^-, p^+] \mid 0 \leq p^- \leq p^+ \leq 1\}.$$

We note that $[p,p] = \{p\}$ for all $p \in [0,1]$. For $p = 0$ or 1, we shall denote $[0,0]$ by $\overline{0}$ and $[1,1]$ by $\overline{1}$.

Let $\overline{\omega} = [\omega^-, \omega^+]$ and $\overline{\varpi} = [\varpi^-, \varpi^+] \in \Omega[0,1]$. Define the operations $\preceq$, $=$, $\curlywedge$ and $\curlyvee$ as follows:

(1) $\overline{\omega} \preceq \overline{\varpi}$ if and only if $\omega^- \leq \varpi^-$ and $\omega^+ \leq \varpi^+$
(2) $\overline{\omega} = \overline{\varpi}$ if and only if $\omega^- = \varpi^-$ and $\omega^+ = \varpi^+$
(3) $\overline{\omega} \curlywedge \overline{\varpi} = [(\omega^- \wedge \varpi^-), (\omega^+ \wedge \varpi^+)]$
(4) $\overline{\omega} \curlyvee \overline{\varpi} = [(\omega^- \vee \varpi^-), (\omega^+ \vee \varpi^+)]$.

If $\overline{\omega} \succeq \overline{\varpi}$, we mean $\overline{\varpi} \preceq \overline{\omega}$.

For each interval $\overline{\omega}_i = [\omega_i^-, \omega_i^+] \in \Omega[0,1]$, $i \in \mathcal{A}$ where $\mathcal{A}$ is an index set, we define

$$\curlywedge_{i \in \mathcal{A}} \overline{\omega}_i = [\wedge_{i \in \mathcal{A}} \omega_i^-, \wedge_{i \in \mathcal{A}} \omega_i^+] \quad \text{and} \quad \curlyvee_{i \in \mathcal{A}} \overline{\omega}_i = [\vee_{i \in \mathcal{A}} \omega_i^-, \vee_{i \in \mathcal{A}} \omega_i^+].$$

Definition 2.5. *[17] Let $\mathfrak{P}$ be a non-empty set. Then the function $\overline{\varpi} : \mathfrak{P} \to \Omega[0,1]$ is called* interval valued fuzzy set *(shortly, IVF set) of $\mathfrak{P}$.*

Definition 2.6. *[17] Let $\mathfrak{K}$ be a subset of a non-empty set $\mathfrak{P}$. An* interval valued characteristic function *of $\mathfrak{K}$ is defined to be a function $\overline{\chi}_{\mathfrak{K}} : \mathfrak{P} \to \Omega[0,1]$ by*

$$\overline{\chi}_{\mathfrak{K}}(\mathfrak{e}) = \begin{cases} \overline{1} & \text{if} \quad \mathfrak{e} \in \mathfrak{K}, \\ \overline{0} & \text{if} \quad \mathfrak{e} \notin \mathfrak{K} \end{cases}$$

for all $\mathfrak{e} \in \mathfrak{P}$.

For two IVF sets $\overline{\omega}$ and $\overline{\varpi}$ of a non-empty set $\mathfrak{P}$, define

(1) $\overline{\omega} \sqsubseteq \overline{\varpi} \Leftrightarrow \overline{\omega}(\mathfrak{k}) \preceq \overline{\varpi}(\mathfrak{k})$ for all $\mathfrak{k} \in \mathfrak{P}$,
(2) $\overline{\omega} = \overline{\varpi} \Leftrightarrow \overline{\omega} \sqsubseteq \overline{\varpi}$ and $\overline{\varpi} \sqsubseteq \overline{\omega}$,
(3) $(\overline{\omega} \sqcap \overline{\varpi})(\mathfrak{k}) = \overline{\omega}(\mathfrak{k}) \curlywedge \overline{\varpi}(\mathfrak{k})$ for all $\mathfrak{k} \in \mathfrak{P}$,
(4) $(\overline{\omega} \sqcup \overline{\varpi})(\mathfrak{k}) = \overline{\omega}(\mathfrak{k}) \curlyvee \overline{\varpi}(\mathfrak{k})$ for all $\mathfrak{k} \in \mathfrak{P}$.

For two IVF sets $\overline{\omega}$ and $\overline{\varpi}$ in a semigroup $\mathfrak{P}$, define the product $\overline{\omega} \circ \overline{\varpi}$ as follows : for all $\mathfrak{k} \in \mathfrak{P}$,

$$(\overline{\omega} \circ \overline{\varpi})(\mathfrak{k}) = \begin{cases} \curlyvee_{(\mathfrak{y},\mathfrak{z}) \in F_{\mathfrak{k}}} \{\overline{\omega}(\mathfrak{y}) \curlywedge \overline{\varpi}(\mathfrak{z})\} & \text{if } F_{\mathfrak{k}} \neq \emptyset, \\ \overline{0} & \text{if } F_{\mathfrak{k}} = \emptyset, \end{cases}$$

where $F_{\mathfrak{k}} := \{(\mathfrak{y},\mathfrak{z}) \in \mathfrak{P} \times \mathfrak{P} \mid \mathfrak{k} = \mathfrak{y}\mathfrak{z}\}$.

Next, we shall give definitions of various types of IVF subsemigroups.

Definition 2.7. *[9] An IVF set $\overline{\omega}$ of a semigroup $\mathfrak{P}$ is said to be an IVF subsemigroup of $\mathfrak{P}$ if $\overline{\omega}(\mathfrak{e}_1\mathfrak{e}_2) \succeq \overline{\omega}(\mathfrak{e}_1) \curlywedge \overline{\omega}(\mathfrak{e}_2)$ for all $\mathfrak{e}_1, \mathfrak{e}_2 \in \mathfrak{P}$.*

Definition 2.8. *[9] An IVF set $\overline{\omega}$ of a semigroup $\mathfrak{P}$ is said to be an IVF left (right) ideal of $\mathfrak{P}$ if $\overline{\omega}(\mathfrak{e}_1\mathfrak{e}_2) \succeq \overline{\omega}(\mathfrak{e}_2)$ ($\overline{\omega}(\mathfrak{e}_1\mathfrak{e}_2) \succeq \overline{\omega}(\mathfrak{e}_1)$) for all $\mathfrak{e}_1, \mathfrak{e}_2 \in \mathfrak{P}$. An IVF subset $\overline{\omega}$ of $\mathfrak{P}$ is called an IVF ideal of $\mathfrak{P}$ if it is both an IVF left ideal and an IVF right ideal of $\mathfrak{P}$.*

Next, we shall give definitions of various types of IVF Γ-subsemigroups.

Definition 2.9. *[10] An IVF set $\overline{\omega}$ of a Γ-semigroup $\mathfrak{P}$ is said to be an IVF Γ-subsemigroup of $\mathfrak{P}$ if $\overline{\omega}(\mathfrak{e}_1\alpha\mathfrak{e}_2) \succeq \overline{\omega}(\mathfrak{e}_1) \curlywedge \overline{\omega}(\mathfrak{e}_2)$ for all $\mathfrak{e}_1, \mathfrak{e}_2 \in \mathfrak{P}$ and $\alpha \in \Gamma$.*

Definition 2.10. *[9] An IVF set $\overline{\omega}$ of a Γ-semigroup $\mathfrak{P}$ is said to be an IVF left (right) ideal of $\mathfrak{P}$ if $\overline{\omega}(\mathfrak{e}_1\alpha\mathfrak{e}_2) \succeq \overline{\omega}(\mathfrak{e}_2)$ ($\overline{\omega}(\mathfrak{e}_1\alpha\mathfrak{e}_2) \succeq \overline{\omega}(\mathfrak{e}_1)$) for all $\mathfrak{e}_1, \mathfrak{e}_2 \in \mathfrak{P}$ and $\alpha \in \Gamma$. An IVF subset $\overline{\omega}$ of $\mathfrak{P}$ is called an IVF ideal of $\mathfrak{P}$ if it is both an IVF left ideal and an IVF right ideal of $\mathfrak{P}$.*

III. New Types of Interval valued Fuzzy Ideals

In this section, we define the interval valued fuzzy (α, β)-ideal and study basic properties of it.

Definition 3.1. *Let $\overline{\omega}$ be an IVF set of a Γ-semigroup $\mathfrak{P}$ and $\alpha, \beta \in \Gamma$. Then $\overline{\omega}$ is called*

(1) *an IVF left α-ideal of $\mathfrak{P}$ if $\overline{\omega}(\mathfrak{u}\alpha\mathfrak{v}) \succeq \overline{\omega}(\mathfrak{v})$ for all $\mathfrak{u}, \mathfrak{v} \in \mathfrak{P}$,*
(2) *an IVF right β-ideal of $\mathfrak{P}$ if $\overline{\omega}(\mathfrak{u}\beta\mathfrak{v}) \succeq \overline{\omega}(\mathfrak{u})$ for all $\mathfrak{u}, \mathfrak{v} \in \mathfrak{P}$,*
(3) *an IVF (α, β)-ideal of $\mathfrak{P}$ if it is both an IVF left α-ideal and an IVF right β-ideal of $\mathfrak{P}$.*

Theorem 3.2. *Intersection and union of any two IVF left α-ideals (right β-ideals, (α, β)-ideals) of a Γ-semigroup $\mathfrak{P}$ is an IVF left α-ideal (right β-ideal, (α, β)-ideal) of $\mathfrak{P}$.*

Proof: Let $\overline{\omega}$ and $\overline{\varpi}$ be IVF left α-ideals of $\mathfrak{P}$, and let $\mathfrak{u}, \mathfrak{v} \in \mathfrak{P}$. Then

$$\begin{aligned} (\overline{\omega} \sqcap \overline{\varpi})(\mathfrak{u}\alpha\mathfrak{v}) &= \overline{\omega}(\mathfrak{u}\alpha\mathfrak{v}) \curlywedge \overline{\varpi}(\mathfrak{u}\alpha\mathfrak{v}) \\ &\succeq \overline{\omega}(\mathfrak{v}) \curlywedge \overline{\varpi}(\mathfrak{v}) \\ &= (\overline{\omega} \sqcap \overline{\varpi})(\mathfrak{v}) \end{aligned}$$

and

$$\begin{aligned}(\overline{\omega} \sqcup \overline{\varpi})(\mathfrak{u}\alpha\mathfrak{v}) &= \overline{\omega}(\mathfrak{u}\alpha\mathfrak{v}) \curlyvee \overline{\varpi}(\mathfrak{u}\alpha\mathfrak{v}) \\ &\succeq \overline{\omega}(\mathfrak{v}) \curlyvee \overline{\varpi}(\mathfrak{v}) \\ &= (\overline{\omega} \sqcup \overline{\varpi})(\mathfrak{v}).\end{aligned}$$

Thus, $\overline{\omega} \sqcap \overline{\varpi}$ and $\overline{\omega} \sqcup \overline{\varpi}$ are IVF left α-ideals of $\mathfrak{P}$. ∎

Theorem 3.3. *Let $\mathfrak{K}$ be a non-empty subset of Γ-semigroup $\mathfrak{P}$. Then $\mathfrak{K}$ is a left α-ideal (right β-ideal, (α, β)-ideal) of $\mathfrak{P}$ if and only if the characteristic function $\overline{\chi}_{\mathfrak{K}}$ is an IVF left α-ideal (right β-ideal, (α, β)-ideal) of $\mathfrak{P}$.*

Proof: Suppose that $\mathfrak{K}$ is a left α-ideal of $\mathfrak{P}$ and $\mathfrak{u}, \mathfrak{v} \in \mathfrak{P}$.

If $\mathfrak{v} \in \mathfrak{P}$, then $\mathfrak{u}\alpha\mathfrak{v} \in \mathfrak{K}$. Thus, $\overline{\chi}_{\mathfrak{K}}(\mathfrak{v}) = \overline{\chi}_{\mathfrak{K}}(\mathfrak{u}\alpha\mathfrak{v}) = \overline{1}$. Hence, $\overline{\chi}_{\mathfrak{K}}(\mathfrak{u}\alpha\mathfrak{v}) \succeq \overline{\chi}_{\mathfrak{K}}(\mathfrak{v})$.

If $\mathfrak{v} \notin \mathfrak{P}$, then $\mathfrak{u}\alpha\mathfrak{v} \in \mathfrak{K}$. Thus, $\overline{\chi}_{\mathfrak{K}}(\mathfrak{v}) = \overline{0}$ and $\overline{\chi}_{\mathfrak{K}}(\mathfrak{u}\alpha\mathfrak{v}) = \overline{1}$. Hence, $\overline{\chi}_{\mathfrak{K}}(\mathfrak{u}\alpha\mathfrak{v}) \succeq \overline{\chi}_{\mathfrak{K}}(\mathfrak{v})$. Therefore, $\overline{\chi}_{\mathfrak{K}}$ is an IVF left α-ideal of $\mathfrak{P}$.

Conversely, assume that $\overline{\chi}_{\mathfrak{K}}$ is an IVF left α-ideal of $\mathfrak{P}$ and $\mathfrak{u}, \mathfrak{v} \in \mathfrak{P}$ with $\mathfrak{v} \in \mathfrak{K}$. Then $\overline{\chi}_{\mathfrak{K}}(\mathfrak{v}) = \overline{1}$. By assumption, $\overline{\chi}_{\mathfrak{K}}(\mathfrak{u}\alpha\mathfrak{v}) \succeq \overline{\chi}_{\mathfrak{K}}(\mathfrak{v})$. Thus, $\mathfrak{u}\alpha\mathfrak{v} \in \mathfrak{K}$. Hence, $\mathfrak{K}$ is a left α-ideal of $\mathfrak{P}$. ∎

Theorem 3.4. *Let $\overline{\omega}$ be a IVF set of a Γ-semigroup $\mathfrak{P}$ and $\overline{\omega}_{\overline{p}} = \{\mathfrak{n} \in \mathfrak{P} \mid \overline{\omega}_{\overline{p}}(\mathfrak{n}) \succeq \overline{p}\}$. Then $\overline{\omega}$ is an IVF left α-ideal (right β ideal, (α, β)-ideal) of $\mathfrak{P}$ if and only if $\overline{\omega}_{\overline{p}}$ is a non-empty set, and $\overline{\omega}_{\overline{p}}$ is a left α-ideal (right β ideal, (α, β)-ideal) of $\mathfrak{P}$ for all $\overline{p} \in \Omega[0, 1]$.*

Proof: Suppose that $\overline{\omega}$ is an IVF left α-ideal of $\mathfrak{P}$ and $\overline{p} \in \Omega[0, 1]$, we have $\overline{\omega}_{\overline{p}}(\mathfrak{u}\alpha\mathfrak{m}) \geq \overline{\omega}_{\overline{p}}(\mathfrak{m})$ for all $\mathfrak{u}, \mathfrak{m} \in \mathfrak{P}$. Thus, $\overline{\omega}_{\overline{p}} \neq \emptyset$. Let $\mathfrak{m} \in \overline{\omega}_{\overline{p}}$ and $\mathfrak{u} \in \mathfrak{P}$. Then $\overline{\omega}_{\overline{p}}(\mathfrak{m}) \succeq \overline{p}$. Thus, $\overline{\omega}_{\overline{p}}(\mathfrak{u}\alpha\mathfrak{m}) \geq \overline{\omega}_{\overline{p}}(\mathfrak{m}) \geq p$, so $\mathfrak{u}\alpha\mathfrak{m} \in \overline{\omega}_{\overline{p}}$. Hence, $\overline{\omega}_{\overline{p}}$ is a left α-ideal of $\mathfrak{P}$.

Conversely, assume that $\overline{\omega}_{\overline{p}}$ is a left α-ideal of $\mathfrak{P}$, $\overline{p} \in \Omega[0, 1]$, and $\overline{\omega}_{\overline{p}} \neq \emptyset$. Let $\mathfrak{u}, \mathfrak{v} \in \mathfrak{P}$ and $\overline{p} = \overline{\omega}_{\overline{p}}(\mathfrak{v})$. By assumption, $\overline{\omega}_{\overline{p}}(\mathfrak{v}) \succeq \overline{p}$, we have $\mathfrak{v} \in \overline{\omega}_{\overline{p}}$. Thus, $\overline{\omega}_{\overline{p}} \neq \emptyset$. Hence, $\overline{\omega}_{\overline{p}}$ is a left α-ideal of $\mathfrak{P}$. Since $\mathfrak{v} \in \overline{\omega}_{\overline{p}}$ and $\mathfrak{u} \in \mathfrak{P}$, we have $\mathfrak{u}\alpha\mathfrak{v} \in \overline{\omega}_{\overline{p}}$. Thus, $\overline{\omega}_{\overline{p}}(\mathfrak{u}\alpha\mathfrak{v}) \geq \overline{p} = \overline{\omega}_{\overline{p}}(\mathfrak{v})$. Hence, $\overline{\omega}_{\overline{p}}$ is an IVF left α-ideal of $\mathfrak{P}$. ∎

Next, we define the α-product.

For IVF sets $\overline{\omega} = [\omega^-, \omega^+]$ and $\overline{\varpi} = [\varpi^-, \varpi^+]$ of a Γ-semigroup $\mathfrak{P}$, define the product $\overline{\omega} \circ_\alpha \overline{\varpi}$ as follows: for all $\mathfrak{k} \in \mathfrak{P}$,

$$(\overline{\omega}\,\overline{\circ}_\alpha \overline{\varpi})(\mathfrak{k}) = \begin{cases} \bigvee\limits_{(\mathfrak{y},\alpha,\mathfrak{z}) \in F_{\mathfrak{k}_\alpha}} \{\overline{\omega}(\mathfrak{y}) \curlywedge \overline{\varpi}(\mathfrak{z})\} & \text{if} \quad F_{\mathfrak{k}_\alpha} \neq \emptyset, \\ \overline{0} & \text{if} \quad F_{\mathfrak{k}_\alpha} = \emptyset. \end{cases}$$

For any element $\mathfrak{k}$ in a Γ-semigroup $\mathfrak{P}$, define the set $F_{\mathfrak{k}_\alpha}$ by $F_{\mathfrak{k}_\alpha} := \{(\mathfrak{y}, \alpha, \mathfrak{z}) \in \mathfrak{P} \times \Gamma \times \mathfrak{P} \mid \mathfrak{k} = \mathfrak{y}\alpha\mathfrak{z}\}$.

Next, we define the interval valued fuzzy (α, β)-interior ideal and study its basic properties.

Definition 3.5. *Let $\overline{\omega}$ be an IVF set of a Γ-semigroup $\mathfrak{P}$ and $\alpha, \beta \in \Gamma$. Then $\overline{\omega}$ is called an IVF (α, β)-interior-ideal of $\mathfrak{P}$ if $\overline{\chi}_{\mathfrak{P}} \overline{\circ}_\alpha \overline{\omega} \overline{\circ}_\beta \overline{\chi}_{\mathfrak{P}} \sqsubseteq \overline{\omega}$, where $\overline{\chi}_S$ is an IVF set mapping every element of $\mathfrak{P}$ to $\overline{1}$.*

Theorem 3.6. *Let $\mathfrak{K}$ be a non-empty subset of Γ-semigroup $\mathfrak{P}$. Then $\mathfrak{K}$ is an (α, β)-interior-ideal of $\mathfrak{P}$ if and only if the characteristic function $\overline{\chi}_{\mathfrak{K}}$ is an IVF (α, β)-interior-ideal of $\mathfrak{P}$.*

Proof: Suppose that $\mathfrak{K}$ is an (α, β)-interior-ideal of $\mathfrak{P}$ and $\mathfrak{u}, \mathfrak{v}, \mathfrak{w} \in \mathfrak{P}$.

If $\mathfrak{v} \in \mathfrak{P}$, then $\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w} \in \mathfrak{K}$. Thus, $\overline{\chi}_{\mathfrak{K}}(\mathfrak{v}) = \overline{\chi}_{\mathfrak{K}}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) = \overline{1}$. Hence, $\overline{\chi}_{\mathfrak{K}}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \succeq \overline{\chi}_{\mathfrak{K}}(\mathfrak{v})$.

If $\mathfrak{v} \notin \mathfrak{P}$, then $\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{v} \in \mathfrak{K}$. Thus, $\overline{\chi}_{\mathfrak{K}}(\mathfrak{v}) = \overline{0}$ and $\overline{\chi}_{\mathfrak{K}}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) = \overline{1}$. Hence, $\overline{\chi}_{\mathfrak{K}}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \succeq \overline{\chi}_{\mathfrak{K}}(\mathfrak{v})$. Therefore, $\overline{\chi}_{\mathfrak{K}}$ is an IVF left α-ideal of $\mathfrak{P}$.

Conversely, assume that $\overline{\chi}_{\mathfrak{K}}$ is an IVF left α-ideal of $\mathfrak{P}$ and $\mathfrak{u}, \mathfrak{v} \in \mathfrak{P}$ with $\mathfrak{v}, \mathfrak{w} \in \mathfrak{K}$. Then $\overline{\chi}_{\mathfrak{K}}(\mathfrak{v}) = \overline{1}$. By assumption, $\overline{\chi}_{\mathfrak{K}}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \succeq \overline{\chi}_{\mathfrak{K}}(\mathfrak{v})$. Thus, $\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w} \in \mathfrak{K}$. Hence, $\mathfrak{K}$ is an (α, β)-interior ideal of $\mathfrak{P}$. ∎

Theorem 3.7. *Intersection and union of any two IVF (α, β)-interior ideals of a Γ-semigroup $\mathfrak{P}$ is an IVF (α, β)-interior ideal of $\mathfrak{P}$.*

Proof: Let $\overline{\omega}$ and $\overline{\varpi}$ be IVF (α, β)-interior ideals of $\mathfrak{P}$, and let $\mathfrak{u}, \mathfrak{v}, \mathfrak{w} \in \mathfrak{P}$. Then

$$\begin{aligned}(\overline{\omega} \sqcap \overline{\varpi})(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) &= \overline{\omega}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \curlywedge \overline{\varpi}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \\ &\succeq \overline{\omega}(\mathfrak{v}) \curlywedge \overline{\varpi}(\mathfrak{v}) \\ &= (\overline{\omega} \sqcap \overline{\varpi})(\mathfrak{v})\end{aligned}$$

and

$$\begin{aligned}(\overline{\omega} \sqcup \overline{\varpi})(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) &= \overline{\omega}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \curlyvee \overline{\varpi}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \\ &\succeq \overline{\omega}(\mathfrak{v}) \curlyvee \overline{\varpi}(\mathfrak{v}) \\ &= (\overline{\omega} \sqcup \overline{\varpi})(\mathfrak{v}).\end{aligned}$$

Thus $\overline{\omega} \sqcap \overline{\varpi}$ and $\overline{\omega} \sqcup \overline{\varpi}$ are IVF (α, β)-interior ideals of $\mathfrak{P}$. ∎

Theorem 3.8. *Let $\overline{\omega}$ be a IVF set of a Γ-semigroup $\mathfrak{P}$ and $\overline{\omega}_{\overline{p}} = \{\mathfrak{n} \in \mathfrak{P} \mid \overline{\omega}_{\overline{p}}(\mathfrak{n}) \succeq \overline{p}\}$. Then $\overline{\omega}$ is an IVF (α, β)-interior ideal of $\mathfrak{P}$ if and only if $\overline{\omega}_{\overline{p}}$ is a non-empty set, and $\overline{\omega}_{\overline{p}}$ is an (α, β)-interior ideal of $\mathfrak{P}$ for all $\overline{p} \in \Omega[0, 1]$.*

Proof: Suppose that $\overline{\omega}$ is an IVF (α, β)-interior ideal of $\mathfrak{P}$ and $\overline{p} \in \Omega[0, 1]$, we have $\overline{\omega}_{\overline{p}}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \geq \overline{\omega}_{\overline{p}}(\mathfrak{v})$ for all $\mathfrak{u}, \mathfrak{v}, \mathfrak{w} \in \mathfrak{P}$. Thus, $\overline{\omega}_{\overline{p}} \neq \emptyset$. Let $\mathfrak{v} \in \overline{\omega}_{\overline{p}}$ and $\mathfrak{u}, \mathfrak{w} \in \mathfrak{P}$. Then $\overline{\omega}_{\overline{p}}(\mathfrak{v}) \succeq \overline{p}$. Thus, $\overline{\omega}_{\overline{p}}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \geq \overline{\omega}_{\overline{p}}(\mathfrak{v}) \geq p$, so $(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \in \overline{\omega}_{\overline{p}}$. Hence, $\overline{\omega}_{\overline{p}}$ is an (α, β)-interior ideal of $\mathfrak{P}$.

Conversely, assume that $\overline{\omega}_{\overline{p}}$ is an (α, β)-interior ideal of $\mathfrak{P}$, $\overline{p} \in \Omega[0, 1]$, and $\overline{\omega}_{\overline{p}} \neq \emptyset$. Let $\mathfrak{u}, \mathfrak{v}, \mathfrak{w} \in \mathfrak{P}$ and $\overline{p} = \overline{\omega}_{\overline{p}}(\mathfrak{v})$. By assumption, $\overline{\omega}_{\overline{p}}(\mathfrak{v}) \succeq \overline{p}$, we have $\mathfrak{v} \in \overline{\omega}_{\overline{p}}$. Thus, $\overline{\omega}_{\overline{p}} \neq \emptyset$. Hence, $\overline{\omega}_{\overline{p}}$ is an (α, β)-interior ideal of $\mathfrak{P}$. Since $\mathfrak{v} \in \overline{\omega}_{\overline{p}}$ and $\mathfrak{u}, \mathfrak{m} \in \mathfrak{P}$, we have $\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w} \in \overline{\omega}_{\overline{p}}$. Thus, $\overline{\omega}_{\overline{p}}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \geq \overline{p} = \overline{\omega}_{\overline{p}}(\mathfrak{v})$. Hence, $\overline{\omega}_{\overline{p}}$ is an IVF (α, β)-interior ideal of $\mathfrak{P}$. ∎

Next, we define the interval valued fuzzy (α, β)-bi-ideal and study its basic properties.

Definition 3.9. *Let $\overline{\omega}$ be an IVF set of a Γ-semigroup $\mathfrak{P}$, and $\alpha, \beta \in \Gamma$. Then $\overline{\omega}$ is called an IVF (α, β)-bi-ideal of $\mathfrak{P}$ if $\overline{\omega} \overline{\circ}_\alpha \overline{\chi}_{\mathfrak{P}} \overline{\circ}_\beta \overline{\omega} \sqsubseteq \overline{\omega}$, where $\overline{\chi}_{\mathfrak{P}}$ is an IVF set mapping every element of $\mathfrak{P}$ to $\overline{1}$.*

Theorem 3.10. *Let $\mathfrak{K}$ be a non-empty subset of Γ-semigroup $\mathfrak{P}$. Then $\mathfrak{K}$ is an (α, β)-bi-ideal of $\mathfrak{P}$ if and only if the characteristic function $\overline{\chi}_{\mathfrak{K}}$ is an IVF (α, β)-bi-ideal of $\mathfrak{P}$.*

Proof: Suppose that $\mathfrak{K}$ is an (α, β)-bi-ideal of $\mathfrak{P}$, and $\mathfrak{K}\alpha\mathfrak{P}\beta\mathfrak{K} \subseteq \mathfrak{P}$.

If $\mathfrak{u} \in \mathfrak{K}\alpha\mathfrak{P}\beta\mathfrak{K}$, then $\overline{\chi}_{\mathfrak{K}}(\mathfrak{u}) = (\overline{\chi}_{\mathfrak{K}}\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{K}})(\mathfrak{u}) = \overline{1}$. Hence, $(\overline{\chi}_{\mathfrak{K}}\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{K}})(\mathfrak{u}) \preceq \overline{\chi}_{\mathfrak{K}}(\mathfrak{u})$.

If $\mathfrak{u} \notin \mathfrak{K}\alpha\mathfrak{P}\beta\mathfrak{K}$, then $\overline{\chi}_{\mathfrak{K}}(\mathfrak{u}) = \overline{0}$. Hence, $(\overline{\chi}_{\mathfrak{K}}\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{K}})(\mathfrak{u}) \preceq \overline{\chi}_{\mathfrak{K}}(\mathfrak{u})$.

Therefore, $\overline{\chi}_{\mathfrak{K}}$ is an IVF (α,β)-bi-ideal of $\mathfrak{P}$.

Conversely, assume that $\overline{\chi}_{\mathfrak{K}}$ is an IVF (α,β)-bi-ideal of $\mathfrak{P}$ and $\mathfrak{u} \in \mathfrak{K}\alpha\mathfrak{P}\beta\mathfrak{K}$. Then $(\overline{\chi}_{\mathfrak{K}}\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{K}})(\mathfrak{u}) = \overline{1}$. By assumption, $(\overline{\chi}_{\mathfrak{K}}\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{K}})(\mathfrak{u}) \preceq \overline{\chi}_{\mathfrak{K}}$. Thus, $\mathfrak{u} \in \mathfrak{K}$. Hence, $\mathfrak{K}$ is an (α,β)-bi-ideal of $\mathfrak{P}$. ■

Theorem 3.11. *Intersection of any two IVF (α,β)-bi-ideal of a Γ-semigroup $\mathfrak{P}$ is an IVF (α,β)-bi-ideal of $\mathfrak{P}$.*

Proof: Let $\overline{\omega}$ and $\overline{\varpi}$ be interval valued fuzzy (α,β)-bi-ideals of $\mathfrak{P}$. Then

$$(\overline{\omega} \sqcap \overline{\varpi})\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\beta}(\overline{\omega} \sqcap \overline{\varpi}) \sqsubseteq \overline{\omega}\,\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\beta}\overline{\varpi} \sqsubseteq \overline{\omega} \sqcap \overline{\varpi}.$$

Thus, $\overline{\omega} \sqcap \overline{\varpi}$ is an IVF (α,β)-bi-ideal of $\mathfrak{P}$. ■

Theorem 3.12. *Let $\overline{\omega}$ be a IVF set of a Γ-semigroup $\mathfrak{P}$ and $\overline{\omega}_{\overline{p}} = \{\mathfrak{n} \in \mathfrak{P} \mid \overline{\omega}_{\overline{p}}(\mathfrak{n}) \succeq \overline{p}\}$. Then $\overline{\omega}$ is an IVF (α,β)-bi-ideal of $\mathfrak{P}$ if and only if $\overline{\omega}_{\overline{p}}$ is a non-empty set, and $\overline{\omega}_{\overline{p}}$ is an (α,β)-bi-ideal of $\mathfrak{P}$ for all $\overline{p} \in \Omega[0,1]$.*

Proof: Suppose that $\overline{\omega}$ is an IVF (α,β)-bi-ideal of $\mathfrak{P}$ and $\overline{p} \in \Omega[0,1]$, we have $\overline{\omega}_{\overline{p}}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \geq \overline{\omega}_{\overline{p}}(\mathfrak{u}) \curlywedge \overline{\omega}_{\overline{p}}(\mathfrak{w})$ for all $\mathfrak{u},\mathfrak{v},\mathfrak{w} \in \mathfrak{P}$. Thus, $\overline{\omega}_{\overline{p}} \neq \emptyset$. Let $\mathfrak{u},\mathfrak{w} \in \overline{\omega}_{\overline{p}}$ and $\mathfrak{v} \in \mathfrak{P}$. Then $\overline{\omega}_{\overline{p}}(\mathfrak{u}) \succeq \overline{p}$ and $\overline{\omega}_{\overline{p}}(\mathfrak{w}) \succeq \overline{p}$. Thus, $\overline{\omega}_{\overline{p}}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \geq \overline{\omega}_{\overline{p}}(\mathfrak{u}) \curlywedge \overline{\omega}_{\overline{p}}(\mathfrak{w}) \geq p$, so $(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \in \overline{\omega}_{\overline{p}}$. Hence, $\overline{\omega}_{\overline{p}}$ is an (α,β)-bi-ideal of $\mathfrak{P}$.

Conversely, assume that $\overline{\omega}_{\overline{p}}$ is an (α,β)-bi-ideal of $\mathfrak{P}$, $\overline{p} \in \Omega[0,1]$, and $\overline{\omega}_{\overline{p}} \neq \emptyset$. Let $\mathfrak{u},\mathfrak{v},\mathfrak{w} \in \mathfrak{P}$ and $\overline{p} = \overline{\omega}_{\overline{p}}(\mathfrak{u}) \curlywedge \overline{\omega}_{\overline{p}}(\mathfrak{w})$. By assumption, $\overline{\omega}_{\overline{p}}(\mathfrak{u}) \curlywedge \overline{\omega}_{\overline{p}}(\mathfrak{w}) \succeq \overline{p}$, we have $\mathfrak{u},\mathfrak{w} \in \overline{\omega}_{\overline{p}}$. Thus, $\overline{\omega}_{\overline{p}} \neq \emptyset$. Hence, $\overline{\omega}_{\overline{p}}$ is an (α,β)-bi-ideal of $\mathfrak{P}$. Since $\mathfrak{u},\mathfrak{w} \in \overline{\omega}_{\overline{p}}$ and $\mathfrak{v} \in \mathfrak{P}$, we have $\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w} \in \overline{\omega}_{\overline{p}}$. Thus, $\overline{\omega}_{\overline{p}}(\mathfrak{u}\alpha\mathfrak{v}\beta\mathfrak{w}) \geq \overline{p} = \overline{\omega}_{\overline{p}}(\mathfrak{u}) \curlywedge \overline{\omega}_{\overline{p}}(\mathfrak{w})$. Hence, $\overline{\omega}_{\overline{p}}$ is an IVF (α,β)-bi-ideal of $\mathfrak{P}$. ■

Next, we define the interval valued fuzzy (α,β)-quasi-ideal and study its basic properties.

Definition 3.13. *Let $\overline{\omega}$ be an IVF set of a Γ-semigroup $\mathfrak{P}$, and $\alpha,\beta \in \Gamma$. Then $\overline{\omega}$ is called an IVF (α,β)-quasi-ideal of $\mathfrak{P}$ if $\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\omega} \sqcap \overline{\omega}\,\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{P}} \sqsupseteq \overline{\omega}$.*

Theorem 3.14. *If $\overline{\omega}$ and $\overline{\varpi}$ is an IVF left α-ideal and IVF right α-ideal of a Γ-semigroup $\mathfrak{P}$, respectively, then $\overline{\omega} \sqcap \overline{\varpi}$ is an IVF α-quasi-ideal of $\mathfrak{P}$.*

Proof: Let $\overline{\omega}$ and $\overline{\varpi}$ be an IVF left α-ideal and IVF right α-ideal of $\mathfrak{P}$, respectively. Then $\overline{\varpi}\,\overline{\circ}_{\alpha}\overline{\omega} \sqsubseteq \chi_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\omega} \sqsubseteq \overline{\omega}$ and $\overline{\varpi}\,\overline{\circ}_{\alpha}\overline{\omega} \sqsubseteq \overline{\varpi}\,\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}} \sqsubseteq \overline{\varpi}$. Thus, $\overline{\varpi}\,\overline{\circ}_{\alpha}\overline{\omega} \sqsupseteq \overline{\omega} \sqcap \overline{\varpi}$. So,

$$\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}(\overline{\omega}\sqcap\overline{\varpi})\sqcap(\overline{\omega}\sqcap\overline{\varpi})\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}} \sqsubseteq \overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}(\overline{\omega}\sqcap\overline{\varpi})\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}} \sqsubseteq \overline{\omega}\sqcap\overline{\varpi}.$$

Hence, $\overline{\omega} \sqcap \overline{\varpi}$ is an IVF α-quasi-ideal of $\mathfrak{P}$. ■

Theorem 3.15. *Every IVF (α,β)-quasi-ideal of Γ-semigroup $\mathfrak{P}$ is intersection of an IVF left α-ideal and an IVF right β-ideal of $\mathfrak{P}$*

Proof: Let $\overline{\omega}$ be an IVF (α,β)-quasi-ideal of $\mathfrak{P}$, $\overline{\varpi} = \overline{\omega} \sqcap (\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\omega})$, and $\mathcal{K} = \overline{\omega} \sqcap (\overline{\omega}\,\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{P}})$. Then

$$\begin{aligned}
\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\varpi} &= \overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}(\overline{\omega} \sqcap (\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\omega}))\\
&= (\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\omega}) \sqcap (\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}(\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\omega}))\\
&= (\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\omega}) \sqcap ((\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}})\overline{\circ}_{\alpha}\overline{\omega})\\
&= (\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\omega}) \sqcap (\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\omega})\\
&\sqsubseteq \overline{\omega} \sqcap (\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\omega}) = \overline{\varpi}
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{K}\,\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{P}} &= (\overline{\omega} \sqcap (\overline{\omega}\text{-} \circ_{\beta} \overline{\chi}_{\mathfrak{P}}))\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}\\
&= (\overline{\omega}\,\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}) \sqcap (\overline{\omega}\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}})\\
&= (\overline{\omega}\,\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}) \sqcap \overline{\omega}\overline{\circ}_{\beta}(\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}})\\
&= (\overline{\omega}\,\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}) \sqcap (\overline{\omega}\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{P}})\\
&\sqsubseteq \overline{\omega} \sqcap (\overline{\omega}\,\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{P}}) = \mathcal{K}.
\end{aligned}$$

Thus, $\overline{\varpi}$ and $\mathcal{K}$ is an IVF left α-ideal and an IVF right β-ideal of $\mathfrak{P}$. Now,

$$\overline{\omega} \sqsubseteq (\overline{\omega} \sqcap (\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\omega})) \sqcap (\overline{\omega} \sqcap (\overline{\omega}\,\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{P}})) = \overline{\varpi} \sqcap \mathcal{K}$$

and

$$\begin{aligned}
\overline{\varpi} \sqcap \mathcal{K} &= \overline{\omega} \sqcap (\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\omega}) \sqcap \overline{\omega} \sqcap (\overline{\omega}\,\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{P}})\\
&\sqsubseteq \overline{\omega} \sqcap ((\overline{\chi}_{\mathfrak{p}}\overline{\circ}_{\alpha}\overline{\omega}) \sqcap (\overline{\omega}\,\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{P}}))\\
&\sqsubseteq \overline{\omega} \sqcap \overline{\omega} = \overline{\omega}.
\end{aligned}$$

Hence, $\overline{\omega} = \overline{\varpi} \sqcap \mathcal{K}$. ■

Theorem 3.16. *Let $\mathfrak{K}$ be a non-empty subset of Γ-semigroup $\mathfrak{P}$. Then $\mathfrak{K}$ is a (α,β)-quasi-ideal of $\mathfrak{P}$ if and only if the characteristic function $\overline{\chi}_{\mathfrak{K}}$ is an IVF (α,β)-quasi-ideal of $\mathfrak{P}$.*

Proof: Suppose that $\mathfrak{K}$ is a (α,β)-quasi-ideal of $\mathfrak{P}$, and $\mathfrak{u} \in \mathfrak{P}$.

If $\mathfrak{u} \in (\mathfrak{P}\alpha\mathfrak{K}) \sqcap (\mathfrak{K}\beta\mathfrak{P})$, then $\mathfrak{u} \in \mathfrak{K}$. Thus, $\overline{\chi}_{\mathfrak{K}}(\mathfrak{u}) = \overline{1}$. Hence, $((\overline{\chi}_{\mathfrak{K}}\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}) \sqcap (\overline{\chi}_{\mathfrak{K}}\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{P}}))(\mathfrak{u}) \succeq \overline{\chi}_{\mathfrak{K}}(\mathfrak{u})$.

If $\mathfrak{u} \notin \mathfrak{P}$, then $((\overline{\chi}_{\mathfrak{K}}\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}) \sqcap (\overline{\chi}_{\mathfrak{K}}\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{P}}))(\mathfrak{u}) = \overline{0}$. Hence, $((\overline{\chi}_{\mathfrak{K}}\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}) \sqcap (\overline{\chi}_{\mathfrak{K}}\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{P}}))(\mathfrak{u}) \succeq \overline{\chi}_{\mathfrak{K}}(\mathfrak{u})$.

Therefore, $\overline{\chi}_{\mathfrak{K}}$ is an IVF (α,β)-quasi-ideal of $\mathfrak{P}$.

Conversely, assume that $\overline{\chi}_{\mathfrak{K}}$ is an IVF (α,β)-quasi-ideal of $\mathfrak{P}$, and $\mathfrak{u} \in (\mathfrak{P}\alpha\mathfrak{K}) \sqcap (\mathfrak{K}\beta\mathfrak{P})$. Then $(\overline{\chi}_{\mathfrak{P}}\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{K}}) \sqcap (\overline{\chi}_{\mathfrak{K}}\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{P}})(\mathfrak{u}) = \overline{1}$. By assumption, $(\overline{\chi}_{\mathfrak{K}}\overline{\circ}_{\alpha}\overline{\chi}_{\mathfrak{P}}) \sqcap (\overline{\chi}_{\mathfrak{K}}\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{P}})(\mathfrak{u}) \succeq \overline{\chi}_{\mathfrak{K}}(\mathfrak{u})$. Thus, $\mathfrak{u} \in \mathfrak{K}$. Hence, $\mathfrak{K}$ is an (α,β)-quasi-ideal of $\mathfrak{P}$. ■

IV. New Types of Interval Valued Fuzzy Almost Ideals

Before we conduct a study in this topic, we will define the point of an interval valued fuzzy set.

For $\overline{p} \in \Omega[0,1]$ and $\overline{\omega}$, an IVF set of a Γ-semigroup $\mathfrak{P}$ is defined by

$$\overline{\chi}_{\mathfrak{p}}(\mathfrak{p}) = \begin{cases} \overline{p} & \text{if} \quad \mathfrak{p} \in \mathfrak{K},\\ \overline{0} & \text{if} \quad \mathfrak{p} \notin \mathfrak{K}, \end{cases}$$

where $\mathfrak{K}$ is a non-empty subset of $\mathfrak{P}$.

Definition 4.1. *Let $\overline{\omega}$ be an IVF set of a Γ-semigroup $\mathfrak{P}$, and $\alpha,\beta \in \Gamma$ is said to be*

(1) *an IVF almost left α-ideal of $\mathfrak{P}$ if $(\overline{\chi}_{\mathfrak{p}}\,\overline{\circ}_{\alpha}\overline{\omega}) \sqcap \overline{\omega} \neq \overline{0}$ for all $\mathfrak{p} \in \mathfrak{P}$.*
(2) *an IVF almost right β-ideal of $\mathfrak{P}$ if $(\overline{\omega}\,\overline{\circ}_{\beta}\overline{\chi}_{\mathfrak{p}}) \sqcap \overline{\omega} \neq \overline{0}$, for all $\mathfrak{p} \in \mathfrak{P}$.*
(3) *an IVF almost (α,β)-ideal of $\mathfrak{P}$ if it is both an IVF almost left α-ideal and an IVF almost right β-ideal of $\mathfrak{P}$,*

where $\overline{\chi}_{\mathfrak{p}}$ is an IVF set of $\mathfrak{P}$ mapping every element of $\mathfrak{P}$ to $\overline{p}$.

Theorem 4.2. *If $\overline{\omega}$ is an IVF almost left α-ideal (right β-ideal, (α,β)-ideal) of a Γ-semigroup $\mathfrak{P}$, and $\overline{\varpi}$ is an IVF set of $\mathfrak{P}$ such that $\overline{\omega} \sqsubseteq \overline{\varpi}$, then $\overline{\varpi}$ is an IVF left almost α-ideal (right β-ideal, (α,β)-ideal) of $\mathfrak{P}$.*

Proof: Suppose that $\overline{\omega}$ is an IVF almost left α-ideal of $\mathfrak{P}$, and $\overline{\varpi}$ is an IVF fuzzy set of $\mathfrak{P}$ such that $\overline{\omega} \sqsubseteq \overline{\varpi}$. Then $(\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\omega}) \sqcap \overline{\omega} \neq \emptyset$. Thus, $(\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\omega}) \sqcap \overline{\omega} \sqsubseteq (\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\varpi}) \sqcap \overline{\varpi} \neq \emptyset$. Hence, $\overline{\varpi}$ is an IVF left almost α-ideal of $\mathfrak{P}$. ∎

Theorem 4.3. *Let $\mathfrak{K}$ be a non-empty subset of Γ-semigroup $\mathfrak{P}$. Then $\mathfrak{K}$ is an almost left α-ideal (right β-ideal, (α, β)-ideal) of $\mathfrak{P}$ if and only if the characteristic function $\overline{\chi}_{\mathfrak{K}}$ is an IVF almost left α-ideal (right β-ideal, (α, β)-ideal) of $\mathfrak{P}$.*

Proof: Suppose that $\mathfrak{K}$ is an almost left α-ideal of $\mathfrak{P}$. Then $\mathfrak{u}\alpha\mathfrak{K} \cap \mathfrak{K} \neq \emptyset$ for all $\mathfrak{u} \in \mathfrak{P}$. Thus, there exists $\mathfrak{v} \in \mathfrak{u}\alpha\mathfrak{K}$ and $\mathfrak{v} \in \mathfrak{K}$. So, $(\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\chi}_{\mathfrak{K}})(\mathfrak{v}) = \overline{\chi}_{\mathfrak{K}}(\mathfrak{v}) = \overline{1}$. Hence, $(\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\chi}_{\mathfrak{K}}) \sqcap \overline{\chi}_{\mathfrak{K}} \neq \overline{0}$. Therefore, $\overline{\chi}_{\mathfrak{K}}$ is an IVF almost left α-ideal of $\mathfrak{P}$.

Conversely, assume that $\overline{\chi}_{\mathfrak{K}}$ is an IVF almost left α-ideal of $\mathfrak{P}$ and $\mathfrak{u} \in \mathfrak{P}$. Then $(\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\chi}_{\mathfrak{K}}) \sqcap \overline{\chi}_{\mathfrak{K}} \neq \overline{0}$. Thus, there exists $\mathfrak{v} \in \mathfrak{P}$ such that $(\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\chi}_{\mathfrak{K}}) \sqcap \overline{\chi}_{\mathfrak{K}}(\mathfrak{v}) \neq \overline{0}$. Hence, $\mathfrak{v} \in \mathfrak{u}\alpha\mathfrak{K} \cap \mathfrak{K}$ implies $\mathfrak{u}\alpha\mathfrak{K} \cap \mathfrak{K} \neq \emptyset$.
Therefore, $\mathfrak{K}$ is an almost left α-ideal of $\mathfrak{P}$. ∎

Next, we review the definition of $\mathrm{supp}(\overline{\varpi})$, and we study properties between $\mathrm{supp}(\overline{\varpi})$ and IVF almost left α-ideal (right β-ideal, (α, β)-ideal) of Γ-semigroups.

Let $\overline{\varpi}$ be a IVF sets of a non-empty of $\mathfrak{K}$. Then the *support* of $\overline{\varpi}$ instead of $\mathrm{supp}(\overline{\varpi}) = \{\mathfrak{u} \in \mathfrak{K} \mid \overline{\varpi}(\mathfrak{u}) \neq \overline{0}\}$.

Example 4.4. *Let $\mathfrak{P} = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}\}$ be a Γ-semigroup with $\Gamma = \{\alpha\}$ defined by the following table:*

α	$\mathfrak{a}$	$\mathfrak{b}$	$\mathfrak{c}$	$\mathfrak{d}$	$\mathfrak{e}$
$\mathfrak{a}$	$\mathfrak{a}$	$\mathfrak{b}$	$\mathfrak{c}$	$\mathfrak{d}$	$\mathfrak{e}$
$\mathfrak{b}$	$\mathfrak{b}$	$\mathfrak{c}$	$\mathfrak{d}$	$\mathfrak{e}$	$\mathfrak{a}$
$\mathfrak{c}$	$\mathfrak{c}$	$\mathfrak{d}$	$\mathfrak{e}$	$\mathfrak{a}$	$\mathfrak{b}$
$\mathfrak{d}$	$\mathfrak{d}$	$\mathfrak{e}$	$\mathfrak{a}$	$\mathfrak{b}$	$\mathfrak{c}$
$\mathfrak{e}$	$\mathfrak{e}$	$\mathfrak{a}$	$\mathfrak{b}$	$\mathfrak{c}$	$\mathfrak{d}$

Define a IVF set $\overline{\omega}$ on $\mathfrak{P}$ as follows : $\overline{\omega}(\mathfrak{a}) = 0.9$, $\overline{\omega}(\mathfrak{b}) = 0.8$ $\overline{\omega}(\mathfrak{c}) = 0$, $\overline{\omega}(\mathfrak{d}) = 0.3$ and $\overline{\omega}(\mathfrak{e}) = 0$. Then $\mathrm{supp}(\overline{\omega}) = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{d}\}$

Theorem 4.5. *Let $\overline{\varpi}$ be a IVF sets of a non-empty of a Γ-semigroup $\mathfrak{P}$. Then $\overline{\varpi}$ is an IVF almost left α-ideal (right β-ideal, (α, β)-ideal) of $\mathfrak{P}$ if and only if* $\mathrm{supp}(\overline{\varpi})$ *is an almost left α-ideal (right β-ideal, (α, β)-ideal) of $\mathfrak{P}$.*

Proof: Let $\overline{\varpi}$ be an IVF almost left α-ideal of $\mathfrak{P}$ and $\mathfrak{u} \in \mathfrak{P}$. Then $(\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\varpi}) \sqcap \overline{\varpi} \neq \emptyset$. Thus, there exists $\mathfrak{r} \in S$ such that $((\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\varpi}) \sqcap \overline{\varpi})(\mathfrak{r}) \neq \overline{0}$. So, there exists $\mathfrak{k} \in \mathfrak{P}$ such that $\mathfrak{r} = \mathfrak{u}\alpha\mathfrak{k}$, $\overline{\varpi}(\mathfrak{r}) \neq \overline{0}$ and $\overline{\varpi}(\mathfrak{k}) \neq \overline{0}$. It implies that $\mathfrak{r}, \mathfrak{k} \in \mathfrak{P}\,\mathrm{supp}(\overline{\varpi})$. Thus, $(\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\chi}_{\mathrm{supp}(\overline{\varpi})})(\mathfrak{r}) \neq \overline{0}$ and $\overline{\chi}_{\mathrm{supp}(\overline{\varpi})} \neq \emptyset$. Hence, $(\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\chi}_{\mathrm{supp}(\overline{\varpi})}) \sqcap \overline{\chi}_{\mathrm{supp}(\overline{\varpi})} \neq \emptyset$. Therefore, $\overline{\chi}_{\mathrm{supp}(\overline{\varpi})}$ is an IVF almost left α-ideal of $\mathfrak{P}$. This shows that $\mathrm{supp}(\overline{\varpi})$ is an almost left α-ideal of $\mathfrak{P}$.

Conversely, let $\mathrm{supp}(\overline{\varpi})$ be an almost left α-ideal of $\mathfrak{P}$. Then, by Theorem 4.3, $\overline{\chi}_{\mathrm{supp}(\overline{\varpi})}$ is an IVF almost left α-ideal of $\mathfrak{P}$. Thus, $(\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\chi}_{\mathrm{supp}(\overline{\varpi})}) \sqcap \overline{\chi}_{\mathrm{supp}(\overline{\varpi})} \neq \emptyset$. So, there exists $\mathfrak{r} \in \mathfrak{P}$ such that $(\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\varpi}) \sqcap \overline{\varpi}(\mathfrak{r}) \neq \overline{0}$. It implies that $(\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\varpi})(\mathfrak{r}) \neq \overline{0}$ and $\overline{\varpi}(\mathfrak{r}) \neq \overline{0}$. Thus, there exists $\mathfrak{k} \in \mathfrak{P}$ such that $\mathfrak{r} = \mathfrak{u}\alpha\mathfrak{k}$, $\overline{\varpi}(\mathfrak{r}) \neq \overline{0}$ and $\varpi(\mathfrak{k}) \neq \overline{0}$. Hence, $(\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\alpha} \overline{\varpi}) \sqcap \overline{\varpi} \neq \overline{0}$. Therefore, $\overline{\varpi}$ is an IVF almost left α-ideal of $\mathfrak{P}$. ∎

Definition 4.6. *An ideal $\mathfrak{I}$ of a Γ-semigroup $\mathfrak{P}$ is called*

(1) *a* minimal *if for every ideal of $\mathfrak{J}$ of $\mathfrak{P}$ such that $\mathfrak{J} \subseteq \mathfrak{I}$, we have $\mathfrak{J} = \mathfrak{I}$,*
(2) *a* maximal *if for every ideal of $\mathfrak{J}$ of $\mathfrak{P}$ such that $\mathfrak{I} \subseteq \mathfrak{J}$, we have $\mathfrak{J} = \mathfrak{I}$.*

Definition 4.7. *An almost ideal $\mathfrak{I}$ of a Γ-semigroup $\mathfrak{P}$ is called*

(1) *a* minimal *if for every almost ideal of $\mathfrak{J}$ of $\mathfrak{P}$ such that $\mathfrak{J} \subseteq \mathfrak{I}$, we have $\mathfrak{J} = \mathfrak{I}$,*
(2) *a* maximal *if for every almost ideal of $\mathfrak{J}$ of $\mathfrak{P}$ such that $\mathfrak{I} \subseteq \mathfrak{J}$, we have $\mathfrak{J} = \mathfrak{I}$.*

Definition 4.8. *An IVF almost left α-ideal (right β-ideal, (α, β)-ideal) $\overline{\omega}$ of a Γ-semigroup $\mathfrak{P}$ is*

(1) *a* minimal *if for all IVF almost left α-ideal (right β-ideal, (α, β)-ideal) $\overline{\varpi}$ of $\mathfrak{P}$ such that $\overline{\varpi} \sqsubseteq \overline{\omega}$, then* $\mathrm{supp}(\overline{\varpi}) = \mathrm{supp}(\overline{\omega})$,
(2) *a* maximal *if for all IVF almost left α-ideal (right β-ideal, (α, β)-ideal) $\overline{\varpi}$ of $\mathfrak{P}$ such that $\overline{\omega} \sqsubseteq \overline{\varpi}$, then* $\mathrm{supp}(\overline{\varpi}) = \mathrm{supp}(\overline{\omega})$,

Theorem 4.9. *Let $\mathfrak{K}$ be a non-empty subset of a Γ-semigroup $\mathfrak{P}$ Then the following statements holds*

(1) *$\mathfrak{K}$ is a minimal almost left α-ideal (right β-ideal, (α, β)-ideal) if and only if $\overline{\chi}_{\mathfrak{K}}$ is a minimal IVF almost left α-ideal (right β-ideal, (α, β)-ideal) of $\mathfrak{P}$.*
(2) *$\mathfrak{K}$ is a maximal almost left α-ideal (right β-ideal, (α, β)-ideal) if and only if $\overline{\chi}_{\mathfrak{K}}$ is a maximal IVF almost left α-ideal (right β-ideal, (α, β)-ideal) of $\mathfrak{P}$.*

Proof:

(1) Suppose that $\mathfrak{K}$ is a minimal almost left α-ideal of $\mathfrak{P}$. Then $\mathfrak{K}$ is an almost left α-ideal of $\mathfrak{P}$. Thus, by Theorem 4.3, $\overline{\chi}_{\mathfrak{K}}$ is an IVF left α-ideal of $\mathfrak{P}$. Let $\overline{\varpi}$ be an IVF left α-ideal of $\mathfrak{P}$ such that $\overline{\varpi} \sqsubseteq \overline{\chi}_{\mathfrak{K}}$. Then, by Theorem 4.5, $\mathrm{supp}(\overline{\varpi})$ is an almost left α-ideal of $\mathfrak{P}$. Thus, $\mathrm{supp}(\overline{\varpi}) \subseteq \mathrm{supp}(\overline{\chi}_{\mathfrak{K}}) = \mathfrak{K}$. By assumption, $\mathrm{supp}(\overline{\varpi}) = \mathfrak{K} = \mathrm{supp}(\overline{\chi}_{\mathfrak{K}})$. Thus $\overline{\chi}_{\mathfrak{K}}$ is a minimal IVF fuzzy almost left α-ideal of $\mathfrak{P}$.
Conversely, suppose that $\overline{\chi}_{\mathfrak{K}}$ is a minimal IVF almost left α-ideal of $\mathfrak{P}$. Then by Theorem 4.3, $\mathfrak{K}$ is an almost left α-ideal of $\mathfrak{P}$. Let $\mathfrak{J}$ be an almost left α-ideal of $\mathfrak{P}$ such that $\mathfrak{J} \subseteq \mathfrak{K}$. Then, by Theorem 4.3, $\overline{\chi}_{\mathfrak{J}}$ is an IVF left α-ideal of $\mathfrak{P}$ such that $\overline{\chi}_{\mathfrak{J}} \sqsubseteq \overline{\chi}_{\mathfrak{K}}$. Thus, $\mathfrak{J} = \mathrm{supp}(\overline{\chi}_{\mathfrak{J}}) = \mathrm{supp}(\overline{\chi}_{\mathfrak{K}}) = \mathfrak{K}$. Hence, $\mathfrak{K}$ is a minimal almost left α-ideal of $\mathfrak{P}$.
(2) Similar to (1).

∎

Corollary 4.10. *Let $\mathfrak{P}$ be a Γ-semigroup $\mathfrak{P}$. Then $\mathfrak{P}$ has no proper almost left α-ideal (right β-ideal, (α, β)-ideal) if and only if for any IVF almost left α-ideal (right β-ideal, (α, β)-ideal) $\overline{\varpi}$ of $\mathfrak{P}$,* $\mathrm{supp}(\overline{\varpi}) = \mathfrak{P}$.

Next, we define the interval valued fuzzy almost (α, β)-quasi-ideal and we study its properties.

Definition 4.11. *Let $\overline{\omega}$ be an IVF set of a Γ-semigroup $\mathfrak{P}$, and $\alpha, \beta \in \Gamma$ is said to be* IVF almost (α, β)-quasi-ideal *of $\mathfrak{P}$ if $(\overline{\omega} \overline{\circ}_{\alpha} \overline{\chi}_{\mathfrak{p}}) \sqcap (\overline{\chi}_{\mathfrak{p}} \overline{\circ}_{\beta} \overline{\omega}) \sqcap \overline{\omega} \neq \overline{0}$.*

Theorem 4.12. *If $\overline{\omega}$ is an IVF almost (α, β)-quasi-ideal of a Γ-semigroup $\mathfrak{P}$, and $\overline{\varpi}$ is an IVF set of $\mathfrak{P}$ such that $\overline{\omega} \sqsubseteq \overline{\varpi}$, then $\overline{\varpi}$ is an IVF (α, β)-quasi-ideal of $\mathfrak{P}$.*

Proof: Suppose that $\overline{\omega}$ is an IVF almost (α, β)-quasi-ideal of $\mathfrak{P}$, and $\overline{\varpi}$ is an IVF set of $\mathfrak{P}$ such that $\overline{\omega} \sqsubseteq \overline{\varpi}$. Then

$(\overline{\omega}\,\overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}) \sqcap (\overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\omega}) \sqcap \overline{\omega} \neq \emptyset$. Thus, $(\overline{\omega}\,\overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}) \sqcap (\overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\omega}) \sqcap \overline{\omega} \subseteq (\overline{\varpi}\,\overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}) \sqcap (\overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\varpi}) \sqcap \overline{\varpi} \neq \emptyset$. Hence, $\overline{\varpi}$ is an IVF (α, β)-quasi-ideal of $\mathfrak{P}$. ■

Theorem 4.13. *Let $\mathfrak{K}$ be a non-empty subset of Γ-semigroup $\mathfrak{P}$. Then $\mathfrak{K}$ is an almost (α, β)-quasi-ideal of $\mathfrak{P}$ if and only if the characteristic function $\overline{\chi}_\mathfrak{K}$ is an IVF almost (α, β)-quasi-ideal of $\mathfrak{P}$.*

Proof: Suppose that $\mathfrak{K}$ is an almost (α, β)-quasi-ideal of $\mathfrak{P}$. Then $(\mathfrak{K}\alpha\mathfrak{u}) \cap (\mathfrak{u}\beta\mathfrak{K}) \cap \mathfrak{K} \neq \emptyset$ for all $\mathfrak{u} \in \mathfrak{P}$. Thus, there exists $\mathfrak{v} \in (\mathfrak{K}\alpha\mathfrak{u}) \cap (\mathfrak{u}\beta\mathfrak{K}) \cap \mathfrak{K}$ and $\mathfrak{u} \in \mathfrak{K}$.
So, $((\overline{\chi}_\mathfrak{K} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}) \sqcap (\overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\chi}_\mathfrak{K}))(\mathfrak{v}) \neq \overline{0}$ and $\overline{\chi}_\mathfrak{K}(\mathfrak{v}) = \overline{1}$. Hence, $(\overline{\chi}_\mathfrak{K} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}) \sqcap (\overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\chi}_\mathfrak{K}) \sqcap \overline{\chi}_\mathfrak{K} \neq \overline{0}$. Therefore, $\overline{\chi}_\mathfrak{K}$ is an IVF almost (α, β)-quasi-ideal of $\mathfrak{P}$.

Conversely, assume that $\overline{\chi}_\mathfrak{K}$ is an IVF almost (α, β)-quasi-ideal of $\mathfrak{P}$ and $\mathfrak{u} \in \mathfrak{P}$. Then $[(\overline{\chi}_\mathfrak{K} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}) \sqcap (\overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\chi}_\mathfrak{K})]$ $\neq \overline{0}$. Thus, there exists $\mathfrak{r} \in \mathfrak{P}$ such that $((\overline{\chi}_\mathfrak{K} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}) \sqcap (\overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\chi}_\mathfrak{p}))(\mathfrak{r}) \neq \overline{0}$. Hence, $\mathfrak{r} \in (\mathfrak{K}\alpha\mathfrak{u}) \cap (\mathfrak{u}\beta\mathfrak{K}) \cap \mathfrak{K}$ implies $(\mathfrak{K}\alpha\mathfrak{u}) \cap (\mathfrak{u}\beta\mathfrak{K}) \cap \mathfrak{K} \neq \emptyset$. Therefore, $\mathfrak{K}$ is an almost (α, β)-quasi-ideal of $\mathfrak{P}$. ■

Next, we study properties between $\text{supp}(\overline{\varpi})$ and IVF fuzzy almost (α, β)-quasi-ideal of Γ-semigroups.

Theorem 4.14. *Let $\overline{\varpi}$ be a IVF sets of a non-empty of a Γ-semigroup $\mathfrak{P}$. Then $\overline{\varpi}$ is an IVF almost (α, β)-quasi-ideal of $\mathfrak{P}$ if and only if $\text{supp}(\overline{\varpi})$ is an almost (α, β)-quasi-ideal of $\mathfrak{P}$.*

Proof: Let $\overline{\varpi}$ be an IVF almost (α, β)-quasi-ideal of $\mathfrak{P}$ and $\mathfrak{u} \in \mathfrak{P}$. Then, $(\overline{\varpi} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}) \sqcap (\overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\varpi}) \sqcap \overline{\varpi} \neq \emptyset$. Thus, there exists $\mathfrak{r} \in \mathfrak{P}$ such that $((\overline{\varpi} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}) \sqcap (\overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\varpi}) \sqcap \overline{\varpi})(\mathfrak{r}) \neq \overline{0}$. So there exists $\mathfrak{k}_1, \mathfrak{k}_2 \in \mathfrak{P}$ such that $\mathfrak{r} = \mathfrak{k}_1\alpha\mathfrak{u} = \mathfrak{u}\beta\mathfrak{k}_2$, $\overline{\varpi}(\mathfrak{r}) \neq \overline{0}$, $\overline{\varpi}(\mathfrak{k}_1) \neq \overline{0}$ and $\overline{\varpi}(\mathfrak{k}_2) \neq \overline{0}$. It implies that $\mathfrak{r}, \mathfrak{k}_1, \mathfrak{k}_2 \in \text{supp}(\overline{\varpi})$. Thus, $((\overline{\chi}_{\text{supp}(\overline{\varpi})} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}) \sqcap (\overline{\chi}_\mathfrak{P} \overline{\circ}_\beta \overline{\chi}_{\text{supp}(\overline{\varpi})}))(\mathfrak{r}) \neq \overline{0}$ and $\overline{\chi}_{\text{supp}(\overline{\varpi})}(\mathfrak{r}) \neq \overline{0}$. Hence, $[(\overline{\chi}_{\text{supp}(\overline{\varpi})} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}) \sqcap (\overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\chi}_{\text{supp}(\overline{\varpi})})] \sqcap \overline{\chi}_{\text{supp}(\overline{\varpi})} \neq \overline{0}$.
Therefore, $\overline{\chi}_{\text{supp}(\overline{\varpi})}$ is an IVF almost (α, β)-quasi-ideal of $\mathfrak{P}$. This shows that $\text{supp}(\overline{\varpi})$ is an almost (α, β)-quasi-ideal of $\mathfrak{P}$.

Conversely, let $\text{supp}(\overline{\varpi})$ be an almost (α, β)-quasi-ideal of $\mathfrak{P}$. Then, by Theorem 4.13, $\overline{\chi}_{\text{supp}(\overline{\varpi})}$ is an IVF almost (α, β)-quasi-ideal of $\mathfrak{P}$. Thus, $[(\overline{\chi}_{\text{supp}(\overline{\varpi})} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}) \sqcap (\overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \chi_{\text{supp}(\overline{\varpi})})] \sqcap \overline{\chi}_{\text{supp}(\overline{\varpi})} \neq \overline{0}$. So, there exists $\mathfrak{r} \in \mathfrak{P}$ such that $[(\overline{\chi}_{\text{supp}(\overline{\varpi})} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}) \sqcap (\overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\chi}_{\text{supp}(\overline{\varpi})})] \sqcap \overline{\chi}_{\text{supp}(\overline{\varpi})}(\mathfrak{r}) \neq \overline{0}$. It implies that $[(\overline{\chi}_{\text{supp}(\overline{\varpi})} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}) \sqcap (\overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\chi}_{\text{supp}(\overline{\varpi})})](\mathfrak{r}) \neq \overline{0}$ and $\overline{\chi}_{\text{supp}(\overline{\varpi})}(\mathfrak{r}) \neq \overline{0}$. Thus, there exist $\mathfrak{k}_1, \mathfrak{k}_2 \in \mathfrak{P}$ such that $\mathfrak{r} = \mathfrak{k}_1\alpha\mathfrak{u} = \mathfrak{u}\beta\mathfrak{k}_2$, $\overline{\varpi}(\mathfrak{r}) \neq \overline{0}$, $\overline{\varpi}(\mathfrak{k}_1) \neq \overline{0}$ and $\overline{\varpi}(\mathfrak{k}_2) \neq \overline{0}$. Hence, $(\overline{\varpi} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{P}) \sqcap (\overline{\chi}_\mathfrak{P} \overline{\circ}_\beta \overline{\varpi}) \sqcap \overline{\varpi} \neq \overline{0}$. Therefore, $\overline{\varpi}$ is an IVF almost (α, β)-quasi-ideal of $\mathfrak{P}$. ■

Definition 4.15. *An almost quasi-ideal $\mathfrak{I}$ of a Γ-semigroup $\mathfrak{P}$ is called*
(1) *a* minimal *if for every almost quasi-ideal of $\mathfrak{J}$ of $\mathfrak{P}$ such that $\mathfrak{J} \subseteq \mathfrak{I}$, we have $\mathfrak{J} = \mathfrak{I}$,*
(2) *a* maximal *if for every almost quasi-ideal of $\mathfrak{J}$ of $\mathfrak{P}$ such that $\mathfrak{I} \subseteq \mathfrak{J}$, we have $\mathfrak{J} = \mathfrak{I}$.*

Definition 4.16. *An IVF almost (α, β)-quasi-ideal $\overline{\omega}$ of a Γ-semigroup $\mathfrak{P}$ is*
(1) *a* minimal *if for all IVF almost (α, β)-quasi-ideal $\overline{\varpi}$ of $\mathfrak{P}$ such that $\overline{\varpi} \sqsubseteq \overline{\omega}$, then* $\text{supp}(\overline{\varpi}) = \text{supp}(\overline{\omega})$,
(2) *a* maximal *if for all IVF almost (α, β)-quasi-ideal $\overline{\varpi}$ of $\mathfrak{P}$ such that $\overline{\omega} \sqsubseteq \overline{\varpi}$, then* $\text{supp}(\overline{\varpi}) = \text{supp}(\overline{\omega})$,

Theorem 4.17. *Let $\mathfrak{K}$ be a non-empty subset of a Γ-semigroup $\mathfrak{P}$. Then the following statements holds*
(1) *$\mathfrak{K}$ is a minimal almost (α, β)-quasi-ideal if and only if $\overline{\chi}_\mathfrak{K}$ is a minimal IVF almost (α, β)-quasi-ideal of $\mathfrak{P}$.*
(2) *$\mathfrak{K}$ is a maximal almost (α, β)-quasi-ideal if and only if $\overline{\chi}_\mathfrak{K}$ is a maximal IVF almost (α, β)-quasi-ideal of $\mathfrak{P}$.*

Proof:
(1) Suppose that $\mathfrak{K}$ is a minimal almost (α, β)-quasi-ideal of $\mathfrak{P}$. Then $\mathfrak{K}$ is an almost left α-ideal of $\mathfrak{P}$. Thus, by Theorem 4.13, $\overline{\chi}_\mathfrak{K}$ is an IVF (α, β)-quasi-ideal of $\mathfrak{P}$. Let $\overline{\varpi}$ be an IVF (α, β)-quasi-ideal of $\mathfrak{P}$ such that $\overline{\varpi} \subseteq \overline{\chi}_\mathfrak{K}$. Then, by Theorem 4.14, $\text{supp}(\overline{\varpi})$ is an almost (α, β)-quasi-ideal of $\mathfrak{P}$. Thus, $\text{supp}(\overline{\varpi}) \subseteq \text{supp}(\overline{\chi}_\mathfrak{K}) = \mathfrak{K}$. By assumption, $\text{supp}(\overline{\varpi}) = \mathfrak{K} = \text{supp}(\overline{\chi}_\mathfrak{K})$. Thus, $\overline{\chi}_\mathfrak{K}$ is a minimal IVF fuzzy almost (α, β)-quasi-ideal of $\mathfrak{P}$.
Conversely, suppose that $\overline{\chi}_\mathfrak{K}$ is a minimal IVF almost (α, β)-quasi-ideal of $\mathfrak{P}$. Then, by Theorem 4.13, $\mathfrak{K}$ is an almost (α, β)-quasi-ideal of $\mathfrak{P}$. Let $\mathfrak{J}$ be an almost (α, β)-quasi-ideal of $\mathfrak{P}$ such that $\mathfrak{J} \subseteq \mathfrak{K}$. Then, by Theorem 4.13, $\overline{\chi}_\mathfrak{J}$ is an IVF (α, β)-quasi-ideal of $\mathfrak{P}$ such that $\overline{\chi}_\mathfrak{J} \sqsubseteq \overline{\chi}_\mathfrak{K}$. Thus, $\mathfrak{J} = \text{supp}(\overline{\chi}_\mathfrak{J}) = \text{supp}(\overline{\chi}_\mathfrak{K}) = \mathfrak{K}$. Hence, $\mathfrak{K}$ is a minimal almost (α, β)-quasi-ideal of $\mathfrak{P}$.
(2) Similar to (1).

■

Corollary 4.18. *Let $\mathfrak{P}$ be a Γ-semigroup $\mathfrak{P}$. Then $\mathfrak{P}$ has no proper almost (α, β)-quasi-ideal if and only if for any IVF almost (α, β)-quasi-ideal $\overline{\varpi}$ of $\mathfrak{P}$, supp$(\overline{\varpi}) = \mathfrak{P}$.*

Next, we define IVF almost (α, β)-bi-ideal, and we study its properties.

Definition 4.19. *Let $\overline{\omega}$ be an IVF set of a Γ-semigroup $\mathfrak{P}$, and $\alpha, \beta \in \Gamma$ is said to be* IVF almost (α, β)-bi-ideal *of $\mathfrak{P}$ if $(\overline{\omega}\,\overline{\circ}_\alpha \overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\omega}) \sqcap \overline{\omega} \neq \overline{0}$.*

Theorem 4.20. *If $\overline{\omega}$ is an IVF almost (α, β)-bi-ideal of a Γ-semigroup $\mathfrak{P}$, and $\overline{\varpi}$ is an IVF set of $\mathfrak{P}$ such that $\overline{\omega} \sqsubseteq \overline{\varpi}$, then $\overline{\varpi}$ is an IVF (α, β)-bi-ideal of $\mathfrak{P}$.*

Proof: Suppose that $\overline{\omega}$ is an IVF almost (α, β)-bi-ideal of $\mathfrak{P}$, and $\overline{\varpi}$ is an IVF set of $\mathfrak{P}$ such that $\overline{\omega} \sqsubseteq \overline{\varpi}$. Then $(\overline{\omega}\,\overline{\circ}_\alpha \overline{\chi}_\mathfrak{p}\,\overline{\circ}_\beta \overline{\omega}) \sqcap \overline{\omega} \neq \overline{0}$. Thus, $(\overline{\omega}\,\overline{\circ}_\alpha \overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\omega}) \sqcap \overline{\omega} \sqsubseteq (\overline{\varpi}\,\overline{\circ}_\alpha \overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\varpi}) \sqcap \overline{\varpi} \neq \overline{0}$. Hence, $\overline{\varpi}$ is an IVF (α, β)-bi-ideal of $\mathfrak{P}$. ■

Theorem 4.21. *Let $\mathfrak{K}$ be a non-empty subset of Γ-semigroup $\mathfrak{P}$. Then K is an almost (α, β)-bi-ideal of $\mathfrak{P}$ if and only if the characteristic function $\overline{\chi}_\mathfrak{K}$ is an IVF almost (α, β)-bi-ideal of $\mathfrak{P}$.*

Proof: Suppose that $\mathfrak{K}$ is an almost (α, β)-bi-ideal of $\mathfrak{P}$. Then $\mathfrak{K}\alpha\mathfrak{p}\beta\mathfrak{K} \cap \mathfrak{K} \neq \emptyset$ for all $\mathfrak{p} \in \mathfrak{P}$. Thus, there exists $\mathfrak{v} \in \mathfrak{K}\alpha\mathfrak{p}\beta\mathfrak{K}$ and $\mathfrak{v} \in \mathfrak{K}$. So, $(\overline{\chi}_\mathfrak{K} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\chi}_\mathfrak{K})(\mathfrak{v}) = \overline{\chi}_\mathfrak{K}(\mathfrak{v}) = \overline{1}$. Hence, $(\overline{\chi}_\mathfrak{K} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\chi}_\mathfrak{K}) \sqcap \overline{\chi}_\mathfrak{K} \neq \overline{0}$. Therefore, $\overline{\chi}_\mathfrak{K}$ is an IVF almost (α, β)-bi-ideal of $\mathfrak{P}$.

Conversely, assume that $\overline{\chi}_\mathfrak{K}$ is an IVF almost (α, β)-bi-ideal of $\mathfrak{P}$ and $\mathfrak{p} \in \mathfrak{P}$. Then $(\overline{\chi}_\mathfrak{K} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\chi}_\mathfrak{K}) \sqcap \overline{\chi}_\mathfrak{K} \neq \overline{0}$. Thus, there exists $\mathfrak{r} \in \mathfrak{P}$ such that $((\overline{\chi}_\mathfrak{K} \overline{\circ}_\alpha \overline{\chi}_\mathfrak{p} \overline{\circ}_\beta \overline{\chi}_\mathfrak{K}) \sqcap \overline{\chi}_\mathfrak{K})(\mathfrak{r}) \neq \overline{0}$. Hence, $\mathfrak{r} \in \mathfrak{K}\alpha\mathfrak{u}\beta\mathfrak{K} \cap \mathfrak{K}$ implies that $\mathfrak{K}\alpha\mathfrak{u}\beta\mathfrak{K} \cap \mathfrak{K} \neq \emptyset$. Therefore, $\mathfrak{K}$ is an almost (α, β)-bi-ideal of $\mathfrak{P}$. ■

Next, we study properties between $\text{supp}(\overline{\varpi})$ and IVF almost (α, β)-bi-ideal of Γ-semigroups.

Theorem 4.22. *Let $\overline{\varpi}$ be an IVF sets of a non-empty of a Γ-semigroup $\mathfrak{P}$. Then $\overline{\varpi}$ is an IVF almost (α,β)-bi-ideal of $\mathfrak{P}$ if and only if* $\text{supp}(\overline{\varpi})$ *is an almost (α,β)-bi-ideal of $\mathfrak{P}$.*

Proof: Let ϖ be an IVF almost (α,β)-bi-ideal of $\mathfrak{P}$ and $\mathfrak{p}\in\mathfrak{P}$. Then $(\overline{\varpi}\overline{\circ}_\alpha\overline{\chi}_\mathfrak{p}\overline{\circ}_\beta\overline{\varpi})\sqcap\overline{\varpi}\neq\emptyset$. Thus, there exists $\mathfrak{r}\in\mathfrak{P}$ such that $(\overline{\varpi}\overline{\circ}_\alpha\overline{\chi}_\mathfrak{p}\overline{\circ}_\beta\overline{\varpi})(\mathfrak{r})\neq\overline{0}$. So, there exists $\mathfrak{k}_1,\mathfrak{k}_2\in\mathfrak{P}$ such that $\mathfrak{r}=\mathfrak{k}_1\alpha\beta\mathfrak{k}_2$, $\varpi(r)\neq\overline{0}$, $\overline{\varpi}(\mathfrak{k}_1)\neq\overline{0}$ and $\overline{\varpi}(\mathfrak{k}_2)\neq\overline{0}$. It implies that $\mathfrak{r},\mathfrak{k}_1,\mathfrak{k}_2\in\text{supp}(\overline{\varpi})$. Thus, $(\overline{\chi}_{\text{supp}(\overline{\varpi})}\overline{\circ}_\alpha\overline{\chi}_\mathfrak{p}\overline{\circ}_\beta\chi_{\text{supp}(\overline{\varpi})})(\mathfrak{r})\neq\overline{0}$ and $\overline{\chi}_{\text{supp}(\overline{\varpi})}(\mathfrak{r})\neq\overline{0}$. Hence $[(\overline{\chi}_{\text{supp}(\overline{\varpi})}\overline{\circ}_\alpha\overline{\chi}_\mathfrak{p}\overline{\circ}_\beta\overline{\chi}_{\text{supp}(\overline{\varpi})})]\sqcap\overline{\chi}_{\text{supp}(\overline{\varpi})}\neq\overline{0}$. Therefore, $\overline{\chi}_{\text{supp}(\overline{\varpi})}$ is an IVF almost (α,β)-bi-ideal of $\mathfrak{P}$. This shows that $\text{supp}(\overline{\varpi})$ is an almost (α,β)-bi-ideal of $\mathfrak{P}$.

Conversely, let $\text{supp}(\overline{\varpi})$ be an almost (α,β)-bi-ideal of $\mathfrak{P}$. Then, by Theorem 4.21, $\overline{\chi}_{\text{supp}(\overline{\varpi})}$ is an IVF almost (α,β)-bi-ideal of $\mathfrak{P}$. Thus, $[(\overline{\chi}_{\text{supp}(\overline{\varpi})}\overline{\circ}_\alpha\overline{\chi}_\mathfrak{p}\overline{\circ}_\beta\overline{\chi}_{\text{supp}(\overline{\varpi})})]\sqcap\overline{\chi}_{\text{supp}(\overline{\varpi})}\neq\overline{0}$. So, there exists $\mathfrak{r}\in\mathfrak{P}$ such that $([(\overline{\chi}_{\text{supp}(\overline{\varpi})}\overline{\circ}_\alpha\overline{\chi}_\mathfrak{p}\overline{\circ}_\beta\overline{\chi}_{\text{supp}(\overline{\varpi})})]\sqcap\overline{\chi}_{\text{supp}(\overline{\varpi})})(\mathfrak{r})\neq\overline{0}$. It implies that $[(\overline{\chi}_{\text{supp}(\overline{\varpi})}\overline{\circ}_\alpha\overline{\chi}_\mathfrak{P}\overline{\circ}_\beta\overline{\chi}_{\text{supp}(\overline{\varpi})})](\mathfrak{r})\neq\overline{0}$ and $\chi_\mathfrak{p}(\mathfrak{r})\neq\overline{0}$. Thus, there exist $\mathfrak{k}_1,\mathfrak{k}_2\in\mathfrak{P}$ such that $\mathfrak{r}=\mathfrak{k}_1\alpha\mathfrak{u}\beta\mathfrak{k}_2$, $\overline{\varpi}(\mathfrak{r})\neq\overline{0}$, $\overline{\varpi}(\mathfrak{k}_1)\neq\overline{0}$ and $\overline{\varpi}(\mathfrak{k}_2)\neq\overline{0}$. Hence, $(\overline{\varpi}\overline{\circ}_\alpha\overline{\chi}_\mathfrak{P}\overline{\circ}_\beta\overline{\varpi})\sqcap\overline{\varpi}\neq\overline{0}$. Therefore, ϖ is an IVF almost (α,β)-bi-ideal of $\mathfrak{P}$. ■

Definition 4.23. *An almost bi-ideal $\mathfrak{I}$ of a Γ-semigroup $\mathfrak{P}$ is called*

(1) *a* minimal *if for every almost bi-ideal of $\mathfrak{J}$ of $\mathfrak{P}$ such that $\mathfrak{J}\subseteq\mathfrak{I}$, we have $\mathfrak{J}=\mathfrak{I}$,*
(2) *a* maximal *if for every almost bi-ideal of $\mathfrak{J}$ of $\mathfrak{P}$ such that $\mathfrak{I}\subseteq\mathfrak{J}$, we have $\mathfrak{J}=\mathfrak{I}$.*

Definition 4.24. *An IVF almost (α,β)-bi-ideal $\overline{\omega}$ of a Γ-semigroup $\mathfrak{P}$ is*

(1) *a* minimal *if for all IVF almost (α,β)-bi-ideal $\overline{\varpi}$ of $\mathfrak{P}$ such that $\overline{\varpi}\sqsubseteq\overline{\omega}$, then* $\text{supp}(\overline{\varpi})\subseteq\text{supp}(\overline{\omega})$,
(2) *a* maximal *if for all IVF almost (α,β)-bi-ideal $\overline{\varpi}$ of $\mathfrak{P}$ such that $\overline{\omega}\sqsubseteq\overline{\varpi}$, then* $\text{supp}(\overline{\varpi})\subseteq\text{supp}(\overline{\omega})$,

Theorem 4.25. *Let $\mathfrak{K}$ be a non-empty subset of a Γ-semigroup $\mathfrak{P}$. Then the following statements holds*

(1) *$\mathfrak{K}$ is a minimal almost (α,β)-bi-ideal if and only if $\overline{\chi}_\mathfrak{K}$ is a minimal IVF almost (α,β)-bi-ideal of $\mathfrak{P}$.*
(2) *$\mathfrak{K}$ is a maximal almost (α,β)-bi-ideal if and only if $\overline{\chi}_\mathfrak{K}$ is a maximal IVF almost (α,β)-bi-ideal of $\mathfrak{P}$.*

Proof:

(1) Suppose that $\mathfrak{K}$ is a minimal almost (α,β)-bi-ideal of $\mathfrak{P}$. Then $\mathfrak{K}$ is an almost left α-ideal of $\mathfrak{P}$. Thus, by Theorem 4.21, $\overline{\chi}_\mathfrak{K}$ is an IVF (α,β)-bi-ideal of $\mathfrak{P}$. Let $\overline{\varpi}$ be an IVF (α,β)-bi-ideal of $\mathfrak{P}$ such that $\overline{\varpi}\sqsubseteq\overline{\chi}_\mathfrak{K}$. Then, by Theorem 4.22, $\text{supp}(\overline{\varpi})$ is an almost (α,β)-bi-ideal of $\mathfrak{P}$. Thus, $\text{supp}(\overline{\varpi})\subseteq\text{supp}(\overline{\chi}_\mathfrak{K})=\mathfrak{K}$. By assumption, $\text{supp}(\overline{\varpi})=\mathfrak{K}=\text{supp}(\overline{\chi}_\mathfrak{K})$. Thus,$\overline{\chi}_\mathfrak{K}$ is a minimal IVF fuzzy almost (α,β)-bi-ideal of $\mathfrak{P}$.
Conversely, suppose that $\overline{\chi}_\mathfrak{K}$ is a minimal IVF almost (α,β)-bi-ideal of $\mathfrak{P}$. Then, by Theorem 4.21, $\mathfrak{K}$ is an almost (α,β)-bi-ideal of $\mathfrak{P}$. Let $\mathfrak{J}$ be an almost (α,β)-bi-ideal of $\mathfrak{P}$ such that $\mathfrak{J}\subseteq\mathfrak{K}$. Then, by Theorem 4.21, $\overline{\chi}_\mathfrak{J}$ is an IVF (α,β)-bi-ideal of $\mathfrak{P}$ such that $\overline{\chi}_\mathfrak{J}\sqsubseteq\overline{\chi}_\mathfrak{K}$. Thus, $\mathfrak{J}=\text{supp}(\overline{\chi}_\mathfrak{J})=\text{supp}(\overline{\chi}_\mathfrak{K})=\mathfrak{K}$. Hence, $\mathfrak{K}$ is a minimal almost (α,β)-bi-ideal of $\mathfrak{P}$.
(2) Similar to (1).

■

Corollary 4.26. *Let $\mathfrak{P}$ be a Γ-semigroup. Then $\mathfrak{P}$ has no proper almost (α,β)-bi-ideal if and only if for any IVF almost (α,β)-bi-ideal $\overline{\varpi}$ of $\mathfrak{P}$,* $\text{supp}(\overline{\varpi})=\mathfrak{P}$.

Next, we define IVF almost (α,β)-interior ideal, and we study its properties.

Definition 4.27. *Let $\overline{\omega}$ be an IVF set of a Γ-semigroup $\mathfrak{P}$, and $\alpha,\beta\in\Gamma$ is said to be* IVF almost (α,β)-interior ideal *of $\mathfrak{P}$ if $(\overline{\chi}_\mathfrak{p}\,\overline{\circ}_\alpha\overline{\omega}\,\overline{\circ}_\beta\overline{\chi}_\mathfrak{p})\sqcap\overline{\omega}\neq\overline{0}$.*

Theorem 4.28. *If $\overline{\omega}$ is an IVF almost (α,β)-interior ideal of a Γ-semigroup $\mathfrak{P}$, and $\overline{\varpi}$ is an IVF set of $\mathfrak{P}$ such that $\overline{\omega}\sqsubseteq\overline{\varpi}$, then $\overline{\varpi}$ is an IVF (α,β)-interior ideal of $\mathfrak{P}$.*

Proof: Suppose that $\overline{\omega}$ is an IVF almost (α,β)-interior ideal of $\mathfrak{P}$, and $\overline{\varpi}$ is an IVF set of $\mathfrak{P}$ such that $\overline{\omega}\sqsubseteq\overline{\varpi}$. Then $(\overline{\chi}_\mathfrak{P}\,\overline{\circ}_\alpha\overline{\omega}\,\overline{\circ}_\beta\overline{\chi}_\mathfrak{P})\sqcap\overline{\omega}\neq\overline{0}$. Thus, $(\overline{\chi}_\mathfrak{P}\,\overline{\circ}_\alpha\overline{\omega}\,\overline{\circ}_\beta\overline{\chi}_\mathfrak{P})\sqcap\overline{\omega}\sqsubseteq(\overline{\chi}_\mathfrak{P}\,\overline{\circ}_\alpha\overline{\varpi}\,\overline{\circ}_\beta\overline{\chi}_\mathfrak{P})\sqcap\overline{\varpi}\neq\overline{0}$. Hence, $\overline{\varpi}$ is an IVF (α,β)-interior ideal of $\mathfrak{P}$. ■

Theorem 4.29. *Let $\mathfrak{K}$ be a non-empty subset of Γ-semigroup $\mathfrak{P}$. Then $\mathfrak{K}$ is an almost (α,β)-interior ideal of $\mathfrak{P}$ if and only if the characteristic function $\overline{\chi}_\mathfrak{K}$ is an IVF almost (α,β)-interior ideal of $\mathfrak{P}$.*

Proof: Suppose that $\mathfrak{K}$ is an almost (α,β)-interior ideal of $\mathfrak{P}$. Then $\mathfrak{u}\alpha\mathfrak{K}\beta\mathfrak{u}\cap\mathfrak{K}\neq\emptyset$ for all $\mathfrak{u}\in\mathfrak{P}$. Thus, there exists $\mathfrak{u}\alpha\mathfrak{K}\beta\mathfrak{u}\cap\mathfrak{K}$ and $\mathfrak{v}\in\mathfrak{K}$. So $(\overline{\chi}_\mathfrak{p}\,\overline{\circ}_\alpha\overline{\chi}_\mathfrak{K}\,\overline{\circ}_\beta\overline{\chi}_\mathfrak{p})(\mathfrak{v})=\overline{\chi}_\mathfrak{K}(\mathfrak{v})=\overline{1}$. Hence, $(\overline{\chi}_\mathfrak{p}\,\overline{\circ}_\alpha\overline{\chi}_\mathfrak{K}\,\overline{\circ}_\beta\overline{\chi}_\mathfrak{p})\sqcap\overline{\chi}_\mathfrak{K}\neq\overline{0}$. Therefore, $\overline{\chi}_\mathfrak{K}$ is an IVF almost (α,β)-interior ideal of $\mathfrak{P}$.

To prove the converse, assume that $\overline{\chi}_\mathfrak{K}$ is an IVF almost (α,β)-interior ideal of $\mathfrak{P}$ and $\mathfrak{u}\in\mathfrak{P}$. Then $(\overline{\chi}_\mathfrak{p}\,\overline{\circ}_\alpha\overline{\chi}_\mathfrak{K}\,\overline{\circ}_\beta\overline{\chi}_\mathfrak{p})\sqcap\overline{\chi}_\mathfrak{K}\neq\overline{0}$. Thus, there exists $\mathfrak{r}\in\mathfrak{P}$ such that $((\overline{\chi}_\mathfrak{p}\,\overline{\circ}_\alpha\overline{\chi}_\mathfrak{K}\,\overline{\circ}_\beta\overline{\chi}_\mathfrak{p})\sqcap\overline{\chi}_\mathfrak{K})(\mathfrak{r})\neq\overline{0}$. Hence, $\mathfrak{r}\in\mathfrak{u}\alpha\mathfrak{K}\beta\mathfrak{u}\cap\mathfrak{K}$ implies that $\mathfrak{u}\alpha\mathfrak{K}\beta\mathfrak{u}\cap\mathfrak{K}\neq\emptyset$. Therefore, $\mathfrak{K}$ is an almost (α,β)-interior ideal of $\mathfrak{P}$. ■

Next, we study properties between $\text{supp}(\overline{\varpi})$ and IVF almost (α,β)-interior ideal of Γ-semigroups.

Theorem 4.30. *Let $\overline{\varpi}$ be an IVF sets of a non-empty of a Γ-semigroup $\mathfrak{P}$. Then $\overline{\varpi}$ is an IVF almost (α,β)-interior ideal of $\mathfrak{P}$ if and only if* $\text{supp}(\varpi)$ *is an almost (α,β)-interior ideal of $\mathfrak{P}$.*

Proof: Let $\overline{\varpi}$ be an IVF almost (α,β)-interior ideal of $\mathfrak{P}$ and $\mathfrak{u}\in\mathfrak{P}$. Then $(\overline{\chi}_\mathfrak{p}\overline{\circ}_\alpha\overline{\varpi}\overline{\circ}_\beta\overline{\chi}_\mathfrak{p})\sqcap\overline{\varpi}\neq\overline{0}$. Thus, there exists $\mathfrak{r}\in\mathfrak{P}$ such that $(\overline{\chi}_\mathfrak{p}\overline{\circ}_\alpha\overline{\varpi}\overline{\circ}_\beta\overline{\chi}_\mathfrak{p})(\mathfrak{r})\neq\overline{0}$. So, there exists $\mathfrak{k}_1,\mathfrak{k}_2\in\mathfrak{P}$ such that $\mathfrak{r}=\mathfrak{k}_1\alpha\beta\mathfrak{k}_2$, $\overline{\varpi}(r)\neq\overline{0}$, $\overline{\varpi}(\mathfrak{k}_1)\neq\overline{0}$ and $\overline{\varpi}(\mathfrak{k}_2)\neq\overline{0}$. It implies that $\mathfrak{r},\mathfrak{k}_1,\mathfrak{k}_2\in\text{supp}(\overline{\varpi})$. Thus, $(\overline{\chi}_\mathfrak{p}\overline{\circ}_\alpha\overline{\chi}_{\text{supp}(\overline{\varpi})}\overline{\circ}_\beta\chi_\mathfrak{p})(\mathfrak{r})\neq\overline{0}$ and $\overline{\chi}_{\text{supp}(\overline{\varpi})}(\mathfrak{r})\neq\overline{0}$. Hence, $[(\overline{\chi}_\mathfrak{p}\overline{\circ}_\alpha\overline{\chi}_{\text{supp}(\overline{\varpi})}\overline{\circ}_\beta\overline{\chi}_\mathfrak{p})]\sqcap\overline{\chi}_{\text{supp}(\overline{\varpi})}\neq\emptyset$. Therefore, $\overline{\chi}_{\text{supp}(\overline{\varpi})}$ is an IVF almost (α,β)-interior ideal of $\mathfrak{P}$. This shows that $\text{supp}(\overline{\varpi})$ is an almost (α,β)-interior ideal of $\mathfrak{P}$.

Conversely, let $\text{supp}(\overline{\varpi})$ be an almost (α,β)-interior ideal of $\mathfrak{P}$. Then, by Theorem 4.29, $\overline{\chi}_{\text{supp}(\overline{\varpi})}$ is an IVF almost (α,β)-interior ideal of $\mathfrak{P}$. Thus, $[(\overline{\chi}_\mathfrak{p}\overline{\circ}_\alpha\overline{\chi}_{\text{supp}(\overline{\varpi})}\overline{\circ}_\beta\overline{\chi}_\mathfrak{p})]\sqcap\overline{\chi}_{\text{supp}(\overline{\varpi})}\neq\overline{0}$. So, there exists $\mathfrak{r}\in\mathfrak{P}$ such that $([(\overline{\chi}_\mathfrak{p}\overline{\circ}_\alpha\overline{\chi}_{\text{supp}(\overline{\varpi})}\overline{\circ}_\beta\overline{\chi}_\mathfrak{p}]\sqcap\overline{\chi}_{\text{supp}(\overline{\varpi})})(\mathfrak{r})\neq\overline{0}$. It implies that $(\overline{\chi}_\mathfrak{p}\overline{\circ}_\alpha\overline{\chi}_{\text{supp}(\overline{\varpi})}\overline{\circ}_\beta\overline{\chi}_\mathfrak{p})(\mathfrak{r})\neq\overline{0}$ and $\chi_\mathfrak{K}(\mathfrak{r})\neq\overline{0}$. Thus, there exist $\mathfrak{k}_1,\mathfrak{k}_2\in\mathfrak{P}$ such that $\mathfrak{r}=\mathfrak{k}_1\alpha\mathfrak{u}\beta\mathfrak{k}_2$, $\overline{\varpi}\varpi(\mathfrak{r})\neq\overline{0}$, $\overline{\varpi}(\mathfrak{k}_1)\neq\overline{0}$ and $\overline{\varpi}(\mathfrak{k}_2)\neq\overline{0}$. Hence, $(\overline{\chi}_\mathfrak{p}\overline{\circ}_\alpha\overline{\varpi}\overline{\circ}_\beta\overline{\chi}_\mathfrak{p})\sqcap\overline{\varpi}\neq\overline{0}$. Therefore, $\overline{\varpi}$ is an IVF almost (α,β)-interior ideal of $\mathfrak{P}$. ■

Definition 4.31. *An almost interior ideal $\mathfrak{I}$ of a Γ-semigroup $\mathfrak{P}$ is called*

(1) *a* minimal *if for every almost interior ideal of $\mathfrak{J}$ of $\mathfrak{P}$ such that $\mathfrak{J} \subseteq \mathfrak{I}$, we have $\mathfrak{J} = \mathfrak{I}$,*
(2) *a* maximal *if for every almost interior ideal of $\mathfrak{J}$ of $\mathfrak{P}$ such that $\mathfrak{I} \subseteq \mathfrak{J}$, we have $\mathfrak{J} = \mathfrak{I}$.*

Definition 4.32. *An IVF almost (α, β)-interior ideal $\overline{\omega}$ of a Γ-semigroup $\mathfrak{P}$ is*

(1) *a* minimal *if for all IVF almost (α, β)-interior ideal $\overline{\varpi}$ of $\mathfrak{P}$ such that $\overline{\varpi} \sqsubseteq \overline{\omega}$, then supp$(\overline{\varpi}) = \operatorname{supp}(\overline{\omega})$,*
(2) *a* maximal *if for all IVF almost (α, β)-interior ideal $\overline{\varpi}$ of $\mathfrak{P}$ such that $\overline{\omega} \sqsubseteq \overline{\varpi}$, then $\operatorname{supp}(\overline{\varpi}) = \operatorname{supp}(\overline{\omega})$.*

Theorem 4.33. *Let $\mathfrak{K}$ be a non-empty subset of a Γ-semigroup $\mathfrak{P}$. Then the following statements holds*

(1) *$\mathfrak{K}$ is a minimal almost (α, β)-interior ideal if and only if $\overline{\chi}_{\mathfrak{K}}$ is a minimal IVF almost (α, β)-interior ideal of $\mathfrak{P}$.*
(2) *$\mathfrak{K}$ is a maximal almost (α, β)-interior ideal if and only if $\overline{\chi}_{\mathfrak{K}}$ is a maximal IVF almost (α, β)-interior ideal of $\mathfrak{P}$.*

Proof:

(1) Suppose that $\mathfrak{K}$ is a minimal almost (α, β)-interior ideal of $\mathfrak{P}$. Then $\mathfrak{K}$ is an almost left α-ideal of $\mathfrak{P}$. Thus, by Theorem 4.29, $\overline{\chi}_{\mathfrak{K}}$ is an IVF (α, β)-interior ideal of $\mathfrak{P}$. Let $\overline{\varpi}$ be an IVF (α, β)-bi-ideal of $\mathfrak{P}$ such that $\overline{\varpi} \sqsubseteq \overline{\chi}_{\mathfrak{K}}$. Then, by Theorem 4.22, $\operatorname{supp}(\overline{\varpi})$ is an almost (α, β)interior ideal of $\mathfrak{P}$. Thus, $\operatorname{supp}(\overline{\varpi}) \subseteq \operatorname{supp}(\overline{\chi}_{\mathfrak{K}}) = \mathfrak{K}$. By assumption, $\operatorname{supp}(\overline{\varpi}) = \mathfrak{K} = \operatorname{supp}(\overline{\chi}_{\mathfrak{K}})$. Thus, $\overline{\chi}_{\mathfrak{K}}$ is a minimal IVF fuzzy almost (α, β)-interior ideal of $\mathfrak{P}$.
Conversely, suppose that $\overline{\chi}_{\mathfrak{K}}$ is a minimal IVF almost (α, β)-interior ideal of $\mathfrak{P}$. Then, by Theorem 4.29, $\mathfrak{K}$ is an almost (α, β)-interior ideal of $\mathfrak{P}$. Let $\mathfrak{J}$ be an almost (α, β)-interior ideal of $\mathfrak{P}$ such that $\mathfrak{J} \subseteq \mathfrak{K}$. Then, by Theorem 4.21, $\overline{\chi}_{\mathfrak{J}}$ is an IVF (α, β)-interior ideal of $\mathfrak{P}$ such that $\overline{\chi}_{\mathfrak{J}} \sqsubseteq \overline{\chi}_{\mathfrak{K}}$. Thus, $\mathfrak{J} = \operatorname{supp}(\overline{\chi}_{\mathfrak{J}}) = \operatorname{supp}(\overline{\chi}_{\mathfrak{K}}) = \mathfrak{K}$. Hence, $\mathfrak{K}$ is a minimal almost (α, β)-interior ideal of $\mathfrak{P}$.
(2) Similar to (1).

■

Corollary 4.34. *Let $\mathfrak{P}$ be a Γ-semigroup. Then $\mathfrak{P}$ has no proper almost (α, β)-interior ideal if and only if for any IVF almost (α, β)-interior ideal $\overline{\varpi}$ of $\mathfrak{P}$, supp$(\overline{\varpi}) = \mathfrak{P}$.*

V. Conclusion

In this article, we propose the concept of new interval valued fuzzy ideals and interval valued fuzzy almost ideals in a Γ-semigroups. We study properites of new interval valued fuzzy ideals and an interval valued fuzzy almost ideals. The relation between interval valued fuzzy almost ideals and almost ideals are proved. In the further, we extend the new typer bipolar fuzzy ideals and bipolar fuzzy almost ideals in a Γ-semigroup or algebraic system.

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