Abstract—The aim of blind image deconvolution is to reconstruct a clear image from a noisy blurred image when the blur kernel is unknown. The approach for blind image deconvolution could be divided into two steps, including blur kernel estimation as well as non-blind image reconstruction. Especially, the quality of the restored image is significantly influenced by the estimation of the blur kernel. To acquire a more accurate blur kernel, it is essential to get the edge information of the image accurately. In this paper, a novel blind image deconvolution model via an adaptive weighted $L_0$ gradient prior is proposed. Due to the incorporation of the adaptive weighted matrix, our proposed model can more effectively describe the edge information of the image so as to make the estimated blur kernel more accurate. In addition, an efficient algorithm is designed to leverage the sparsity of patch-wise minimal pixels (PMPs) in deblurring. Experimental results demonstrate the superiority of the proposed method when compared to other related blind deconvolution methods.

Index Terms—Blind image deconvolution, Adaptive weighted matrix, $L_0$ gradient prior, Patch-wise minimal pixels.

1. INTRODUCTION

In the process of image acquisition, the precision of the acquisition equipment and external factors can cause a degree of blurriness to the obtained image. Consequently, the recovery of a high-quality image from its blurred counterpart has emerged as a formidable challenge within the field of image processing. Typically, we conceptualize the blurred image as the outcome of a linear convolution involving the original image as well as a blur kernel. The blur kernel is also referred to as the point spread function (PSF). Mathematically, the process of image degradation could be expressed as

$$ B = k * I + \eta. $$

In this equation, $I$ stands for the original image, $B$ denotes the corrupted image, $k$ represents the blur kernel, and $\eta$ signifies additive Gaussian noise. Depending on whether the blur kernel is known, image deconvolution could be categorized into two distinct types: non-blind deconvolution and blind deconvolution. Recently, there has been notable progress in the research on non-blind image deconvolution [1]–[5]. However, in numerous practical applications, the blur kernel is commonly unknown. Under these circumstances, one needs to estimate both $I$ and $k$ from $B$. This problem is commonly referred to as blind image deconvolution. In contrast to non-blind deconvolution, the blind one exhibits sensitivity to noise and has the potential to generate multiple solutions [6]. Therefore, research on blind image deconvolution has consistently remained a hotspot in the field of image processing [7]–[10]. To handle the inherent ill-posed feature of blind image deconvolution, it becomes crucial to apply regularization to both the image and the blur kernel by integrating prior knowledge. When employing regularization methods to address the issue of blind image deconvolution, the choice of an appropriate regularization term plays a pivotal role [11], [12].

To address the problem of blind deconvolution, numerous regularization techniques have undergone extensive research. In [13], a blind deconvolution approach was introduced, using the $H^1$ norm to optimize the image $I$ as well as the blur kernel $k$. However, the $H^1$ norm exhibits robust isotropic smoothing properties, which enables the approach falls short in effectively preserving image edges. To overcome this shortcoming, an innovative blind image deconvolution technique was presented by employing TV regularization rather than $H^1$ norm [14]. Although TV regularization can better process the edge information of the image, TV norm readily transforms the smooth signal into a signal that is piecewise constant, resulting in staircase effects in the flat region of the image. Subsequently, to alleviate the staircase effects, a model based on high-order TV was introduced in [15].

In recent years, numerous models that leverage alternative sparse prior constraints have been employed in the context of blind image deconvolution. In [16], the authors discovered that the gradient of natural images follows the heavy-tailed distribution. Consequently, they introduced a mixed Gaussian model to emulate this characteristic when estimating blur kernels. However, this method is time-consuming and inefficient. Therefore, a normalized sparse prior $L_1/L_2$ was presented in [17], which can significantly reduce computational costs compared with the approach in [16]. To further describe the sparsity of image gradients, in [18], Xu et al. introduced a technique in their work that employs a piecewise function to approximate the $L_0$ norm. This approach enhances the deblurring performance. In addition, Pan et al. [19] presented a new method for blur kernel estimation by directly applying the $L_0$ norm of image gradients as the constraint term. This method can better highlight the edge information of the image and enhance the...
accuracy of the blur kernel estimation. To further improve
the restoration effect of text images, in [20], Pan et al.
presented a blind image deconvolution model by integrating a
$L_0$ sparse prior framework with a combination of image and
image gradient. This approach was developed based on an
analysis of the structural attributes inherent to text images. In
[21], they extended this method to the restoration of non-text
images and achieved satisfactory results. Moreover, inspired
by the dark channel prior in image defogging algorithms
[22], Pan et al. [23] proposed a blind image deconvolution
approach, using the dark channel prior as a foundation.
The experimental outcomes demonstrate that this approach
exhibits enhanced restoration performance across a range
of image scenarios. However, Yan et al. [24] pointed out
that in the absence of conspicuous dark pixels in images,
the approach in [23] may not always achieve satisfactory
results. According to this factor, they proposed an extreme
channel prior by merging the dark channel with the bright
channel. Afterwards, Chen et al. [25] found that when there
are not enough extreme pixels in images, the method in [24]
cannot accurately estimate the blur kernel. Therefore, they
presented a blind image deconvolution approach based on
local maximum gradient prior, which improved the accuracy
of blur kernel estimation. Nevertheless, these methods have
higher computational costs. For the purpose of enhancing
algorithmic efficiency, Wen et al. [26] put forward a sparse
prior model rooted in patch-wise minimal pixel (PMP). They
further devised a novel algorithm to efficiently address the
formulated model. The experimental results illustrate that this
method can significantly enhance computational efficiency
and achieve better restoration results.

While many of the methods studied earlier can yield
reasonably satisfactory results, they use $\|\nabla I\|_0$ as non-
natural image prior so as to find salient edges in the image
[27]. In more recent work, Pang et al. [28], [29] introduced
some novel regularization approaches for image denoising.
These involved integrating an adaptive weighted matrix with
the gradient operator. This special matrix has the capability
to change the orientation of the gradient operator, causing
it to lean towards a larger weight. Thus, it could more
effectively characterize the image local features. Inspired by
this observation, we incorporate the adaptive weighted matrix
into the realm of blind deconvolution. This article has three
main contributions:

1. We construct a blind image deconvolution model by
employing an adaptive weighted $L_0$ gradient prior. With
the incorporation of the adaptive weighted matrix, our
proposed model can better highlight edge information of
the image, resulting in the estimated blur kernel more
accurately.

2. We design an efficient algorithm to solve the proposed
model, which can sparsely induce the PMP of latent
image during the blur kernel estimation process.

3. Extensive experimental results concerning blind image
deconvolution have been furnished to showcase the
cutting-edge performance of the proposed method.

The rest of this paper is organized as follows. In Section
2, some related preliminary work are first introduced, and
then a blind image deconvolution model based on an adap-
tive weighted $L_0$ gradient prior is proposed. The solution
method for the proposed model is provided in Section 3. In
Section 4, some relevant experimental results are presented to
demonstrate the effectiveness and superiority of the proposed
method. Finally, some conclusions are drawn in Section 5.

2. THE PROPOSED MODEL

In general, approaches for blind image deconvolution can
be categorized into two main groups, as discussed in [30].
The first category involves the joint estimation of both blur
kernel and potentially clear image. The second category
focuses on initially estimating the blur kernel and then
applying non-blind deconvolution using this estimated kernel
to obtain potentially clear image. In our work, we adopt the
second approach, and its success relies on emphasizing the
edge details of the image during the blur kernel estimation
phase.

More specifically, Pang et al. [29] introduced an
anisotropic total variation (ATV) model designed for
denoising Gaussian images. This model exhibits improved
diffusion characteristics along the orientation of local fea-
tures’ tangents, thereby enhancing its denoising capabilities.
To couple more efficiently with the local structures, they
implemented a more effective approach by incorporating the
adaptive weighting matrix $T$ to construct different weights.
The specific form of $T$ is

$$
T(i,j) = \begin{bmatrix}
0 & 0 & 0 \\
1/2 & 0 & 1/2 \\
0 & 1 & 0
\end{bmatrix}
$$

(2)

Here, $G_δ(·)$ represents a Gaussian convolution kernel. $\nabla_y B$ and $\nabla_\beta B$ refer to the horizontal and vertical gradients of $B$,
respectively. $δ$ and $ν$ serve as two adjustable parameters.

In the context of blind image deconvolution, we explore a
method that involves integrating the image gradient operator
$\nabla$ with an adaptive weighted matrix $T$. We utilize $L_0$ norm
as a means of imposing a non-natural image prior. Further-
more, we incorporate the patch-wise minimal pixel (PMP)
technique as a representation of the natural image prior, and
we enforce a constraint of $\|k\|_2^2$ on the blur kernel. As a
result of these considerations, we propose the following blind
image deconvolution model, termed the adaptive weighted
$L_0$ gradient prior model:

$$
\min_{k,I} \|k*I-B\|^2_2 + \gamma\|k\|^2_2 + \mu\|T\nabla I\|_0,
$$

s.t. $\mathcal{P}(I)(o) \sim p(x)$, for $o \in \{1, \ldots, P\}.
$$

(3)

Here, $γ$ and $µ$ stand for two weighting parameters, and $p(x)$
signifies the probability density function subject to super
Laplacian distribution under the set threshold. $\mathcal{P}(I)(o)$
represents a collection of PMPs over non-overlapping patches,
which is denoted as

$$
\mathcal{P}(I)(o) = \min_{(i,j) \in \Omega_o} \left( \min_{c \in \{r,g,b\}} I(i,j,c) \right).
$$

(4)

In this way, the image $I \in \mathbb{R}^{M \times N \times c}$ can be partitioned
into $P$ non-overlapping blocks, each of size $r \times r$, where
$P = \left[ \frac{M}{r} \right] \cdot \left[ \frac{N}{r} \right]$. $\Omega_o$ refers to the $o$-th non-overlapping block.
3. ALGORITHM

The blind image deconvolution algorithm proposed in this paper can be divided into two steps. The first step is to estimate the blur kernel, and the second one is to utilize this estimated blur kernel to carry out non-blind image deconvolution, ultimately yielding the final estimated image.

In the first step, an alternating iterative algorithm is employed to solve the intermediate latent image and the blur kernel. To be more precise, the intermediate latent image could be acquired by solving the following constrained problem:

\[
\begin{aligned}
\min_{I} & \| k^\nu \ast I - B \|_2^2 + \mu\| T \nabla I \|_0, \\
\text{s.t.} & \quad \mathcal{P}(I)(o) \sim p(x), \text{ for } o \in \{1, \ldots, P\}, \\
\end{aligned}
\]  

(5)

where \( k^\nu \) represents the blur kernel of the temporary estimation. By introducing two auxiliary variables \( p \) and \( q \), the problem (5) can be approximated by

\[
\begin{aligned}
\min_{I, p, q} & \| k^\nu \ast I - B \|_2^2 + \mu\| q \|_0 + \omega\| \nabla I - p \|_2^2 \\
+ & \beta\| Tp - q \|_2^2, \\
\text{s.t.} & \quad \mathcal{P}(I)(o) \sim p(x), \text{ for } o \in \{1, \ldots, P\}, \\
\end{aligned}
\]  

(6)

where \( \omega \) and \( \beta \) are two penalty parameters. Then, we need to minimize three subproblems \( p, q \) and \( I \) with other variables fixed.

It is noteworthy that during the solving process, we use the threshold shrinkage step to promote sparsity within the PMP of the image. To be specific, let \( I_s(o) \) represent the PMP subset of the image \( I \). Within the iteration process, we impose the following threshold constraint on this subset:

\[
I^{t+1,n+1}_s(o) = \begin{cases} 
0, & |I^{t+1,n+1}_s(o)| < \lambda, \\
I^{t+1,n+1}_s(o), & \text{otherwise},
\end{cases}
\]  

(7)

where \( I^{t+1,n+1}_s(o) \) signifies the PMP subset following the application of the threshold constraint, and \( \lambda > 0 \) is the threshold parameter. Then \( I^{t+1,n} \) can be updated as follows:

\[
I^{t+1,n} = I^{t+1,n} \circ (1 - M^{t+1,n} \circ \mathcal{P}) \circ (\hat{I}^{t+1,n}).
\]  

(8)

Here, \( \mathcal{P}^T \) denotes the inverse operation of \( \mathcal{P} \), and \( M \in \mathbb{R}^{M \times N} \) stands for the binary mask of the PMP subset corresponding to image \( I \).

The \( p \)-subproblem could be represented as

\[
p^{t+1,n+1} = \arg \min_{p} \omega\| \nabla I^{t+1,n} - p \|_2^2 + \beta\| Tp - q^{t+1,n} \|_2^2.
\]  

(9)

We delve into its optimality condition. Let \( p = [p_1, p_2] \), \( q = [q_1, q_2] \). Then the associated linear equation can be organized as

\[
\begin{bmatrix}
\omega + \beta t_1^2 & 0 \\
0 & \omega + \beta t_2^2
\end{bmatrix}
\begin{bmatrix}
p_1^{t+1,n+1} \\
p_2^{t+1,n+1}
\end{bmatrix}
= 
\begin{bmatrix}
\omega \nabla_x \hat{I}^{t+1,n} + \beta_1 q_1^{t+1,n} \\
\omega \nabla_y \hat{I}^{t+1,n} + \beta_2 q_2^{t+1,n}
\end{bmatrix}.
\]  

(10)

Evidently, the subproblem (9) is a smooth optimization problem. Consequently, the explicit solution of \( p^{t+1,n+1} \) can be achieved from (10) through a simple calculation, namely

\[
\begin{aligned}
p_1^{t+1,n+1} = \frac{\omega \nabla_x \hat{I}^{t+1,n} + \beta_1 q_1^{t+1,n}}{\omega + \beta t_1^2}, \\
p_2^{t+1,n+1} = \frac{\omega \nabla_y \hat{I}^{t+1,n} + \beta_2 q_2^{t+1,n}}{\omega + \beta t_2^2}.
\end{aligned}
\]  

(11)

The \( q \)-subproblem could be formulated as

\[
qu^{t+1,n+1} = \arg \min_{q} \| q \|_0 + \beta\| Tp^{t+1,n+1} - q \|_2^2.
\]  

(12)

This problem is solved via the approximate minimization method [31]. Then we have

\[
qu^{t+1,n+1} = \begin{cases} 
0, & (Tp^{t+1,n+1})^2 < \frac{\mu}{\beta}, \\
T_{p^{t+1,n+1}}, & \text{otherwise}.
\end{cases}
\]  

(13)

The \( I \)-subproblem can be articulated as

\[
I^{t+1,n+1} = \arg \min_{I} \| k^\nu \ast I - B \|_2^2 + \omega\| \nabla I - p^{t+1,n+1} \|_2^2.
\]  

(14)

In terms of the optimality condition, we can readily deduce the Euler-Lagrange equation of (14). Under periodic boundary condition, it can be efficiently computed using the fast Fourier transform (FFT). The updated scheme can be found in (15), where \( \nabla_x \) and \( \nabla_y \) represent difference operators in the horizontal and vertical directions, respectively. \( \mathcal{F}(\cdot) \) stands for conjugate operator. \( \mathcal{F} \) and \( \mathcal{F}^{-1} \) respectively denote the FFT as well as its inverse transform.

To solve the blur kernel \( k \), the estimation method based on the gradient domain can be employed, which has been shown to be more stable and accurate in [30]. Then we get

\[
k^{t+1} = \arg \min_{k} \| k \ast (\nabla I^{t}) - \nabla B \|_2^2 + \gamma\| k \|_2^2.
\]  

(15)

The solution of (16) can also be obtained through FFT:

\[
k^{t+1} = \mathcal{F}^{-1}\left(\frac{\mathcal{F}(\nabla I^{t}) \circ \mathcal{F}(\nabla B) + \mathcal{F}(\nabla y I^{t}) \circ \mathcal{F}(\nabla y B)}{\mathcal{F}(\nabla y I^{t}) \circ \mathcal{F}(\nabla y B) + \gamma} \right).
\]  

(16)

By iteratively solving the intermediate latent image and the blur kernel, the final estimated blur kernel can be obtained. We give the blur kernel estimation algorithm for solving (3) in Algorithm 3.1. In the experiments, we set \( \alpha = 2, J = 3, \beta_{\max} = 10^5, \beta_0 = 2\mu \).

Algorithm 3.1 Blur kernel estimation algorithm

**Input and initialization:** degraded image \( B \), initial blur kernel \( k^1 \).

**for** \( n = 0 : J - 1 \) **do**

\[
\begin{aligned}
&\beta \leftarrow \beta_0, I^{0} \leftarrow B, t \leftarrow 0, \\
&\text{while } \beta < \beta_{\max} \text{ do} \\
&\quad I^{t+1,0} \leftarrow I^{t}, \\
&\quad \beta \leftarrow \alpha \beta, \omega \leftarrow \alpha \omega, \\
&\quad t \leftarrow t + 1.
\end{aligned}
\]  

**end while**

\[
I^{t+1} \leftarrow I^{t+1,J}.
\]

**end for**

\[
k^{t+1} \leftarrow I^{t+1}.\]

**Compute** \( q^{t+1,n+1} \) by (11).

**Compute** \( p^{t+1,n+1} \) by (13).

**Compute** \( I^{t+1,n+1} \) by (15).

**end for**

\[
k \leftarrow k^{t+1}, \hat{I} \leftarrow I^{t+1}.
\]

**Output:** kernel estimation \( k \), intermediate image \( \hat{I} \).

In the second step, namely the stage of non-blind image deconvolution, we employ the same algorithm as described
in [23] and [26] to acquire the ultimate estimation of the clear image.

4. NUMERICAL EXPERIMENTS

In this section, we unveil a wealth of experimental findings showcasing the prowess and superiority of our proposed method over other blind deconvolution approaches in the field: the dark channel algorithm [23] (hereafter referred to as “Pan-16”) and the patch-wise minimal pixels-based blind image deconvolution algorithm [26] (hereafter referred to as “Wen-20”). All the experiments happened in the MATLAB environment, utilizing a PC equipped with a robust 3.20GHz CPU and a generous 16GB RAM. To assess the quality of restoration results, the peak signal-to-noise ratio (PSNR) and the structural similarity (SSIM) are utilized as quantitative indexes, whose definitions are as follows:

\[
\text{PSNR} = 10 \log_{10} \left( \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (u_{i,j} - I_{i,j})^2 \right)^{-1},
\]

\[
\text{SSIM} = \frac{(2\mu_I\mu_u + c_1)(2\sigma_{Iu} + c_2)}{(\mu_I^2 + \mu_u^2 + c_1)(\sigma_I^2 + \sigma_u^2 + c_2)},
\]

Here, $I$ and $u$ respectively stand for the original and restored images, while $\mu$ and $\sigma$ respectively signify the local mean value and the standard deviation of the image. $\sigma_{Iu}$ corresponds to the covariance between $I$ and $u$. The constants $c_1$ and $c_2$ are employed to prevent extremely small denominator values. In general, enhanced values of PSNR and SSIM mean an improved quality for the recovered image.

In experiments, we initially evaluate our approach using four grayscale images sourced from the dataset introduced in [32], as illustrated in Fig. 1. And the blur kernels used for image blurring are displayed in Fig. 2. Moreover, all blurred images are added with zero mean Gaussian noise with a standard deviation of 0.01. Table I presents the numerical comparison results of the PSNR and SSIM indicators. It’s evident that our approach consistently yields the highest PSNR values, and in the majority of cases, it also attains the highest SSIM values. Due to the addition of an adaptive weighted matrix in this paper, which can better highlight the edge information of the intermediate latent image during the blur kernel estimation stage, a more precise blur kernel can be achieved. This leads to an improved quality in the restored image.

In the following experiments, some images of real scenes are tested, including face images and natural images. In the case of the first face image, its primary parameters are configured as follows: $\mu = 0.004$, $\omega = 0.0005$, $\lambda = 0.00001$, $\delta = 1$. As for the second face image, our method’s primary parameters are adjusted to $\mu = 0.00004$, $\omega = 0.0005$, $\lambda = 0.00001$, $\delta = 1$. The absence of distinct edges and textures in face images poses a challenge for achieving accurate kernel estimation. Fig. 3 shows the restoration results of two face images. Compared with the other two approaches, our method obtains higher image quality and estimates the blur kernel more precisely. In particular, when examining the restoration outcomes of the first face image, it becomes apparent that the restored image achieved through our method exhibits superior visual characteristics. This distinction is further highlighted through the locally magnified view in the bottom-left corner, providing a clearer observation. In addition, compared to other approaches, our restored image is more natural. From the restored blur kernel positioned in the upper left corner, we observe that our approach excels in managing isolated noise within the blur kernel, resulting in a more precise estimation of the blur kernel. Regarding the second face image, from the locally enlarged image positioned in the lower-left corner, we observe that the restored images obtained by other approaches exhibit artifacts in a flat region. Fortunately, our method effectively mitigates these artifacts. Moreover, in contrast to the other two methods, our approach produces a more precise estimation of the blur kernel.

For the first natural image, the main parameters of our approach are set to $\mu = 0.004$, $\omega = 0.0005$, $\lambda = 0.00001$, $\delta = 1$. As for the second natural image, the primary parameters of our method are adjusted to $\mu = 0.0004$, $\omega = 0.0005$, $\lambda = 0.00001$, $\delta = 1$. Fig. 4 presents the experimental results of the two natural images. It is obvious that our proposed approach acquires better visual effects compared to the other two approaches. Specifically, from the locally enlarged image placed in the lower-left corner, we observe that our method reduces image artifacts to a certain extent. Furthermore, it can be noted from the estimated blur kernel in the upper-left corner that our method achieves a more precise estimation of the blur kernel. It means that the adaptive weighted matrix can enhance the accuracy of blur kernel estimation, thereby enhancing the quality of restored images. Particularly, for face images and natural images, we provide the CPU time spent by different approaches in Table II. From this table, it can be easily observed that our method is considerably more time-efficient compared to Pan-16, and is closer to the time used by Wen-20.

<table>
<thead>
<tr>
<th>Method</th>
<th>Pan-16</th>
<th>Wen-20</th>
<th>Ours</th>
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<td>Image</td>
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<td>SSIM</td>
<td>PSNR</td>
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<td>im01_ker02</td>
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<td>im01_ker03</td>
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<td>32.31</td>
</tr>
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<td>im02_ker01</td>
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<td>0.931</td>
<td>33.45</td>
</tr>
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</table>
Fig. 1. Test images.

Fig. 2. Blur kernels used in image degradation.

Fig. 3. Blind deconvolution results of two face images achieved by different methods.
Fig. 4. Blind deconvolution results of two natural images gained by different methods.
To achieve more precise blur kernel and raise the quality of restored image, in this paper, we introduced an adaptive weighted matrix and developed a new blind image deconvolution method based on an adaptive weighted $L_0$ gradient prior. Compared with the existing two approaches, in most cases, our proposed approach could get the highest PSNR and SSIM values. Additionally, the experimental results for real scene images indicate that the proposed method can acquire more accurate blur kernels and suppress image artifacts.

Looking ahead, we aim to delve into various acceleration techniques to further diminish the computational cost. In addition, our focus will extend to applying the adaptive strategy to other image processing challenges, such as image segmentation, hyperspectral image fusion and unmixing.

## REFERENCES


