Fixed-Time Fuzzy Adaptive Control for Switched Nonlinear Systems with Input Constraints based on Event-Triggered

Jiangnan Zhao, Xinyu Ouyang, Nannan Zhao and Yaobang Zang

***Abstract*—A fixed-time fuzzy adaptive event-triggered control strategy is proposed for a class of strict-feedback uncertain switched nonlinear systems with input saturation. Firstly, the unknown nonlinear functions in the system are approximated by fuzzy logic systems, and the unknown states of the system are estimated by using a linear state observer. This eliminates the limitation that states must be measured and functions must be known during the controller design process. Secondly, the event-triggered control is introduced to dynamically compensate for the saturated input of the system, which reduces the updating frequency of the control signal and effectively saves the communication resources of the system. And only one adaptive parameter is needed in the design, which not only reduces the computational burden, but also solves the problem of excessive parameterization. Finally, the simulation results verify that the proposed method can ensure that the reference signal can be well tracked within the preset time when input saturation occurs.**

***Index Terms*—event-triggered control, fixed-time control, switched nonlinear systems, input saturation, fuzzy adaptive control**

I. Introduction

WITH the deepening of research on nonlinear system control technology, the importance of switched system has attracted a lot of attention[1-4]. A tracking control method for switched nonlinear systems with unknown time-varying parameters was provided in [5], an adaptive neural network control scheme was given, which ensured that the whole system was semi-global uniformly eventually bounded. In [6], Hespanha generalized the concept of dwell time to mean dwell time and proved that the switched system was exponentially stable under the mean dwell time switching signal. In [7], a robust adaptive control scheme was proposed for strict-feedback uncertain switched nonlinear systems, which effectively reduced the number of adjustable parameters of the controller. However, the above research ignored the phenomenon of input saturation in the system, which reduced the performance of the system.

Compared with time-triggered control, event-triggered control (ETC) [8-9] can significantly reduce the amount of data in the network transmission, as well as reduce the calculation frequency and execution times of the controller, improve the transmission efficiency. In [10], Sahoo et al proposed an optimal control method based on event-triggered mechanism for uncertain nonlinear discrete systems, the ultimate boundedness of the closed-loop system is guaranteed by using Lyapunov technique and event-triggered conditions. In [11], an ETC method based on HDP technique was given, which can make the discrete system asymptotically stable. In [12], the problem of adaptive tracking control for nonlinear systems with time-varying delays was studied, and an event-based adaptive control strategy was provided to reduce the communication burden. However, in the above research, the fixed-time control problem of the system under input constraints is not considered.

In control systems, input saturation can lead to performance degradation and even instability of closed-loop systems [13]. To address these issues, in [14], Li et al studied adaptive fuzzy control scheme for nonlinear systems including input saturation, which guaranteed that all signals in the closed-loop system were bounded. In [15], for the closed-loop nonlinear system with input saturation, an auxiliary control signal was designed to deal with the saturation function to ensure that the closed-loop nonlinear system can maintain stability when input saturation occurs. The adaptive fuzzy control problem for nonstrict feedback systems with input saturation was studied in [16], the auxiliary control signals are introduced to sovle the input saturation problem. However, in the above studies, the fixed-time control scheme for switched nonlinear systems with strict-feedback form is not considered.

In order to overcome the shortcomings of the infinite time control, the finite time control method [17-19] was proposed, which has faster convergence speed and better robustness. However, the control performance of the system largely depends on the initial condition of the system, which makes the finite time control have great application limitations. Therefore Polyakov proposed fixed-time control in [20] to solve this problem. In [21], a tracking controller with fixed-time was given for the strict-feedback nonlinear system to ensure that the system output can track the reference signal in a fixed time. In [22], an adaptive fixed-time controller was provided for the MIMO nonlinear system, which had effectively improved the robustness of the system. In [23], an adaptive fuzzy fixed-time controller based on was provided for the uncertain multi-link robot system to guarantee the rapidity of system's transient response.

Based on the above research, this paper focuses on a class of strict-feedback switched nonlinear systems with saturated

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input constraints. A fixed-time fuzzy adaptive event-triggered control method is proposed. The main contributions include:

(1) By introducing fixed-time control and event-triggered mechanism, the proposed control method not only reduces the large occupation of communication resources caused by compensating the saturation input, but also ensures that the output of the system can track the reference signal within a preset time, and the tracking time is independent of the initial state of the system. When there is saturation phenomenon in the system input, it can maintain a stable state and improve the system performance, which is more conducive to practical applications.

(2) Compared with the control scheme proposed in [14-16], firstly, there is no need to design auxiliary control signals in the process of dealing with saturation input constraints, which makes the designed control scheme more simple and the proposed control scheme has a wider application range. Secondly, only one adaptive parameter is needed in the process of controller design, which reduces the computational burden and solves the problem of over-parameterization. At the same time, the state of the system is estimated by using a linear observer, and the unknown nonlinear functions in the system are approached by using a fuzzy logic system (FLS), which eliminates the restriction that the state must be measurable and the function must be known.

II. System descriptions and basic knowledge

A. Problem formulations

Consider the uncertain switched nonlinear system with strict-feedback form as follows

$$\begin{cases} \dot{x}_1 = x_2 + f_{1,\sigma(t)}(x_1) + d_{1,\sigma(t)}(t) \\ \dot{x}_i = x_{i+1} + f_{i,\sigma(t)}(\bar{x}_i) + d_{i,\sigma(t)}(t) \\ \dot{x}_n = u(v) + f_{n,\sigma(t)}(\bar{x}_n) + d_{n,\sigma(t)}(t) \\ y \ = x_1. \end{cases} \quad (1)$$

Where $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T \in R^i (i = 1, 2, \cdots, n)$, $u \in R$ and $y \in R$ are state vector, input variables and output variables of the system respectively; $\sigma_{(t)} : [0, \infty) \to M = \{1, 2, \cdots, m\}$ denotes switching signal, m is the number of subsystems, $f_{i,k(\cdot)}$ and $d_{i,k}(\cdot)(k \in M)$ represent the unknown nonlinear function and the unknown external disturbance in the system respectively; $v(t)$ is the actual control input of the system, $u(v)$ denotes the system input subject to saturation constraint.

$u(v)$ can be fomulated as

$$u(v) = \begin{cases} u_{\max} sgn(u_c) & |u_c| \geq u_{\max} \\ u_c & |u_c| < u_{\max} \end{cases} \quad (2)$$

Where u_{max} is the maximum value of the control input, and u_c is the control input to be designed.

The main control objective of this paper is to design a fixed-time control scheme for the system shown in (1), which can maintain a stable state when the system has input saturation phenomenon, and ensure that the output of the system can track the reference signal within a preset time.

B. Some preliminaries

Definition 1.[21] A nonlinear system

$$\dot{x}(t) = f(x(t)) \quad (3)$$

Here $x(t) \in R^n$ is the state variable of the system, $f(\cdot)$ denotes a continuous nonlinear function, $x(0) = 0$, $f(0) = 0$. If the system (3) is stable in the sense of $Lyapunov$ and there exists a convergence time T_s makes $x(t) = 0$ for all $t \geq T_s$, then the system (3) is said to be finite time stable.

Lemma 1.[24] If $V(x(t))$ is a continuous function, there exists arguments $\chi > 0$, $c > 0$, $h > 0$, $\beta > 1$, $0 < \gamma < 1$ and $0 < \eta < 1$ holds

$$\dot{V}(x(t)) \leq -(hV^{\gamma} + cV^{\beta})x(t) + \chi \quad (4)$$

Then the system (3) is referred to as fixed time stability and has a convergence time of T_s satisfies the following inequality:

$$T_s \leq T_{max} = \frac{1}{(1-\gamma)\eta h} - \frac{1}{(1-\beta)\eta c} \quad (5)$$

The state of the system satisfies

$$\oslash = \{x(t)|V(x(t)) \leq min\{(\frac{\chi}{(1-\eta)h})^{\frac{1}{\gamma}}, (\frac{\chi}{(1-\eta)c})^{\frac{1}{\beta}}\}\} \quad (6)$$

Lemma 2.[22] $z_i \geq 0$ satisfies the following inequalitys

$$\sum_{i=1}^{n} {z_i}^p \geq (\sum_{i=1}^{n} z_i)^p \ , 0 < p < 1 \quad (7)$$

$$\sum_{i=1}^{n} {z_i}^p \geq n^{1-p}(\sum_{i=1}^{n} z_i)^p \ , p > 1 \quad (8)$$

Lemma 3.[25] For any x, $y \in R^2$, there has

$$xy \leq \frac{\iota^a}{a}|x|^a + \frac{1}{b\iota^b}|y|^b \quad (9)$$

Where $\iota > 1$, $a > 1$, $b > 1$ and $(a-1)(b-1) = 1$.

Lemma 4.[26] For ω, $\xi \in R$ there exists any positive constant κ, ϵ and l satisfying the inequality as follows

$$|\omega|^{\kappa}|\xi|^{\epsilon} \leq \frac{\kappa}{\kappa+\epsilon} l|\omega|^{\kappa+\epsilon} + \frac{\epsilon}{\kappa+\epsilon} l^{-\frac{1}{\epsilon}}|\xi|^{\kappa+\epsilon} \quad (10)$$

Lemma 5[27] $F(x)$ is a continuous function defined in a closed set Ω_x, for $\forall \varepsilon > 0$, there exists a FLS $y(x) = W^T\varphi(x)$ satisfying the inequality

$$\sup_{x\in\Omega_x} \left|f(x) - W^T\varphi(x)\right| \leq \varepsilon \quad (11)$$

Where $W = [w_1, \ w_2, \ \cdots, \ w_n]^T$ is the weight vector, $\varphi_{(x)} = [p_1(x), \ p_2(x), \ \cdots, \ p_n(x)]^T / \sum_{i=1}^{n} p_i(x)$, is the basis function vector, $N > 1$ is the number of fuzzy rule, p_i is the selected Gaussian function, $p_i = exp[-(x-\mu_i)^T(x-\mu_i)/{\gamma_i}^2]$, $\mu_i = [\mu_{i1}, \ \cdots, \ \mu_{in}]^T$ is the center of the basis function with a width of γ_i, i= 1, 2, $\cdots$, n.

Lemma 6.[28] For $\Gamma \in R$, $\sigma > 0$, there has

$$0 \leq |\Gamma| - \Gamma tanh(\frac{\Gamma}{\sigma}) \leq 0.2785\sigma \quad (12)$$

Assumption 1. The reference signal y_d and its nth order derivatives are continuous, known and bounded, then there is a function $k_{a_{i+1}}(t)$, satisfying inequality $k_{a_{i+1}}(t) > |y_d^{(i)}(t)|$ $(i = 0, \ 1, \ \cdots, \ n)$.

Assumption 2. The external disturbance $d_{i,k}(t)$ in the model is bounded, and there exists a constant $\bar{d}_{i,k} > 0$, satisfying inequality $\bar{d}_{i,k} \geq |d_{i,k}(t)|$

Assumption 3. $\forall\ X, Y \in R^i$, there exist positive constants $h_{i,k}(i = 1, 2, \ldots, n)$ such that

$$|f_{i,k}(X) - f_{i,k}(Y)| \leq h_{i,k}\|X - Y\| \tag{13}$$

III. STATE OBSERVER DESIGN

In order to solve the problem of unknown state of the system, a state observer is designed to estimate the system state. Then (1) can be written as:

$$\begin{cases} \dot{x_1} = x_2 + f_{1,k}(\hat{x}_1) + d_{1,k}(t) + \triangle f_{1,k} \\ \dot{x}_i = x_{i+1} + f_{i,k}(\hat{\bar{x}}_i) + d_{i,k}(t) + \triangle f_{i,k} \\ \dot{x}_n = u + f_{n,k}(\hat{\bar{x}}_n) + d_{n,k}(t) + \triangle f_{n,k} \\ y\ = x_1. \end{cases} \tag{14}$$

Where $\hat{\bar{x}}_i$ is the estimated value of $\bar{x}_i$, $\triangle f_{i,k} = f_{i,k}(\bar{x}_i) - f_{i,k}(\hat{\bar{x}}_i)$, $i = 1,\ \cdots,\ n$.

The designed state observer is as follows:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + \eta_{1,k}(y - \hat{x}_1) \\ \dot{\hat{x}}_i = \hat{x}_{i+1} + \eta_{i,k}(y - \hat{x}_1) \\ \dot{\hat{x}}_n = u + \eta_{n,k}(y - \hat{x}_1) \\ \hat{y}\ = \hat{x}_1. \end{cases} \tag{15}$$

According to equation (15), there is

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + L\ y + B_n u \\ \hat{y}\ = C\hat{x}_1. \end{cases} \tag{16}$$

Where $\hat{x} = [\hat{x}_1,\ \cdots, \hat{x}_n]^T$, $B_n = [0, \cdots, 0, 1]^T$, $C = [1, 0, \cdots, 0]^T$, $L = [\eta_{1,k}, \eta_{2,k}, \cdots, \eta_{n,k}]^T$,

$$A = \begin{bmatrix} -\eta_{1,k} & & & \\ -\eta_{2,k} & & I_{n-1} & \\ \vdots & & & \\ -\eta_{n,k} & 0 & \cdots & 0 \end{bmatrix}$$

For a given matrix $Q = Q^T > 0$, there exists a matrix $P = P^T > 0$ that satisfies

$$A^T P + PA = -Q \tag{17}$$

Where A is a strict Hurwitz matrix, define $e = x - \hat{x} = [e_1,\ e_2,\ \cdots,\ e_n]^T$, combining (14) and (16), there has

$$\dot{e} = Ae + \triangle F_k + F_k + D_k \tag{18}$$

Where $\triangle F_k = [\triangle f_{1,k}, \cdots, \triangle f_{n,k}]^T$, $D_k = [d_{1,k}, \cdots, d_{n,k}]^T$, $F_k = [f_{1,k}(\hat{x}_1), \cdots, f_{n,k}(\hat{\bar{x}}_n)]^T$.

Select the Lyapunov function as follows:

$$V_0 = e^T Pe \tag{19}$$

Taking the derivative on both sides of equation (19) yields

$$\begin{aligned} \dot{V}_0 =& - e^T Qe + 2e^T P\triangle F_k + \\ & 2e^T PF_k + 2e^T PD_k \end{aligned} \tag{20}$$

By using FLS, F_k can be expressed as follows:

$$\begin{aligned} F_{k(Z_0)} =& W_{0,k}^T \varphi_0(Z_0) + \varepsilon_{0,k}(Z_0) \\ & \|\varepsilon_{0,k}(Z_0)\| \leq \varepsilon_0 \end{aligned} \tag{21}$$

Where $W_{0,k} = [W_{0,k,1}^T, \cdots, W_{0,k,n}^T]$, $\varepsilon_{0,k}(Z_0) = [\varepsilon_{0,k,1}(Z_0), \cdots, \varepsilon_{0,k,n}(Z_0)]^T$.

According to Lemma 3 and Assumptions 2-3 , we can obtain the inequality as follows:

$$\begin{aligned} 2e^T P\Delta F_k \leq& (n + \|\ P\ \|^2 \sum_{i=1}^{n} h_{i,k}^2)e^T e \\ \leq& (n + \|\ P\ \|^2 \sum_{i=1}^{n} \bar{h}_i^2)e^T e \end{aligned} \tag{22}$$

$$2e^T PD_k\ \leq (ne^T e + \|P\|^2 \sum_{i=1}^{n} \bar{d}_i^2) \tag{23}$$

$$\begin{aligned} 2e^T PF_k\ \leq& e^T e + \|P\|^2 \|F_k\|^2 \\ \leq& e^T e + \|P\|^2 (\theta_0 + \varepsilon_0^2) \end{aligned} \tag{24}$$

Where $\bar{h}_i = max_{k\in M}\{h_{i,k}\}$, $\bar{d}_i = max_{k\in M}\{d_{i,k}\}$, $\theta_0 = max_{k\in M}\{||W_{0,k}||^2\}$.

Taking (22)-(24) into (20) we have:

$$\begin{aligned} \dot{V}_0 \leq& - e^T Qe + (n + \|P\|^2 \sum_{i=1}^{n} \bar{h}_i^2)e^T e + \\ & (ne^T e + \|P\|^2 \sum_{i=1}^{n} \bar{d}_i^2) + \\ & e^T e + \|P\|^2 (\theta_0 + \varepsilon_0^2) \\ & \leq -\mu_0 e^T e + \|P\|^2 (\sum_{i=1}^{n} \bar{d}\ _i^2 + \theta_0 + \varepsilon_0^2) \end{aligned} \tag{25}$$

Let $\mu_0 = (\lambda_{min}(Q) - 2n - 1 - ||P||^2 \sum_{i=1}^{n} \bar{h}_i^2)$.

IV. CONTROLLER DESIGN

In this section, the steps for designing the tracking controler of system (1) will be provided. Firstly, coordinate transformation is defined as

$$\begin{cases} z_1 = x_1 - y_d \\ z_i = \hat{x}_i - \alpha_{i-1} \\ i = 2, \cdots, n. \end{cases} \tag{26}$$

Here y_d is the reference signal, z_i is the tracking error, and α_{i-1} is the control signal at the α_{i-1} step.

Step 1: Combining (1) and (26), we can get:

$$\dot{z}_1 = z_2 + e_2 + \alpha_1 + f_{1,k}(x_1) + d_{1,k} - \dot{y}_d \tag{27}$$

Select the Lyapunov function as follows:

$$V_1 = V_0 + \frac{1}{2}z_1^2 + \frac{1}{2g_1}\tilde{\theta}_1^2 \tag{28}$$

Where $g_1 > 0$ is the design parameter, the derivative of equation (28) can be obtained:

$$\begin{aligned} \dot{V}_1 =& \dot{V}_0 + z_1(z_2 + \alpha_1 + e_2 + f_{1,k}(x_1) + d_{1,k} - \\ & \dot{y_d}) - \frac{1}{g_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 \end{aligned} \tag{29}$$

Where $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ denotes the parameter error, by using FLS, it has

$$f_{1,k}(x_1) = W_{1,k}^T \varphi_1(x_1) + \varepsilon_{1,k}(x_1), |\varepsilon_{1,k}(x_1)| \leq \varepsilon_1 \tag{30}$$

According to Lemma 2, it has

$$z_1 z_2 \leq \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \tag{31}$$

$$z_1 e_2 \leq \frac{1}{2}z_1^2 + \frac{1}{2}e^T e \tag{32}$$

$$\begin{aligned} z_1 f_{1,k}(x_1) =& z_1(W_{1,k}^T \varphi_1(x_1) + \varepsilon_{1,k}(x_1)) \\ \leq& \frac{\theta_1}{2{a_1}^2} z_1^2 \varphi_1^T \varphi_1 + \frac{1}{2}a_1^2 + \frac{1}{2}z_1^2 + \frac{1}{2}\varepsilon_1^2 \end{aligned} \tag{33}$$

Where $\theta_1 = max_{k\in M}\{||W_{1,k}||^2\}$, $a_1 > 0$ is the design parameter.

$$z_1 d_{1,k} \leq \frac{1}{2}z_1^2 + \frac{1}{2}\bar{d_1}^2 \tag{34}$$

Substituting (31)-(34) into (31), we can obtain

$$\begin{aligned} \dot{V_1} \leq& \dot{V_0} + z_1(\alpha_1 - \dot{y_d}) + 2z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}\|e\|^2 + \\ & \frac{\theta_1}{2a_1^2} z_1^2 \varphi_1^T \varphi_1 + \frac{1}{2}{a_1}^2 + \frac{1}{2}{\varepsilon_1}^2 - \frac{1}{g_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 + \frac{1}{2}\bar{d_1}^2 \end{aligned} \tag{35}$$

The designed virtual control signal α_1 and adaptive rate $\dot{\hat{\theta}}_1$ are

$$\begin{aligned} \alpha_1 =& -2z_1 + \dot{y}_d - \frac{\hat{\theta}_1}{2a_1^2} z_1 \varphi_1^T \varphi_1 - \\ & h_1 z_1^{2\gamma-1} - c_1 z_1^{2\beta-1} \end{aligned} \tag{36}$$

$$\dot{\hat{\theta}}_1 = g_1\left(\frac{1}{2a_1^2} z_1^2 \varphi_1^T \varphi_1 - \sigma_1 \hat{\theta}_1\right) \tag{37}$$

Where the design parameters σ_1, a_1 and h_i, $c_i (i = 1, \cdots, n)$ satisfies $\sigma_1 > 0$, $a_1 > 0$, $h_i > 0$, $c_i > 0$.

Substituting (36) and (37) into (35) yields

$$\begin{aligned} \dot{V_1} \leq& \dot{V_0} + \frac{1}{2}{z_2}^2 + \frac{1}{2}\|e\|^2 + \frac{1}{2}a_1^2 + \\ & \frac{1}{2}{\varepsilon_1}^2 + \sigma_1\tilde{\theta}_1\hat{\theta}_1 + \frac{1}{2}\bar{d_1}^2 \end{aligned} \tag{38}$$

Referring to Lemma 3, we can get

$$\sigma_1\tilde{\theta}_1\hat{\theta}_1 \leq -\frac{1}{2}\sigma_1\tilde{\theta}_1\tilde{\theta}_1 + \frac{1}{2}\sigma_1\theta_1\theta_1 \tag{39}$$

Substituting (39) into (38), there has

$$\begin{aligned} \dot{V_1} \leq& -\mu_0\|e\|^2 + \|p\|^2\left(\sum_{i=1}^{n}\bar{d}_i^2 + \theta_0 + \varepsilon_0^2\right) + \\ & \frac{1}{2}{z_2}^2 + \frac{1}{2}\|e\|^2 + \frac{1}{2}a_1^2 + \frac{1}{2}\varepsilon_1^2 - \frac{1}{2}\sigma_1\tilde{\theta}_1\tilde{\theta}_1 + \\ & \frac{1}{2}\sigma_1\theta_1\theta_1 + \frac{1}{2}\bar{d_1}^2 - h_1 z_1^{2\gamma} - c_1 z_1^{2\beta} \end{aligned} \tag{40}$$

Step 2: According to (15) and (26), we can get

$$\begin{aligned} \dot{z}_2 &= \dot{\hat{x}}_2 - \dot{\alpha}_1 \\ &= z_3 + \alpha_2 + \eta_{2,k}e_1 - \dot{\alpha}_1 \end{aligned} \tag{41}$$

Select the following Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_2^2 \tag{42}$$

$\dot{V}_2$ can be get

$$\dot{V_2} = \dot{V_1} + z_2(z_3 + \alpha_2 + \eta_{2,k}e_1 - \dot{\alpha}_1) \tag{43}$$

According to the Lemma 3:

$$z_2 z_3 \leq \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2 \tag{44}$$

$$-z_2\frac{\partial\alpha_1}{\partial x_1}e_2 \leq \frac{1}{2}z_2^2\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 + \frac{1}{2}\|e\|^2 \tag{45}$$

$$\begin{aligned} -z_2\frac{\partial\alpha_1}{\partial x_1}f_{1,k} \leq& -z_2\frac{\partial\alpha_1}{\partial x_1}(W_{1,k}^T\varphi_1 + \varepsilon_{1,k}) \\ \leq& \frac{1}{4}z_2^2\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2{\varphi_1}^T\varphi_1 + \theta_1 + \\ & \frac{1}{2}z_2^2\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 + \frac{1}{2}\varepsilon_1^2 \end{aligned} \tag{46}$$

$$-z_2\frac{\partial\alpha_1}{\partial x_1}d_{1,k} \leq \frac{1}{2}z_2^2\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 + \frac{1}{2}\bar{d_1}^2 \tag{47}$$

Substituting $(44) - (47)$ into (43), we can obtain:

$$\begin{aligned} \dot{V_2} \leq& \dot{V_1} + \frac{1}{2}z_2^2 + \frac{1}{2}{z_3}^2 + \frac{3}{2}z_2^2\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 + \frac{1}{2}\|e\|^2 + \\ & \frac{1}{4}z_2^2\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2{\varphi_1}^T\varphi_1 + \theta_1 + +\frac{1}{2}\varepsilon_1^2 + \frac{1}{2}\bar{d_1}^2 + z_2(\alpha_2 + \\ & \eta_{2,k}e_1 - \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 - \sum_{j=1}^{2}\frac{\partial\alpha_1}{\partial {y_d}^{(j-1)}}y_d^{(j)} - \frac{\partial\alpha_1}{\partial x_1}\hat{x}_2) \end{aligned} \tag{48}$$

Design the virtual control signal α_2 as follows:

$$\begin{aligned} \alpha_2 =& -\frac{3}{2}z_2\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 - \eta_{2,k}e_1 + \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 + \\ & \sum_{j=1}^{2}\frac{\partial\alpha_1}{\partial {y_d}^{(j-1)}}y_d^{(j)} + \frac{\partial\alpha_1}{\partial x_1}\hat{x}_2 - h_2 z_2^{2\gamma-1} - \\ & c_2 z_2^{2\beta-1} - \frac{1}{4}z_2\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2{\varphi_1}^T\varphi_1 - z_2 \end{aligned} \tag{49}$$

Substituting (49) into (48), we can get:

$$\begin{aligned} \dot{V_2} \leq& \dot{V_1} + \frac{1}{2}{z_3}^2 + \frac{1}{2}\|e\|^2 + \theta_1 + \frac{1}{2}{\varepsilon_1}^2 + \\ & \frac{1}{2}\bar{d_1}^2 - \frac{1}{2}{z_2}^2 - h_2 z_2^{2\gamma} - c_2 z_2^{2\beta} \end{aligned} \tag{50}$$

Step i (2<i<n): Combining (15) and (26), we can obtain:

$$\begin{aligned} \dot{z}_i &= \dot{\hat{x}}_i - \dot{\alpha}_{i-1} \\ &= z_{i+1} + \alpha_i + \eta_{i,k}e_1 - \dot{\alpha}_{i-1} \end{aligned} \tag{51}$$

Select the Lyapunov function in the following form

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 \tag{52}$$

$\dot{V}_i$ can be get

$$\dot{V_i} = \dot{V}_{i-1} + z_i(z_{i+1} + \alpha_i + \eta_{i,k}e_1 - \dot{\alpha}_{i-1}) \tag{53}$$

According to Lemma 3, one has:

$$z_i z_{i+1} \leq \frac{1}{2}{z_i}^2 + \frac{1}{2}{z_{i+1}}^2 \tag{54}$$

$$-z_i\frac{\partial\alpha_{i-1}}{\partial x_1}e_2 \leq \frac{1}{2}z_i^2\left(\frac{\partial\alpha_{i-1}}{\partial x_1}\right)^2 + \frac{1}{2}\|e\|^2 \tag{55}$$

$$\begin{aligned} -z_i\frac{\partial\alpha_{i-1}}{\partial x_1}f_{1,k} \leq& \frac{1}{4}z_i^2\left(\frac{\partial\alpha_{i-1}}{\partial x_1}\right)^2{\varphi_1}^T\varphi_1 + \\ & \theta_1 + \frac{1}{2}z_i^2\left(\frac{\partial\alpha_{i-1}}{\partial x_1}\right)^2 + \frac{1}{2}\varepsilon_1^2 \end{aligned} \tag{56}$$

$$-z_i\frac{\partial\alpha_{i-1}}{\partial x_1}d_{1,k} \leq \frac{1}{2}z_i^2\left(\frac{\partial\alpha_{i-1}}{\partial x_1}\right)^2 + \frac{1}{2}\bar{d_1}^2 \tag{57}$$

Substituting (54)-(57) into (53) produces:

$$\begin{aligned}\dot{V}_i \leq& \dot{V}_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}z_{i+1}^2 + \frac{3}{2}z_i^2(\frac{\partial\alpha_{i-1}}{\partial x_1})^2 + \\ & \frac{1}{2}\|e\|^2 + \frac{1}{4}z_i^2(\frac{\partial\alpha_{i-1}}{\partial x_1})^2{\varphi_1}^T\varphi_1 + \theta_1 + \\ & \frac{1}{2}\varepsilon_1^2 + \frac{1}{2}{\bar{d}_1}^2 + z_i(\alpha_i + \eta_{i,k}e_1 - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 - \\ & \sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j - \sum_{j=1}^{i}\frac{\partial\alpha_{i-1}}{\partial {y_d}^{(j-1)}}y_d^{(j)} - \frac{\partial\alpha_{i-1}}{\partial x_1}\hat{x}_2)\end{aligned} \tag{58}$$

The virtual control signal α_i is designed as follows:

$$\begin{aligned}\alpha_i =& -\frac{3}{2}z_i(\frac{\partial\alpha_{i-1}}{\partial x_1})^2 - \eta_{i,k}e_1 + \sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j + \\ & \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 - z_i + \sum_{j=1}^{i}\frac{\partial\alpha_{i-1}}{\partial {y_d}^{(j-1)}}y_d^{(j)} - h_i z_i^{2\gamma-1} - \\ & c_i z_i^{2\beta-1} + \frac{\partial\alpha_{i-1}}{\partial x_1}\hat{x}_2 - \frac{1}{4}z_i(\frac{\partial\alpha_{i-1}}{\partial x_1})^2{\varphi_1}^T\varphi_1\end{aligned} \tag{59}$$

Substituting (59) into (58) results in

$$\begin{aligned}\dot{V}_i \leq& \dot{V}_{i-1} - \frac{1}{2}z_i^2 + \frac{1}{2}z_{i+1}^2 + \frac{1}{2}\|e\|^2 + \\ & \frac{1}{2}{\varepsilon_1}^2 + \frac{1}{2}{\bar{d}_1}^2 + \theta_1 - h_i z_i^{2\gamma} - c_i z_i^{2\beta}\end{aligned} \tag{60}$$

Step n: From (15) and (26), we have

$$\begin{aligned}\dot{z}_n &= \dot{\hat{x}}_n - \dot{\alpha}_{n-1} \\ &= u + \eta_{n,k}e_1 - \dot{\alpha}_{n-1} \\ &= \triangle u + u_c + \eta_{n,k}e_1 - \dot{\alpha}_{n-1}\end{aligned} \tag{61}$$

Where $\triangle u = u - u_c$, there must be a positive number D satisfied $D \geq |\triangle u|$.

Choose the Lyapunov function as:

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 \tag{62}$$

it has

$$\dot{V}_n = \dot{V}_{n-1} + z_n(\triangle u + u_c + \eta_{n,k}e_1 - \dot{\alpha}_{n-1}) \tag{63}$$

The triggering event is defined as follows

$$\begin{aligned}& w(t) = \alpha_n - \bar{\mu}\tanh\frac{z_n\bar{\mu}}{r} \\ & u_c = w(t_s), \forall t \in [t_s, t_{s+1}), s \in Z^+ \\ & t_{s+1} = \inf\{t \in R^+ | \, |e(t) \geq \mu|\}\end{aligned} \tag{64}$$

Where μ, r and $\bar{\mu}$ are positive design parameters, $\mu \leq \bar{\mu}$, $e(t) = w(t) - u_c$.

According to [29], when $t \in [t_s, t_{s+1})$, we can obtain $|w(t) - u_c| \leq \mu$, so there is a continuous function $\chi(t)$ with $\chi(t) \leq 1$ when $\chi(t_s) = 0$ and $\chi(t_{s+1}) = \pm 1$, and it has

$$u_c = w(t) - \chi(t)\mu \tag{65}$$

Combining (61) and (63)-(65), we can get:

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + z_n(\triangle u + u_c + \eta_{n,k}e_1 - \dot{\alpha}_{n-1}) \\ &= \dot{V}_{n-1} + z_n(w(t) - \chi(t)\mu + \triangle u + \eta_{n,k}e_1 - \dot{\alpha}_{n-1}) \\ &= \dot{V}_{n-1} + z_n(\alpha_n - \bar{\mu}tanh\frac{z_n\bar{\mu}}{r} - \chi(t)\mu + \triangle u + \\ &\quad \eta_{n,k}e_1 - \dot{\alpha}_{n-1})\end{aligned} \tag{66}$$

According to Lemma 6, one has:

$$-z_n\bar{\mu}tanh\frac{z_n\bar{\mu}}{r} + |z_n\mu| \leq 0.2785r \tag{67}$$

Substituting (67) into (66) produces:

$$\begin{aligned}\dot{V}_n \leq& \dot{V}_{n-1} + 0.2785r + z_n\triangle u + \\ & z_n(\alpha_n + \eta_{n,k}e_1 - \dot{\alpha}_{n-1})\end{aligned} \tag{68}$$

Further, (68) can be rewritten as

$$\begin{aligned}\dot{V}_n \leq& \dot{V}_{n-1} + 0.2785r + z_n\triangle u + z_n(\alpha_n + \\ & \eta_{n,k}e_1 - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 - \sum_{j=2}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j - \\ & \sum_{j=1}^{n}\frac{\partial\alpha_{n-1}}{\partial {y_d}^{(j-1)}}y_d^{(j)} - \frac{\partial\alpha_{n-1}}{\partial x_1}\hat{x}_2 - \\ & \frac{\partial\alpha_{n-1}}{\partial x_1}e_2 - \frac{\partial\alpha_{n-1}}{\partial x_1}f_{1,k} - \frac{\partial\alpha_{n-1}}{\partial x_1}d_{1,k})\end{aligned} \tag{69}$$

According to Lemma 3, we can get

$$z_n\triangle u \leq \frac{1}{2}{z_n}^2 + \frac{1}{2}D^2 \tag{70}$$

$$-z_n\frac{\partial\alpha_{n-1}}{\partial x_1}e_2 \leq \frac{1}{2}z_n^2(\frac{\partial\alpha_{n-1}}{\partial x_1})^2 + \frac{1}{2}\|e\|^2 \tag{71}$$

$$\begin{aligned}-z_n\frac{\partial\alpha_{n-1}}{\partial x_1}f_{1,k} \leq& \frac{1}{4}z_n^2(\frac{\partial\alpha_{n-1}}{\partial x_1})^2{\varphi_1}^T\varphi_1 \\ & + \theta_1 + \frac{1}{2}z_n^2(\frac{\partial\alpha_{n-1}}{\partial x_1})^2 + \frac{1}{2}\varepsilon_1^2\end{aligned} \tag{72}$$

$$-z_n\frac{\partial\alpha_{n-1}}{\partial x_1}d_{1,k} \leq \frac{1}{2}z_n^2(\frac{\partial\alpha_{n-1}}{\partial x_1})^2 + \frac{1}{2}{\bar{d}_1}^2 \tag{73}$$

Substituting $(70)-(73)$ into (69) produces

$$\begin{aligned}\dot{V}_n \leq& \dot{V}_{n-1} + 0.2785r + \frac{1}{2}z_n^2 + \frac{1}{2}D^2 + \frac{1}{2}\varepsilon_1^2 + \\ & \frac{3}{2}z_n^2(\frac{\partial\alpha_{n-1}}{\partial x_1})^2 + \frac{1}{2}\|e\|^2 + \theta_1 + \frac{1}{2}{\bar{d}_1}^2 + \\ & z_n(\alpha_n + \eta_{n,k}e_1 - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 - \sum_{j=2}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j - \\ & \sum_{j=1}^{n}\frac{\partial\alpha_{n-1}}{\partial {y_d}^{(j-1)}}y_d^{(j)} - \frac{\partial\alpha_{n-1}}{\partial x_1}\hat{x}_2) + \frac{1}{4}z_n^2(\frac{\partial\alpha_{n-1}}{\partial x_1})^2{\varphi_1}^T\varphi_1\end{aligned} \tag{74}$$

The final control signal α_n is designed as follows:

$$\begin{aligned}\alpha_n =& -\frac{3}{2}z_n(\frac{\partial\alpha_{n-1}}{\partial x_1})^2 - \eta_{n,k}e_1 + \sum_{j=2}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j + \\ & \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 + \sum_{j=1}^{n}\frac{\partial\alpha_{n-1}}{\partial {y_d}^{(j-1)}}y_d^{(j)} + \frac{\partial\alpha_{n-1}}{\partial x_1}\hat{x}_2 - \\ & z_n - \frac{1}{4}z_n(\frac{\partial\alpha_{n-1}}{\partial x_1})^2{\varphi_1}^T\varphi_1 - h_n z_n^{2\gamma-1} - c_n z_n^{2\beta-1}\end{aligned} \tag{75}$$

Substituting (75) into (74) produces:

$$\begin{aligned}\dot{V}_n \leq& \dot{V}_{n-1} + 0.2785r - \frac{1}{2}z_n^2 + \frac{1}{2}D^2 + \\ & \frac{1}{2}\|e\|^2 + \frac{1}{2}{\varepsilon_1}^2 + \frac{1}{2}{\bar{d}_1}^2 + \theta_1 - h_n z_n^{2\gamma} - \\ & c_n z_n^{2\beta}\end{aligned} \tag{76}$$

Theorem 1. Under assumptions 1-3, the virtual control rates of system (1) are designed as (36), (49), (59), and (75), and the adaptive rate is (40). Then, by adjusting the design parameters h_i, c_i, $(i=1,\cdots,n)$, g_1, σ_1, a_1, the output of the closed-loop system can track the reference signal within a preset time, and the tracking time is independent of the initial state of the system.

Proof. Combining (25), (40), (50), (60) and (76) yields

$$\begin{aligned}\dot{V}_n \leq &-(\mu_0-\frac{n}{2})\|e\|^2+\frac{n}{2}(\bar{d}_1^2+\varepsilon_1^2+\frac{\sigma_1}{n}\theta_1\theta_1+\\&\frac{2(n-1)}{n}\theta_1+\frac{{a_1}^2}{n})-\frac{1}{2}\sigma_1\tilde{\theta}_1^2+0.2785r+\\&\frac{1}{2}D^2+\|p\|^2(\sum_{i=1}^{n}\bar{d}\,_i^2+\theta_0+\varepsilon_0^2)+0.2785r-\\&\sum_{i=1}^{n}h_iz_i^{2\gamma}-\sum_{i=1}^{n}c_iz_i^{2\beta}\end{aligned} \tag{77}$$

Choose $\mu_1=\mu_0-\frac{n}{2}$, $F=min\{2c_i,2h_i,\ i=1,\cdots,n\}$, we can get

$$\begin{aligned}\dot{V}_n \leq &-\frac{1}{2}\mu_1\|e\|^2-\frac{1}{2}\mu_1\|e\|^2-\frac{1}{2}\mu_1(\|e\|^2)^\beta+\\&\frac{1}{2}\mu_1(\|e\|^2)^\beta+\frac{n}{2}(\bar{d}_1^2+\varepsilon_1^2+\frac{\sigma_1}{n}\theta_1\theta_1+\\&\frac{2(n-1)}{n}\theta_1+\frac{a_1^2}{n})+\|p\|^2(\sum_{i=1}^{n}\bar{d}\,_i^2+\theta_0+\\&\varepsilon_0^2)-\frac{\sigma_1\tilde{\theta}_1^2}{4}-\frac{\sigma_1\tilde{\theta}_1^2}{4}-\sigma_1(\frac{\tilde{\theta}_1^2}{4})^\beta+\sigma_1(\frac{\tilde{\theta}_1^2}{4})^\beta+\\&0.2785r+\frac{1}{2}D^2-F\sum_{i=1}^{n}\frac{1}{2}z_i^{2\gamma}-F\sum_{i=1}^{n}\frac{1}{2}z_i^{2\beta}\end{aligned} \tag{78}$$

Because of $\sigma_1>0$, $\beta>1$, $n\geq 1$, it has

$$-\sigma_1(\frac{\tilde{\theta}_1^2}{4})^\beta \leq -n^{1-\beta}\sigma_1(\frac{\tilde{\theta}_1^2}{4})^\beta \tag{79}$$

According to Lemma 2, we can obtain

$$-F\sum_{i=1}^{n}\frac{1}{2}z_i^{2\gamma} \leq -2^{\gamma-1}F(\sum_{i=1}^{n}\frac{1}{2}z_i^2)^\gamma \tag{80}$$

$$-F\sum_{i=1}^{n}\frac{1}{2}z_i^{2\beta} \leq -2^{\beta-1}n^{1-\beta}F(\sum_{i=1}^{n}\frac{1}{2}z_i^2)^\beta \tag{81}$$

According to the Lemma 4 , when $\omega=1$, $\xi=\parallel e\parallel^2$, $\kappa=1-\gamma$, $l=\gamma^{\frac{\gamma}{1-\gamma}}$, $\epsilon=\gamma$, it has:

$$\frac{\mu_1}{2}(\|e\|^2)^\gamma \leq \frac{1}{2}\mu_1\|e\|^2+\frac{\mu_1}{2}(1-\gamma)l \tag{82}$$

Similarly, it can be concluded that

$$(\frac{\sigma_1\tilde{\theta}_1^2}{4})^\gamma \leq \frac{\sigma_1\tilde{\theta}_1^2}{4}+(1-\gamma)l \tag{83}$$

Substituting (79)-(83) into (78) produces

$$\begin{aligned}\dot{V}_n \leq &-\frac{\mu_1}{2}(\|e\|^2)^\gamma-\frac{\mu_1}{2}(\|e\|^2)^\beta-\frac{\mu_1}{2}\|e\|^2-\\&(\frac{\sigma_1\tilde{\theta}_1^2}{4})^\gamma-n^{1-\beta}\sigma_1(\frac{\tilde{\theta}_1^2}{4})^\beta-\sigma_1\frac{\tilde{\theta}_1^2}{4}+\sigma_1(\frac{\tilde{\theta}_1^2}{4})^\beta+\\&\frac{1}{2}\mu_1(\|e\|^2)^\beta+\frac{\mu_1}{2}(1-\gamma)l+(1-\gamma)l-\\&2^{\gamma-1}F(\sum_{i=1}^{n}\frac{1}{2}z_i^2)^\gamma+\|p\|^2(\sum_{i=1}^{n}\bar{d}_i^2+\theta_0+\varepsilon_0^2)+\\&\frac{n}{2}(\bar{d}_1^2+\varepsilon_1^2+\frac{\sigma_1}{n}\theta_1\theta_1+\frac{2(n-1)}{n}\theta_1+\frac{a_1^2}{n})+\\&0.2785r+\frac{1}{2}D^2-2^{\beta-1}n^{1-\beta}F(\sum_{i=1}^{n}\frac{1}{2}z_i^2)^\beta\end{aligned} \tag{84}$$

Further, there has

$$\begin{aligned}\dot{V}_n \leq &-\bar{h}((e^Tpe)^\gamma+(\frac{\tilde{\theta}_1^2}{2g_1})^\gamma+(\sum_{i=1}^{n}\frac{1}{2}z_i^2)^\gamma)\\&-\bar{c}((e^Tpe)^\beta+(\frac{\tilde{\theta}_1^2}{2g_1})^\beta+(\sum_{i=1}^{n}\frac{1}{2}z_i^2)^\beta)\\&-\frac{1}{2}\mu_1\|e\|^2+\frac{1}{2}\mu_1(\|e\|^2)^\beta+\bar{\chi}+\\&\sigma_1(\frac{\tilde{\theta}_1^2}{4})^\beta-\sigma_1\frac{\tilde{\theta}_1^2}{4}\end{aligned} \tag{85}$$

Where $\bar{h}=\min\left\{\frac{\mu_1}{2(\lambda_{max}(P))^\gamma},\ (\frac{g_1\sigma_1}{2})^\gamma,2^{\gamma-1}F\right\}$, $\bar{c}=\min\{\frac{\mu_1}{2(\lambda_{max}(P))^\beta},n^{1-\beta}\sigma_1(\frac{g_1}{2})^\beta,2^{\beta-1}n^{1-\beta}F\}$, $\bar{\chi}=\frac{\mu_1}{2}(1-\gamma)l+(1-\gamma)l+\|P\|^2(\sum_{i=1}^{n}\bar{d}\,_i^2+\theta_0+\varepsilon_0^2)+\frac{n}{2}(\bar{d}_1^2+\varepsilon_1^2+\frac{\sigma_1}{n}\theta_1\theta_1+\frac{2(n-1)}{n}\theta_1+\frac{a_1^2}{n})+0.2785r+\frac{1}{2}D^2$.

The common Lyapunov function is chosen as follows:

$$V_n=e^Tpe+\sum_{i=1}^{n}\frac{1}{2}z_i^2+\frac{1}{2g_1}\tilde{\theta}_1^2 \tag{86}$$

Substituting (86) into (85) one has:

$$\begin{aligned}\dot{V}_n \leq &-\bar{h}V_n^\gamma-\bar{c}V_n^\beta+\frac{\mu_1}{2}(\|e\|^2)^\beta-\frac{\mu_1}{2}\|e\|^2+\\&\sigma_1(\frac{\tilde{\theta}_1^2}{4})^\beta-\sigma_1\frac{\tilde{\theta}_1^2}{4}+\bar{\chi}\end{aligned} \tag{87}$$

Suppose there are arbitrary constants Θ_1, Θ_2, such that $||e||<\Theta_1$, $||\tilde{\theta}_1||<\Theta_2$.

The following inequalities hold when $\Theta_1<1$, $\Theta_2<2$:

$$\frac{\mu_1}{2}(\|e\|^2)^\beta-\frac{\mu_1}{2}\|e\|^2<0 \tag{88}$$

$$\sigma_1(\frac{\tilde{\theta}_1^2}{4})^\beta-\sigma_1\frac{\tilde{\theta}_1^2}{4}<0 \tag{89}$$

Substituting (88)-(89) into (90) produces

$$\dot{V}_n \leq -\bar{h}V_n^\gamma-\bar{c}V_n^\beta+\bar{\chi} \tag{90}$$

When $\Theta_1\geq 1$, $\Theta_2\geq 2$, we can obtain

$$\frac{\mu_1}{2}(\|e\|^2)^\beta-\frac{\mu_1}{2}\|e\|^2 \leq \frac{\mu_1}{2}\Theta_1^2[(\Theta_1^2)^{\beta-1}-1] \tag{91}$$

$$\begin{aligned}\sigma_1(\frac{\tilde{\theta}_1^2}{4})^\beta-\sigma_1\frac{\tilde{\theta}_1^2}{4}&=\sigma_1\frac{\tilde{\theta}_1^2}{4}[(\frac{\tilde{\theta}_1^2}{4})^{\beta-1}-1]\\&\leq\sigma_1\frac{\Theta_2^2}{4}[(\frac{\Theta_2^2}{4})^{\beta-1}-1]\end{aligned} \tag{92}$$

Substituting (91)-(92) into (87) we have

$$\dot{V}_n \le -\bar{h}V_n^{\gamma} - \bar{c}V_n^{\beta} + \bar{\bar{\chi}} \tag{93}$$

Where $\bar{\bar{\chi}} = \bar{\chi} + \bar{\chi}_1 + \bar{\chi}_2$, $\bar{\chi}_1 = \frac{\mu_1}{2}\Theta_1^2[(\Theta_1^2)^{\beta-1} - 1]$, $\bar{\chi}_2 = \sigma_1\frac{\Theta_2^2}{4}[(\frac{\Theta_2^2}{4})^{\beta-1} - 1]$.

Summarizing the above two cases, we can obtain

$$\dot{V}_n \le -\bar{h}V_n^{\gamma} - \bar{c}V_n^{\beta} + \chi \tag{94}$$

Where $\chi = \bar{\chi} + \chi_1 + \chi_2$.

$$\chi_1 = \begin{cases} 0 & \Theta_1 < 1 \\ \bar{\chi}_1 & \Theta_1 \ge 1 \end{cases} \quad \chi_2 = \begin{cases} 0 & \Theta_2 < 2 \\ \bar{\chi}_2 & \Theta_2 \ge 2 \end{cases} \tag{95}$$

By using the definition of $\oslash$ in Lemma 1 and V_n, it has

$$\sum_{i=1}^{n}\frac{1}{2}z_i^2 \le (\frac{\chi}{(1-\eta)\bar{h}})^{\frac{1}{\gamma}} \tag{96}$$

$$\sum_{i=1}^{n}\frac{1}{2}z_i^2 \le (\frac{\chi}{(1-\eta)\bar{c}})^{\frac{1}{\beta}} \tag{97}$$

Combining (96) and (97), $z_i \in \Omega_z$ can be obtained

$$\Omega_z = \{z_i \mid |z_i| \le \min\left\{\sqrt{\frac{\chi}{(1-\eta)\bar{h}}}^{\frac{1}{\gamma}}, \sqrt{\frac{\chi}{(1-\eta)\bar{c}}}^{\frac{1}{\beta}}\right\}\} \tag{98}$$

According to (98), we can get $z_1 = y - y_d \in \Omega_z$, So the tracking error can be aggregated into the Ω_z region by adjusting the design parameters.

V. Simulation Examples

Consider strict-feedback switched nonlinear system as follows

$$\begin{cases} \dot{x}_1 = x_2 + f_{1,\sigma(t)}(x_1) + d_{1,\sigma(t)}(t) \\ \dot{x}_2 = u + f_{2,\sigma(t)}(\bar{x}_2) + d_{n,\sigma(t)}(t) \\ y \ = x_1 \end{cases} \tag{99}$$

The designed state observer is

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + \eta_{1,\sigma(t)}(y - \hat{x}_1) \\ \dot{\hat{x}}_2 = u + \eta_{2,\sigma(t)}(y - \hat{x}_1) \\ \hat{y} = \hat{x}_1 \end{cases} \tag{100}$$

When $\sigma(t) = 1$, we choose $f_{1,1} = 0.01x_1e^{-0.5x_1}$, $f_{2,1} = 0.1x_2 + x_1^3$, $d_{1,1} = 0.02cos(t)$, $d_{2,1} = 0.16sin(t)$, when $\sigma(t) = 2$, we choose $f_{1,2} = 0.02x_1e^{-x_1^2}$, $f_{2,2} = 0.015e^{-x_1} + 0.01x_2$, $d_{1,2} = 0.01sin(t)$, $d_{2,2} = 0.02sin(0.2t)$.

In this simulation, $x_1 = 0.21$, $x_2 = 0.2$, $\hat{x}_1 = 0$, $\hat{x}_2 = 0$, $\hat{\theta}_1 = 0.01$. $r = 1$, $\bar{\mu} = 7$, $\mu = 0.1$, $u_{max} = 150$, $u_{min} = -150$, $h1 = 150$, $h2 = 150$, $c1 = 150$, $c2 = 100$, $a_1 = 0.01$, $\sigma_1 = 30$, $g_1 = 10$, $\gamma = 100/101$, $\beta = 2$, $\eta_{1,1} = 150$, $\eta_{2,1} = 100$, $\eta_{1,2} = 30$, $\eta_{2,2} = 200$. Reference signal $y_d = sin(0.6t)$. Here only a FLS is needed to approach the unknown nonlinear function, and the fuzzy membership function is $\mu_{F_1^j} = [e^{(-0.5*(x_1-(-4+l))^T(x_1-(-4+l)))}]$, $l = -1.5, \cdots, 1.5$, $j = 1, \cdots, 7$.

The simulation results are shown in Fig.1- Fig.7. From Fig.1 and Fig.2, it can be seen that the designed state observer can estimate the unknown state x_1 and x_2 of the system very well. Fig.3 shows that the system's output y can effectively track y_d. Fig.4 shows $v(t)$ and $\omega(t)$, respectively. Combining Fig.6 and Fig.3, it can be seen that the system meets the phenomenon of input saturation, but the system can maintain a stable state. Fig.5 is the holding time of the trigger execution interval, the number of events triggered in 30s is 2859, the trigger rate of the system is 9.5% (the simulation step size is 0.001), therefore, the communication resources are effectively saved, Fig.7 shows the switching curve of the function $\sigma(t)$.

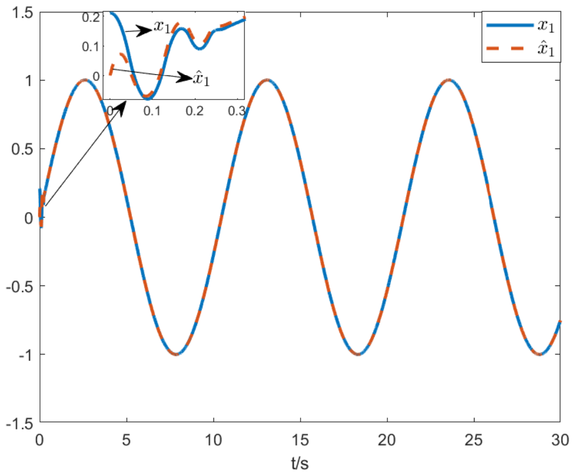

Fig. 1. State variable x_1 and variable estimate $\hat{x}_1$.

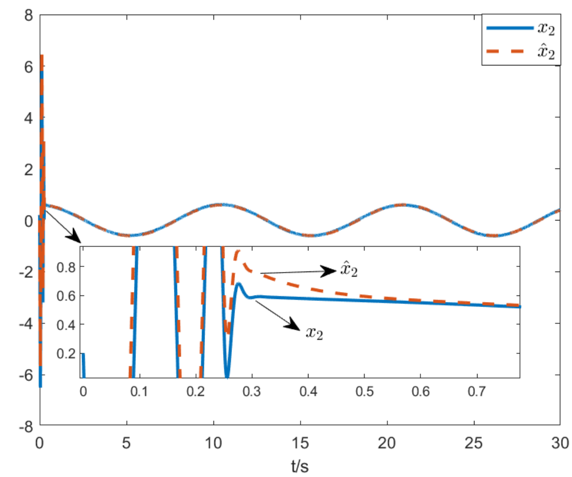

Fig. 2. State variable x_2 and its estimate $\hat{x}_2$.

VI. Conclusion

A fixed-time fuzzy adaptive event-triggered control method is given for strict-feedback uncertain switched nonlinear systems including input saturation. This method can ensure that the system has good signal tracking performance within the prescribed time, and the initial state of the system will not affect the tracking time. Meanwhile, by introducing event triggered control strategies, the communication resources of the system can be effectively saved, and even when the system experiences input saturation, it can still maintain good performance.

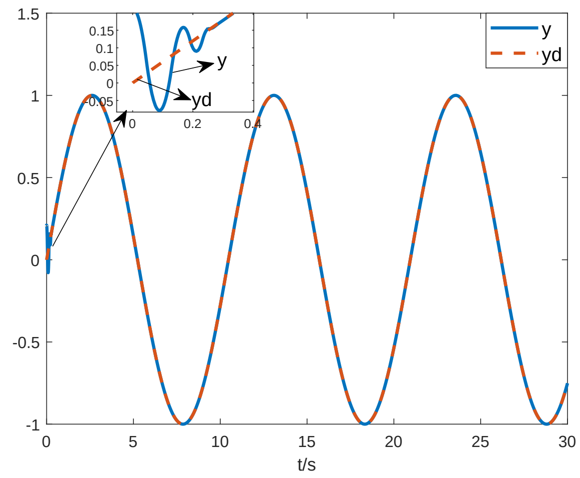

Fig. 3. System output y and reference signal y_d.

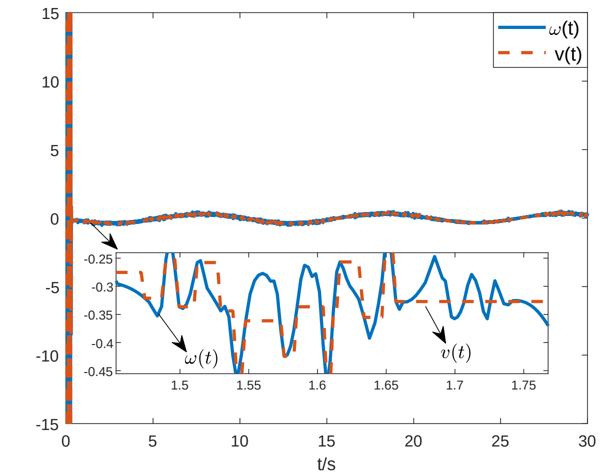

Fig. 4. The trajectories of signals $v(t)$ and $\omega(t)$.

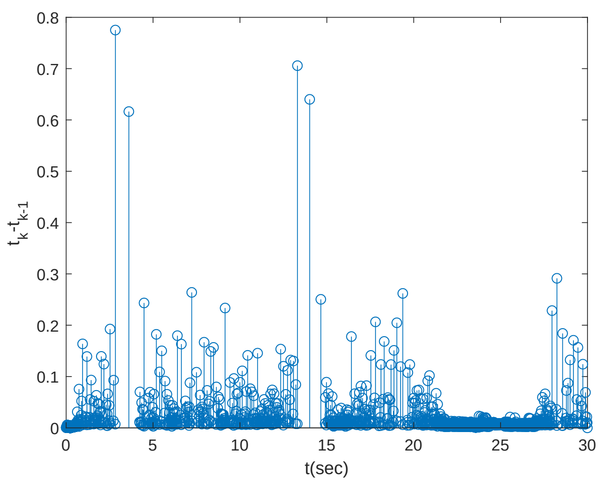

Fig. 5. Time interval of triggering events.

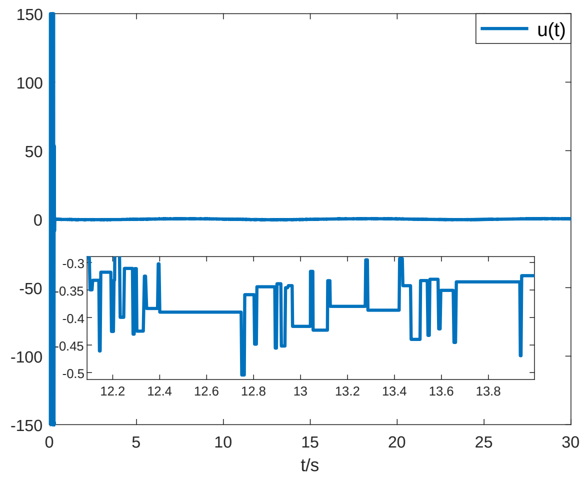

Fig. 6. Actual control signal $u(t)$ with saturation constraints.

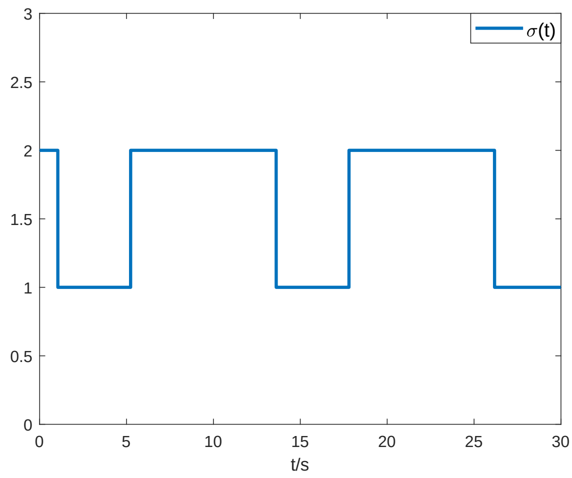

Fig. 7. Swithing signal $\sigma(t)$.

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