Further Study on Goal Programming Models

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***Abstract*—In this study, we investigate various goal programming models and highlight their problematic outcomes, offering our revised versions. While previous publications relied on numerical examples to determine the optimal alternative, we provide theoretical explanations for these examples to enhance researchers' understanding of goal programming models. Additionally, we employ a fast integration approach to solve a challenging calculus problem concerning the centroid of trapezoidal fuzzy numbers. Our findings aim to assist researchers in comprehending intricate theoretical concepts and facilitate the application of these results in their own research endeavors.**

***Index Terms*—Centroid, Mathematical derivations, Fuzzy numbers, Goal programming model, Fuzzy preference relation, Decision-making**

I. Introduction

WE will directly point out those questionable results in Xu [1]. In his incomplete fuzzy preference relation for those incomplete information, for example in Equation (1), a_{13}, a_{25} and a_{36}, and those corresponding relations a_{31}, a_{52} and a_{63}, Xu used the same variable *x* to represent the unknown value. However, his approach violates two fundamental assumptions in fuzzy preference relation: (a) a_{ij} and a_{st} are related, but in general, they are not equal, and (b) $a_{ij} + a_{ji} = 1$, for the additive relation. It indicates that his LOT2 model and LOP 4 model are evaluated on a false foundation. In the following, we will provide two revisions. First, base on his assumption, we revise his approach to suggest a simplified method. Second, we provide an extension to accommodate the shortcomings of his approach such that the two important criteria for fuzzy preference relation are satisfied. A revision of the approach of Xu [1] with greater efficiency in terms of time-saving and identical effectiveness in terms of accuracy is presented in the paper to tackle group decision-making problems. With which Xu put forth the fuzzy judgment associative matrix takes the form of uncertain additive or multiplicative linguistic preference relations given by experts. The complicate computations of goal programming developed by Xu [1] are significantly simplified with a concise and fast problem-solving algorithm for attainment of the best alternative. Meanwhile, some of his assumptions with weak rationale in mathematical and practical applications are pinpointed and modified without giving rise to the alternation of pivotal outcome. Two illustrative numerical examples are developed to illustrate our presented process. Our easy method will undoubtedly result in improvement in theoretic and practical development. This paper makes a important modification to the technique proposed by Xu [1] for dealing incomplete preference relations. Rather than representing incomplete information as .5 (no preference), incomplete information is represented by x and 1-x, with different variables for each incomplete pair. The row sum of preferences is then computed and used to determine priorities. The research goal is to new solution approach that will produce. For the accuracy problem, this topic should have no corrected answer. Different method may imply different the most important alternative such that different methods should only compare with their speed. According to this principle, we have an efficient approach which is a better result than other known algorithms. Our important modification results in a very efficient method to derive priority vector for alternatives. If derived findings are differed greatly with the previous results our approach provide an easily computed method to found the most important alternative.

Saaty is the original author to use the pairwise comparison procedure to deal with decision-making problems to decide the priority vectors for a given family of possible alternatives. To the best of our knowledge, there are more than twenty thousand papers that had discussed pairwise comparison process in their solution algorithms.

In this article, we will follow this research trend to study preference relationship with fuzzy and incomplete environments by pairwise comparison methods that were constructed by several experts. The goal is to locate the best alternative. Sometimes, the decision-making problem is too complicated that is beyond the ability of some invited experts and then those experts only offer incomplete pairwise comparison matrix for alternatives under some criteria. Moreover, under the fuzzy environments, more experts are hesitant to provide a specific answer for those unclear comparison that will result in fuzzy comparison matrices with incomplete data. Some researchers tried to fulfill those missing data and then handle the decision-making problems under complete data. On the other hand, other researchers tried to develop new solution procedure to deal with those incomplete data environments.

In this paper, we study the goal programming models of Xu [1]. There are 15 papers, namely Alonso et al. [2], Xu, [3, 4], Dopaizo et al. [5], Wang and Parkan [6], Xu [7-9], Alonso et al. [10], Wang and Parkan [11], Xu [12-16], that have referred Xu [1] in their references.

We run a detailed examination of above mentioned fifteen papers to find that those papers only mentioned Xu [1] in

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their introduction but they did not provide a careful study for the priority weights proposed by Xu [1]. Hence, our revisions for Xu [1] will present those shortcoming in his derivations and provide a different solution approach for future researchers.

Consequently, our findings will provide assistance to subsequent researchers in revision of their procedure for solving practical problems.

II. Brief Reviewing for Previous Findings

Xu [1] based on that an additive consistent incomplete fuzzy preference relation does not hold in the general case, to develop a multi-objective programming model (MOP1) and then he constructed goal programming model (LOP1) to solve the minimization problem of MOP1. When the goal functions are fair, Xu revised LOP1 to LOP2. He considered that a multiplicative consistent incomplete fuzzy preference relation does not hold in the general case in order to construct another multi-objective programming model (MOP2). And then he constructed goal programming model (LOP3) to solve the minimization problem of MOP2. When the goal functions are fair, he modified LOP3 to LOP4.

For the collective priority vector of two or more incomplete fuzzy preference relations, Xu [1] extended LOP3 and LOP4 to LOP5 and LOP6 to incorporate multiple incomplete fuzzy preference relations. On the contrary, Xu [1] first discussed the fuzzy preference relation under incomplete data.

Next, he considered the consistent additive fuzzy preference relation under incomplete data. Third, he examined consistent multiplicative fuzzy preference relation under incomplete data and pointed out that the incomplete fuzzy preference relations provided by experts are generally not consistent.

Hence, Xu [1] established some goal programming systems to reduce the inconsistency and then based on these models directly to select the priority weights for fuzzy preference relationship under incomplete data. Thus, Xu [1] claimed that his results are reasonable and logical. We will point out his assumptions of pairwise relationships are questionable to show that a pair should denoted as (x, 1-x) for preference relationship and then his previous findings must contain questionable results.

The severe questionable derivations in Xu [1] is that he didn't provide any details and explanations on acquisitions of minimum solutions while constructing several new models. We try to in the paper use the row arithmetic mean method to find the best alternative. There is no any explanations as to why they use the row sum to derive the priority vector of an incomplete fuzzy preference relation. The effectiveness and feasibility of the process are demonstrated by two numerical examples quoted from Xu [1].

We may suggest the decision-makers to use the row arithmetic sum in derivation of priority vector.

III. Our Revisions

Based on the numerical example of Xu [1], we examine the following decision-making procedure. Among six alternatives, which are expressed as A_k, for k = 1,2,3,4,5, and 6, the goal is to select the best alternative.

The decision-maker provided an incomplete fuzzy preference relation as the next matrix:

$$R=\begin{bmatrix}0.5&0.4&x&0.3&0.8&0.3\\0.6&0.5&0.6&0.5&x&0.4\\x&0.4&0.5&0.3&0.6&x\\0.7&0.5&0.7&0.5&0.4&0.8\\0.2&x&0.4&0.6&0.5&0.7\\0.7&0.6&x&0.2&0.3&0.5\end{bmatrix}. \quad (2.1)$$

Xu [1] used his model (LOP 2) to select the priority weights w as

$$w=(0.144,0.192,0.133,0.267,0.142,0.122) \quad (2.2)$$

such that the ranking of these six alternatives is

$$A_4 \succ A_2 \succ A_1 \succ A_5 \succ A_3 \succ A_6. \quad (2.3)$$

On the other hand, he used another model (LOP 4) to select the priority weights w as

$$w=(0.156,0.179,0.155,0.200,0.154,0.156) \quad (2.4)$$

such that the ranking of these six alternatives is

$$A_4 \succ A_2 \succ A_1 = A_6 \succ A_3 \succ A_5. \quad (2.5)$$

For both models, A_4 is the best alternative.

Motivated by Xu [1], we here only concern the selection of the best alternative.

With the same matrix, we directly compute the row sum as

$$x+2.3, \quad (2.6)$$

$$x+2.6, \quad (2.7)$$

$$1.8+2x, \quad (2.8)$$

$$3.6,\ 2.4+x, \quad (2.9)$$

and

$$2.3+x. \quad (2.10)$$

On the basis of the assumption of Xu [1] for fuzzy element, if we accept that $x \approx 0.5$, then the priority weights for alternatives are

2.8, 3.1, 2.8, 3.6, 2.9 and 2.8 . (2.11)

Based on Equation (2.11), the best alternative is x_4.

The priority weights are examined in the following, for all six alternatives, not just only need to consider the priority weights for two of them. We have considered for all six alternatives.

However we must point out that the arbitrary assumption of an identical variable x to all fuzzy elements is unreasonable both in mathematical and practical applications. Hence under the condition of $r_{ij}+r_{ji}=1$ in the incomplete fuzzy preference relation, we assign three variables: x, y, and z for corresponding fuzzy elements and rewrite above matrix R as follow:

$$R=\begin{bmatrix} 0.5 & 0.4 & x & 0.3 & 0.8 & 0.3 \\ 0.6 & 0.5 & 0.6 & 0.5 & y & 0.4 \\ 1-x & 0.4 & 0.5 & 0.3 & 0.6 & z \\ 0.7 & 0.5 & 0.7 & 0.5 & 0.4 & 0.8 \\ 0.2 & 1-y & 0.4 & 0.6 & 0.5 & 0.7 \\ 0.7 & 0.6 & 1-z & 0.2 & 0.3 & 0.5 \end{bmatrix}. \quad (2.12)$$

With the same calculation of row sum, Equations (2.6-2.10) can be improved as

$$x+2.3, \quad (2.13)$$

$$2.6+y, \quad (2.14)$$

$$2.8-x+z, \quad (2.15)$$

$$3.6,\ 3.4-y, \quad (2.16)$$

and

$$3.3-z. \quad (2.17)$$

Since $0\le z,y,x\le 1$, we only have to consider the priority weights for A_3 and A_4 with $2.8-x+z$ and 3.6, respectively.

By observing the certain numerical values in matrix of Equation (2.8) that

$$0.2\le r_{ij}\le 0.8 . \quad (2.18)$$

The range of the element r_{ij} in Equation (2.18) should be [0, 1] instead of [0.2, 0.8].

Five referees first individually decided the price, and then the first referee will consider the five price to revise his previous price. The second referee will run the same procedure, and then the other referee will reconsidered their price again. Finally, the standard derivation among prices will decrease.

We may accept this fact as a rule of thumb that will be valid for those uncertain values falling in between

$$0.2\le x,y,z\le 0.8. \quad (2.19)$$

Using Equation (2.19) it yields that

$$2.8-x+z\le 2.8-0.2+0.8=3.4<3.6. \quad (2.20)$$

Obviously A_4 is the best alternative.

Next, we quote his numerical example 4.2 for problem with three decision makers and four alternatives which are denoted as A_i, $i=1,2,,4$.

The three decision-makers with incomplete fuzzy preference relation as the following matrices R_i, $i=1.2.3$, respectively.

$$R_1=\begin{bmatrix} 0.5 & 0.6 & x & 0.7 \\ 0.4 & 0.5 & 0.2 & 0.8 \\ x & 0.8 & 0.5 & 0.4 \\ 0.3 & 0.2 & 0.6 & 0.5 \end{bmatrix}, \quad (2.21)$$

$$R_2=\begin{bmatrix} 0.5 & 0.8 & 0.4 & x \\ 0.2 & 0.5 & 0.3 & 0.6 \\ 0.6 & 0.7 & 0.5 & 0.3 \\ x & 0.4 & 0.7 & 0.5 \end{bmatrix}, \quad (2.22)$$

and

$$R_3=\begin{bmatrix} 0.5 & 0.3 & 0.4 & 0.6 \\ 0.7 & 0.5 & x & 0.5 \\ 0.6 & x & 0.5 & 0.7 \\ 0.4 & 0.5 & 0.3 & 0.5 \end{bmatrix}. \quad (2.23)$$

Xu [1] used his model (LOP 5) to obtain the priority vector v as

$$v=(0.276,0.225,0.300,0.199) \quad (2.24)$$

such that the ordering for proposed four alternatives is listed below,

$$A_3\succ A_1\succ A_2\succ A_4. \quad (2.25)$$

On the other hand, he used another model (LOP 6) to obtain the priority vector v as

$$v=(0.265,0.236,0.276,0.223), \quad (2.26)$$

such that the ordering for proposed four alternatives is listed below,

$$A_3\succ A_1\succ A_2\succ A_4. \quad (2.27)$$

For both models, A_3 is the best alternative.

At the outset, we still follow his rationale. For example 4.2, we compute the row sum of three matrices that satisfies fuzzy preference relation as

$$5.3+2x, \quad (2.28)$$

$$5.2+x, \quad (2.29)$$

$$5.6+2x, \quad (2.30)$$

and

$$4.9+x. \quad (2.31)$$

It's obvious that for any value x, A_3 is the best alternative. The ranking still satisfies

$$A_3\succ A_1\succ A_2\succ A_4. \quad (2.32)$$

With our manner, by rationally assigning distinct variables to fuzzy elements we modify matrices R_1, R_2, and R_3 as

$$R_1=\begin{bmatrix} 0.5 & 0.6 & x & 0.7 \\ 0.4 & 0.5 & 0.2 & 0.8 \\ 1-x & 0.8 & 0.5 & 0.4 \\ 0.3 & 0.2 & 0.6 & 0.5 \end{bmatrix}, \quad (2.33)$$

$$R_2=\begin{bmatrix} 0.5 & 0.8 & 0.4 & y \\ 0.2 & 0.5 & 0.3 & 0.6 \\ 0.6 & 0.7 & 0.5 & 0.3 \\ 1-y & 0.4 & 0.7 & 0.5 \end{bmatrix}, \quad (2.34)$$

and

$$R_3 = \begin{bmatrix} 0.5 & 0.3 & 0.4 & 0.6 \\ 0.7 & 0.5 & z & 0.5 \\ 0.6 & 1-z & 0.5 & 0.7 \\ 0.4 & 0.5 & 0.3 & 0.5 \end{bmatrix}. \quad (2.35)$$

Row sum of three matrices that satisfied fuzzy preference relation can be obtained as

$$5.3 + x + y, \quad (2.36)$$

$$5.2 + z, \quad (2.37)$$

$$7.6 - x - z, \quad (2.38)$$

and

$$5.9 - y. \quad (2.39)$$

Since there are three decision-makers, we may use the certain values in matrices to provide the exact value or lower and upper bound for incomplete elements as below: no any arguments can be used to support this statement,

$$x = 0.4, \quad (2.40)$$

$$0.6 \le y \le 0.7, \quad (2.41)$$

and

$$0.2 \le z \le 0.3. \quad (2.42)$$

Consequently, based on Equations (2.36-2.39), we can infer the following restrictions,

$$6.3 \le 5.3 + x + y \le 6.4, \quad (2.43)$$

$$5.4 \le 5.2 + z \le 5.5, \quad (2.44)$$

$$6.9 \le 7.6 - x - z \le 7, \quad (2.45)$$

and

$$5.2 \le 5.9 - y \le 5.3. \quad (2.46)$$

From Equations (2.43-2.46), the best alternative A_3 can be found.

We thus demonstrate that the row arithmetic method can derive the same results for both numerical examples. Our approach with much efficient computation improves the findings of Xu [1] which was lack of logic that did not satisfy the basic rule for fuzzy preference relation.

Building upon above analysis and refinement, an exciting aftermath is that two problem-solving processes can be combined and become complement each other in pursuit of optimal goals while both effectiveness and greater efficiency can be attained simultaneously. That is, the row quick arithmetic method takes the major role as in most cases that all uncertain or fuzzy variables fall between maximum and minimum elements of linguistic preference matrix. is false. In this sentence, "linguistic preference matrix" should be "fuzzy preference relation", and all the elements (including the known and the unknown elements) fall between 0 and 1 rather than between maximum and minimum elements. The goal programming approach of Xu [1] will be applied only in the scenarios that fuzzy variables are found not in between max-and-min interval.

Why is the given algorithm only motivated by two simple numerical examples in Xu [1] and without any theoretical analysis. It is clear that this algorithm is unjustifiable and untenable.

IV. Algorithm

In the following algorithm, we denoted by R that is a n by n incomplete fuzzy preference relation matrix. Fuzzy element or variable: Any element of R whose exact value is uncertain or deemed as a fuzzy variable between 0 and 1.

We hence organize the proposed approach as a systemized algorithm with five concise steps as below:

Step 1: Confirmation of Applicability

Go to next step if for any two elements of R, r_{ij} and r_{ji}, the relation, $r_{ij}+r_{ji}=1$, is held. Otherwise the algorithm is not applicable and the whole process terminates.

Step 2: Computation for row sums

By summing up all the elements in each row of R, we obtain a total of n row sums. Each of these row sums represents a real number or may contain one or more fuzzy variables.

Step 3: Evaluation of fuzzy variables

Find value or interval of value for each of fuzzy variables through observation and comparison of elements in matrix R with perceivable assumption that the value of each fuzzy variable falls in the interval [A,B] in which $A = \min\{r_{ij}, i, j = 1, 2, \ldots, n\}$ and $B = \max\{r_{ij}, i, j = 1, 2, \ldots, n\}$. Next.

Step 4: Finding of local optimal alternative

Find local optimal alternative by putting all types of fuzzy variables back into row sums secured in step 2 and carrying out the comparisons and simple computations.

Step 5: Testing and Finding of global optimal alternative

If local optimal alternative keeps unchanged by assigning any values out of interval [A, B] to fuzzy variables and redoing step 4, the global optimal alternative has been found. Otherwise employ the goal programming model other than Xu [1] to derive the optimal alternative.

V. Review of a Related Problem

In this section, we try to provide a short review for Wang et al. [17], Shieh [18] and Dat et al. [19] with respect to a difficult derivation of centroid. Up to now, there are twenty seven related papers and then we may classify them in the following categories:

(i) Theoretical approach: Dat et al. [20], Mei et al. [21], Gavina et al. [22], Bakar and Gegov [23], Momeni and Gildeh [24], Hesamian and Akbari [25].

(ii) Comparison among different ranking methods: Gegov and Bakar [26], Hajjari [27, 28], Sotoudeh-Anvari [29,30], Bai et al. [31], El-Kholy et al. [32], Song et al. [33].

(iii) Application orient: Bakar and Gegov [34], Bakar et al. [35], Wang and Wang [36], Das and Guha [37].

(iv) New methods for ranking fuzzy numbers: Dhanasekar et al. [38], Zhang et al. [39], Chai et al. [40], Shahsavari-Pour et al. [41], Yanbing et al. [42], Yang et al. [43], Nasseri et al. [44], Yu et al. [45].

However, none of them had presented a clear derivation for the centroid proposed by Wang et al. [17]. Hence, we will try to show a detailed derivation for a hard mathematical derivation to assist practitioners absorb the content mentioned by Wang et al. [17].

VI. Centroid-index for Trapezoidal Fuzzy Numbers

For a fuzzy number, A, its membership function is defined as

$$A(x)=\begin{cases}A_L(x), & a\le x\le b\\ w, & b\le x\le c\\ A_R(x), & c\le x\le d\\ 0, & o.w.\end{cases}, \tag{6.1}$$

where $0<w\le 1$ is a constant, $A_R:[c,d]\to[0,w]$ is a monotonic decreasing continuous function (right wing), and $A_L:[a,b]\to[0,w]$ is a monotonic increasing continuous function (left wing).

For a fuzzy number A with an invertible A_L and A_R left and right wings, Wang et al. [17] defined the following centroids formulae for fuzzy numbers:

$$\bar{x}_0(A)=\frac{\int_{-\infty}^{\infty}xA(x)dx}{\int_{-\infty}^{\infty}A(x)dx}, \tag{6.2}$$

and

$$\bar{y}_0(A)=\frac{\int_0^w y\left[A_R^{-1}(y)-A_L^{-1}(y)\right]dy}{\int_0^w\left[A_R^{-1}(y)-A_L^{-1}(y)\right]dy}. \tag{6.3}$$

For a trapezoidal fuzzy number $A=[a,b,c,d;w]$ with its membership function

$$A(x)=\begin{cases}w\left(\dfrac{x-a}{b-a}\right), & a\le x\le b\\ w, & b\le x\le c\\ w\left(\dfrac{d-x}{d-c}\right), & c\le x\le d\\ 0, & o.w.\end{cases}, \tag{6.4}$$

where $0<w\le 1$, Wang et al. [17] derived

$$\bar{x}_0(A)=\frac{1}{3}\left[\frac{ba-cd}{(c+d)-(b+a)}+d+c+b+a\right], \tag{6.5}$$

and

$$\bar{y}_0(A)=\frac{w}{3}\left[1-\frac{c-b}{(b+a)-(d+c)}\right]. \tag{6.6}$$

In Shieh [18] and Dat et al. [19] both of them cited Equations (6.5) and (6.6) in their paper. Shieh [18] derived the result of Equation (6.6) again by his new approach but he did not obtain Equation (6.5) once more to indicate that the derivation of Equation (6.5) is difficult.

The goal of our discussion is to present a simpler derivation of Equation (6.5).

VII. Our Approach

Based on Equation (6.2), and $A(x)$ are piece-wisely defined, we divide the integration into three parts as

$$\bar{x}_0(A)=\frac{\int_a^b xw\left(\frac{x-a}{b-a}\right)dx+\int_b^c xw\,dx+\int_c^d xw\left(\frac{d-x}{d-c}\right)dx}{\int_a^b w\left(\frac{x-a}{b-a}\right)dx+\int_b^c w\,dx+\int_c^d w\left(\frac{d-x}{d-c}\right)dx}. \tag{7.1}$$

We recall Horowitz [46] to solve $\int_a^b x(x-a)dx$ as follows,

$$\begin{aligned}\int_a^b(x-a)xdx&=\left[\frac{(a-x)^2}{2}x+\frac{(a-x)^3}{6}\right]\Bigg|_a^b,\\ &=\frac{(a-b)^2}{2}b+\frac{(a-b)^3}{6},\\ &=\frac{(a+2b)(a-b)^2}{6}.\end{aligned} \tag{7.2}$$

Similarly, we derive that $\int_c^d(d-x)xdx$ as follows,

$$\begin{aligned}\int_c^d(d-x)xdx&=\left[x\frac{(d-x)^2}{(-2)}-(1)\frac{(d-x)^3}{(-2)(-3)}\right]\Bigg|_c^d,\\ &=\frac{c(c-d)^2}{2}-\frac{(c-d)^3}{6},\\ &=\frac{(2c+d)(d-c)^2}{6}.\end{aligned} \tag{7.3}$$

According to our results of Equations (7.2) and (7.3), we find the numerator of Equation (7.1) as follows

$$\frac{w}{6}(a+2b)(b-a)+ w(c-b)+\frac{w}{6}(2c+d)(d-c). \tag{7.4}$$

For the denominator of Equation (7.1), we know that represents for the area between $A(x)$ and the x-axis such that we can directly find the denominator of Equation (7.1) as

$$\frac{w}{2}(b-a)+w(c-b)+\frac{w}{2}(d-c). \tag{7.5}$$

From our results of Equations (7.4) and (7.5), we derive that

$$\bar{x}_0(A)=\frac{(a+2b)(b-a)+3\left(c^2-b^2\right)+(2c+d)(d-c)}{3[(d-c)+2(c-b)+(b-a)]}. \tag{7.6}$$

We simplify the denominator of Equation (7.6),

$$(d-c)+2(c-b)+(b-a)=d+c-b-a, \quad (7.7)$$

and the numerator of Equation (7.5),

$$(a+2b)(b-a)-3(b^2-c^2)+(2c+d)(d-c),$$
$$=-2ba-b^2-a^2+2dc+d^2+c^2-cd+ab$$
$$=(d+c)^2-(b+a)^2-cd+ab$$
$$=(d+c-b-a)(d+c+b+a)-cd+ab. \quad (7.8)$$

Based on our derivations of Equations (7.7) and (7.5), we imply that

$$\bar{x}_0(A)=\left(\frac{1}{3}\right)\left[(d+c+b+a)-\frac{cd-ab}{c+d-a-b}\right], \quad (7.9)$$

which is the desired result as Equation (6.5).

VIII. A Related Problem

In this section, we will proposed an unsolved problem in inventory models that was appeared in Bose et al. [47] and Moon and Giri [48]. Moon and Giri [48] examined Bose et al. [47] to provide several improvements. Moon and Giri [48] recalled that the objective function denoted as $\mathrm{TC}(n,K)$, where n is a discrete variable and K is a continuous variable with $0<K<1$. Moon and Giri [48] assumed that

$$\frac{\partial}{\partial K}\mathrm{TC}(n,K)=G(n,K). \quad (8.1)$$

For a given values on n, Moon and Giri [48] knew that $G(n,1)$ is positive, such that for the existence of a solution for

$$G(n,K)=0, \quad (8.2)$$

it has a solution. Based on the Intermediated Value Theorem, Moon and Giri [48] wanted that

$$G(n,0)<0, \quad (8.3)$$

that is the following relationship,

$$\left(p-\frac{c_{22}}{R_2}\right)\left(1-e^{-R_2\frac{n-1}{n}H}\right)<\frac{c_{21}}{R_1}\left(1-e^{-R_1\frac{n-1}{n}H}\right), \quad (8.4)$$

under the restriction $n\ge 2$, and the condition,

$$R_1>R_2, \quad (8.5)$$

with two abbreviations,

$$R_1=r-i_1=0.2-0.08=0.12, \quad (8.6)$$

and

$$R_2=r-i_2=0.2-0.14=0.06. \quad (8.7)$$

Based on Equation (8.4), we assume an auxiliary, denoted as f(x), as follows,

$$f(x)=\frac{c_{21}}{R_1}\left(1-e^{-R_1\frac{n-1}{n}H}\right),$$
$$-\left(x-\frac{c_{22}}{R_2}\right)\left(1-e^{-R_2\frac{n-1}{n}H}\right). \quad (8.8)$$

According to Equation (8.8), we derive that

$$f\left(x=\frac{c_{22}+c_{21}}{R_2}\right)=c_{21}\left(g(R_1)-g(R_2)\right), \quad (8.9)$$

with our second auxiliary function,

$$g(x)=\frac{1}{x}\left(1-e^{-x\frac{n-1}{n}H}\right). \quad (8.10)$$

Based on Equation (8.10), we obtain that

$$g'(x)=\frac{-1}{x^2}e^{-x\frac{n-1}{n}H}\left(e^{x\frac{n-1}{n}H}-1-x\frac{n-1}{n}H\right)<0, \quad (8.11)$$

under the condition of Equation (8.5), and then we derive that

$$g(R_1)-g(R_2)<0. \quad (8.12)$$

We find that

$$f\left(\frac{c_{22}}{R_2}\right)=\frac{c_{21}}{R_1}\left(1-e^{-R_1\frac{n-1}{n}H}\right)>0, \quad (8.13)$$

and we combine results of Equations (8.9) and (8.12), we verify that

$$f\left(\frac{c_{22}+c_{21}}{R_2}\right)<0. \quad (8.14)$$

Based on our above discussions, we can provide the following directions for future research. Motivated by Equations (8.13) and (8.14), we can predict that the first future goal is to locate some sub-domain, denoted as Ω, such that $f(x)>0$ is valid. Moreover, the second future goal is to show that

$$p\in\Omega. \quad (8.15)$$

IX. A Further Discussion of Fuzzy Inventory Models

In this section, we will examine a recently published paper of Wu [49] that studied Glock et al. [50] to remove an extra criteria proposed by Glock et al. [50]. Recently in the research field of inventory models, there is a trend in which inventory models are solved using algebraic methods to help practitioners without calculus backgrounds easily absorb the concept of inventory models in business management. Consequently, algebraic methods are used to prove that the optimal strategy is to adopt the assumption of equal order quantities.

We try to introduce this kind of inventory models to those practitioners who are not familiar to analytic approach and then we will use algebraic methods to handle the unsolved problem left by Glock et al. [50].

Algebraic methods will be used to verify that the equal order quantity policy for each replenishment cycle is the optimal strategy.

Hence, two consecutive replenishment cycles are considered without loss of generality. The first and second replenishment cycles with order quantities will be shown as Q_1 and Q_2. The average cost for these two cycles is denoted as

$$f(Q_1,Q_2)=\frac{1}{\frac{Q_1}{D}+\frac{Q_2}{D}}\left[A+\frac{h}{2D}Q_1^2+\right.$$
$$\left.+\frac{A}{3D}(\Delta_2-\Delta_1)+A+\frac{h}{2D}Q_2^2+\frac{A}{3D}(\Delta_2-\Delta_1)\right]. \quad (9.1)$$

The first goal is to prove that $Q_1^*=Q_2^*$, while the second goal is to derive the results by an algebraic method.

We simplify the expression of Equation (9.1) as

$$f\left(Q_1,Q_2\right)=\alpha\frac{Q_1^2+Q_2^2}{Q_1+Q_2}+\frac{\beta}{Q_1+Q_2}, \quad (9.2)$$

with two abbreviations,

$$\alpha=\frac{h}{2}, \quad (9.3)$$

and

$$\beta=2A\left(D+\frac{\Delta_2-\Delta_1}{3}\right), \quad (9.4)$$

to simplify the expressions.

If we follow the algebraic method mentioned in Lan et al. [51] to decompose one as

$$1=a\frac{b}{a(a+b)}+b\frac{a}{b(a+b)}, \quad (9.5)$$

and then we can apply Equation (9.5) to derive that

$$c=a\frac{b}{a(a+b)}c+b\frac{a}{b(a+b)}c. \quad (9.6)$$

We rewrite a two-variable quantic polynomial to complete their square to obtain

$$\begin{aligned} &ax^2+by^2+c= \\ &a\left(x^2+\frac{bc}{a(a+b)}\right)+b\left(y^2+\frac{ac}{b(a+b)}\right) \\ &=2(x+y)\sqrt{\frac{abc}{(a+b)}} \\ &+a\left(x-\sqrt{\frac{bc}{a(a+b)}}\right)^2+b\left(y-\sqrt{\frac{ac}{b(a+b)}}\right)^2. \quad (9.7)\end{aligned}$$

Hence, we transfer Equation (9.2) in a new combination as follows

$$\begin{aligned} &f\left(Q_1,Q_2\right)=\sqrt{2\alpha\beta}+ \\ &\frac{\alpha}{Q_1+Q_2}\left[\left(Q_1-\sqrt{\frac{\beta}{2\alpha}}\right)^2+\left(Q_2-\sqrt{\frac{\beta}{2\alpha}}\right)^2\right], \quad (9.8)\end{aligned}$$

to yield that

$$Q_1^*=\sqrt{\frac{\beta}{2\alpha}}=Q_2^*, \quad (9.9)$$

and

$$f\left(Q_1^*,Q_2^*\right)=\sqrt{2\alpha\beta}. \quad (9.10)$$

This approach is elegant but it is based on the fascinating decomposition of Equation (9.6) which is equivalent to apply Equation (9.5).

The most well know decomposition formula for 1 is the triangular formula,

$$1=\cos^2\theta+\sin^2\theta, \quad (9.11)$$

and its transformation

$$1=\left(\frac{t^2-1}{t^2+1}\right)^2+\left(\frac{2t}{t^2+1}\right)^2. \quad (9.12)$$

Selecting the suitable decomposition formula for 1 will be a challenge for researchers such that the above solution approach may be difficult to absorb for ordinary practitioners.

Therefore, the algebraic method proposed by Chang et al. [52] is adopted to use a new parameter Q_3 with

$$Q_3=Q_1+Q_2. \quad (9.13)$$

Hence, converting $f\left(Q_1,Q_2\right)$ to $f\left(Q_1,Q_3\right)$ as

$$f\left(Q_1,Q_3\right)=\frac{2\alpha}{Q_3}Q_1^2-2\alpha Q_1+\alpha Q_3+\frac{\beta}{Q_3}. \quad (9.14)$$

We complete the square of the variable Q_1 in Equation (9.14) to derive that

$$\begin{aligned} f\left(Q_1,Q_3\right)&=\frac{2\alpha}{Q_3}\left(Q_1-\frac{Q_3}{2}\right)^2+\frac{\beta}{Q_3}+\frac{\alpha}{2}Q_3 \\ &=\frac{2\alpha}{Q_3}\left(Q_1-\frac{Q_3}{2}\right)^2+\left(\sqrt{\frac{\beta}{Q_3}}-\sqrt{\frac{\alpha}{2}Q_3}\right)^2 \\ &+\sqrt{2\alpha\beta}, \quad (9.15)\end{aligned}$$

to locate the optimal point

$$Q_1^*=\frac{Q_3^*}{2}, \quad (9.16)$$

$$Q_3^*=\sqrt{\frac{2\beta}{\alpha}}, \quad (9.17)$$

and

$$f\left(Q_1^*,Q_3^*\right)=\sqrt{2\alpha\beta}. \quad (9.18)$$

After showing that

$$Q_1^*=\sqrt{\frac{\beta}{2\alpha}}=Q_2^*, \quad (9.19)$$

by using a similar approach,

$$Q_i^*=Q_j^*, \quad (9.20)$$

for $i, j=1,2,...,n$ can be derived.

Hence, we prove wanted results as

$$Q_1=Q_2=...=Q_n=\frac{D}{n}, \quad (9.21)$$

is obtained by algebraic methods.

X. Direction for Potential Study

In this section, we list several recently published papers to help researchers knowing the hot spot of studies. Yang and Chen [53] considered Yen [54], Çalışkan [55, 56], and Osler [57], to provide revisions. Wang and Chen [58] showed that Aguaron and Moreno-Jimenez [59] contained questionable findings and then presented amendment. Wang and Chen [58] also provided improvements for Yen [54]. Yen [60] examined Minner [61], Çalışkan [62], Wee [63], and Çalışkan [64, 65] to showed that their shortcomings and then offer enhancements. Yen [54] developed a new algebraic procedure to solve an inventory model that had been studied by Ronald et al. [66], Chang et al. [52], Luo and Chou [67], Cárdenas-Barrón [68], and Grubbström and Erdem [69].

We cite more recently published papers to point out possible directions for future studies. Referring to additional information and multiple hyper-planes twin support vector machine, Chu et al. [70] examined the steel plate surface defects classification method. Based on unconstrained optimization with sufficient descent property, Fang et al. [71] developed a new modified nonlinear conjugate gradient method. With outsourcing optimization problems, Kusuma and Dirgantara [72] constructed a new meta-heuristic, and then applied to run-catch optimizer. During Covid-19 Pandemic, for electricity strategy business, Chaerani et al. [73] considered optimization modeling through a systematic literature review. According to student retention, Deng and Chaudhury [74] constructed strategic knowledge bases by adaptive data mining. Based on fractional-order back stepping strategy and input saturation, Tian et al. [75] developed finite-time control for engineering systems. Referring to weighted loss and transfer learning, Oktavian et al. [76] examined the convolutional neural network to classify Alzheimer's patient. With two-dimensional frameworks and evidential reasoning approaches, Huang et al. [77] considered to decide the weights of experts.

XI. Conclusion

In conclusion, this study focused on the decision-making process under fuzzy preference relationships, particularly when dealing with incomplete fuzzy preference data. Through our research, we have made significant improvements to Xu's goal programming models [1]. Specifically, we have enhanced the computation by introducing the row arithmetic mean method, simplifying the previously complex calculations.

Furthermore, our revisions address the weaknesses and unreasonable construction of fuzzy elements in Xu's approach, particularly in relation to incomplete fuzzy preference relations. By providing these improvements, we aim to offer researchers a more reliable and robust methodology.

Additionally, we have presented a clear and systematic algorithm for efficiently applying our proposed approach, allowing subsequent researchers with relevant interests to employ it effectively.

Moreover, we have tackled a challenging formula proposed by Wang et al. [17], providing a precise mathematical procedure that will aid future researchers in their own derivations within a calculus environment.

Lastly, our study has also contributed to the work of Moon and Giri [48], building upon the examination conducted by Bose et al. [47]. We have proposed a potential solution for an open question that remained unresolved in Moon and Giri's work, thereby expanding the knowledge base in this area.

In summary, our research has made valuable contributions by enhancing existing models, providing efficient algorithms, offering mathematical procedures for complex formulas, and addressing open questions in the field. We believe these findings will inspire further investigations and advancements in decision-making under fuzzy preference relationships.

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