Neural Adaptive Control for a Class of Uncertain Switched Nonlinear Systems with Input and Output Constraints

Lei Zhou, Lidong Wang, Yonghui Yang

***Abstract*—A neural network adaptive control scheme for a class of uncertain switched nonlinear systems with input and output constraints is presented in this paper. A smooth function is adopted to approximate the input dead zone and saturation function, which can be used to solve the non differentiable phenomenon in system equations. Radial basis function (RBF) based neural networks are introduced to estimate the nonlinear functions of the system and the barrier Lyapunov function is selected to solve the problem of system output constraint, based on which a neural adaptive controller is designed by backstepping technique. Under the condition that the switching system meets a certain average dwell time, the designed controller can ensure that all signals in the closed-loop system are bounded and the tracking error of the system can converge to a compact set. The effectiveness of the designed controller is verified by the simulation example.**

***Index Terms*—Switched Nonlinear Systems, Input dead zone and saturation, Output constraint, Average dwell time, Adaptive control.**

I. Introduction

THE adaptive control problem for a class of uncertain switched nonlinear systems with input and output constraints is discussed in this paper. Switched nonlinear systems are extremely complex and important hybrid systems. Adaptive backstepping technology is widely used in nonlinear systems, such as documents [1]–[10]. Input dead zone and saturation were also discussed in reference [1], but they were not extended to the case of switched systems. A class of nonlinear systems with unknown dead zone are studied in literature [2], and the prescribed performance control method was given in [2]. The authors in reference [3] introduced the input dead zones in the nonlinear system. For reference [4], the author considered the unknown dead-zone output of the system, they preprocessed the non-affine function by using the mean value theorem and finally kept the system stable by designing a suitable fuzzy tracking controller. however, the case of switched nonlinear systems did not be considered in [4]. Similar to article [4], the authors in literature [5] also introduced the unknown dead zone, both of them used the fuzzy control strategy; although [4] is the pure-feedback nonlinear system and [5] is the strict-feedback nonlinear system, because there are unmeasurable state variables in literature [5], the design of state observer was introduced. The case of input dead zones was also mentioned in the paper [6], which described the input dead zones function in a more concise linear function form. All the systems in the literature [7]–[9] have dead zones; these systems studied belong to the nonlinear systems, and all of them used the neural network. A class of uncertain nonlinear systems with input Saturation was studied in the literature [10], where a smoothing function was introduced to similar the function of input. In this paper, both input dead zone and input saturation are considered, which is more complicated and universal than only considering input dead zone and input saturation. After years of intensive research, some scholars have achieved many research results on switched nonlinear systems. A solution to the adaptive control problem of switched nonlinear systems with unmodeled dynamics was proposed in [11]. The work of [12] mentioned the fuzzy command filter control and designed a unique event triggering strategy. The authors in [13] also used the average dwell time method. The study of [14] put forward the control scheme of stochastic switched nonlinear systems with asymmetric out-constrained. An uncertain nonlinear switching system with prescribed performance was studied in document [15]. For switched non-strict feedback nonlinear systems with state-constrained, a controller was designed based on event triggering in [16]. When designing the adaptive controller, the integral barrier Lyapunov function was selected in [17]. The neural networks were used to estimate the nonlinear functions [18]. It can be seen that neural network and fuzzy logic are often used as tools to analyze the control problems in many literatures. An adaptive fuzzy control method for a class of nonstrict feedback switched nonlinear systems with state constraint was designed in [19]. Referring to the paper [20], the input saturation function is approximated by a smooth function, but the authors in [20] only discuss the case of single input saturation. In the literature [21], a fuzzy tracking controller was designed based on fuzzy control method and common backstepping method, and a small gain method was used in the stability analysis. Because unknown hysteresis control input in the system, a smooth function is used to approximate the system input in [22]. In [23]–[25], uncertain terms were added to switched nonlinear systems, but the authors of they didn't consider the dead zone and saturation of system input. The authors in reference [26] introduced the control of discrete-time switched systems under this circumstance of input saturation. The control method of neural network in the design of controller was introduced in [27]. Time-varying

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delay was added to the nonlinear uncertain switched system in [28]. The research of [29] is a class of switched pure feedback nonlinear system, but it is not a conventional lower triangular system, the mean-value theorem was used in dealing with input saturation, an adaptive tracking controller was designed based on neural network and adaptive backstepping technology. A nonlinear switching system with asymmetric actuator dead zones was studied in [30].

Through a comprehensive consideration of the above literature, a class of uncertain switched nonlinear systems with input and output constraints are studied in this paper. The input dead zone and saturation function is firstly approximated by a smooth function in the paper, then based on backstepping method and neural network, an adaptive controller is designed to make the output signal track the reference signal effectively. Through some inequalities, it is proved that the output satisfies the constraint conditions. About the stability, the average dwell time method is used in this paper. Compared with a large number of literatures about switched nonlinear systems and nonlinear systems, the results of this paper have three clear characters.

1. A class of uncertain nonlinear switched systems with input and output constraints are studied and the system studied in this paper is more complicated. The input dead zone, input saturation and output constraint are considered in the uncertain switched nonlinear system.

2. Then the input dead zone and saturation function is approximated by a smooth function to solve the non differentiable phenomenon in system equations. Although the authors in reference [1] also discusses the case of input dead zone and saturation, it does not consider the case of switching nonlinear systems and the output constraint. The authors in literature [20] considers the case of input saturation, but the input does not involve the more complicated case of both saturation and dead zone.

3. Considering the situation of system output, the barrier Lyapunov function is adopted to solve the problem of system output constraint, which is more conducive in designing controllers. The condition of average dwell time is also used, the average dwell time method effectively limits the switching times of the switched system. Under the condition of average dwell time, an adaptive controller is designed to make all signals in the closed-loop system bounded. The switched nonlinear system was studied in literature [17], but the average dwell time method was not used.

II. Problem formulation and preliminaries

A class of uncertain switched nonlinear systems with input and output constraints are considered.

$$\begin{cases} \dot{x}_1 = x_2 + f_{1,\sigma}(x_1) + h_{1,\sigma}(t) \\ \dot{x}_k = x_{k+1} + f_{k,\sigma}(\overline{x}_k) + h_{k,\sigma}(t) \\ \dot{x}_n = u_\sigma + f_{n,\sigma}(x) + h_{n,\sigma}(t) \\ y = x_1, \end{cases} \tag{1}$$

where $\overline{x}_k = [x_1, x_2, x_3, \cdots, x_k]^{\mathrm{T}} \in \mathbb{R}^k, k = 1, 2, \cdots, n-1$, then $x = [x_1, x_2, x_3, \cdots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$, and y $\in \mathbb{R}$. They are the state variables and output variable of the above system. $\sigma(t)$:$[0, +\infty) \to \sum = \{1, 2, \cdots, M\}$, this function expresses the switching signal used in the above system, it is matched with the corresponding subsystem $\{(j_{i_0}, t_0), \cdots (j_{i_s}, t_s), \cdots\}, j_{i_s} \in \sum, s = 0, 1, \cdots$. The j_{i_s} subsystem is active under this circumstance of $t \in [t_s, t_{s+1})$, and $h_{k,\sigma}(t)$ are smooth and unknown functions, for convenience: $h_{k,\sigma} = h_{k,\sigma}(t) \quad k = 1, 2, \cdots, n$. $f_{k,\sigma}(\cdot)$, $k = 1, 2, \cdots, n$; $\sigma \in \sum$ are smooth unknown nonlinear functions, then the dead zone and saturation input of the system (1) is expressed as follows:

$$u_j = ds(m_j) = \begin{cases} d_{m_j} & m_j \le b_{l1_j} \\ p_{l_j}(m_j) & b_{l1_j} < m_j \le b_{l0_j} \\ 0 & b_{l0_j} < m_j \le b_{r0_j} \\ p_{r_j}(m_j) & b_{r0_j} < m_j \le b_{r1_j} \\ d_{M_j} & m_j > b_{r1_j}, \end{cases} \tag{2}$$

where m_j is the input of the dead zone and saturation nonlinear function with $j \in \sum$; $p_{l_j}(m_j)$ and $p_{r_j}(m_j)$ are unknown but differentiable nonlinear functions. $p_{r_j}(b_{r0_j}) = p_{l_j}(b_{l0_j}) = 0$, saturation: $p_{r,j}(b_{r1_j}) = d_{M_j} > 0$, $p_{l_j}(b_{l1_j}) = d_{m_j} < 0$, where d_{M_j} and d_{m_j} are the unknown saturation values of the input function. $b_{l1_j} < b_{l0_j} < 0$, $0 < b_{r0_j} < b_{r1_j}$ and they are unknown parameters. In the article, the output variable is constrained and satisfies this set $\{x_1 \,||x_1| \le K_{c1} \quad K_{c1} > 0\}$, K_{c1} is the known upper bound.

In order to stabilize the system (1) while ensuring that the signals of the closed-loop system are bounded, it is necessary to give some assumptions and lemmas.

Assumption 1. [1] There are the following constraints:

$$\begin{aligned} &0 < b_{lm_j} < \frac{dp_{l_j}}{dm_j} < b_{lM_j} < \infty \qquad \forall m_j \in [b_{l1_j}, b_{l0_j}] \\ &0 < b_{rm_j} < \frac{dp_{r_j}}{dm_j} < b_{rM_j} < \infty \qquad \forall m_j \in [b_{r0_j}, b_{r1_j}], \end{aligned} \tag{3}$$

where all the $b_{lm_j}, b_{lM_j}; b_{rm_j}, b_{rM_j}$ are unknown positive constants, $\frac{dp_{l_j}}{dm_j}$ and $\frac{dp_{r_j}}{dm_j}$ are the derivative of p_{l_j} and p_{r_j} to m_j.

Assumption 2. The studied system (1) satisfies the condition: input-to-state stable. The signal to be tracked in this paper is bounded and known, y_d is continuously derivable of order n, and its derivatives are bounded from the first derivative to the n derivative: $|y_d| \le \bar{y}_d, \cdots, \left|y_d^{(n)}\right| \le \bar{y}_{nd}$; for the external disturbance h_{kj}, there are positive constants $\bar{h}_{kj}^*$ with the following formula: $|h_{kj}| \le \bar{h}_{kj}^*$, $k = 1, 2, \cdots, n$; $j = 1, 2, \cdots, M$.

Definition 1. The average dwell time method was used in the literature [26]. The switching signal σ need to satisfy the following inequality

$$N_\sigma(T, t) \le N_0 + \frac{T - t}{\tau_a} \qquad \forall T \ge t \ge 0, \tag{4}$$

with $N_\sigma(T, t)$ being the switching numbers on the interval $[t, T]$ for any $T \ge t \ge 0$, τ_a is the average dwell time parameter, N_0 is a positive constant.

Lemma 1. Similar to the document [13], the RBF neural network is expressed as follows:

$$F_{rbf}(Z) = W^T S(Z), \tag{5}$$

where $Z \in \Omega_Z \subset \mathbb{R}^n$ is this input variable, $W = [w_1, w_{2,} \cdots w_l]^T$ is the weight parameter of the function, $S(Z) = [s_1(Z), s_2(Z), \cdots s_l(Z)]^T$, $l > 1$ is the number of neural network nodes with $W \in \mathbb{R}^l$ and $S(Z) \in \mathbb{R}^l$. $s_i(Z)$ is selected as Gaussian function in this paper:

$$s_i(Z) = \exp\left[\frac{-(Z-\mu_i)^T(Z-\mu_i)}{{\phi_i}^2}\right] \quad i = 1, 2, \cdots l, \quad (6)$$

with $\mu_i = [\mu_{i1}, \mu_{i2}, \cdots, \mu_{in}]^T$ is the center of the function, ϕ_i is the width of the function.

Using neural network to approximate the nonlinear function

$$F(Z) = W^{*T}S(Z) + \delta(Z), \quad (7)$$

where $|\delta(Z)| \leq \varepsilon$, the ideal constant weight W^*:

$$W^* = \arg \min_{W\in\mathbb{R}^l}\left\{\sup_{Z\in\Omega_Z}\left|F(Z) - W^TS(Z)\right|\right\}.$$

Lemma 2. [20] Such a kind of scaling inequality is given

$$\frac{z^2}{B_m^2} \leq \log\frac{B_m^2}{B_m^2 - z^2} \leq \frac{z^2}{B_m^2 - z^2}, \quad (8)$$

assume that there is an upper bound $B_m > 0$, so that the error variable z satisfies $|z| < B_m$ for any $z \in \mathbb{R}$. The logarithmic function used in this paper is natural logarithm.

Remark 1. It is inconvenient to directly process the input with dead zone and saturation, so it is necessary to choose a suitable smoothing function to fit it, similar to the literature [1].

$$\begin{aligned} g_j(m_j) = & -\frac{d_{m_j}}{2}\tanh l_{1_j}(m_j - \frac{b_{l0_j}}{l_{1_j}} + t_{l_j}) \\ & + \frac{d_{M_j}}{2}\tanh l_{2_j}\left(m_j - \frac{b_{r0_j}}{l_{2_j}} - t_{r_j}\right) \\ & - \frac{d_{m_j}}{2}\tanh\left(b_{l0_j} - t_{l_j}l_{1_j}\right) \\ & + \frac{d_{M_j}}{2}\tanh\left(b_{r0_j} + t_{r_j}l_{2_j}\right), \end{aligned} \quad (9)$$

where l_{1_j}, l_{2_j}; t_{l_j}, t_{r_j} are all adjustable positive parameters used to approximation the input dead zone and saturation function; then, there is $u_j(m_j) = g_j(m_j) + p_j(m_j)$, where $p_j(m_j)$ is the approximation error and bounded. An example of the approximation input function is shown in Fig. 1:

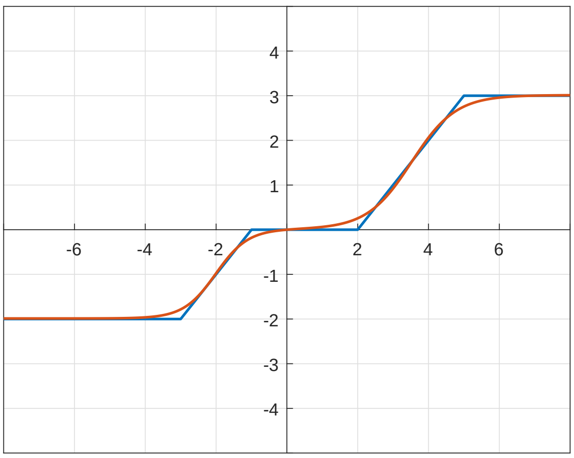

Fig. 1. Approximation example

Assumption 3. The approximation error $|p_j(m_j)| \leq p$, p is the unknown positive constant. Using the mean-value theorem for $g(m_j)$.

$$g_j(m_j) = g_j(m_{0_j}) + \dot{g}_j(m_{a_j})(m_j - m_{0_j}), \quad (10)$$

where $m_{a_j} = a_jm_j + (1-a_j)m_{0_j}$, $a_j \in (0,1)$. m_{0_j} is selected as 0, $g_j(0) = 0$, and $g_j(m_j)$ can be expressed as $g_j(m_j) = \dot{g}_j(m_{a_j})m_j$, $\dot{g}_j(m_{a_j}) = \left.\frac{\partial g_j}{\partial m_j}\right|_{m_j=m_{a_j}}$, $\dot{g}_j(m_{a_j})$ is expressed as follows:

$$\begin{aligned} \dot{g}_j(m_{a_j}) = & \frac{-d_{m_j}l_{1_j}}{2}\frac{1}{\cosh^2 l_{1_j}\left(m_{a_j} - \frac{b_{l0_j}}{l_{1_j}} + t_{l_j}\right)} \\ & + \frac{d_{M_j}l_{2_j}}{2}\frac{1}{\cosh^2 l_{2_j}\left(m_{a_j} - \frac{b_{r0_j}}{l_{2_j}} - t_{r_j}\right)}, \end{aligned} \quad (11)$$

b_{l1_j}, b_{l0_j}, b_{r0_j}, b_{r1_j}, d_{m_j}, d_{M_j}, l_{1_j}, l_{2_j}, t_{l_j} and t_{r_j} are all bounded, $\dot{g}_j(m_{a_j})$ is bounded.

Remark 2. There are the following formula conversions

$$\begin{aligned} u_j(m_j) &= \dot{g}_j(m_{a_j})m_j + p_j(m_j) \\ &= \dot{g}_j(m_{a_j})m_j - r_jm_j + r_jm_j + p_j(m_j), \end{aligned} \quad (12)$$

where r_j is the design positive parameter, $\dot{x}_n$ can be converted into

$$\dot{x}_n = r_jm_j + \eta_j(x) + p_j(m_j) + h_{n,j}, \quad (13)$$

where $\eta_j(x) = \dot{g}_j(m_{a_j})m_j - r_jm_j + f_{n,j}(x), j \in \sum$.

III. ADAPTIVE NEURAL NETWORK CONTROLLER DESIGN

The main part of this section is to introduce the design steps of neural network adaptive controller. The controller is designed based on the RBF neural network and backstepping method. Before designing the controller, the following coordinate transformations are introduced.

$$\begin{aligned} z_1 &= x_1 - y_d \\ z_k &= x_k - \alpha_{k-1,j}, \end{aligned} \quad (14)$$

with k and j belong to $k = 2, 3, \cdots, n$; $j = 1, 2, \cdots, M$, $\alpha_{k-1,j}$ is the designed virtual control law. In the design process, $\hat{\theta}_i$ is the estimated value of θ_i:

$$\tilde{\theta}_i = \theta_i - \hat{\theta}_i \quad i = 1, 2, \cdots, n. \quad (15)$$

According to the design scheme, $\hat{\theta}$ is designed as follows:

$$\dot{\hat{\theta}}_1 = \frac{\lambda_0 z_1^2 S_{1,j}^T(Z_1)S_{1,j}(Z_1)}{2\tau_{1,j}^2\left(B_{m_{1,j}}^2 - z_1^2\right)^2} - \gamma_0\hat{\theta}_1$$

$$\dot{\hat{\theta}}_k = \frac{\lambda_0 z_k^2 S_{k,j}^T(Z_k)S_{k,j}(Z_k)}{2\tau_{k,j}^2} - \gamma_0\hat{\theta}_k, \quad (16)$$

$\tau_{1,j}$, $\tau_{k,j}$, λ_0 and γ_0 are the design positive parameters, $k = 2, 3, \cdots n$, $j \in \sum$.

Step 1: Now, introduce the coordinate transformation as follows $z_1 = x_1 - y_d$ we can obtain

$$\dot{z}_1 = x_2 + f_{1,j} + h_{1,j} - \dot{y}_d. \quad (17)$$

In the scheme of designing the controller, such a barrier Lyapunov function is introduced to deal with the system with constraints more effectively.

$$V_{1,j} = \frac{1}{2}\log\left(\frac{B_{m_{1,j}}^2}{B_{m_{1,j}}^2 - z_1^2}\right) + \frac{1}{2\lambda_0}\tilde{\theta}_1^2, \quad (18)$$

where $B_{m_{1,j}}$ is the upper bound of z_1, then it satisfies the following inequality: $\Psi_{1,j} = \left\{ z_1 \middle| |z_1| < B_{m_{1,j}} \right\}$, and $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ is the error of estimating θ_1. The time derivative of the above function is expressed as follows:

$$\dot{V}_{1,j} = \frac{z_1}{B_{m_{1,j}}^2 - z_1^2}\left(x_2 + f_{1,j} + h_{1,j} - \dot{y}_d\right) - \frac{1}{\lambda_0}\tilde{\theta}_1\dot{\hat{\theta}}_1. \quad (19)$$

By using the Youngs inequality: $\left[(z_1 h_{1,j}) \Big/ \left(B_{m_{1,j}}^2 - z_1^2\right)\right] \le \left(z_1^2 \Big/ \left[2\left(B_{m_{1,j}}^2 - z_1^2\right)^2\right]\right) + \frac{1}{2}\overline{h}_{1,j}^{*2}$, combining formula (17) and (19), we can obtain

$$\begin{aligned}\dot{V}_{1,j} \le & \frac{z_1}{B_{m_{1,j}}^2 - z_1^2}\left(z_2 + \alpha_{1,j} + f_{1,j} - \dot{y}_d\right) \\ & + \frac{z_1}{B_{m_{1,j}}^2 - z_1^2}\frac{z_1}{\left(B_{m_{1,j}}^2 - z_1^2\right)} \\ & - \frac{z_1^2}{2\left(B_{m_{1,j}}^2 - z_1^2\right)^2} + \frac{1}{2}\overline{h}_{1,j}^{*2} - \frac{1}{\lambda_0}\tilde{\theta}_1\dot{\hat{\theta}}_1. \quad (20)\end{aligned}$$

Remark 3. Based on the Lemma 1, neural network is designed to approximate the above partial formula, $F_{1,j}(Z_1) = f_{1,j} + \frac{z_1}{\left(B_{m_{1,j}}^2 - z_1^2\right)} - \dot{y}_d$, $Z_1 = [x_1, y_d, \dot{y}_d]^T$, $F_{1,j}$ is denoted as $F_{1,j}(Z_1) = W_{1,j}^{*T} S_{1,j}(Z_1) + \delta_{1,j}(Z_1)$, then the approximation error $\delta_{1,j}(Z_1)$ satisfies $|\delta_{1,j}(Z_1)| \le \varepsilon_{1,j}$ with $\varepsilon_{1,j} > 0$.

Using the inequalities as

$$\begin{aligned}\frac{z_1 W_{1,j}^{*T} S_{1,j}}{B_{m_{1,j}}^2 - z_1^2} &\le \frac{\theta_1 z_1^2 S_{1,j}^T S_{1,j}}{2\tau_{1,j}^2\left(B_{m_{1,j}}^2 - z_1^2\right)^2} + \frac{1}{2}\tau_{1,j}^2 \\ \frac{z_1 \delta_{1,j}}{B_{m_{1,j}}^2 - z_1^2} &\le \frac{z_1^2}{2\left(B_{m_{1,j}}^2 - z_1^2\right)^2} + \frac{\varepsilon_{1,j}^2}{2}, \quad (21)\end{aligned}$$

where $\theta_1 = \max\left\{\left\|W_{1,j}^*\right\|^2, j \in \sum\right\}$. It follows that:

$$\begin{aligned}\dot{V}_{1,j} \le & \frac{z_1}{B_{m_{1,j}}^2 - z_1^2}\left(z_2 + \alpha_{1,j}\right) + \frac{1}{2}\overline{h}_{1,j}^{*2} - \frac{1}{\lambda_0}\tilde{\theta}_1\dot{\hat{\theta}}_1 \\ & + \frac{z_1}{B_{m_{1,j}}^2 - z_1^2}\frac{\theta_1 z_1 S_{1,j}^T S_{1,j}}{2\tau_{1,j}^2\left(B_{m_{1,j}}^2 - z_1^2\right)} + \frac{1}{2}\tau_{1,j}^2 + \frac{1}{2}\varepsilon_{1,j}^2. \quad (22)\end{aligned}$$

Choose the virtual control law $\alpha_{1,j}$ as

$$\alpha_{1,j} = -\left(c_{1,j} + \frac{1}{2}\right)z_1 - \frac{\hat{\theta}_1 z_1 S_{1,j}^T S_{1,j}}{2\tau_{1,j}^2\left(B_{m_{1,j}}^2 - z_1^2\right)}, \quad (23)$$

where the design parameter $c_{1,j} > 0$, using the square inequality $z_1 z_2 \le \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2$, combining (22), (23) can obtain the following expression:

$$\begin{aligned}\dot{V}_{1,j} \le & \frac{-c_{1,j}{z_1}^2}{B_{m_{1,j}}^2 - z_1^2} + \frac{{z_2}^2}{2\left(B_{m_{1,j}}^2 - z_1^2\right)} - \frac{1}{\lambda_0}\tilde{\theta}_1\dot{\hat{\theta}}_1 + \frac{1}{2}\overline{h}_{1,j}^{*2} \\ & + \frac{\tilde{\theta}_1 {z_2}^2 S_{1,j}^T S_{1,j}}{2\tau_{1,j}^2\left(B_{m_{1,j}}^2 - z_1^2\right)^2} + \frac{1}{2}\tau_{1,j}^2 + \frac{1}{2}\varepsilon_{1,j}^2, \quad (24)\end{aligned}$$

substituting $\dot{\hat{\theta}}_1$ to (24):

$$\begin{aligned}\dot{V}_{1,j} \le & \frac{-c_{1,j}{z_1}^2}{B_{m_{1,j}}^2 - z_1^2} + \frac{{z_2}^2}{2\left(B_{m_{1,j}}^2 - z_1^2\right)} \\ & + \gamma_0{\lambda_0}^{-1}\tilde{\theta}_1\hat{\theta}_1 + \Delta_{1,j}^*, \quad (25)\end{aligned}$$

with $\Delta_{1,j}^* = \frac{1}{2}\overline{h}_{1,j}^{*2} + \frac{1}{2}\tau_{1,j}^2 + \frac{1}{2}\varepsilon_{1,j}^2$ with $j = 1, 2, \cdots, M$.

Step 2: Based on coordinate transformation $z_2 = x_2 - \alpha_{1,j}$, select the following Lyapunov function:

$$V_{2,j} = V_{1,j} + \frac{1}{2}z_2^2 + \frac{1}{2\lambda_0}\tilde{\theta}_2^2, \quad (26)$$

the time derivative of $V_{2,j}$ is described as follows

$$\begin{aligned}\dot{V}_{2,j} = & \dot{V}_{1,j} + z_2\left(x_3 + f_{2,j} + h_{2,j}\right) - z_2\sum_{i=0}^{1}\frac{\partial\alpha_{1,j}}{\partial {y_d}^{(i)}}{y_d}^{(i+1)} \\ & - z_2\frac{\partial\alpha_{1,j}}{\partial x_1}\left(x_2 + f_{1,j} + h_{1,j}\right) - z_2\frac{\partial\alpha_{1,j}}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 - \frac{1}{\lambda_0}\tilde{\theta}_2\dot{\hat{\theta}}_2, \quad (27)\end{aligned}$$

using inequalities

$$\begin{aligned}z_2 h_{2,j} &\le \frac{z_2^2}{2} + \frac{\overline{h}_{2,j}^{*2}}{2} \\ -z_2\frac{\partial\alpha_{1,j}}{\partial x_1}h_{1,j} &\le \frac{1}{2}z_2^2\left(\frac{\partial\alpha_{1,j}}{\partial x_1}\right)^2 + \frac{1}{2}\overline{h}_{1,j}^{*2}, \quad (28)\end{aligned}$$

by combining $\dot{V}_{1,j}$ and the above inequality, we can get

$$\begin{aligned}\dot{V}_{2,j} \le & \frac{-c_{1,j}{z_1}^2}{B_{m_{1,j}}^2 - z_1^2} + \frac{{z_2}^2}{2\left(B_{m_{1,j}}^2 - z_1^2\right)} + z_2\left(x_3 + f_{2,j}\right) \\ & - z_2\frac{\partial\alpha_{1,j}}{\partial x_1}\left(x_2 + f_{1,j}\right) - z_2\sum_{i=0}^{1}\frac{\partial\alpha_{1,j}}{\partial {y_d}^{(i)}}{y_d}^{(i+1)} \\ & - z_2\frac{\partial\alpha_{1,j}}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 - \frac{1}{\lambda_0}\tilde{\theta}_2\dot{\hat{\theta}}_2 + \frac{z_2^2}{2} + \frac{\overline{h}_{2,j}^{*2}}{2} \\ & + \gamma_0{\lambda_0}^{-1}\tilde{\theta}_1\hat{\theta}_1 + \Delta_{1,j}^* + \frac{1}{2}z_2^2\left(\frac{\partial\alpha_{1,j}}{\partial x_1}\right)^2 + \frac{1}{2}\overline{h}_{1,j}^{*2}, \quad (29)\end{aligned}$$

approximation by neural network

$$\begin{aligned}F_{2,j}(Z_2) = & f_{2,j} - \frac{\partial\alpha_{1,j}}{\partial x_1}\left(x_2 + f_{1,j}\right) + \frac{z_2}{2\left(B_{m_{1,j}}^2 - z_1^2\right)} \\ & - \sum_{i=0}^{1}\frac{\partial\alpha_{1,j}}{\partial {y_d}^{(i)}}{y_d}^{(i+1)} - \frac{\partial\alpha_{1,j}}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 \\ & + \frac{1}{2}z_2\left(\frac{\partial\alpha_{1,j}}{\partial x_1}\right)^2 + \frac{1}{2}z_2, \quad (30)\end{aligned}$$

where $Z_2 = \left[x_1, x_2, \hat{\theta}_1, y_d, \dot{y}_d, y_d^{(2)}\right]^T$, $F_{2,j}$ can be expressed as $F_{2,j}(Z_2) = W_{2,j}^{*T} S_{2,j}(Z_2) + \delta_{2,j}(Z_2)$, $|\delta_{2,j}(Z_2)| \le \varepsilon_{2,j}$ with $\varepsilon_{2,j} > 0$, following inequalities exist

$$\begin{aligned}z_2 W_{2,j}^{*T} S_{2,j} &\le \frac{\theta_2 z_2^2 S_{2,j}^T S_{2j}}{2\tau_{2,j}^2} + \frac{1}{2}\tau_{2,j}^2 \\ z_2\delta_{2,j} &\le \frac{z_2^2}{2} + \frac{1}{2}\varepsilon_{2,j}^2, \quad (31)\end{aligned}$$

where $\theta_2 = \max\left\{\left\|W_{2,j}^*\right\|^2, j \in \sum\right\}$. Combining (29) (30) and (31) can get:

$$\begin{aligned}\dot{V}_{2,j} \leq & \frac{-c_{1j}{z_1}^2}{B_{m_{1,j}}^2 - z_1^2} + z_2\left(z_3 + \alpha_{2,j} + \frac{\theta_2 z_2 S_{2,j}^T S_{2,j}}{2\tau_{2,j}^2}\right) \\ & - \frac{1}{\lambda_0}\tilde{\theta}_2\dot{\hat{\theta}}_2 + \frac{\bar{h}_{2,j}^{*2}}{2} + \frac{1}{2}\bar{h}_{1,j}^{*2} + \frac{z_2^2}{2} \\ & + \frac{1}{2}\tau_{2,j}^2 + \frac{1}{2}\varepsilon_{2,j}^2 + \gamma_0\lambda_0^{-1}\tilde{\theta}_1\hat{\theta}_1 + \Delta_{1,j}^*, \end{aligned} \tag{32}$$

where $x_3{=}z_3 + \alpha_{2,j}$, then substituting the virtual control law $\alpha_{2,j} = -\left(c_{2,j} + \frac{1}{2}\right) z_2 - \frac{\hat{\theta}_2 z_2 S_{2,j}^T S_{2,j}}{2\tau_{2,j}^2}$ into (32), $c_{2,j}$ is a design positive parameter.

$$\begin{aligned}\dot{V}_{2,j} \leq & \frac{-c_{1,j}{z_1}^2}{B_{m_{1,j}}^2 - z_1^2} + z_2\left(z_3 - \left(c_{2,j} + \frac{1}{2}\right) z_2\right) \\ & + \frac{\tilde{\theta}_2 z_2^2 S_{2,j}^T S_{2,j}}{2\tau_{2,j}^2} - \frac{\tilde{\theta}_2\dot{\hat{\theta}}_2}{\lambda_0} + \frac{\gamma_0}{\lambda_0}\tilde{\theta}_1\hat{\theta}_1 \\ & + \frac{1}{2}\bar{h}_{1,j}^{*2} + \Delta_{1,j}^* + \Delta_{2,j}^* + \frac{z_2^2}{2}, \end{aligned} \tag{33}$$

where $\Delta_{2,j}^* = \frac{\bar{h}_{2,j}^{*2}}{2} + \frac{1}{2}\tau_{2,j}^2 + \frac{1}{2}\varepsilon_{2,j}^2$, substituting $\dot{\hat{\theta}}_2$ get:

$$\begin{aligned}\dot{V}_{2,j} \leq & \frac{-c_{1,j}{z_1}^2}{B_{m_{1,j}}^2 - z_1^2} - c_{2,j}z_2^2 + z_2 z_3 \\ & + \frac{\gamma_0}{\lambda_0}\tilde{\theta}_1\hat{\theta}_1 + \frac{\gamma_0}{\lambda_0}\tilde{\theta}_2\hat{\theta}_2 + \frac{1}{2}\bar{h}_{1,j}^{*2} + \Delta_{1,j}^* + \Delta_{2,j}^*. \end{aligned} \tag{34}$$

Step k ($k = 3, 4, \cdots, n-1$): According to the backstepping design method, we also introduce the coordinate transformation $z_k = x_k - \alpha_{k-1,j}$, we can obtain easily that:

$$\begin{aligned}\dot{z}_k = & x_{k+1} + f_{k,j} + h_{k,j} - \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1,j}}{\partial x_i}\left(x_{i+1} + f_{i,j} + h_{i,j}\right) \\ & - \sum_{i=0}^{k-1}\frac{\partial\alpha_{k-1,j}}{\partial y_d^{(i)}}y_d^{(i+1)} - \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1,j}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i. \end{aligned} \tag{35}$$

Select the following Lyapunov function:

$$V_{k,j} = V_{k-1,j} + \frac{1}{2}z_k^2 + \frac{1}{2\lambda_0}\tilde{\theta}_k^2, \tag{36}$$

the next, combining (35) and (36), it is not difficult to get:

$$\begin{aligned}\dot{V}_{k,j} = & \dot{V}_{k-1,j} + z_k\left(x_{k+1} + f_{k,j} + h_{k,j}\right) \\ & - z_k\sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1,j}}{\partial x_i}\left(x_{i+1} + f_{i,j} + h_{i,j}\right) \\ & - z_k\sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1,j}}{\partial y_d^{(i)}}y_d^{(i+1)} - z_k\sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1,j}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i, \end{aligned} \tag{37}$$

similar to the above steps:

$$\begin{aligned}\dot{V}_{k-1,j} \leq & \frac{-c_{1j}z_1^2}{B_{m_{1,j}}^2 - z_1^2} - \sum_{i=2}^{k-1}c_{i,j}z_i^2 + z_{k-1}z_k \\ & + \sum_{i=1}^{k-1}\frac{\gamma_0}{\lambda_0}\tilde{\theta}_i\hat{\theta}_i + \frac{1}{2}\sum_{i=1}^{k-2}\sum_{l=1}^{i}\bar{h}_{l,j}^{*2} + \sum_{i=1}^{k-1}\Delta_{i,j}^*, \end{aligned} \tag{38}$$

where $\Delta_{k-1,j}^* = \frac{1}{2}\bar{h}_{k-1,j}^{*2} + \frac{1}{2}\tau_{k-1,j}^2 + \frac{1}{2}\varepsilon_{k-1,j}^2$. Then some necessary inequalities are introduced:

$$\begin{aligned} z_k h_{k,j} \leq & \frac{z_k^2}{2} + \frac{1}{2}\bar{h}_{k,j}^{*2} \\ -z_k\sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1,j}}{\partial x_i}h_{i,j} \leq & \frac{z_k^2}{2}\sum_{i=1}^{k-1}\left(\frac{\partial\alpha_{k-1,j}}{\partial x_i}\right)^2 + \frac{1}{2}\sum_{i=1}^{k-1}\bar{h}_{i,j}^{*2}. \end{aligned} \tag{39}$$

We can easily get the following expression

$$\begin{aligned}\dot{V}_{k,j} \leq & \frac{-c_{1,j}z_1^2}{B_{m_{1,j}}^2 - z_1^2} - \sum_{i=2}^{k-1}c_{i,j}z_i^2 + z_{k-1}z_k \\ & + \sum_{i=1}^{k-1}\frac{\gamma_0}{\lambda_0}\tilde{\theta}_i\hat{\theta}_i + \frac{1}{2}\sum_{i=1}^{k-2}\sum_{l=1}^{i}\bar{h}_{l,j}^{*2} + \sum_{i=1}^{k-1}\Delta_{i,j}^* \\ & + z_k\left(x_{k+1} + f_{k,j}\right) - \lambda_0^{-1}\tilde{\theta}_k\dot{\hat{\theta}}_k \\ & - z_k\sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1,j}}{\partial x_i}\left(x_{i+1} + f_{i,j}\right) \\ & - z_k\sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1,j}}{\partial y_d^{(i)}}y_d^{(i+1)} - z_k\sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1,j}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i \\ & + \frac{z_k^2}{2} + \frac{1}{2}\bar{h}_{k,j}^{*2} + \frac{z_k^2}{2}\sum_{i=1}^{k-1}\left(\frac{\partial\alpha_{k-1,j}}{\partial x_i}\right)^2 + \frac{1}{2}\sum_{i=1}^{k-1}\bar{h}_{i,j}^{*2}. \end{aligned} \tag{40}$$

For the analysis of the above inequalities, we use a suitable RBF neural network to approximate some of the above nonlinear functions

$$\begin{aligned} F_{k,j}\left(Z_k\right) = & f_{k,j} + \frac{z_k}{2} - \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1,j}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i + z_{k-1} \\ & - \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1,j}}{\partial x_i}\left(x_{i+1} + f_{i,j}\right) - \sum_{i=0}^{k-1}\frac{\partial\alpha_{k-1,j}}{\partial y_d^{(i)}}y_d^{(i+1)} \\ & + \frac{1}{2}z_k\sum_{i=1}^{k-1}\left(\frac{\partial\alpha_{k-1,j}}{\partial x_i}\right)^2, \end{aligned} \tag{41}$$

with $Z_k = \left[\bar{x}_k^T, \hat{\theta}_1, \cdots\hat{\theta}_{k-1}, \underline{y_d}^{(k)T}\right]^T$, and $\underline{y_d}^{(k)} = \left[y_d, \dot{y}_d, \cdots, y_d^{(k)}\right]^T$. Consequently, $F_{k,j}\left(Z_k\right)$ can be expressed as $F_{k,j}\left(Z_k\right) = W_{k,j}^{*T}S_{k,j}\left(Z_k\right) + \delta_{k,j}\left(Z_k\right)$, where the approximation error $\delta_{k,j}\left(Z_k\right)$ meets $\left|\delta_{k,j}\left(Z_k\right)\right| \leq \varepsilon_{k,j}$ with $\varepsilon_{k,j} > 0$ being the upper boundary of approximation error, by using the inequalities:

$$\begin{aligned} z_k W_{k,j}^{*T}S_{k,j} \leq & \frac{\theta_k z_k^2 S_{k,j}^T S_{k,j}}{2\tau_{k,j}^2} + \frac{1}{2}\tau_{k,j}^2 \\ z_k\delta_{k,j} \leq & \frac{z_k^2}{2} + \frac{1}{2}\varepsilon_{k,j}^2, \end{aligned} \tag{42}$$

where $\theta_k = \max\left\{\left\|W_{k,j}^*\right\|^2, j \in \sum\right\}$. Combining (40), (41) and (42), we obtain:

$$\dot{V}_{k,j} \leq \frac{-c_{1,j}z_1^2}{B_{m_{1,j}}^2 - z_1^2} - \sum_{i=2}^{k-1} c_{i,j}z_i^2 + \frac{z_k^2}{2} + z_k \times \left(z_{k+1} + \alpha_{k,j} + \frac{\theta_k z_k S_{k,j}^T S_{k,j}}{2\tau_{k,j}^2} \right) + \sum_{i=1}^{k-1} \frac{\gamma_0}{\lambda_0} \tilde{\theta}_i \hat{\theta}_i - \lambda_0^{-1} \tilde{\theta}_k \dot{\hat{\theta}}_k + \sum_{i=1}^{k} \Delta_{i,j}^* + \frac{1}{2} \sum\nolimits_{i=1}^{k-1} \sum\nolimits_{l=1}^{i} \bar{h}_{l,j}^{*2}. \quad (43)$$

Design the following virtual control law

$$\alpha_{k,j} = -\left(c_{k,j} + \frac{1}{2} \right) z_k - \hat{\theta}_k z_k S_{k,j}^T S_{k,j} \Big/ 2\tau_{k,j}^2, \quad (44)$$

where $c_{k,j} > 0$ is the design positive parameter. In the same way, substituting $\alpha_{k,j}$, $\dot{\hat{\theta}}_k$, it is not difficult to get

$$\dot{V}_{k,j} \leq \frac{-c_{1,j}z_1^2}{B_{m_{1,j}}^2 - z_1^2} - \sum_{i=2}^{k} c_{i,j}z_i^2 + z_k z_{k+1} + \sum_{i=1}^{k} \frac{\gamma_0}{\lambda_0} \tilde{\theta}_i \hat{\theta}_i + \sum_{i=1}^{k} \Delta_{i,j}^* + \frac{1}{2} \sum\nolimits_{i=1}^{k-1} \sum\nolimits_{l=1}^{i} \bar{h}_{l,j}^{*2}, \quad (45)$$

where $\Delta_{k,j}^* = \frac{1}{2}\bar{h}_{k,j}^{*2} + \frac{1}{2}\tau_{k,j}^2 + \frac{1}{2}\varepsilon_{k,j}^2$.

Step n: Also, by introducing the estimated error equation $z_n = x_n - \alpha_{n-1,j}$, there is the following expression

$$\dot{z}_n = u_\sigma + f_{n,\sigma}(x) + h_{n,\sigma} - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial x_i} (x_{i+1} + f_{i,j} + h_{i,j}) - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial y_d^{(i)}} y_d^{(i+1)} - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i, \quad (46)$$

then combining (13) can obtain

$$\dot{z}_n = r_j m_j + \eta_j(x) + p_j(m_j) + h_{n,j} - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial x_i} (x_{i+1} + f_{i,j} + h_{i,j}) - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial y_d^{(i)}} y_d^{(i+1)} - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i. \quad (47)$$

Design a positive definite Lyapunov function:

$$V_{n,j} = V_{n-1,j} + \frac{1}{2} z_n^2 + \frac{\tilde{\theta}_n^2}{2\lambda_0}, \quad (48)$$

where γ_0, λ_0 are the design positive parameters, $|p_j(m_j)| \leq p$. (45) is the derivative of time, and then combining (47) can get

$$\dot{V}_{n,j} \leq \frac{-c_{1,j}z_1^2}{B_{m_{1,j}}^2 - z_1^2} - \sum_{i=2}^{n-1} c_{i,j}z_i^2 + z_{n-1}z_n + \sum_{i=1}^{n-1} \frac{\gamma_0}{\lambda_0} \tilde{\theta}_i \hat{\theta}_i + \frac{1}{2} \sum_{i=1}^{n-2} \sum_{l=1}^{i} \bar{h}_{l,j}^{*2} + \sum_{i=1}^{n-1} \Delta_{i,j}^* - \frac{\tilde{\theta}_n \dot{\hat{\theta}}_n}{\lambda_0} + z_n (r_j m_j + \eta_j(x) + p_j(m_j) + h_{n,j}) - z_n \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial x_i} (x_{i+1} + f_{i,j} + h_{i,j}) - z_n \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial y_d^{(i)}} y_d^{(i+1)} - z_n \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i. \quad (49)$$

Use some inequalities

$$z_n h_{n,j} \leq \frac{1}{2} z_n^2 + \frac{1}{2} \bar{h}_{n,j}^{*2}$$

$$-z_n \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial x_i} h_{i,j} \leq \frac{1}{2} z_n^2 \sum_{i=1}^{n-1} \left(\frac{\partial \alpha_{n-1,j}}{\partial x_i} \right)^2 + \frac{1}{2} \sum_{i=1}^{n-1} \bar{h}_{i,j}^{*2}$$

$$z_n p(m_j) \leq \frac{1}{2} {z_n}^2 + \frac{1}{2} p^2.$$

Using this neural network:

$$F_{n,j}(Z_n) = \eta_j(x) - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial x_i} (x_{i+1} + f_{i,j}) - \left(\sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial y_d^{(i)}} y_d^{(i+1)} + \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1,j}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i \right) + z_n + z_{n-1} + \frac{1}{2} z_n \sum_{i=1}^{n-1} \left(\frac{\partial \alpha_{n-1,j}}{\partial x_i} \right)^2, \quad (50)$$

the overall approximation form is

$$F_{n,j}(Z_n) = W_{n,j}^{*T} S_{n,j}(Z_n) + \sigma_{n,j}(Z_n), \quad (51)$$

where $Z_n = \left[x^T, \hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_n, \underline{y_d}^{(n)T} \right]^T$ and $\underline{y_d}^{(n)} = \left[y_d, \dot{y}_d, \cdots, y_d^{(n)} \right]^T$, then this error $\sigma_{nj}(Z_n)$ of neural network function approximation satisfies $|\sigma_{n,j}(Z_n)| \leq \varepsilon_{n,j}$, where $\varepsilon_{n,j}$ is the upper bound of this approximation error and $\varepsilon_{n,j} > 0$.

Remark 4. As in Step 1 and Step k above, for the nonlinear functions in the system, the approximation scheme is adopted when designing the controller.

In the same analytical way, using the inequalities

$$z_n W_{n,j}^{*T} S_{n,j} \leq \frac{\theta_n z_n^2 S_{n,j}^T S_{n,j}}{2\tau_{n,j}^2} + \frac{1}{2} \tau_{n,j}^2$$

$$z_n \sigma_{n,j} \leq \frac{1}{2} {z_n}^2 + \frac{1}{2} \varepsilon_{n,j}^2, \quad (52)$$

where $\theta_n = \max\left\{ \left\| W_{n,j}^* \right\|^2, j \in \sum \right\}$. By substituting (52) can obtain:

$$\dot{V}_{n,j} \leq \frac{-c_{1,j}z_1^2}{B_{m_{1,j}}^2 - z_1^2} - \sum_{i=2}^{n-1} c_{i,j}z_i^2 + z_n \left(r_j m_j + \frac{\theta_n z_n S_{n,j}^T S_{n,j}}{2\tau_{n,j}^2} \right) + \sum_{i=1}^{n-1} \frac{\gamma_0}{\lambda_0} \tilde{\theta}_i \hat{\theta}_i - \frac{1}{\lambda_0} \tilde{\theta}_n \dot{\hat{\theta}}_n + \frac{1}{2} {z_n}^2 + \frac{1}{2} \sum_{i=1}^{n-1} \sum_{l=1}^{i} \bar{h}_{l,j}^{*2} + \sum_{i=1}^{n} \Delta_{i,j}^* + \frac{1}{2} p^2, \quad (53)$$

select the actual control input:

$$m_j = \frac{1}{r_j} \left[-\left(c_{n,j} + \frac{1}{2} \right) z_n - \frac{\hat{\theta}_n z_n S_{n,j}^T S_{n,j}}{2\tau_{n,j}^2} \right], \quad (54)$$

where the $c_{n,j}$ and r_j are the design positive parameters. Substituting (54) into (53):

$$\begin{aligned}\dot{V}_{n,j} \leq & \frac{-c_{1,j}z_1^2}{B_{m_{1,j}}^2 - z_1^2} - \sum_{i=2}^{n} c_{i,j}z_i^2 \\ & + \frac{\tilde{\theta}_n z_n^2 S_{n,j}^T S_{n,j}}{2\tau_{n,j}^2} + \sum_{i=1}^{n-1} \frac{\gamma_0}{\lambda_0}\tilde{\theta}_i\hat{\theta}_i - \frac{1}{\lambda_0}\tilde{\theta}_n\dot{\hat{\theta}}_n \\ & + \frac{1}{2}\sum_{i=1}^{n-1}\sum_{l=1}^{i}\bar{h}_{l,j}^{*2} + \sum_{i=1}^{n}\Delta_{i,j}^* + \frac{1}{2}p^2, \end{aligned} \quad (55)$$

substituting $\dot{\hat{\theta}}_n$ and inequality $\tilde{\theta}\hat{\theta} = -\tilde{\theta}^2 + \theta\tilde{\theta} \leq -\frac{1}{2}\tilde{\theta}^2 + \frac{1}{2}\theta^2$, it is not difficult to obtain

$$\begin{aligned}\dot{V}_{n,j} \leq & -\frac{c_{1,j}z_1^2}{B_{m_{1,j}}^2 - z_1^2} - \sum_{i=2}^{n} c_{i,j}z_i^2 \\ & - \frac{1}{2}\sum_{i=1}^{n}\lambda_0^{-1}\gamma_0\tilde{\theta}_i^2 + \nu_0, \end{aligned} \quad (56)$$

where $\nu_0 = \frac{1}{2}\sum\limits_{i=1}^{n}\lambda_0^{-1}\gamma_0\theta_i^2 + \frac{1}{2}\sum\limits_{i=1}^{n-1}\sum\limits_{l=1}^{i}\bar{h}_{l,j}^{*2} + \sum\limits_{i=1}^{n}\Delta_{i,j}^* + \frac{1}{2}p^2$. Furthermore, according to Lemma 2:

$$\log\left(\frac{B_{m_{1,j}}^2}{B_{m_{1,j}}^2 - z_1^2}\right) \leq \frac{z_1^2}{B_{m_{1,j}}^2 - z_1^2},$$

where $j = 1, 2, \cdots, M$, we can get that

$$\dot{V}_{n,j} \leq -\mu_0 V_{n,j} + \nu_0, \quad (57)$$

where $\mu_0 = \min\{2c_{i,j}, \gamma_0 \quad j = 1, 2, \cdots, M\}$, $i = 1, 2, \cdots, n$.

IV. Stability analysis

Theorem 1. Aiming at the output constraint of uncertain switched nonlinear systems, the average dwell time $\tau_a > \frac{\log\rho}{\mu_0}$ is given in combination with Definition 1, the design of adaptive neural network controller (16), (23), $\alpha_{2,j}$, (44), (54) need to ensure that all signals in the system are bounded under the condition of average dwell time. From the above conclusions and conditions, it can be concluded that the tracking error of the system will also converge to a limited compact set and the system output satisfies the constraint conditions.

In this section, it is necessary to prove that the system output meets the constraint and the final tracking error will converge to a compact set, then the following proofs are given:

$$\rho = \frac{B_{m_{1M}}^2}{B_{m_{1m}}^2 - B_{m_{10}}^{*2}}, \quad (58)$$

where $B_{m_{1M}} = \max\limits_{1\leq j\leq M}\{B_{m_{1,j}}\}$, $B_{m_{1m}} = \min\limits_{1\leq j\leq M}\{B_{m_{1,j}}\}$, then $|z_1| \leq B_{m_{10}}^* < B_{m_{1m}}$ with $B_{m_{10}}^* > 0$; initial condition: the initial value $x_1(0)$ of the output variable is bounded and satisfies $\{x_1 \,||x_1(0)| \leq K_{c1}\}$, with $K_{c1} = \min\limits_{1\leq j\leq M}\{K_{c1,j}\}$.

proof. Combining equations (18), (26), (36) and (48) we can get

$$V_{n,j} = \frac{1}{2}\log\left(\frac{B_{m_{1,j}}^2}{B_{m_{1,j}}^2 - z_1^2}\right) + \frac{1}{2}\sum_{i=2}^{n} z_n^2 + \sum_{i=1}^{n}\frac{\tilde{\theta}_i^2}{2\lambda_0}, \quad (59)$$

with Lemma 2 and (58), the equation (59) becomes

$$\begin{aligned} V_{n,j} & \leq \frac{1}{2}\frac{z_1^2}{B_{m_{1,j}}^2 - z_1^2} + \frac{1}{2}\sum_{i=2}^{n} z_i^2 + \sum_{i=1}^{n}\frac{1}{2\lambda_0}\tilde{\theta}_i^2 \\ & \leq \frac{1}{2}\frac{B_{m_{1,l}}^2}{B_{m_{1m}}^2 - B_{m_{10}}^{*2}}\frac{z_1^2}{B_{m_{1,l}}^2} + \frac{1}{2}\sum_{i=2}^{n} z_n^2 + \sum_{i=1}^{n}\frac{1}{2\lambda_0}\tilde{\theta}_i^2 \\ & \leq \rho\left(\frac{1}{2}\frac{z_1^2}{B_{m_{1,l}}^2} + \frac{1}{2}\sum_{i=2}^{n} z_i^2 + \sum_{i=1}^{n}\frac{1}{2\lambda_0}\tilde{\theta}_i^2\right) \\ & \leq \rho\left(\frac{1}{2}\log\left(\frac{B_{m_{1,l}}^2}{B_{m_{1,l}}^2 - z_1^2}\right) + \frac{1}{2}\sum_{i=2}^{n} z_i^2 + \sum_{i=1}^{n}\frac{1}{2\lambda_0}\tilde{\theta}_i^2\right) \\ & \leq \rho V_{n,l}, \end{aligned} \quad (60)$$

for any $k, l \in \sum$. Then there are such functions $\overline{\alpha}$, $\underline{\alpha} \in \kappa_\infty$, the following inequality $\underline{\alpha}(\|X\|) \leq V_{n,j}(X) \leq \overline{\alpha}(\|X\|)$ exists, where $X = \left[z_1, z_2, \cdots, z_n, \tilde{\theta}_1, \tilde{\theta}_2, \cdots \tilde{\theta}_n\right]^T$. Creating an auxiliary function $L(t) = e^{\mu_0 t}V_{n,\sigma(t)}(X(t))$ which is a segmented differentiable function, one each interval $[t_s, t_{s+1})$, according to (57) can get

$$\begin{aligned}\dot{L}(t) &= \mu_0 e^{\mu_0 t}V_{n,\sigma}(X(t)) + e^{\mu_0 t}\dot{V}_{n,\sigma}(X(t)) \\ &\leq \nu_0 e^{\mu_0 t}, \qquad t \in [t_s, t_{s+1}). \end{aligned} \quad (61)$$

Combining (60), (61) can easily obtain

$$\begin{aligned} L(t_{s+1}) &= e^{\mu_0 t_{s+1}}V_{n,\sigma(t_{s+1})}(X(t_{s+1})) \\ &\leq \rho e^{\mu_0 t_{s+1}}V_{n,\sigma(t_s)}(X(t_{s+1})) = \rho L(t_{s+1}^-) \\ &\leq \rho\left[L(t_s) + \int_{t_s}^{t_{s+1}}\nu_0 e^{\mu_0 t}dt\right]. \end{aligned} \quad (62)$$

With arbitrary $T > t_0$ and $t_0 = 0$, there is inequality (62) from $s = 0$ to $s = N_\sigma(T, 0) - 1$, yields

$$\begin{aligned} L(T^-) &\leq L\left(t_{N_{\sigma(T,0)}}\right) + \int_{t_{N_{\sigma(T,0)}}}^{T}\nu_0 e^{\mu_0 t}dt \\ &\leq \rho\left[L\left(t_{N_{\sigma(T,0)}-1}\right) + \int_{t_{N_{\sigma(T,0)}-1}}^{t_{N_{\sigma(T,0)}}}\nu_0 e^{\mu_0 t}dt \right. \\ &\left. + \rho^{-1}\int_{t_{N_{\sigma(T,0)}}}^{T}\nu_0 e^{\mu_0 t}dt\right] \\ &\leq \cdots \\ &\leq \rho^{N_{\sigma(T,0)}}\left[L(0) + \sum_{s=0}^{N_{\sigma(T,0)}-1}\rho^{-s}\int_{t_s}^{t_{s+1}}\nu_0 e^{\mu_0 t}dt \right. \\ &\left. + \rho^{-N_{\sigma(T,0)}}\int_{t_{N_{\sigma(T,0)}}}^{T}\nu_0 e^{\mu_0 t}dt\right]. \end{aligned} \quad (63)$$

It is worth emphasizing that $\tau_a > \frac{\log\rho}{\mu_0}$, for any $o \in \left(0, \mu_0 - \frac{\log\rho}{\tau_a}\right)$, there is $\tau_a > [(\log\rho)/(\mu_0 - o)]$, according to (4) can get

$$N_\sigma(T, t) \leq N_0 + \frac{(\mu_0 - o)(T - t)}{\log\rho} \quad \forall T \geq t \geq 0, \quad (64)$$

since this inequality $N_\sigma(T, 0) - s \leq 1 + N_\sigma(T, t_{s+1})$, $s = 0, 1, \cdots, N_\sigma(T, 0)$, there is the following formula

$$\rho^{N_\sigma(T,0)-s} \leq \rho^{1+N_0}e^{(\mu_0 - o)(T - t_{s+1})}. \quad (65)$$

In addition, since $o < \mu_0$, we have

$$\int_{t_s}^{t_{s+1}} \nu_0 e^{\mu_0 t} dt \leq e^{(\mu_0 - o)t_{s+1}} \int_{t_s}^{t_{s+1}} \nu_0 e^{ot} dt, \tag{66}$$

then it follows (65) and (66), the following formula is get

$$L\left(T^-\right) \leq \rho^{N_{\sigma(T,0)}} L\left(0\right) + \rho^{1+N_0} e^{(\mu_0 - o)T} \int_0^T \nu_0 e^{ot} dt. \tag{67}$$

Which indicates that

$$\begin{aligned} \underline{\alpha}\left(\|X\left(T\right)\|\right) &\leq V_{n,\sigma(T^-)}\left(X\left(T^-\right)\right) \\ &\leq e^{N_0 \log \rho} e^{\left(\frac{\log \rho}{\tau_a} - \mu_0\right)T} \overline{\alpha}\left(\|X\left(0\right)\|\right) \\ &+ \rho^{1+N_0} \frac{\nu_0}{o}\left(1 - e^{-oT}\right) \\ &\leq e^{N_0 \log \rho} e^{\left(\frac{\log \rho}{\tau_a} - \mu_0\right)T} \overline{\alpha}\left(\|X\left(0\right)\|\right) \\ &+ \rho^{1+N_0} \frac{\nu_0}{o} \quad \forall T > 0. \end{aligned} \tag{68}$$

By observing the inequality (68), it is not difficult to conclude that the error z_k in this system $k = 1, 2, \cdots, n$ and $\left\{\tilde{\theta}_1, \tilde{\theta}_2, \cdots \tilde{\theta}_n\right\}$ in the closed loop system (1) are bounded in the condition of satisfying the average dwell time $\tau_a > \frac{\log \rho}{\mu_0}$. Judging from the definition and assumptions at the beginning of the article θ_i are constants, so $\hat{\theta}_i$ are bounded $i = 1, 2, \cdots, n$, then according to (14) $\alpha_{k-1,j}$ $(k = 2, 3, \cdots, n)$ and y_d are bounded, so x_i, $i = 1, 2, \cdots, n$ are bounded. Therefore, all signals in the system are bounded under the condition of switching signals.

Another aspect, due to the $|z_1| \leq B^*_{m_{1,j}} < B_{m_{1,j}}$ and $|y_d| \leq \bar{y}_d$, they are bounded; so x_1 is bounded, the next, $|y| = |x_1| \leq |z_1| + |y_d| \leq B^*_{m_{1,j}} + \bar{y}_d = K_{c1,j}$, $|x_1| \leq K_{c1}$, $K_{c1} = \min\limits_{1 \leq j \leq M} \{K_{c1,j}\}$, output constraint condition is proved.

Using the above inequalities (59) and (68), with the average dwell time condition $\tau_a > (\log \rho)/\mu_0$, we can obtain

$$\frac{1}{2} \log\left(\frac{B^2_{m_{1,j}}}{B^2_{m_{1,j}} - z_1^2}\right) \leq e^{N_0 \log \rho} \overline{\alpha}\left(\|X\left(0\right)\|\right) + \rho^{1+N_0} \frac{\nu_0}{o}, \tag{69}$$

it follows that:

$$|z_1| \leq B_{m_{1,j}} \sqrt{1 - e^{-2\left(e^{N_0 \log \rho} \overline{\alpha}(\|X(0)\|) + \rho^{1+N_0} \frac{\nu_0}{o}\right)}}, \tag{70}$$

according to the condition of (58), we further obtain that tracking error z_1 will converge to a suitable set

$$\xi_1 = \left\{|z_1| \leq B_{m_{1m}} \sqrt{1 - e^{-2\left(e^{N_0 \log \rho} \overline{\alpha}(\|X(0)\|) + \rho^{1+N_0} \frac{\nu_0}{o}\right)}}\right\}. \tag{71}$$

It is proved that the tracking error converges to a compact set.

Remark 5. It is noted here that the system in this paper has the input dead zone and saturation, and by means of equations (9) and (10), the input is replaced by a smooth function. Lemma 1 needs to be used when constructing neural network adaptive controller. When dealing with the output constraint problem, prove that the output is within the boundary by using Assumption 2 and Lemma 2. Then by Definition 1 and Lemma 2, this new constraint rule $\tau_a > \frac{\log \rho}{\mu_0}$ is designed for system (1). Finally, it is shown by stability analysis that the tracking error converges to a very small set.

V. Example results

After selecting appropriate parameters, the feasibility of the control scheme is verified by simulation example.

Example A second-order uncertain switched nonlinear system with input constraint and output constraint is given

$$\begin{cases} \dot{x}_1 = x_2 + f_{1,\sigma}\left(x_1\right) + h_{1,\sigma}\left(t\right) \\ \dot{x}_2 = u_\sigma + f_{2,\sigma}\left(\bar{x}_2\right) + h_{2,\sigma}\left(t\right) \\ y = x_1, \end{cases} \tag{72}$$

where x_1 and x_2 are the state variables, then the input dead zone and saturation functions are given.

$$u_1 = \begin{cases} -5 & m_1 \leq -5 \\ 5(m_1 + 1)/4 & -5 < m_1 \leq -1 \\ 0 & -1 < m_1 \leq 2 \\ 6(m_1 - 2)/4 & 2 < m_1 \leq 6 \\ 6 & m_1 > 6, \end{cases} \tag{73}$$

$$u_2 = \begin{cases} -3 & m_2 \leq -3 \\ 3(m_2 + 1.5)/1.5 & -3 < m_2 \leq -1.5 \\ 0 & -1.5 < m_2 \leq 1.5 \\ 4(m_2 - 1.5)/2.5 & 1.5 < m_1 \leq 4 \\ 4 & m_2 > 4, \end{cases} \tag{74}$$

$\sigma = \{1, 2\}$, the nonlinear functions are chosen as $f_{1,1}\left(x_1\right) = x_1 \sin\left(x_1\right)$, $f_{2,1}\left(x_1, x_2\right) = 2\left(x_2 + x_1^2\right)$, $f_{1,2}\left(x_1\right) = x_1 \cos\left(2x_1\right)$, $f_{2,2}\left(x_1, x_2\right) = x_1^2 + x_2^2$, then $h_{1,1}\left(t\right) = 0.03 \cos\left(t\right)$, $h_{2,1}\left(t\right) = -0.03 \sin\left(t\right)$, $h_{1,2}\left(t\right) = 0.01 \sin\left(t\right)$, $h_{2,2}\left(t\right) = 0.02 \cos\left(t\right)$, y_d is given: $y_d = 0.2 \sin\left(t\right)$. Based on the (16) (23) and (54), we can obtain

$$\begin{aligned} \dot{\hat{\theta}}_1 &= \frac{\lambda_0 z_1^2 S^T_{1,j}\left(Z_1\right) S_{1,j}\left(Z_1\right)}{2\tau^2_{1,j}\left(B^2_{m_{1,j}} - z_1^2\right)^2} - \gamma_0 \hat{\theta}_1 \\ \dot{\hat{\theta}}_2 &= \frac{\lambda_0 z_2^2 S^T_{2,j}\left(Z_2\right) S_{2,j}\left(Z_2\right)}{2\tau^2_{2,j}} - \gamma_0 \hat{\theta}_2 \\ \alpha_{1,j} &= -\left(c_{1,j} + \frac{1}{2}\right) z_1 - \frac{\hat{\theta}_1 z_1 S^T_{1,j} S_{1,j}}{2\tau^2_{1,j}\left(B^2_{m_{1,j}} - z_1^2\right)} \\ m_j &= \frac{1}{r_j}\left[-\left(c_{n,j} + \frac{1}{2}\right) z_n - \frac{\hat{\theta}_n z_n S^T_{n,j} S_{n,j}}{2\tau^2_{n,j}}\right], \end{aligned} \tag{75}$$

in the above equations, the appropriate parameter values are selected, $c_{1,1} = 17$, $c_{1,2} = 18$, $\tau_{1,1} = 1$, $\tau_{1,2} = 2$, $B_{m1,1} = 1.3$, $B_{m1,2} = 0.5$, $\lambda_0 = 0.5$, $\gamma_0 = 0.5$, $c_{2,1} = 8$, $c_{2,2} = 9$, $r_1 = 1$, $r_2 = 1$, $\tau_{2,1} = 3$, $\tau_{2,2} = 4$, the initial value of $\hat{\theta}_1$ and $\hat{\theta}_2$ are 0, $x_1\left(0\right) = 0.2$, $x_2\left(0\right) = 0.3$; $|y_d| \leq 0.2$, the output constraint is chosen as $|x_1| \leq 0.5$, $\mu_0 = 0.5$ and select the value of $B^*_{m_{10}}$:$B^*_{m_{10}} = 0.3$. The average dwell time $\tau_a > [\log(10.5625)]/0.5 = 4.7146$.

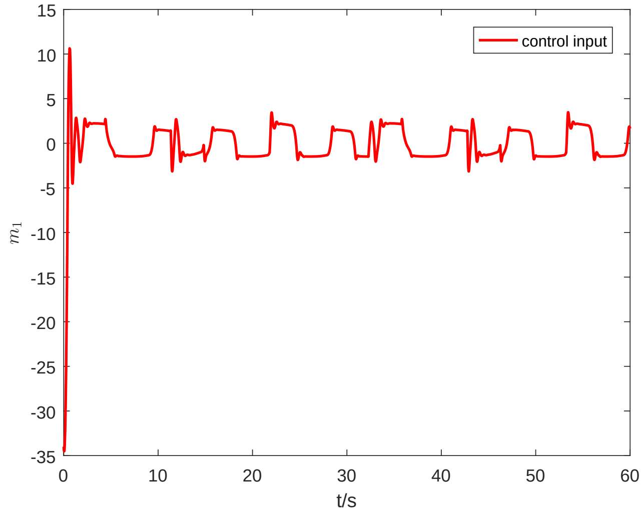

Fig. 2. Control input m_1 of **Example**.

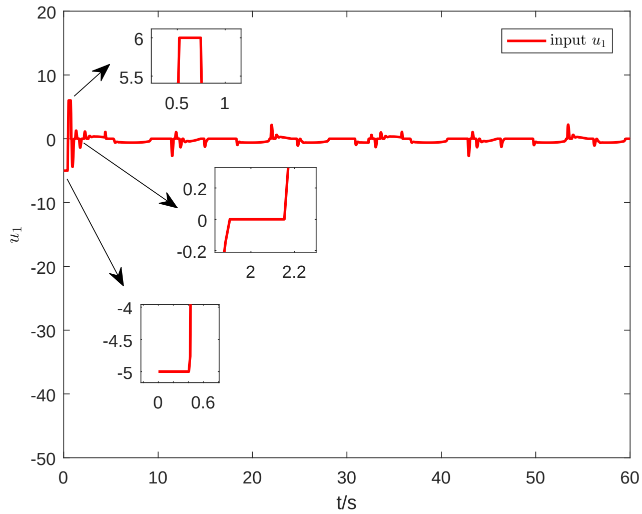

Fig. 3. Input dead zone and saturation function u_1.

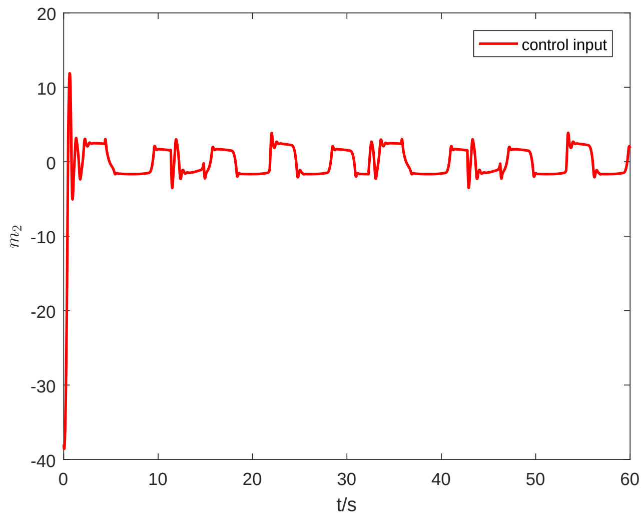

Fig. 4. Control input m_2 of **Example**.

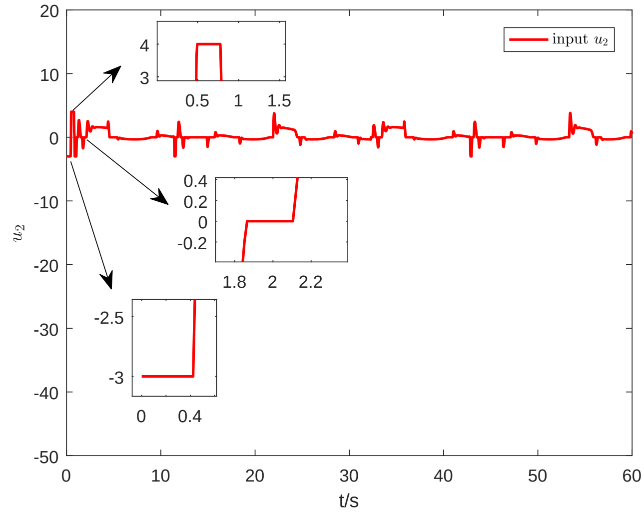

Fig. 5. Input dead zone and saturation function u_2.

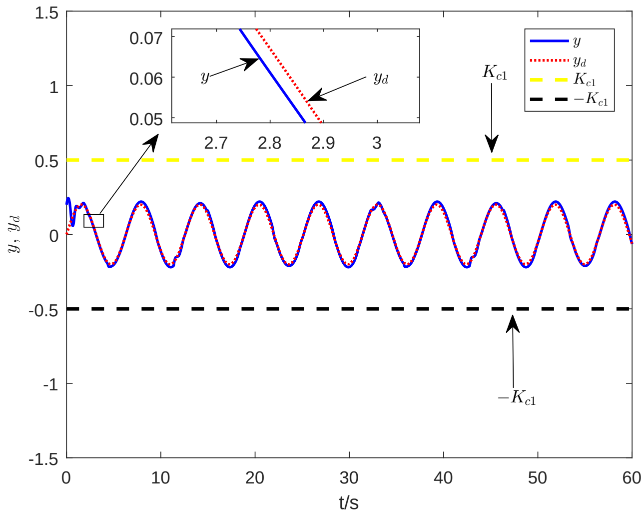

Fig. 6. Output y and y_d.

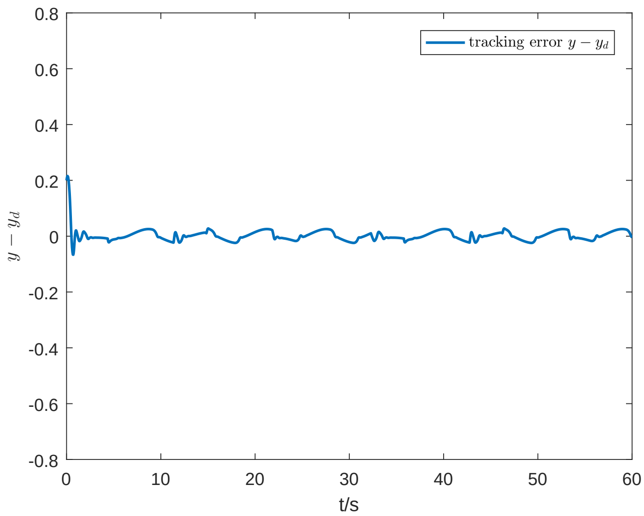

Fig. 7. Tracking error of **Example**.

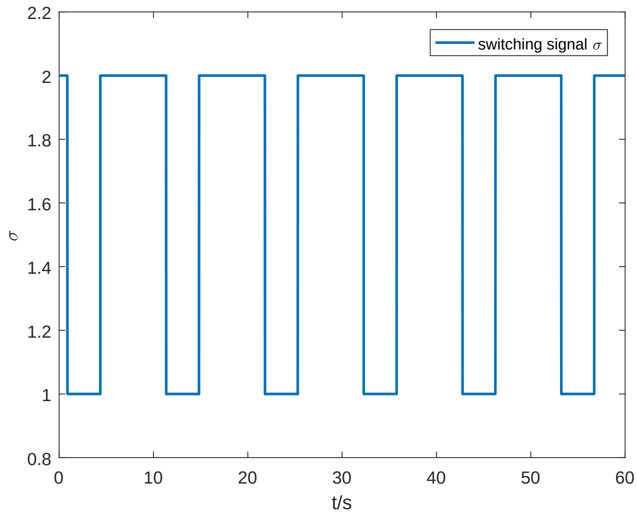

Fig. 8. Switching signal σ.

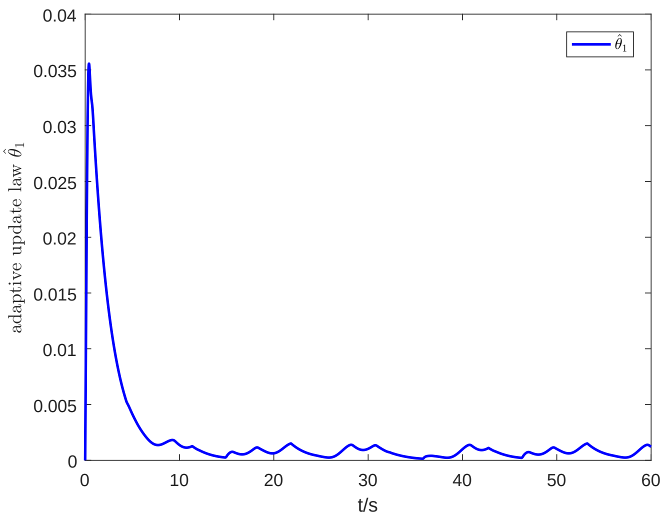

Fig. 9. Adaptive update law 1 $\hat{\theta}_1$ of **Example**.

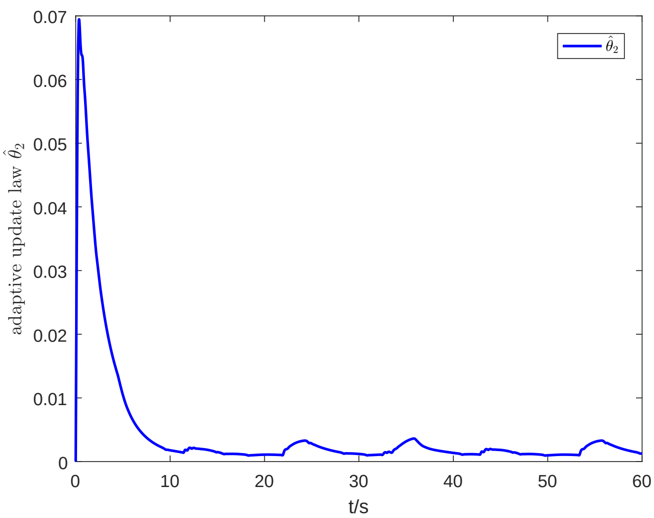

Fig. 10. Adaptive update law 2 $\hat{\theta}_2$ of **Example**.

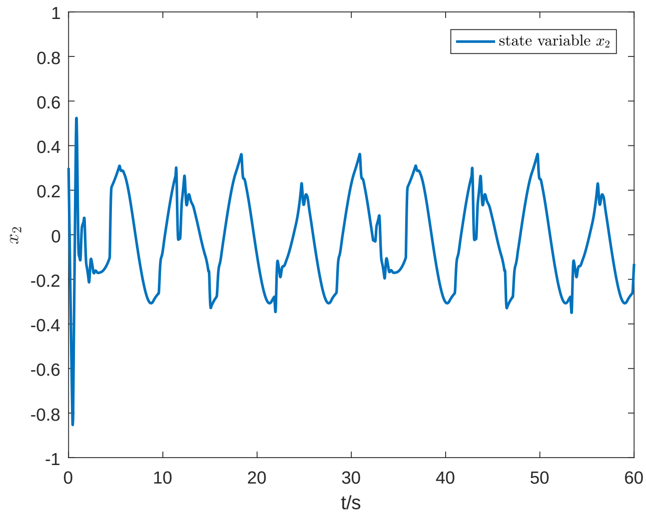

Fig. 11. System state x_2.

In the above simulation graphs, the graphs of the state variables and the output of the system as a function of time can be seen, the above graphs are more based on the second-order system and the given parameters simulated. In Figs. 2 and 4, the control input m_1 of subsystem 1 and the control input m_2 of subsystem 2 are shown, respectively. The input dead zone and saturation functions of the subsystem 1 and subsystem 2 are described in the Figs. 3 and 5, respectively, the dead zone and saturation of the input function can be seen in the figures. From Fig. 6, we can see the output signal and reference signal, the system output is constrained within the boundary. About Fig. 7, the tracking error varies around 0. The square switching signal is described in Fig. 8. The adaptive update law $\hat{\theta}_1$ and $\hat{\theta}_2$ are depicted in Fig. 9 and 10.

VI. Conclusion

The neural network adaptive control problem for a class of uncertain switched nonlinear systems with input and output constraints is studied in this paper, input constraint and output constraint are added to make system more complicated, because the given input dead zone and saturation function is a piecewise function, it is not easy to handle in the process of designing the controller, using the smooth function to approximate the input dead zone and saturation function to solve this problem. The adaptive controller is designed based on the neural network and adaptive back-stepping method, because of the existence of the output constraint, the barrier Lyapunov function is used. In the stability analysis, the average dwell time method is used and inequality $N_\sigma(T,t) \leq N_0 + \frac{T-t}{\tau_a}$ is used to limit the number of switching in the uncertain switched nonlinear system. Finally, the system output constraint is proved and the system tracking error converges to a compact set.

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(1)Date of modification: November 28, 2023.

(2)Brief description of the changes 1.Fixed some grammatical errors. 2.Assumption 3 was modified and a sentence was added. 3.Modified the $\alpha_{1,j}$, $c_{2,1}$, $c_{2,2}$, the initial value of $\hat{\theta}_1$ and $\hat{\theta}_2$. Modified the simulation figures.