Abstract—A soft set is a mapping from a parameter to a universe's power set. Molodtsov suggested soft sets as a method for simulating ambiguous situations. Sandhya and Baiju describe soft L-topological spaces over a soft lattice $L$ with a definite set of parameters $M$, and they have also looked into the continuity of soft L-topological space mappings. Soft L-$T_i$-space ($i = 0; 1; 2; 2\frac{1}{2}; 3; 3\frac{1}{2}; 4; 5; 6$) on a soft L-topological space was explored with some of their properties by the same authors. In this paper, we introduce the concept of connectedness and compactness in soft L-topological spaces. We also analyze how soft L-connected spaces behave when subjected to soft L-continuous mappings, soft L-boundary, and soft L-closure.

Index Terms—Soft L-set, soft L-topology, soft L-connected space, soft L-compact space.

I. INTRODUCTION

Molodtsov [15] was the first to describe soft set theory in 1999. The approach to modeling, ambiguity and uncertainty is altogether novel. Molodtsov [15] demonstrated several applications of soft set theory in various fields. Maji et.al [13], [19] conducted a study on Molodtsov soft sets, wherein they established definitions that revolved around the equivalence of two soft sets, soft set inclusion and containment, the soft set complement, the null soft set, and the absolute soft set. They complemented these definitions with illustrative examples and explanations of fundamental properties. Additionally, various researchers have explored the algebraic aspects of set theory in managing uncertain situations [2], [3], [4], [10], [11], [28], [16], [6], [21], [11], [32], [14], [33], [34]. In 2010, Li F [12] expanded the definition of a soft set to include soft lattices and soft fuzzy sets. Shabir and Naz [29] introduced the concept of soft topological spaces in the year 2011 and studied some basic properties. Sandhya and Baiju [23] used the notion of a soft set initiated by Molodtsov [15] and extended this idea to the field of soft lattices and obtained the topological properties of soft lattices. In 2016, Cigdem Gunduz Aras, Ayse Sonmez, and Huseyin Cakalli [5] presented soft continuous mappings. They also looked at some of its properties. The notion of continuity for soft mappings was provided in 2012 by Hazra, Majumdar, and Samanta [7]; numerous other authors [9], [18] have also explored soft mappings. Furthermore, in 2015, Yang et al. [8] first looked at the concept of soft continuous mapping between two soft topological spaces. In 2015, Tantawy et al. [31] established and studied the separation axioms $T_i$ ($i = 0; 1; 2; 3; 4; 5$) on a soft L-topological space. They clarified that these axioms are soft topological properties under certain soft mapping. Soft separation axioms are also studied in [30]. Ramkumar et al. [20] researched soft Urysohn space.

One of the most important topological characteristics for distinguishing topological spaces is connectedness. The concept of compactness was introduced into topology to generalize the characteristics of closed and bounded subsets of Euclidean space. In 2012, Peyghan et al. [17] introduced and researched soft connected topological spaces. Zorlutuna et al. [34] initially introduced soft compactness in 2012.

Sandhya and Baiju [23] propose Soft Lattice topological spaces (Soft L-topological spaces or Soft L-space) over an initial universe $X$ with a predefined set of parameters $M$. They elucidated key attributes of soft L-topological spaces while providing explanations for what constitute soft L-open and soft L-closed sets. Furthermore, they introduced a generalized concept, the soft L-closure of a soft lattice, as an extension of set closure. In the context of parameterized topologies within an initial universe, the role of parameters was highlighted. Each parameter was assigned its individual topological space, underscoring their significance in the overall framework. The authors [23] established that a soft L-topological space generates a parameterized family of topologies in the initial universe, although the reverse may not hold true. This suggests that constructing a soft L-topological space is not feasible if specific topologies are given for each parameter. Sandhya and Baiju [24] defined properties related to the continuity of soft L-continuous mappings, encompassing aspects such as injectivity, surjectivity, bijectivity, and the composition of soft L-mappings. These mappings maintained a stable set of parameters across the initial universe. Intriguing findings included the exploration of soft open and closed L-mappings, soft L-homeomorphism, and various other concepts. Additionally, they examined soft L-continuous mappings between two soft L-topological spaces, the Cartesian product of soft L-topological spaces [25], and Soft L-separation axioms i.e., soft L-$T_i$-spaces ($i = 0; 1; 2; 2\frac{1}{2}; 3; 3\frac{1}{2}; 4; 5; 6$) for soft L-topological spaces [26], [27]. Also, the requirement for a soft L-topological space to be a soft L-$T_i$-spaces ($i = 0; 1; 2; 2\frac{1}{2}; 3; 3\frac{1}{2}; 4; 5; 6$) was proved. Soft L-invariant properties such as soft L-hereditary and soft L-topological properties were investigated in [26] and [27].

The concept and characteristics of soft L-connectedness and compactness in soft L-topological spaces are discussed in this paper. We also examine the behavior of soft L-connected spaces when soft L-continuous mappings, soft L-boundary and soft L-closure are applied. To demonstrate that several key findings in general topology do not apply to soft L-topological spaces, such as every compact Hausdorff space...
does not have to be normal, we construct a specific soft L-topological space.

II. PRELIMINARIES AND BASIC DEFINITIONS

If we assume the consistency of L, we will consistently regard L as a complete lattice in the context of this investigation.

We define a unary operation denoted as \( r: L \rightarrow L \) as a quasi complementation, provided it exhibits two key properties: first, it is an involution, which means that for all elements \( \alpha \in L \), we have \( \alpha'' = \alpha \); second, it reverses the ordering, meaning that if \( \alpha \leq \beta \), then \( \beta' \leq \alpha' \).

**Definition 1:** [15] “Let’s consider that we have an initial universe set represented by \( X \), and a set of parameters represented by \( M \). We denote the power set of \( X \) as \( \wp(X) \), and if we have a subset \( A \) contained within \( M \), we can define a pair \( (F, A) \) as a soft set over \( X \). In this context, the mapping \( F \) can be described as \( F: A \rightarrow \wp(X) \).

In simpler terms, a soft set over \( X \) can be thought of as a collection of subsets of the universe \( X \), and each subset is associated with a parameter from the set \( A \). To be precise, for any element \( \alpha \in A \), the collection of approximate elements of the soft set \( (F, A) \) is denoted as \( F(\alpha) \).

**Definition 2:** [12] “Let’s consider a triple \( P = (f, X, L) \), where:

(i) \( L \) represents a complete lattice, (ii) \( f: X \rightarrow \wp(L) \) is a mapping, (iii) \( X \) denotes a universe set. For each element \( x \) in the set \( X \), we can define \( f_M \) as a soft lattice over \( L \) if the image of \( f(x) \) under \( f \) is a sublattice of \( L \).

In such a context, we refer to the triple \( P = (f, X, L) \) as the soft lattice denoted by \( f_M \).

**Definition 3:** [23] Consider an initial universe set denoted as \( X \) and a non-empty set of parameters denoted as \( M \).

Let \( T \) be defined as the collection of complete and uniquely complemented soft lattices over \( L \). This collection satisfies the following conditions:

(i) If \( \phi \) and \( L \) belong to \( T \).

(ii) If you take the arbitrary union of soft lattices from \( T \), the result also belongs to \( T \).

(iii) If you take the finite intersection of soft lattices from \( T \), the result is a member of \( T \).

Thus, these collections depending on each parameter result in a soft topological lattice space (also known as a soft L-space) over \( L \).

**Definition 4:** [23] \( (f_M)' = (f_M') \) where \( f': P \rightarrow \wp(L) \) is a mapping given by \( f'(m) = L - (f(m)) \) for all \( m \in M \) is called as the relative complement of a soft lattice \( f_M \) is denoted by \( f_M' \).

**Definition 5:** [23] \( (L, T, M) \) is a soft lattice topological space over \( L \). Then soft L-open sets in \( L \) are members of \( T \).

**Definition 6:** [23] “Consider a triple \( (L, T, M) \) constitutes as a soft lattice topological space defined over \( L \). A soft lattice denoted as \( f_M \) over \( L \) is considered to be a soft L-closed set in \( L \) if its relative complement, denoted as \( (f_M') \), belongs to the set \( T \).

**Definition 7:** [23] “Consider a lattice denoted as \( L \) and a set of parameters represented by \( M \). Now, let’s define \( T \) as the collection encompassing all possible soft lattices that can be constructed over \( L \). In this context, we refer to \( T \) as the soft discrete lattice topology on \( L \), and the triple \( (L, T, M) \) is termed a soft discrete lattice topological space defined over \( L \).

**Definition 8:** [23] “Let’s consider a lattice denoted as \( L \) and a set of parameters represented by \( M \). Now, if we define a collection \( T \) as \( \phi, L \), we refer to \( T \) as the soft indiscrete lattice topology on \( L \). In this context, the triple \( (L, T, M) \) is characterized as a soft indiscrete topological space established over \( L \).

**Definition 9:** [23] “Let’s consider a soft lattice topological space denoted as \( (L, T, M) \) over \( L \), and suppose we have a non-empty subset of \( L \) represented as \( Y \). In this context, we can define a collection \( T_Y \) as follows: \( T_Y = Y f_M \mid f_M \in T \). We refer to \( T_Y \) as the soft relative lattice topology on \( Y \), and the triple \( (Y, T_Y, M) \) is termed a soft \( L \)-subspace of \( f_M \).

**Definition 10:** [24] \( f_M \) is a soft lattice over \( L \). The soft lattice \( f_M \) is called a soft lattice point, denoted by \( (m, M) \), for the element \( m \in M \), if \( f(m) = \{l\} \) and \( f(m') = \phi \) for all \( m' \in M - \{l\} \).

**Definition 11:** [23] “Suppose we have a soft lattice topological space represented as \( (L, T, M) \) over \( L \), and let \( f_M \) be a soft lattice defined over \( L \). In this context, we can define the soft lattice closure of \( f_M \) and denote it as \( \bar{T}M \). This closure is obtained by taking the intersection of all soft L-closed sets that contain \( f_M \).

**Definition 12:** [23] “Let’s consider a soft lattice topological space denoted as \( (L, T, M) \) over \( L \), and suppose we have a soft lattice \( f_M \) defined over \( L \). With \( f_M \), we can associate another soft lattice, denoted as \( f_M^L \), defined as
follows: $\overline{f(m)} = \overline{f(m)}$. In this definition, $\overline{f(m)}$ represents the soft L-closure of $f(m)$ within $T_M$, for each parameter $m$ in the set $M$.

**Definition 13:** [23] “Consider the soft lattice topological space $(L, T, M)$ defined over $L$, and let $g_M^L$ be a soft lattice over $L$ Additionally, suppose we have an element $x \in L$. We say that $x$ qualifies as a soft $L$-interior point of $g_M^L$ if there exists a soft L-open set $f_M^L$ such that $x$ belongs to $f_M^L$ and $f_M^L$ is a subset of $g_M^L$ denoted as $(f_M^L)^o$.”

**Definition 14:** [23] “Let’s consider the soft lattice topological space $(L, T, M)$ defined over $L$, and suppose we have a soft lattice $g_M^L$ over $L$. Additionally, let $x$ be an element belonging to $L$. We say that $g_M^L x$ qualifies as a soft lattice neighborhood of $x$ if there exists a soft L-open set $f_M^L$ such that $x$ is an element of $f_M^L$ and $f_M^L$ is a subset of $g_M^L$.

**Definition 15:** [24] “Take $(L_1, T_1, M_1)$ and $(L_2, T_2, M_2)$ as two soft lattice topological spaces. The mapping $g_f$ is called a soft L-mapping from $L_1$ to $L_2$ denoted by $g_f: (L_1, T_1, M_1) \rightarrow (L_2, T_2, M_2)$, where $f: L_1 \rightarrow L_2$ and $g: M \rightarrow M$ are two mappings. For each soft L-neighbourhood $g_M^L$ of $(f(l), M)$, if there exist a soft L-neighbourhood $f_M^L$ of $(l, M)$ such that $f(g_M^L) \subset f_M^L$, then $g_f$ is a soft lattice continuous mapping at $(l, M)$.

**Definition 16:** [24] “$(L_1, T_1, M_1)$ and $(L_2, T_2, M_2)$ be two soft lattice topological spaces, and $g_f: (L_1, T_1, M_1) \rightarrow (L_2, T_2, M_2)$ . Then
(a) If $g_f(f_M^L)$ of each soft L-open set $f_M^L$ over $L_1$ is a soft L-open set in $L_2$, then $g_f$ is known as a soft L-mapping.
(b) If $g_f(h_M^L)$ of each soft L-closed set $h_M^L$ over $L_1$ is a soft L-closed set in $L_2$, then $g_f$ is called a soft L-closed mapping.”

**Definition 17:** [24] “$(L_1, T_1, M_1)$ and $(L_2, T_2, M_2)$ as two soft lattice topological spaces, $g_f: (L_1, T_1, M_1) \rightarrow (L_2, T_2, M_2)$ be a mapping. If $g_f$ is a bijection, soft L-continuous and $g_f^{-1}$ is a soft L-continuous mapping, then $g_f$ is said to be soft L-homeomorphism from $L_1$ to $L_2$. When a soft homeomorphism $g_f$ exists between $L_1$ and $L_2$, we say that $L_1$ is soft L-homeomorphic to $L_2$.”

**Definition 18:** [26] “$(L, T, M)$ be a soft lattice topological space over $L$ and $l_1, l_2 \in L$ such that $l_1 \neq l_2$. If there exist soft L-open sets $f_M^L$ and $g_M^L$ such that $l_1 \in f_M^L$ and $l_2 \notin f_M^L$ or $l_2 \in g_M^L$ and $l_1 \notin g_M^L$, then $(L, T, M)$ is called a soft L-disconnected space.”

**Definition 20:** [26] “$l \in L$, therefore $f_M^L$ denotes the soft L-set over $L$ for which $l(m) = \{l\}$ for all $m \in M$.”

**Definition 21:** [26] “$(L, T, M)$ be a soft lattice topological space over $L$, and $l_1, l_2 \in L$ s.t. $l_1 \neq l_2$. If $\exists$ soft L-open sets $f_M^L$ and $g_M^L$ s.t. $l_1 \in f_M^L$ and $l_2 \notin f_M^L$ and $l_2 \in g_M^L$ and $l_1 \notin g_M^L$, then $(L, T, M)$ is said to be a soft L-T2-space or soft L-Hausdorff space.”

**Definition 22:** [26] “$(L, T, M)$ is a soft lattice topological space over $L$ Then $(L, T, M)$ is a soft L-T2-space or soft L-Urysohn space if for $l_1, l_2 \in L$ such that $l_1 \neq l_2$, there exist two soft L-open sets $f_M^L$ and $g_M^L$ such that $l_1 \in f_M^L$ and $l_2 \in g_M^L$ and $f_M^L \cap g_M^L = \emptyset$.

**Definition 23:** [26] “$(L, T, M)$ is a soft lattice topological space over $L$, $g_M^L$ be a soft L-closed set in $L$ and $l_1 \in L$ such that $l_2 \notin g_M^L$. If there exist soft L-open sets $f_M^L$ and $g_M^L$ such that $l_1 \in f_M^L$, $g_M^L \subset f_M^L$ and $f_M^L \cap g_M^L = \emptyset$, then $(L, T, M)$ is called a soft L-regular space.”

**Definition 24:** [26] “$(L, T, M)$ be a soft lattice topological space over $L$. Then $(L, T, M)$ is said to be a soft L-T3-space if it soft L-regular and soft L-T1-space.

**Definition 25:** [26] “$(L, T, M)$ be a soft lattice topological space over $L$, then $(L, T, M)$ is called a soft L-completely regular space if every soft L-closed subset $f_M^L$ and any given soft L-point $l_1^L \notin f_M^L$, then there is a soft L-continuous function $f_g: (L, T, M) \rightarrow (L, T, M)$ such that $f(l_1) = \emptyset$ and $f(f_M^L) = \emptyset$. Otherwise, we say $l$ and $f_M^L$ can be separated by a soft L-continuous function.”

**Definition 26:** [26] “A soft L-topological space, denoted as $(L, T, M)$, is characterized as a soft L-T3,-space if it possesses both the properties of being a soft L-completely regular space and a soft L-T1-space.”

**Definition 27:** [27] “$(L, T, M)$ be a soft lattice topological space over $L$, $f_M^L$ and $g_M^L$ be a soft L-closed set s.t. $f_M^L \cap g_M^L = \emptyset$. If $\exists$ soft L-open sets $f_M^L$ and $g_M^L$ s.t. $f_M^L \subset f_M^L$, $g_M^L \subset f_M^L$ and $f_M^L \cap g_M^L = \emptyset$, then $(L, T, M)$ is called a soft L-normal space.”

**Definition 28:** [27] “Suppose we have a soft lattice topological space represented as $(L, T, M)$ defined over $L$. In such a case, we classify $(L, T, M)$ as a soft L-T3-space provided that it satisfies both the conditions of being a soft L-normal space and a soft L-T1-space.”

**Definition 29:** [27] “$(L, T, M)$ be a soft lattice topological space over $L$ and $f_M^L$, $g_M^L$ be two non-empty soft L-subset over $L$. Then we say $A_M^L$, $B_M^L$ are two separated soft L-sets if $A_M^L \cap B_M^L = \emptyset$ and $A_M^L \cap B_M^L = \emptyset$.”

**Definition 30:** [27] “$(L, T, M)$ be a soft lattice topological space over $L$. A soft L-topological space $(L, T, M)$ is said to be soft L-completely normal if for any two non-empty separate soft L-sets $A_M^L$, $B_M^L$, $\exists$ $F_M^L$, $G_M^L \subset \subset T$ s.t. $A_M^L \subset F_M^L$, $B_M^L \subset G_M^L$ and $F_M^L \cap G_M^L = \emptyset$.”

**Definition 31:** [27] “A soft L-topological space denoted as $(L, T, M)$ is classified as a soft L-T3-space when it exhibits the combined characteristics of being a soft L-completely normal space and a soft L-T1-space.”

**Definition 32:** [27] “$(L, T, M)$ be a soft lattice topological space over $L$. A soft L-topological space $(L, T, M)$ is said to be soft L-perfectly normal space if it is soft L-normal and every soft L-closed subset has countable intersection of soft L-open subsets.”

**Definition 33:** [27] “$(L, T, M)$ be a soft lattice topological space over $L$. A soft L-topological space $(L, T, M)$ is called soft L-T0-space when it is a soft L-perfectly T0-space.”

### III. SOFT L-CONNECTNESS

**Definition 34:** Consider $(L, T, M)$ as a soft lattice topological space over $L$. If $\exists$ no $f_M^L$, $g_M^L \in L - \{m\}$ s.t. $f_M^L \cap g_M^L = \emptyset$ and $f_M^L \cup g_M^L = L$, then $(L, T, M)$ is called soft L-connected, otherwise $(L, T, M)$ is called soft L-disconnected.

**Definition 35:** Let $(L, T, M)$ be a soft lattice topological space over $L$. A soft L-topological space $(L, T, M)$ is called soft L-connected if there does not have a separation of $L$. 

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Example 2: Let \( L = \{l_1, l_2\}, M = \{m_1 = \text{expensive,} m_2 = \text{cheap}\} \) and \( T = \{L, f_{M1}, f_{M2}, f_{M3}, f_{M4}, f_{M5}\} \) is a soft L-topological space over \( L \), where \( f_{M1}, f_{M2}, f_{M3}, f_{M4}, f_{M5} \) are soft lattices over \( L \), defined as follows:
\[
\begin{align*}
  f_{M1}(m_1) &= \{l_1\}, f_{M1}(m_2) = \{l_2\}, \\
  f_{M2}(m_1) &= \{l_1, l_2\}, f_{M2}(m_2) = \{l_1\}, \\
  f_{M3}(m_1) &= \{l_1, l_2\}, f_{M3}(m_2) = \{l_1\}, \\
  f_{M4}(m_1) &= \{l_1, l_2\}, f_{M4}(m_2) = \{l_1\}, \\
  f_{M5}(m_1) &= \{l_1\}, f_{M5}(m_2) = \{l_1, l_2\}.
\end{align*}
\]
Then \( T \) is a soft L-topology on \( L \) and \( (L, T, M) \) is a soft L-connected space.

Definition 36: Let \( (L, T, M) \) be a soft lattice topological space over \( L \) and \( f_{M} \) be a soft L-subset over \( L \). Then \( f_{M} \) is soft L-connected, if it is soft L-connected as a soft \( L \)-subspace.

Theorem 4: Let \( (L, T, M) \) be a soft lattice topological space over \( L \) and \( f_{M} \) be a soft L-subset of \( L \). Then \( f_{M} \) is soft L-closed if and only if \( f_{M}^{\uparrow} \subseteq f_{M}^{\uparrow} \).

Proof: \( f_{M} \) is soft L-closed iff \( f_{M} \subseteq f_{M}^{\uparrow} \).
\( f_{M}^{\uparrow} \) is soft L-closed iff \( f_{M} \subseteq (f_{M}^{\uparrow})^{\uparrow} \).
i.e., \( f_{M} \subseteq (f_{M}^{\uparrow})^{\uparrow} \).

Theorem 5: Let \( (L, T, M) \) be a soft lattice topological space over \( L \) and \( f_{M} \) be two soft L-subsets of \( L \). Then
\( (i) f_{M} \cap f_{M}^{\uparrow} \subseteq f_{M} \Rightarrow f_{M} \subseteq (f_{M}^{\uparrow})^{\uparrow} \),
\( (ii) f_{M} \cap (f_{M}^{\uparrow})^{\uparrow} \subseteq (f_{M}^{\uparrow})^{\uparrow} \),
\( (iii) f_{M} \cap (f_{M}^{\uparrow})^{\uparrow} \subseteq f_{M} \),
\( (iv) ((f_{M}^{\uparrow})^{\uparrow})^{\uparrow} \subseteq (f_{M}^{\uparrow})^{\uparrow} \).

Proof: (i) \( f_{M} \subseteq f_{M}^{\uparrow} \). Since \( (f_{M}^{\uparrow})^{\uparrow} \subseteq (f_{M}^{\uparrow})^{\uparrow} \), we get \( f_{M} \subseteq (f_{M}^{\uparrow})^{\uparrow} \).

(ii) \( f_{M} \subseteq (f_{M}^{\uparrow})^{\uparrow} \). Then by \( (i) \), \( f_{M} \subseteq (f_{M}^{\uparrow})^{\uparrow} \).

(iii) \( f_{M} \subseteq (f_{M}^{\uparrow})^{\uparrow} \). Hence \( f_{M} \subseteq (f_{M}^{\uparrow})^{\uparrow} \).

(iv) \( (f_{M}^{\uparrow})^{\uparrow} \subseteq (f_{M}^{\uparrow})^{\uparrow} \). This implies that \( \exists \) a soft L-open set \( g_{M}^{L} \) s.t. \( l \in g_{M}^{L} \) and \( g_{M}^{L} \cap (f_{M}^{\uparrow})^{\uparrow} \).

Assume on the contrary that \( l \in ((f_{M}^{\uparrow})^{\uparrow})^{\uparrow} \), then \( f_{M} \subseteq (f_{M}^{\uparrow})^{\uparrow} \).

Theorem 6: Let \( (L, T, M) \) be a soft L-T2-space and \( Y \) be the non-empty subset of \( L \) containing finite number of points, then \( Y \) is soft L-closed.

Proof: Let \( Y = \{l_1\} \). Now we prove that \( Y \) is soft L-closed. If \( l_2 \) is a point of \( L \) different from \( l_1 \), then \( l_1 \) and \( l_2 \) have distinct soft L-neighbourhoods \( f_{M}^{L} \) and \( g_{M}^{L} \) respectively.
Since \( f_{M}^{L} \) does not soften L-intersect \( l_2 \), point \( l_1 \) does not belong to the soft L-closure of \( \{l_2\} \). As a result, the soft L-closure of \( \{l_2\} \) is \( \{l_1\} \) itself and hence it is soft L-closed. Since \( Y \) is arbitrary, it is true for all subsets of \( L \) containing a finite number of points, follows that \( Y \) is soft L-closed.

Definition 38: Let \( (L, T, M) \) be a soft lattice topological space over \( L \). Then \( f_{M} \) denotes the soft L-boundary of soft lattice \( f_{M}^{L} \) over \( L \) and defined by \( f_{M}^{L} = (f_{M}^{L})^{\uparrow} \).

Theorem 7: A soft L-topological space \((L, T, M)\) is soft L-connected if and only if every non-empty soft L-subspace has a non-empty soft L-boundary.

Proof: The proof is by contradiction. Assume that a non-empty soft L-subspace \( f_{M}^{L} \) of a soft L-connected space
(L, T, M) has empty soft L-boundary. Then \( f^L_M \) is soft L-open and \((f^L_M) \cap (L \cap L(M)') = \phi\). Let \( l \) be a soft L-limit point of \( f^L_M \). Then \( l \in (f^L_M) \) but \( l \notin ((f^L_M)')\). Particularly, \( l \notin (f^L_M)' \) and \( l \in f^L_M \). So \( f^L_M \) is both soft L-open and closed. By theorem 4, \((L, T, M)\) is soft L-disconnected. This contradiction shows that \( f^L_M \) has a non-empty soft L-boundary. Conversely, suppose that \( L \) is soft L-disconnected. Then by theorem 1, \((L, T, M)\) has a soft L-subset \( f^L_M \) which is both soft L-open and soft L-closed. Then \((f^L_M) = f^L_M\), \((f^L_M) = (f^L_M)'\), and \((f^L_M) \cap (f^L_M) = \phi\). So \( f^L_M \) has empty soft L-boundary, a contradiction, thereby \((L, T, M)\) is soft L-connected.

**Theorem 8:** Let two soft lattices \( f^L_M \) and \( g^L_M \) be a soft L-disconnected in soft l-topological space \((L, T, M)\) and \( h^L_M \subseteq \ f^L_M \) or \( g^L_M \). Then 

\[
\text{Proof:} \ \text{By contradiction, let} \ h^L_M \text{ be neither contained in} \ f^L_M \text{ nor in} \ g^L_M. \ \text{Then} \ (h^L_M \cap f^L_M) \text{ and} \ (h^L_M \cap g^L_M) \text{ are both non-empty soft L-open subsets of} \ h^L_M \text{ s.t.} \ (h^L_M \cap f^L_M) \cap (h^L_M \cap g^L_M) = \phi \text{ and} \ (h^L_M \cap f^L_M) \cup (h^L_M \cap g^L_M) = h^L_M. \ \text{This implies that} \ (h^L_M \cap f^L_M) \text{ and} \ (h^L_M \cap g^L_M) \text{ is a soft L-disconnection of} \ h^L_M. \]

**Theorem 9:** Let \( g^L_M \) be a soft L-connected subset of a soft l-topological space \((L, T, M)\) and \( f^L_M \) be a soft L-subset of \( L \) s.t. \( g^L_M \subseteq f^L_M \subseteq g^L_M \). Then \( f^L_M \) is soft L-connected.

\[
\text{Proof: } \text{It is sufficient to prove} \ g^L_M \text{ is soft L-connected. By contradiction, let us assume that} \ g^L_M \text{ is soft L-disconnected. Then} \exists \ \text{a soft L-disconnection} \ (h^L_M, k^L_M) \text{ of} \ g^L_M. \ \text{That is} \ h^L_M \cap g^L_M = \phi \text{ and} \ h^L_M \cap g^L_M \neq \phi. \text{There are two soft L-open subsets in} \ g^L_M \text{ s.t.} \ (h^L_M \cap g^L_M) \text{ and} \ (h^L_M \cap g^L_M) \text{ is a soft L-disconnection of} \ g^L_M. \]

**Corollary 1:** If \( f^L_M \) is a soft L-connected and soft L-subspace of a soft l-topological space \((L, T, M)\), then \((f^L_M)\) is soft L-connected.

### IV. SOFT L-COMPACTNESS

**Definition 39:** A family \( A = \{ f^L_{i, \alpha} \}_{\alpha \in I} \) of a soft L-set is a cover of a soft L-set \( f^L_M \) if \( f^L_M \subseteq \bigcup_{\alpha \in I} f^L_{i, \alpha} \). If each element of \( A \) is a soft L-open set, then it is a soft L-open cover. A subcover of \( A \) is a subfamily of \( A \) which is also a cover.

**Definition 40:** A soft L-topological space \((L, T, M)\) is called soft L-compact if each soft L-open cover of \( L \) has a finite subcover.

Also let \((L, T, M)\) and \((L, T, M)\) be two soft L-topological spaces. If \( T_1 \subseteq T_2 \), then \( T_2 \) is finer than \( T_1 \). We say \( T_1 \) is soft L-comparable with \( T_2 \) if \( T_1 \subseteq T_2 \) or \( T_2 \subseteq T_1 \). Then we have the following.

**Proposition 10:** Consider \((L, T, M)\) to be a soft L-compact and \( T_1 \subseteq T_2 \). Then \((L, T, M)\) is soft L-compact.

**Proof:** Let \( \{ f^L_{i, \alpha} \}_{\alpha \in I} \) be a soft L-open cover of \( L \) by soft L-open sets of \((L, T, M)\). Since \( T_1 \subseteq T_2 \), then \( \{ f^L_{i, \alpha} \}_{\alpha \in I} \) is a soft L-open cover of \( L \) by soft L-open sets of \((L, T, M)\). But \((L, T, M)\) is soft L-compact. Therefore \( L^L_M \subseteq \bigcup_{\alpha \in I} f^L_{i, \alpha} \) for some \( \alpha_1, \ldots, \alpha_n \in I \). Hence \((L, T, M)\) is soft L-compact.
soft L-basis element

Next, we characterize soft L-compact spaces in terms of basis element in the following theorem.

**Theorem 16:** A soft L-topological space \((L, T, M)\) is soft L-compact if and only if there is a soft L-basis \(B_L^0\) for \(T\) that has a finite subcover for every cover of \(L\) by elements of \(B_L^0\).

**Proof:** Let \((L, T, M)\) be a soft L-compact. Then \(T\) is a soft L-basis for \(T\) trivially. As a result, any \(L\) cover by the members of \(T\) has a finite subcover.

Conversely, let \(\{f_{L_{1,2},M}\} \subseteq I\) be a soft L-open cover of \(L\). After that, \(\{f_{L_{1,2},M}\}\) may be represented as a union of basis elements for each \(a \in I\). They combine to produce a soft L-open cover of \(L\), such as \(\{g_{B_{1,2},M}^n\} \subseteq J\).

Therefore \(L_M^I \subseteq \{g_{B_{1,2},M}^n\} \cup \cdots \cup \{g_{B_{m,n},M}^n\}\), for some \(B_1, \ldots, B_m \in J\).

Let \(\{g_{B_i,M}^n\} \subseteq \{f_{L_{1,2},M}\}\) for each \(i = 1, \ldots, n\).

This implies \(\{f_{L_{1,2},M}\} \subseteq \{f_{L_{1,2},M}\}\) is a finite subcover of \(L\). Hence \((L, T, M)\) is a soft L-compact.

\[\blacksquare\]

**V. CONCLUSION**

A method for dealing with uncertainty called soft set theory has been extended in this work. By utilizing soft lattice in a soft L-topology setting, we broaden the concept of soft set. This study focuses on the soft L-connected space and soft L-compact space on soft L-topological spaces. We come to the conclusion that this work is just the start of a new framework and that we have investigated a number of novel ideas that will be useful in future theoretical studies. This work has future research in the field of noncompact covering properties, viz., paracompactness, metacompactness, subparacompactness, submetacompactness, Lindelofness, Para-Lindelofness, etc. in soft L-topological spaces. We also suggest future directions that one can extend this work towards new directions like Linear Diophantine Fuzzy sets and Spherical linear Diophantine Fuzzy sets.

**REFERENCES**


