Variance Reduction Techniques in Variance Gamma Option Pricing

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Abstract—Research on options models is still relevant to help buyers determine the fairness of option prices. The Black-Scholes model assumes that the stock price is lognormally distributed, whereas, in real applications, the stock price data does not match this assumption because it has different skewness and kurtosis values from the normal distribution. This condition is more suitable to be solved by non-normal models such as the Gram-Charlier and Variance Gamma. To reduce the variances, there are Antithetic Variate and Importance Sampling techniques. In this paper, we discuss an empirical study of option prices under skewness and kurtosis conditions using reduced variance techniques from two options of automotive stock (NIO) and technology stock (INTC), where we want to investigate the performance of those methods in the estimation of the stock call option price model in those two stocks. From the analysis, we found that those techniques can reduce the option price variance and give a more accurate price, where the Variance Gamma models produce the smallest MAPE compared to the other models used.

Index Terms—Black-Scholes, Call Option, Gram Charlier Expansion, Variance-Gamma, Variance Reduction

I. INTRODUCTION

Option is the right to buy or sell securities or commodities at an agreed price at a specific date during the contract period [1-3]. It can be an alternative to buying securities that have high risk so that investors can buy them at relatively low prices and can control the risk. The model can help the buyer to determine the fairness of an option price. One of the commonly used option pricing methods is the Black-Scholes (B-S) model [3] which is used in determining the price of European options. This model assumes that the log return price of stocks has a normal distribution. However, this assumption cannot be fully met in empirical data, where empirical data often have skewness and kurtosis [4-8]. As a result, the prices obtained with the B-S model are often inconsistent with market prices.

Considering skewness and kurtosis in an option price are accommodated in many alternative models. One model that can be an alternative to the B-S model is the Gram-Charlier (G-C) expansion and Variance Gamma (VG) models. The G-C expansion method is used to include option price adjustments for abnormal skewness and kurtosis in the B-S formula [8].

On the other hand, the VG model was introduced as a development of Geometric Brownian Motion to overcome the skewness and kurtosis of the distribution of return. As a development of the VG model, several methods can reduce the variance value of the call price simulation. These models are the Antithetic Variance Reduction (AVR) and the Importance Sampling (IS) VG methods.

The B-S model [1] was developed to determine option prices, with several assumptions used in the model as follows (See [2]).
1. The option used is a European type;
2. The stock price variance is constant over the life of the option;
3. The stock prices follow a random process and central interest rates.

The call option price formula using the B-S model is [9]:

\[ C_{BS} = S_0 N(d_1) - e^{-r(T-t)} KN(d_2) \]  (1)

Where \( N(z) \) is cumulative normal standard distribution, 

\[ d_1 = \frac{\log(S_0/E) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma \sqrt{T-t}} \quad \text{and} \quad d_2 = d_1 - \sigma \sqrt{T-t}. \]

The inputs needed to calculate the option price with the B-S model are [10]:
1. \( S_0 \), the options market price
2. \( K \), the strike price at maturity time
3. \( T \), maturity time of the option (year)
4. \( R \), the risk-free interest rate
5. \( \sigma \), the yearly volatility which is obtained by the standard deviation of the stock’s return price times the square root value of the trading days in 1 year.

The contribution of this paper is as follows.
1. We conduct the comparison study of the B-S model and VG model using Gram-Charlier expansion (for the B-S model), Antithetic Variate and Importance Sampling variance reduction method (for the VG model) in option pricing theory.
2. The data used for this analysis in this paper are NIO and INTC stock.
3. To compare the model performances, we use MAPE as the evaluation metric.

II. GRAM CHARLIER EXPANSION

Gram-Charlier expansion is a distribution with a density in the form of a polynomial multiplied by a normal distribution [6]. In determining the price of options using this distribution, it will maintain the nature of the normal distribution by also considering the non-zero skewness and kurtosis. The G-C expansion formula for normal distribution is [8]:

\[ f(x) = \phi(a;b;x) \left[ 1 + c_1 He_1 \left( \frac{x-a}{b} \right) + c_2 He_2 \left( \frac{x-a}{b} \right) + \ldots \right] \]  (2)

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where \( \phi(a, b; x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-a)^2}{2b^2}}, x \in \mathbb{R} \) is the general normal density and \( H_{kn} \) is the Hermite polynomial with order \( k \). The \( c \) constant is obtained from the moment of distribution function \( f(x) \). In a real-world application, this series is truncated for the values above the fourth moment. After standardized to a normal distribution with mean 0 and constant variance, the density function of \( g(z) \) that accounts for the skewness and kurtosis could be obtained as below:

\[
g(z) = n(z) \left[ 1 + \frac{3}{2} \left( z^2 - 3z + \frac{3}{2} \right) \right]^{-\frac{3}{4}}
\]

where \( n(z) \) is normal pdf, \( z = \frac{\ln(S_t) - \mu t}{\sigma \sqrt{t}} \), \( \mu \) is the skewness coefficient, and \( \mu_4 \) is the kurtosis coefficient. Using the present value of the expected payment at the time of the option's maturity, the option pricing formula is:

\[
C_{OG} = e^{-rt} \int_0^T (S_t - K) g(z) dz(S_t)
\]

Solving the integral in Equation 4, the formula for the option pricing with the G-C expansion could be obtained as follows [7-8]:

\[
C_{OG} = C_{BS} + \mu_3 Q_3 + (\mu_4 - 3)Q_4
\]

where

\[
Q_3 = \frac{1}{3!} \int_0^T (2x \sqrt{t} - d) n(d) - \sigma^2 t N(d)
\]

is the marginal effect of skewness,

\[
Q_4 = \frac{1}{4!} \int_0^T (d^2 - 1 - 3 \sqrt{t} (d - \sqrt{t})) n(d) + \sigma^2 t^2 N(d)
\]

is the marginal effect of kurtosis, and

\[
d = \frac{\ln(S_t) + (\mu + \frac{\alpha^2}{2}) t}{\sigma \sqrt{t}}.
\]

### III. VARIANCE GAMMA MODEL

The VG process is obtained by evaluating the Brownian Motion (BM) in random time change with the gamma process [11-13]. In this process, the return time unit, which is continuously combined, is normally distributed, depending on the random time realization. This random time has a gamma density. The result of the stochastic process and the option pricing model that is obtained would result in 3 robust parameters as follows.

\( \sigma \) = the volatility of BM

\( \nu \) = variance rate of the gamma

\( \theta \) = drift in the BM

These parameters would give control to the skewness and kurtosis, which is the symmetrical increase in the left and right tail of the return distribution. Variance gamma process is defined concerning the BM with drift \( (b(t;\theta, \sigma)) \) and the gamma process with the unit mean rate \( \{\gamma(t;1, \nu)\} \) as:

\[
X(t; \sigma, \nu, \theta) = b\gamma(t; 1, \nu) + \nu 
\]

This process gives two dimensions of control to the distribution, that is, the skewness with the parameter \( \theta \) and the kurtosis with the parameter \( \nu \). Define \( X(t) \) at time interval \( t \) as a VG random variable with the normal distribution, which can be written as:

\[
X_{VG}(t) = \theta g + \sigma \sqrt{g} z
\]

where \( z \) is the standard normal distribution random variable, which is independent of \( g \), the gamma-distributed with mean \( t \) and variance \( vt \). This VG random variable has mean \( \theta g \) and variance \( \sigma^2 \sqrt{g} \). Variance gamma parameters could be estimated with the moment method [11]:

1. \( \sigma = \sqrt{Var(X)} \)
2. \( \theta = \frac{\text{Skewness}(X)}{\sigma} \)
3. \( \nu = \frac{\text{Kurtosis}(X) - 3}{\sigma} \)

and the stock price model with VG becomes:

\[
S_t = S_0 e^{\left(\gamma(t + \omega) + tX_{VG}(t)\right)}
\]

where \( \omega = \frac{1}{2} \ln(1 - \theta \nu + \frac{1}{2} \sigma^2 \nu) \) and \( X_{VG}(t) = \theta g + \sigma \sqrt{g} z \), which is the VG process obtained from the standard normal process of the gamma process.

### IV. MONTE CARLO VARIANCE REDUCTION SIMULATION

Throughout this section, we explain the variance reduction method used for Monte Carlo simulation in this paper, namely, Antithetic Variates (AV) and Importance sampling.

Monte Carlo simulation is a famous computational tool to calculate the option price [14]. It is an approach of stochastic simulation that adopts random variables to resolve practical problems based on the probability theory [15]. This method evaluates the expected value of a random variable by generating many random variable samples that are independent and takes the empirical mean from those samples as the point estimation of an expectation. This method's accuracy is proportional to \( 1/\sqrt{n} \) where \( \sigma^2 \) is the sample variance and \( n \) is the number of samples generated. The advantage of this model is that the computational effort is smaller to reach the intended accuracy. Moreover, the simulation does not necessitate the assumption of asset distribution [16, 17].

Antithetic Variates (AV) is the most familiar variance reduction techniques, usually using complementary random numbers. The AV variance reduction is justified by the negative correlation in paired simulation estimates. In the AV technique, for every sample obtained, given a path \( \epsilon_1, \ldots, \epsilon_M \) also take \( -\epsilon_1, \ldots, -\epsilon_M \), we have

\[
V(Y) = \frac{X_1 + X_2}{2} = \frac{V(X_1) + V(X_2) + 2Cov(X_1, X_2)}{4}
\]

\[
V(Y) = \frac{1}{2} (\sigma^2 + \sigma^2 + 2Cov(X_1, X_2)) = \frac{1}{2} (\sigma^2 + Cov(X_1, X_2))
\]

If \( Cov(X_1, X_2) < 0 \), then \( Var(Y) < \sigma^2/2 \), therefore the variance of \( Y \) would be reduced (see [23]).

Importance sampling in the option pricing method has the base idea of concentrating the simulation on the sample path that gives the highest contribution in the payment estimation [15-18]. Importance sampling estimates the expectation of a random variable by generating sample paths with a probability measure that is different from the original one. This is done by prioritizing the important path of the estimation from a different point of view. If it is done correctly, this model will yield a much smaller variance [18-
In this paper, we use the interest rate-Treasury Bill Rate, which is 0.04.

B. Analysis Step

This study computed the call option for government stock. The steps are as follows:
1. Calculate the return using compounded log return;
2. Test the normality distribution of the return series using Kolmogorov-Smirnov (K-S) or Jarque-Bera (J-B) test;
3. Calculate the call option using the B-S model;
4. Calculate the call option using VG simulation;
5. Compare the results from each model using MAPE.

VI. RESULTS AND DISCUSSIONS

Throughout this section, we discuss the results obtained from the analysis that we conducted in this paper. We tested the Variance Gamma model on NIO and INTC stocks. The results of descriptive statistics related to the average and diversity of data for the two stocks are presented in Table I. It can be seen that the average daily stock price return for one year for Intel Corp. stock is -0.0004 with a variance of 0.1470. As for the NIO stock, the average return is 0.0093, and the variance is 0.8870. A negative value of stock returns for INTC on average means that the stock investment is incurring a loss, even if the value is very small. Conversely, a positive value on the average NIO means that stock investment is profitable. On the other hand, the normality test of the return data, which is time-series data, can be obtained by the J-B test. With the null hypothesis that the log return data is normally distributed, we got p-values for Intel Corp. stocks and NIO stocks is 0, respectively. Due to both p-values being less than alpha 5%, it means that both of the log return data are not normally distributed.

We also see clearly that INTC stock's log return has high kurtosis values (>3), which confirms that it has heavier tails than implied by the normal distribution. Because the coefficient of kurtosis is greater than 3, we can say that the data distribution is leptokurtic and has a sharp peak on the graph. This also applies to NIO stocks where the kurtosis value is so high, which confirms that it has heavier tails than implied by the normal distribution. On the other hand, we can see that the coefficient of skewness of INTC stock's log return is less than 0, so it means the graph is negatively skewed and most of the data values are greater than the mean.

Different from AAPL stock, the coefficient of skewness of NIO stock's log return is greater than 0, so it means the graph is positively skewed and most of the data values are less than the mean.

In accommodating the skewness and kurtosis of the distribution of each stock price return, we would fit the VG distribution. The parameters of the VG distribution and Goodness-of-Fit with the K-S test are presented in Table II. The null hypothesis is that the log return data is really from VG distribution, the significance level of alpha is 0.05, and the critical area that the null hypothesis is rejected if D is larger than the critical value. We get D for Intel Corp. stocks and NIO stocks of 0.0543 and 0.0408, respectively. Both D value is less than the critical point, so the null hypothesis is accepted, which means that both of the log return data are really from the VG distribution.

A. Black Scholes and Gram Charlier Expansion Model

In this section, we check the estimation of the call option price using B-S and G-C expansion models. INTC stock options are used with a strike price (K) of 53 and 34.06 for NIO stock. We can see from Table III that the mean absolute percentage error (MAPE) for INTC and NIO stocks is 65.60164% and 12.01842%, respectively. This shows that the actual and the estimated option selling price using the B-S formula is a bit different. In addition, since the MAPE of NIO stock is smaller than that of INTC’s stock, it can be concluded that based on the B-S model, NIO stock representing the automotive cluster has a more reasonable options price when compared to INTC stock representing the technology cluster.

We can see from Table III that using G-C expansion, the MAPE for INTC and NIO is 22.73977% and 8.68847%, respectively. We can say it’s quite large for INTC and small for NIO. This shows that the MAPE for INTC and NIO resulting from using the G-C expansion formula is lower than

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**TABLE I**

<table>
<thead>
<tr>
<th>Descriptive Statistics</th>
<th>INTC</th>
<th>NIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.0004</td>
<td>0.0093</td>
</tr>
<tr>
<td>Variance</td>
<td>0.1470</td>
<td>0.8870</td>
</tr>
<tr>
<td>Skewness</td>
<td>-2.1747</td>
<td>0.6280</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>16.1575</td>
<td>3.9055</td>
</tr>
<tr>
<td>Jarque Bara</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
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</table>

**TABLE II**

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>NIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sigma (%)</td>
<td>0.021184</td>
<td>0.05109</td>
</tr>
<tr>
<td>Theta (θ)</td>
<td>-0.007613</td>
<td>0.07916</td>
</tr>
<tr>
<td>Nu (v)</td>
<td>0.766856</td>
<td>0.12099</td>
</tr>
<tr>
<td>D</td>
<td>0.05430662</td>
<td>0.04080745</td>
</tr>
<tr>
<td>Critical Value</td>
<td>0.08584244</td>
<td>0.08584244</td>
</tr>
</tbody>
</table>

**TABLE III**

<table>
<thead>
<tr>
<th>Measurement</th>
<th>INTC</th>
<th>NIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0</td>
<td>55.95</td>
<td>27</td>
</tr>
<tr>
<td>K</td>
<td>53</td>
<td>34.06</td>
</tr>
<tr>
<td>Market</td>
<td>3.22</td>
<td>7.83</td>
</tr>
<tr>
<td>B-S</td>
<td>5.332</td>
<td>8.771</td>
</tr>
<tr>
<td>MAPE</td>
<td>65.60%</td>
<td>12.02%</td>
</tr>
<tr>
<td>GC</td>
<td>3.952221</td>
<td>8.510307</td>
</tr>
<tr>
<td>MAPE</td>
<td>22.74%</td>
<td>8.69%</td>
</tr>
</tbody>
</table>

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**TABLE IV**

<table>
<thead>
<tr>
<th>Parameters Estimation and Goodness-of-Fit Kolmogorov Smirnov Test for Variance Gamma Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>Sigma (%)</td>
</tr>
<tr>
<td>Theta (θ)</td>
</tr>
<tr>
<td>Nu (v)</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>Critical Value</td>
</tr>
</tbody>
</table>

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**TABLE V**

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>NIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sigma (%)</td>
<td>0.021184</td>
<td>0.05109</td>
</tr>
<tr>
<td>Theta (θ)</td>
<td>-0.007613</td>
<td>0.07916</td>
</tr>
<tr>
<td>Nu (v)</td>
<td>0.766856</td>
<td>0.12099</td>
</tr>
<tr>
<td>D</td>
<td>0.05430662</td>
<td>0.04080745</td>
</tr>
<tr>
<td>Critical Value</td>
<td>0.08584244</td>
<td>0.08584244</td>
</tr>
</tbody>
</table>
using the B-S. It means that the G-C formula can adjust option prices with abnormal skewness and kurtosis. However, the actual option selling price and the estimated option selling price using the G-C expansion formula are a bit different for INTC, although they are not significantly different for NIO. In addition, because the MAPE of NIO stock is smaller than that of INTC stock, it can be concluded that based on the G-C expansion model, NIO stock representing the automotive cluster has a more reasonable options price when compared to INTC’s shares representing the technology cluster.

B. Call Option: Variance Gamma Simulation

In this section, we compare the standard error of estimating the price of a call option using the Variance Gamma (VG), Antithetic Variate VG (VG AVR), and Importance Sampling VG (VG IS) models with ten different treatments based on the number of simulations. The number of simulations (No.) starts from the smallest, which is 5 to the maximum, which is $5 \times 10^{6}$. Comparison of the value of the call option (price) and standard error (SE) for each stock price is presented in Tables IV and V.

<table>
<thead>
<tr>
<th>No</th>
<th>Price</th>
<th>SE</th>
<th>Price</th>
<th>SE</th>
<th>Price</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3.160</td>
<td>0.228</td>
<td>3.316</td>
<td>0.142</td>
<td>3.484</td>
<td>0.173</td>
</tr>
<tr>
<td>10</td>
<td>2.970</td>
<td>0.163</td>
<td>2.866</td>
<td>0.135</td>
<td>3.096</td>
<td>0.138</td>
</tr>
<tr>
<td>20</td>
<td>3.298</td>
<td>0.112</td>
<td>3.236</td>
<td>0.108</td>
<td>3.085</td>
<td>0.162</td>
</tr>
<tr>
<td>25</td>
<td>3.206</td>
<td>0.071</td>
<td>3.273</td>
<td>0.066</td>
<td>3.399</td>
<td>0.087</td>
</tr>
<tr>
<td>5 × 10</td>
<td>3.271</td>
<td>0.046</td>
<td>3.270</td>
<td>0.036</td>
<td>2.951</td>
<td>0.117</td>
</tr>
<tr>
<td>5 × 10²</td>
<td>3.298</td>
<td>0.023</td>
<td>3.304</td>
<td>0.017</td>
<td>3.278</td>
<td>0.003</td>
</tr>
<tr>
<td>5 × 10³</td>
<td>3.321</td>
<td>0.007</td>
<td>3.312</td>
<td>0.005</td>
<td>3.396</td>
<td>0.014</td>
</tr>
<tr>
<td>5 × 10⁴</td>
<td>3.310</td>
<td>0.002</td>
<td>3.307</td>
<td>0.002</td>
<td>3.370</td>
<td>0.000</td>
</tr>
<tr>
<td>5 × 10⁵</td>
<td>3.311</td>
<td>0.001</td>
<td>3.311</td>
<td>0.001</td>
<td>3.384</td>
<td>0.001</td>
</tr>
<tr>
<td>5 × 10⁶</td>
<td>3.311</td>
<td>0.000</td>
<td>3.311</td>
<td>0.000</td>
<td>3.382</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Option price simulations for INTC stock are carried out using VG, Antithetic Variate VG, and Importance Sampling VG models for 5, 10, …, $50000000$ number of simulations. The INTC option is used with a strike price K of 53. From Table IV, the standard errors get smaller as the number of simulations increases, both for the VG model, Antithetic Variate VG, and Importance Sampling VG for INTC stock options prices. On the other hand, we can see that the standard errors generated by the Importance Sampling VG model are smaller than the other two models and the Antithetic Variate VG’s are always smaller than the VG’s model for any number of simulations. It can be concluded that the fluctuation level of the Importance Sampling VG and Antithetic Variate VG models is lower than the VG model, and the fluctuation level of the Antithetic Variate VG model is lower than the fluctuation level of the VG model. So, in this simulation, it is proven that the Importance Sampling VG and the Antithetic Variate VG models reduce the variance and it can more stable in predicting the option prices market.

Simulations of option prices for NIO stock are carried out using the VG, Antithetic Variate VG, and Importance Sampling VG models with ten values of n. The NIO option is used with a strike price (K) of 27. We can see from the table above that, in general, the more simulations conducted, the standard errors produced are smaller. On the other hand, we can see that the standard errors generated by the Importance Sampling VG model are very often smaller than the other two models, and the Antithetic Variate VG’s are always smaller than the VG’s model (sometimes even smaller than the Importance Sampling VG’s model) for any number of simulations. From this simulation, it can be concluded that, in general, the fluctuation level of the Importance Sampling VG model is lower than the other two models.

C. Model Comparison

In this section, we compare the MAPE of call option price estimation using B-S, G-C expansion, VG, Antithetic Variate VG, and Importance Sampling VG models to find out which model can estimate the closest call price to the real market price, 3.22 for INTC stock and 7.83 for NIO stock, respectively. The comparison is presented in Table VI.

<table>
<thead>
<tr>
<th>Stock</th>
<th>BS</th>
<th>BS GC</th>
<th>VG</th>
<th>VG AVR</th>
<th>VG IS</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTC</td>
<td>65.60%</td>
<td>22.74%</td>
<td>2.80%</td>
<td>3.14%</td>
<td>5.22%</td>
</tr>
<tr>
<td>NIO</td>
<td>12.02%</td>
<td>8.69%</td>
<td>7.83%</td>
<td>7.89%</td>
<td>7.57%</td>
</tr>
</tbody>
</table>
Overall, the MAPE of all VG models is much lower than all of the B-S models, both for INTC and NIO stocks. This shows that all VG models can give a call option price that is closer to the market price than all B-S models. This is reinforced by stock characteristics where each stock return log is not normally distributed and the VG model could accommodate those characteristics better than the B-S model. Moreover, in this situation, the VG model gives a better fit than the normal distribution. In this case, of course, all VG models will be better than all B-S models. More specifically, the best model which gives the closest price to the real market price seen by the MAPE value on NIO stock is the Importance Sampling VG model. Theoretically, it makes sense because the Importance Sampling VG model could reduce the simulations’ standard errors from the original VG simulations model. On the other hand, the best model that gives the closest price to the real market price on INTC stock is the original VG model. This could happen because the price volatility in INTC stock tends to be stable throughout the year, which means it has a small variance. In NIO, the high variance makes the VG model unable to follow the pattern. This statement is supported by the standard error value movement in Figure 1. For INTC stock prices, the SE value that is not smooth is in the VG IS model which indicates the highest MAPE value compared to other VG models. As for the NIO stock price, the model chart is different compared to the others, namely the VG IS chart which shows that it has the lowest MAPE value.

![Graph showing SE comparison of VG for INTC and NIO](image1)

![Graph showing SE comparison of VG AVR for INTC and NIO](image2)

![Graph showing SE comparison of VG IS for INTC and NIO](image3)

**Fig. 1.** Standard error (SE) comparison graph between VG model types for INTC and NIO option prices based on the number of simulations.

**VII. CONCLUSION**

We found that the MAPE for INTC and NIO resulting from using the G-C expansion formula is lower than using the B-S. This means that the G-C expansion formula can adjust the abnormal skewness and kurtosis in the option pricing using the B-S formula. In the simulation, generally, we get the result that the standard errors get smaller as the number of simulations increases. On the other hand, for both stocks, the fluctuation level of the Importance Sampling VG model is generally lower than the other two models, and the fluctuation level of the Antithetic Variate VG model is lower than the VG model. So, the Importance Sampling VG and the Antithetic Variate VG models can reduce the variance and give a more accurate option price compared to the VG model. Finally, the MAPE from both VG models is always smaller than all B-S models. This shows that all VG models can give a call option price closer to the market price than all B-S models.

REFERENCES


