Three-way Decision Based on TODIM Method with Single-valued Neutrosophic Sets

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Abstract—Three-way decision models have received substantial interest grounded in decision-theoretic rough sets and Bayesian decision theory. Single-valued neutrosophic sets are extremely useful for handling uncertain and inconsistent information, making them a valuable tool that is commonly applied in decision-making. In a three-way decision problem involving a piece of single-value neutrosophic information, the losses of each equivalence class under different actions can usually be identified with some accuracy. A critical aspect of the three-way decision problem centers around appropriate handling of the loss function. This paper proposes a novel approach to rank loss functions in each equivalence class of three-way decisions, based on the TODIM method and operates within a single-valued neutrosophic environment. Furthermore, a numerical experiment on the location of a breakfast restaurant is used to assess the model compared to some existing related models, with the aim of demonstrating its validity and soundness.

Index Terms—Three-way decision, single-valued neutrosophic sets, decision-theoretic rough sets, TODIM method.

I. INTRODUCTION

ROUGH set theory is a theory proposed by the Polish scientist Z. Pawlak[1], which has gained wide attention because it can effectively analyze various kinds of incomplete information such as imprecision, inconsistency and incompleteness. However, the limited applicability in classical rough set theory is attributed to be deficiency in consideration for fault tolerance in dealing with uncertainty. To perfect this classical theory, Yao et al.[2] combined it with the Bayesian decision process and introduced the decision-theoretic rough sets (DTRSs). Afterward, Yao[3] introduced three-way decision making (3WD) using DTRSs, which focuses on decision-making process of individuals. In Yao’s view, the 3WD divides a feasible domain into positive, negative, and boundary regions, each of which is pairwise disjoint. Then the decision rules can be generated for each of these regions, that are acceptance, rejection and non-commitment decisions.

Up until now, the 3WD has been developed with the contribution of numerous researchers. They have focused their research on 3WD with respect to conditional probability and loss function. Research on conditional probability is mainly to find new approaches for estimating conditional probability. Among these studies, Yao and Zhou[4] developed the model of Naive Bayesian decision-theoretic rough sets (NBRS), which combines the concepts of Naive Bayesian classification and DTRSs, and can be estimated in accordance with Bayesian theory and the assumption of naive probability independence. In a similar vein, Liu et al.[5] suggested a novel discriminant analysis method that computes the DTRSs’ conditional probability through logistic regression, with thresholds calculated using DTRSs and a Bayesian decision procedure. The new method also provides a suitable mechanism for the interpretation of the thresholds. In the study of the 3WD’s loss function, Herbert and Yao ([6],[7],[8]) incorporated the game theory of classification metric into the loss function of DTRSs to identify the equilibrium point of the game and optimize the size objective of the decision domain. Deng and Yao[9] suggested an information-theoretic method that employs uncertainty as the objective function to explain and determine the threshold value. This approach introduced information theory into probabilistic rough sets. Liang et al.[10] and Liu ([11],[12],[13]) introduced an uncertainty assessment form into the loss function. They considered different forms of uncertainty for each loss value in the DTRSs and proposed new DTRSs models that broaden the range of possible loss values.

In uncertain problems, there are some biases in using exact numerical sets to describe uncertain phenomena. The American scholar Professor Zadeh[14] established fuzzy sets theory (Fuzzy Sets), considering the degree of membership. After that, some scholars refined the membership degree. Among them, Smarandache([15],[16]) proposed the neutrosophic sets. The elements in the neutrosophic sets can be real numbers, interval values or some mergers and intersections of both, so that the neutrosophic sets can be generalized to single-valued neutrosophic sets (SVNSs)[17]. Fewer studies have been conducted on 3WD in single-valued neutrosophic environments. By utilizing an evaluation function, Abdel Basset et al.[18] conducted research on 3WD that was based on SVNSs. Furthermore, Singh ([19],[20]) succeeded in establishing a correlation between 3WD, SVNSs, and conceptual lattice in his research. The handling of loss is a critical aspect when dealing with the 3WD problem that involves SVNSs information. Using cosine similarity measures and Euclidean distances, Jiao et al. ([21],[22]) developed a 3WD model with SVNSs information. By utilizing these measures, the 3WD model can calculate the metric between the loss functions of different actions and the ideal loss. This enables the determination of the expected behavior for each equivalence class. However, in Jiao et al’s model, the risk preferences of decision-makers are somewhat overlooked, while this method also has the
drewback of relying on pre-defined identification of the ideal solution. The TODIM (Tomada de Decisão Iterativa Multicritério) method [23] is a multi-attribute decision-making method informed by prospect theory, in which the relative dominance degree is obtained by the comparison of the relative dominance of two solutions, which can avoid the artificial prior confirmation of the reference point and adapt to the risk appetite of decision-makers to a certain degree. Xu [24] incorporated the cumulative prospect theory into the TODIM method, and took full advantage of the TODIM method to contemplate decision-makers’ psychological state. Gong [25] used the TODIM to solve the decision-making problem of interval type II fuzzy sets considering decision-makers’ psychological behavioral state. It’s clear that the TODIM method has a greater advantage in solving decision-making problem.

Therefore, in this paper, we apply the relative dominance degree in the TODIM method to rank the losses of performing different actions of the 3W and propose a 3W model based on the TODIM method with single-valued neutrosophic information.

II. PRELIMINARIES

A. SVNS

Definition 1: Assuming X is a given domain, x is in the domain, the SVNS defined on X is composed of the truth subordinate function T_x (x), the uncertainty subordinate function \( I_x (x) \) and the distortion subordinate function \( F_x (x) \).

\[ Z = \{(x, T_x (x), I_x (x), F_x (x)) | x \in X \}, \]

and all three membership functions of A have values between 0 and 1, 0 \leq T_x (x) + I_x (x) + F_x (x) \leq 3. To facilitate writing, Z can be simply expressed as \((T_x, I_x, F_x)\).

Definition 2: Suppose Z is a SVNN on X, and \( Z' \) be complement of Z. \( Z' \) satisfies:

\[ T_x' = F_x, I_x' = 1 - I_x, F_x' = T_x. \] (1)

Definition 3: On X. Let \( Z_1 \) and \( Z_2 \) be two SVNNs, the SVNN’s calculation rules for addition, number multiplication and the normalized Hamming distance are as follows:

1) \( Z_1 \oplus Z_2 = (T_{z_1} \oplus T_{z_2}, T_{z_1} \cdot T_{z_2}, I_{z_1} \cdot I_{z_2}, F_{z_1} \cdot F_{z_2}) \).
2) \( \mu Z_1 = (1 - 1 - T_{z_1})^\mu I_{z_1}^\mu, F_{z_1}^\mu \).
3) \( d (Z_1, Z_2) = \frac{1}{3} (T_{z_1} - T_{z_2}) + |I_{z_1} - I_{z_2}| + |F_{z_1} - F_{z_2}|.\) (2)

Definition 4: [26] Assuming Z is a SVNS, whose score function S(Z) and accuracy function H(Z) are:

\[ S (Z) = \frac{2 + T_x - I_x - F_x}{3}, \]

\[ H (Z) = T_x - F_x, \]

where \( S (Z) \in (0, 1) \) and \( H (Z) \in (-1, 1) \).

Definition 5: Let \( Z_1 \) and \( Z_2 \) be two SVNNs,

1) If the score function value of \( Z_1 \) is less than \( Z_2 \), then \( Z_1 < Z_2 \).
2) When the score function value of \( Z_1 \) is equal to that of \( Z_2 \), then:
   - if the accuracy function value of \( Z_1 \) is less than \( Z_2 \), then \( Z_1 < Z_2 \);
   - if the accuracy function value of \( Z_1 \) is equal to \( Z_2 \), then \( Z_1 = Z_2 \);

Definition 6: [27] Suppose \( Z_1 = (T_{z_1}, I_{z_1}, F_{z_1}) \) and \( Z_2 = (T_{z_2}, I_{z_2}, F_{z_2}) \) are SVNNs, if satisfying the following four axiomatization conditions:

1) \( E (Z_1) = 0 \leftrightarrow T_{z_1}, I_{z_1}, F_{z_1} = 0 \) or \( T_{z_1}, I_{z_1}, F_{z_1} = 1 \);
2) \( E (Z_1) = 1 \leftrightarrow (T_{z_1}, I_{z_1}, F_{z_1}) = (0.5, 0.5, 0.5) \);
3) \( E (Z_1) = E (Z_1) \);
4) If \( Z_2 \) is more uncertain than \( Z_1 \), then for \( T_{z_2} - T_{z_2}' \leq 0, I_{z_2} - I_{z_2}' \leq 0, F_{z_2} - F_{z_2}' \leq 0 \), or when \( T_{z_2} > T_{z_2}' \geq 0, I_{z_2} - I_{z_2}' \geq 0, F_{z_2} - F_{z_2}' \geq 0 \), have \( T_{z_1} - T_{z_1}' \geq 0, I_{z_2} - I_{z_2}' \geq 0, F_{z_2} - F_{z_2}' \geq 0 \), \( E (Z_1) \leq E (Z_2) \), the E is called SVNN’s entropy.

B. THE TODIM METHOD

Here, we consider such a multi-attribute decision problem. In this problem, \( A_i (i \in [1, m]) \) is option. \( G_j (j \in [1, n]) \) is the attribute, whose weight is \( \omega_j, \omega_j \in [0, 1] \) and \( \sum \omega_j = 1 \). \( z_{ij} \) is the evaluation value of \( A_i \) under the \( G_j \), then we can obtain an evaluation matrix \( Z = (z_{ij})_{m \times n} \). The TODIM method entails the following steps:

Step 1: Normalize \( Z \) into \( U = (u_{ij})_{m \times n} \).

Step 2: In accordance with the weights of the attributes, calculate the relative weight of \( G_j \) with respect to \( G_r \):

\[ \omega_{jr} = \frac{\omega_j}{\omega_r}. \]

The \( \omega_r \) is the maximum value of \( \omega_j \).

Step 3: In accordance with normalisation results and relative weights, compute the dominance of \( A_i \) over every alternative \( A_k \) under attribute \( G_j \):

\[ \phi_j (A_i, A_k) = \begin{cases} \frac{\omega_{jr} (u_{ij} - u_{kj})}{\sum_{j=1}^{n} \omega_{jr}}, & u_{ij} - u_{kj} > 0, \\ 0, & u_{ij} - u_{kj} = 0, \\ \frac{1}{\theta} \sum_{j=1}^{n} \frac{\omega_{jr} (u_{ij} - u_{kj})}{\omega_{jr}}, & u_{ij} - u_{kj} < 0, \end{cases} \]

where \( i, k = 1, 2, \ldots, m \).

Step 4: Determine the extent to which alternative \( A_i \) is dominant over all other alternatives.

\[ \delta (A_i, A_k) = \sum_j \phi_j (A_i, A_k). \]

Step 5: In accordance with the results of the previous step, compute the overall dominance degree of each alternative \( A_i \):

\[ \phi (A_i) = \frac{\sum_k \delta (A_i, A_k) - \min_i \left\{ \sum_k \delta (A_i, A_k) \right\}}{\max_i \left\{ \sum_k \delta (A_i, A_k) \right\} - \min_i \left\{ \sum_k \delta (A_i, A_k) \right\}}. \]
Step 6: Rank these solutions based on the magnitude of their corresponding \( \phi(A_i) \) values. The solution with the largest \( \phi(A_i) \) is considered the best.

C. 3WD and DTRS model

The 3WD approach divides a universe into three parts: the "positive", "negative", and "boundary" regions, which represent the decision rules of acceptance, noncommitment, and rejection in sequence. These three parts are pairwise disjoint and are represented by Pos, Neg, and Bnd.[21]

The state sets \( \Omega = \{X, \neg X\} \) and action sets \( \{a_P, a_B, a_N\} \) together form the DTRS model. When \( x \) belong to \( X \), which can be expressed as \( x \in X \), it can be accompanied by three actions in the action sets: \( Pos(X) \), \( Bnd(X) \), \( Neg(X) \), resulting in respective losses denoted as \( \lambda_P, \lambda_B, \lambda_N \). Similarly, when \( x \) is not in \( X \), denoted as \( x \in \neg X \), it can also face three actions, whose loss is denoted as \( \lambda_P, \lambda_B, \lambda_N \), respectively. The losses for each action are summarized in Table 1.

The expected loss resulting from performing three different behaviors \( (\cdot = P, B, N) \) is determined by:

\[
E(\cdot ||x||_R) = \lambda_P Pr(X||x||_R) + \lambda_N Pr(\neg X||x||_R),
\]

where \( Pr(\cdot||x||_R) = \frac{Pr(x||x||_R)}{Pr(x)} \) is conditional probability. Obviously \( Pr(\cdot||x||_R) \in [0, 1], (\cdot \in \Omega) \). And \( U \) represent a set of all solutions, \( x||x||_R \) is a set in \( U \) which has the equivalence relation \( R \) with \( x \).

The decision rules of the DTRS, in accordance with Bayesian decision theory, are outlined as follows:

- For the expected loss of actions taken on equivalence class \( x||x||_R \), if \( a_P < \min(a_P, a_N) \), it indicates that \( x \in Pos(X) \);
- For the expected loss of actions taken on equivalence class \( x||x||_R \), if \( a_B < \min(a_P, a_N) \), it indicates that \( x \in Bnd(X) \);
- For the expected loss of actions taken on equivalence class \( x||x||_R \), if \( a_N < \min(a_P, a_B) \), it indicates that \( x \in Neg(X) \).

III. THREE-WAY DECISION BASED ON TODIM METHOD WITH SINGLE-VALUED NEUTROSOPHIC SETS

Jiao et al.[21] utilized the Bayesian decision procedure in the development of a single-valued neutrosophic decision-theoretic rough set (SVN-DTRS) method. In the SVN-DTRS model, all loss functions are expressed using SVNN, distinguishing it from the original DTRS model. Then based on the above discussion, we can calculate the average loss of SVN-DTRS under different actions\( (\cdot = P, B, N) \):

\[
\varepsilon(\cdot ||x||_R) = \lambda_P Pr(X||x||_R) + \lambda_N Pr(\neg X||x||_R),
\]

\( \lambda_P \) denotes the evaluation value of performing the corresponding action when \( x \in X \), and \( \lambda_N \) represents the evaluation value of performing the corresponding action when \( x \in \neg X \).

According to Definition 3, the average loss can be calculated as:

\[
\varepsilon(\cdot ||x||_R) = \begin{pmatrix}
\lambda_P Pr(X||x||_R) \\
\lambda_N Pr(\neg X||x||_R)
\end{pmatrix}
\]

Through the SVN-DTRS process, we can get the average loss, as shown in Table II.

In the TODIM method, losses are typically converted into gains. In our case, we normalise losses to gains for each equivalence class under different actions. The normalisation of decision data with SVN information is commonly achieved by finding its complement, so we use Equation 1 for each loss function to obtain Table III.

Here we use the relative advantage degree calculation in the TODIM method to measure the relative advantages of three different actions in the case of the same project. This approach avoids the need for artificial determination of the ideal solution in advance. The TODIM method captures decision-makers’ risk attitudes, making it a more practical approach compared to other multi-attribute decision-making methods. Below are specific steps involved in the 3WD based on the TODIM method:

Step 1: In the 3WD problem, the weights of each equivalence class are usually unknown. However, the classical TODIM method requires the weight of \( A_i \). To solve this problem, Majumdar and Samanta[27] proposed a method for calculating the entropy of a SVNS. Furthermore, Biswas et al.[28] introduced an entropy weighting method to ascertain attribute weights, where the entropy of \( A_i (i = 1, 2, \ldots, n) \) is:

\[
\alpha_i = 1 - \frac{1}{n} \sum_{j=P,B,N} (T_{ji} + F_{ji}) |I_{ji} + I^c_{ji}|, \tag{4}
\]

where \( \alpha_i \geq 0 \), and \( T_{ji}, I_{ji}, F_{ji} \) respectively represent the different degrees of membership of the \( A_i \) adopting the j-th behavior. Then based on the information entropy of each solution, the weight of each solution \( A_i \) can be calculated:

\[
w_i = \frac{1 - \alpha_i}{\sum_i (1 - \alpha_i)}, \tag{5}
\]
Step 2: Compute the relative weight of $A_i$:
\[
\omega_{ir} = \frac{\omega_i}{\omega_r},
\]
where the $\omega_r$ is the maximum value of $\omega_i$.

Step 3: According to the classical TODIM method, calculate $A_i$ in the execution $a_P, a_B, a_N$ relative to the execution $(a_B, a_N), (a_P, a_N), (a_P, a_B)$, whose calculation formula is:
\[
\phi_i(a_j, a_k) = \begin{cases} 
\frac{\omega_{ir} d(E(a_j|A_i), E(a_k|A_i))}{\sum_{r=1}^{R} \omega_{ir}}, & E(a_j|A_i) > E(a_k|A_i), \\
0, & E(a_j|A_i) = E(a_k|A_i), \\
\frac{1}{\theta} \left( \sum_{r=1}^{R} \omega_{ir} \right) - \min_j \left\{ \sum_k \phi_i(a_j, a_k) \right\}, & E(a_j|A_i) < E(a_k|A_i),
\end{cases}
\]
where the $\theta$ is the attenuation coefficient, the $j, k = P, B, N$.

Step 4: Utilizing the results of the calculations from step 3 and referring to the classic TODIM method, we can calculate the overall advantage of performing different actions under the solution $A_i$.
\[
\phi_i(a_j, a_k) = \phi_i(a_j, a_k) - \min_j \left\{ \sum_k \phi_i(a_j, a_k) \right\},
\]

where $j, k = P, B, N$.

Step 5: Repeat steps 3-4 to calculate the overall advantage of performing different actions for all solutions.

Step 6: Make decisions for the different actions of each solution. The decision rule becomes:

- If the overall dominance of the equivalence class $[x]_R$ to perform $a_P$ is greater than that of performing $a_B$ and $a_N$, $[x]_R$ will perform $a_P$, namely $x \in Pos(X)$.
- If the overall dominance of the equivalence class $[x]_R$ to perform $a_B$ is greater than that of performing $a_P$ and $a_N$, $[x]_R$ will perform $a_B$, namely $x \in Bnd(X)$.
- If the overall dominance of the equivalence class $[x]_R$ to perform $a_N$ is greater than that of performing $a_P$ and $a_B$, $[x]_R$ will perform $a_N$, namely $x \in Neg(X)$.

IV. NUMERICAL EXAMPLE AND ANALYSIS

80% of catering industry operators are small-scale entrepreneurs who may not have established social networks or possess significant economic resources. These operators are often faced with the embarrassing fact that they cannot afford to pay rent when choosing locations. Therefore, it is essential for the store to choose the right location. Table IV displays the values of the loss functions corresponding to the four equivalence classes in the 20 recommended locations, where some basic conditions have already been examined.

These loss functions are represented in the form of SVNN. Meanwhile, Table V illustrates the probabilities obtained for these four equivalence classes. [21]

Then we can calculate the loss function of each solution under different actions using Equations 3, as presented in Table VI.

According to Equation 1, we can transform the loss function into a gain function, as shown in Table VII.

A. Using the 3WD based on the score function of the SVNSs

For the loss ranking in the 3WD problem, we can compare the losses from different actions in the 3WD problem for ranking purposes by employing SVNSs’ score and accuracy function, with a focus on the membership aspect of the
SVNS. Here we use this method to obtain the score function for each item in Table VII.

Table VIII shows the scores of the benefit function for each equivalence class under different actions. In Table V, $x_1R$ performs each action in the following order of score: $a_N > a_B > a_P$; $x_2R$ performs each action in the following order of score: $a_P > a_N > a_B$; $x_3R$ performs each action in the following order of score: $a_B > a_N > a_P$; $x_4R$ performs each action in the following order of score: $a_N > a_B > a_P$.

B. Using the 3WD based on Cosine Similarity and Euclidean Distance

Jiao et al. [22] used cosine similarity and Euclidean distance to measure the metric between the loss function and the ideal loss in the 3WD for ranking purposes, and the results using these two methods applied to this data are shown in Tables IX and X, when the ideal loss is chosen as $(0,1,1)$.

From Table IX, $x_1R$ performs each action in the following order of similarity: $a_N > a_B > a_P$; $x_2R$ performs each action in the following order of similarity: $a_P > a_N > a_B$; $x_3R$ performs each action in the following order of similarity: $a_B > a_N > a_P$; $x_4R$ performs each action in the following order of similarity: $a_N > a_B > a_P$.

From Table X, $x_1R$ performs each action in the following order of distance: $a_N > a_B > a_P$; $x_2R$ performs each action in the following order of distance: $a_P > a_N > a_B$; $x_3R$ performs each action in the following order of distance: $a_B > a_N > a_P$; $x_4R$ performs each action in the following order of distance: $a_N > a_B > a_P$.

C. Using the 3WD based on the TORDIM method

Step 1: According to Equations 5-6, the weights in different actions are computed, as shown in Table XI.

Step 2: In accordance with Equation 6, calculate the relative weights under different actions, as shown in Table XII.

Step 3: As per Equation 7, calculate the relative advantage of each solution under different actions, and the results are shown in $\phi_1 - \phi_4$.

Step 4: In accordance with Equation 8, calculate the combined dominance of each action relative to the others, as shown in Table XIII.
TABLE XIV
THE RESULTS OF BOTH METHODS

<table>
<thead>
<tr>
<th></th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(x_3)</th>
<th>(x_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>By the TIDIM method</td>
<td>(a_N)</td>
<td>(a_P)</td>
<td>(a_B)</td>
<td>(a_N)</td>
</tr>
<tr>
<td>By the score function</td>
<td>(a_N)</td>
<td>(a_P)</td>
<td>(a_B)</td>
<td>(a_N)</td>
</tr>
<tr>
<td>By the cosine similarity</td>
<td>(a_N)</td>
<td>(a_P)</td>
<td>(a_B)</td>
<td>(a_N)</td>
</tr>
<tr>
<td>By the euclidean distance</td>
<td>(a_N)</td>
<td>(a_P)</td>
<td>(a_B)</td>
<td>(a_N)</td>
</tr>
</tbody>
</table>

Fig. 1. Comparison of Results from Four Methods Using \([x_2]\) as an example.

D. Comparative Analysis

From the findings in Table XIV, the model ranking results in this paper are consistent with the other three results. Obviously, \([x_2]\) is the most suitable for the decision-makers’ interest. From Fig. 1, it can be observed that all four methods are acceptable when executing \(P\) in \([x_2]\). However, the acceptance level is lower for executing \(B\) and \(N\) compared to \(P\). Among them, the judgment methods based on cosine similarity and score function show relatively small differences for the three behaviors. Although it can be seen that \(P\) is the optimal decision, the intuitive perception for decision-makers is not strong. This finding successfully proves the rationality and effectiveness of the 3WD based on the TIDIM method. Therefore, our method is effective in helping decision-makers make decisions in complex environments.

Compared with the other three methods, our approach enables a more precise capture of the distinct psychological states experienced by decision-makers, when confronted by gains and losses and make the decision more realistic. At the same time, the TIDIM method can avoid the prior selection of reference points compared to Jiao’s methods. In summary, our model has wider applicability and implementability.

E. Sensitivity analysis

At the same time, we perform a sensitivity analysis of the parameter \(\theta\) of this algorithm for \([x_2]\), and the result is shown in Fig.2, which shows that as the \(\theta\) changes, the result does not change. It verifies the stability of the result in terms of the decision-makers’ preferences.

V. Conclusions

3WD uses a pair of state and a trio of behavior for characterizing the decision process by finding the expected loss of different behavior sets under the equivalence class and then making decisions in accordance with the Bayesian minimization risk-cost decision principle. In this paper, we extend the 3WD idea and the classical TIDIM method to propose a three-way decision model based on the TIDIM method. This method factors in the different performances of decision-makers facing losses and gains, and reflects the objective psychological state of decision-makers. Then, we apply our method along with some other validated effective 3WD methods to the 3WD problem with single-value neutrosophic information. Through the results, it is demonstrated that our method is practical and objective. By way of contrast, although the outcomes of our method and other methods are consistent, the three-way decision model based on the TIDIM method increases the spacing of the overall degree of dominance of each action, which can help decision-makers make better judgments. Moreover, our method takes into account the reality that decision-makers have different investment intentions when facing different risks. This is reflected in the introduction of the TIDIM method relative advantage degree function based on prospect theory. Our method differs from using score functions and accuracy functions in that it considers not only the data, but also the risk preferences of decision-makers. Unlike using the Euclidean distance and similarity function to find measures of expected and ideal losses, our method can take decision-makers’ risk preferences into consideration as well as avoid the idea of identifying reference points in advance. At the same time, the sensitivity analysis shows that the parameters have minimal impact on the results, demonstrating the robustness of our proposed method. Overall, our method focuses on decision-makers’ risk preferences while considering on the data itself, which for a three-way decision problem can result in a decision that better caters to the preferences of decision-makers. However, despite our approach having some advantages in the site selection problem discussed in this paper, it still has the shortcoming of ignoring the psychological characteristics of decision-makers. In subsequent endeavors, we will be devoted to examining the psychological factors of decision-makers in fuzzy three-way decision problems.

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