Optimal Path with Interval Value of Intuitionistic Fuzzy Number in Multigraph

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***Abstract*—Determining the shortest path on a network is widely used for improving network performance. In fact, not all network can be generated with a single parameter and are simple graphs. Many networks contain multiple parameters and are not simple graphs (multigraphs). The shortest path in a multigraph is determined by removing the most weight before calculating with the algorithm. Eliminations on different parameters leads to in various shortest paths, hence an optimal path cannot be found in the multigraph. Multigraphs are built in this research using interval-valued intuitionistic fuzzy numbers, which are a combination of multiple parameters. The Floyd-warshall algorithm was modified to create an algorithm that may be applied on interval-valued intuitionistic fuzzy multigraphs. The optimum path for a transportation network made of various parameters in the form of a multigraph was determined as an experiment using an interval-valued intuitionistic fuzzy number approach. The improved Floyd-warshall algorithm generates an optimal path on the transportation network under consideration.**

***Index Terms*—optimal path, multiple parameters, Floyd-warshall modification, multigraphs, fuzzy intuitionistic number, interval value.**

I. Introduction

A Multigraph is a non-simple graph with multiple edges connecting two vertices that share a common edge. Even though it connects the same two points, the weight of each edge varies depending on the parameters applied. Numerous studies [1] – [4] have also used multigraphs to construct network systems due to the realism of multigraphs as system representations. Multiple edges are not permitted in the shortest path problem because the edges must have the minimum weight. As a result, prior to using a particular procedure, the side with the greatest weight must be eliminated. This transforms the multigraph into simple graph, eliminating the opportunity for multiple edges to be chosen as the solution to the shortest path problem.

The shortest path problem entails determining the path with the lowest weight. The edges weight is important because it is the only indicator that depicts the systems. In a transportation network system, for example, we can minimize time, cost, distance, or various other factors to get through the shortest route. One of the challenges that arises with multigraphs is the use of a single parameter to organize edge weights. There are several methods for determining the shortest path in multigraphs, as earlier researchers have proved [5] – [7].

According to [8] a multigraph with three parameters will turn into three simple graphs depending on the parameters used. The three shortest paths that do not overlap will be determined after solving each graph. The ideal solution for this problem's shortest path to resolution cannot be determined

In this study, edge weights were created by combining several parameter values with the desired proportions. The intuitionistic fuzzy number (IFN) approach is used to combine those several parameters. Because it comprises of degrees of membership and degrees of non-membership, IFN is a broad sort of fuzzy number that can be used to characterize data ambiguity and uncertainty. IFN has seen extensive use in graph optimize issues [9] – [12]. The multigraphs of IFN are known as intuitionistic fuzzy multigraphs (IFM).

The shortest path problem becomes an optimal path problem due to the existence of numerous parameters. In contrast to [8], this study will use a linear approach of membership and non-membership functions. A modified Floyd-warshall algorithms will be used to determine the optimal path in this research. Using case studies of transportation network challenges, the best solution is obtained by determining the minimum values for three parameters: road length, travel costs, and travel time.

II. Methods

A. *Intuitionistic Fuzzy Multigraph (IFM)*

Definition 2.1. [13] An intuitionistic fuzzy set is a fuzzy set A in the space E so that

$$A=\{(x,\mu_A(x),\nu_A(x))\mid x\in E\}$$

where $\mu_A:E\to[0,1]$, $\nu_A:E\to[0,1]$ in which $\mu_A(x)$ is the membership function that show the possibility x belongs to set A and $\nu_A(x)$ is the non-membership function that show the possibility x does not belongs to set A, also

$$0\le\mu_A(x)+\nu_A(x)\le 1.$$

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Here the pair value $\left(\mu_A(x), \nu_A(x)\right)$ is called intuitionistic fuzzy number (IFN) of elements x in set A.

A graph that contains IFN is called intuitionistic fuzzy graph (IFG). The IFG as definition 2.2, can be contain at least one vertex (as common definition of vertices set of common graphs) without any edges.

Definition 2.2. [14] An IFG is of the form $G = \langle V, E \rangle$ where

i. $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1 : V \to [0,1]$ and $\nu_1 : V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and

$$0 \le \mu_1(v_i) + \nu_1(v_i) \le 1,$$

For every $v_i \in V, (i = 1, 2, \ldots, n)$,

ii. $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1]$ and $\nu_2 : V \times V \to [0,1]$ are such that

$$\mu_2(v_i, v_j) \le \min\left[\mu_1(v_i), \mu_1(v_j)\right],$$

$$\nu_2(v_i, v_j) \le \max\left(\nu_1(v_i), \nu_1(v_j)\right),$$

$$\text{and } 0 \le \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \le 1$$

for every $(v_i, v_j) \in E, (i, j = 1, 2, \ldots, n)$.

Here we can say that the triple $\langle v_i, \mu_1(v_i), \nu_1(v_i) \rangle$ denotes the degree of membership and non-membership of vertex v_i. The triple $\langle e_{ij}, \mu_2(e_{ij}), \nu_2(e_{ij}) \rangle$ denotes the degree of membership and non-membership of the edge relation $e_{ij} = (v_i, v_j)$ on V. For the next discussion we will use edge weight as $\langle e_{ij}, \mu_2(e_{ij}), \nu_2(e_{ij}) \rangle$.

To find an optimal path in IFG we must have the definition of path and how to find the length of the optimal path. In IFG a path is define as below.

Definition 2.3. [14] A path P in an IFG is a sequence of distinct vertices $v_1, v_2, \ldots, v_n$ such that either one of the following conditions is satisfied:

i. $\mu_2(e_{ij}) > 0$ and $\nu_2(e_{ij}) = 0$ for some *i* and *j*,

ii. $\mu_2(e_{ij}) = 0$ and $\nu_2(e_{ij}) > 0$ for some *i* and *j*,

iii. $\mu_2(e_{ij}) > 0$ and $\nu_2(e_{ij}) > 0$ for some *i* and *j*,

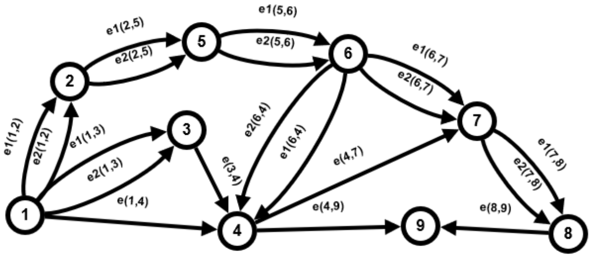

Fig. 1. Multigraph of Transportation Network.

$(i, j = 1, 2, \ldots, n)$.

The length of a path $P = v_1 v_2 \ldots v_{n+1}$ $(n > 0)$ is *n*.

To obtain IFN as a graph weight, the parameter values must be converted into fuzzy values using the appropriate method. Numerous triangular and trapezoidal approaches were utilized by previous researchers to determine fuzzy numbers [15] – [17]. We will use linear approach to determine intuitionistic fuzzy number as an easy illustration here.

A multigraph is a graph with numerous edges. The term "double edged" refers to the presence of at least two identical points connected by more than one edge. The presence of numerous sides in a multigraph suggests that the network established contains a large number of relationships.

Intuitionistic fuzzy multigraph (IFM) indicates that the graph is complex and has IFN weight. As shown in Figure 1, a multigraph in this paper define as

Definition 2.4. [18] let $G = \{V, E, r\}$ be a graph in which $V = \{v_i,\ i = 1, 2, 3, \ldots, n\}$ and

$$E = \left\{(v_i, v_j)_r \,|\, \forall i, j = 1, 2, 3, ..., n,\ r = 1\right\}$$

A multigraph is a graph in which there can be more than one relationship (connection) between two adjacent vertices, i.e., it is a graph for which $r \ge 1$.

B. Combination Function in Intuitionistic Fuzzy Set

Definition 2.5. [13] An IVIFS (interval-valued intuitionistic fuzzy sets) *A* over *E* is defined as an object of the form:

$$A = \left\{\left(x, M_A(x), N_A(x)\right) x \in E \right|\}$$

where $M_A(x) \subset [0,1]$ and $N_A(x) \subset [0,1]$ are intervals, and for all $x \in E$,

$$\sup M_A(x) + \sup N_A(x) \le 1.$$

To combine the tree parameter value that will be used in IFM, we will use weight function as mention in Definition 2.6 below. Let $\alpha, \beta \in [0,1]$, the constant value α, β show the different proportional of the decision maker use to find the optimal path of IFM.

Definition 2.6. [13] given an IFS *A* we define the operator $G_{\alpha,\beta}(A) = \left\{\left(x, \alpha \cdot \mu_A(x), \beta \cdot \nu_A(x)\right) \middle| x \in E\right\}$.

Then for IVIFS, the definition become

$$G_{\alpha,\beta}(A) = \left\{ \left\langle \begin{matrix} x, \left[\alpha \cdot \left(M_A^L(x), M_A^U(x)\right)\right], \\ \left[\beta \cdot \left(N_A^L(x), N_A^U(x)\right)\right] \end{matrix} \right\rangle \middle| x \in E \right\}.$$

To make the definition fits weighted function in discrete event, we need to make sure that $\alpha = \beta$ so that every interval value in membership degree and non-membership degree have equal coefficient value. The coefficient value in every IVIFS show the influence of the parameter to the decision to be taken. Then we will have theorem 2.7 below.

Theorem 2.7. If

$$G_{\alpha,\beta}(A)=\left\{\left\langle \begin{array}{l} x,\left[\alpha\cdot\left(M_A^L(x),M_A^U(x)\right)\right], \\ \left[\beta\cdot\left(N_A^L(x),N_A^U(x)\right)\right] \end{array}\right\rangle \middle| x\in E\right\}$$

for every $\alpha,\beta\in[0,1]$ and $\beta=\alpha$ then $G_{\alpha,\beta}(A)=\alpha.(A)$.

Proof. We will define $G_{\alpha,\alpha}(A)$ as $G_{\alpha,\beta}(A)$ and focused on the IVIFN.

$$\begin{aligned}
&G_{\alpha,\alpha}(A)\\
&=\left\langle\left(\alpha\cdot M_A^L(x),\alpha\cdot M_A^U(x)\right),\left(\alpha\cdot N_A^L(x),\alpha\cdot N_A^U(x)\right)\right\rangle\\
&=\left\langle\alpha\cdot\left(M_A^L(x),M_A^U(x)\right),\alpha\cdot\left(N_A^L(x),N_A^U(x)\right)\right\rangle\\
&=\alpha\cdot\left\langle\left(M_A^L(x),M_A^U(x)\right),\left(N_A^L(x),N_A^U(x)\right)\right\rangle\\
&=\alpha\cdot A \qquad \square
\end{aligned}$$

Several IFNs can be arranged based on theorem 2.7, which is a mapping from parameters to weighting functions, namely

$$\begin{aligned}
\alpha_1(A_1)&=\left\langle\left[\alpha_1 M_{A_1}^L(x),\alpha_1 M_{A_1}^U(x)\right],\left[\alpha_1 N_{A_1}^L(x),\alpha_1 N_{A_1}^U(x)\right]\right\rangle\\
\alpha_2(A_2)&=\left\langle\left[\alpha_2 M_{A_2}^L(x),\alpha_2 M_{A_2}^U(x)\right],\left[\alpha_2 N_{A_2}^L(x),\alpha_2 N_{A_2}^U(x)\right]\right\rangle\\
&\vdots\\
\alpha_n(A_n)&=\left\langle\left[\alpha_n M_{A_n}^L(x),\alpha_n M_{A_n}^U(x)\right],\left[\alpha_n N_{A_n}^L(x),\alpha_n N_{A_n}^U(x)\right]\right\rangle
\end{aligned}$$

Theorem 2.8. For every $\alpha_1,\alpha_2,\ldots,\alpha_n\in[0,1]$, weighting functions [19] $f(A_n)=\sum_{n=\square}\alpha_n A_n$ is an IVIFN if and only if $\alpha_1+\alpha_2+\cdots+\alpha_n\le 1$.

Proof.

1. $f(A_i(x))$ is an IVIFN $\Rightarrow \alpha_1+\alpha_2+\cdots+\alpha_n\le 1$.

$$f(A_n)=\left\langle\left(f_{M^L}(A_n),f_{M^U}(A_n)\right),\left(f_{N^L}(A_n),f_{N^U}(A_n)\right)\right\rangle,$$

where $f_{M^L}(A_n)=\sum_{n\in\square}\alpha_n M_{A_n}^L(x)$, $f_{M^U}(A_n)=\sum_{n\in\square}\alpha_n M_{A_n}^U(x)$, $f_{N^L}(A_n)=\sum_{n\in\square}\alpha_n N_{A_n}^L(x)$, dan $f_{N^U}(A_n)=\sum_{n\in\square}\alpha_n N_{A_n}^U(x)$.

As mention in definition 2.1, the equals of interval-valued membership and non-membership degree have to less than 1, then $f_{M^U}(A_n)+f_{N^U}(A_n)\le 1$.

$$\begin{aligned}
&f_{M^U}(A_n)+f_{N^U}(A_n)=\sum_{n\in\square}\alpha_n M_{A_n}^U(x)+\sum_{n\in\square}\alpha_n N_{A_n}^U(x)\\
&=\left(\alpha_1 M_{A_1}^U(x)+\alpha_2 M_{A_2}^U(x)+\cdots+\alpha_n M_{A_n}^U(x)\right)\\
&\qquad+\left(\alpha_1 N_{A_1}^U(x)+\alpha_2 N_{A_2}^U(x)+\cdots+\alpha_n N_{A_n}^U(x)\right)\\
&=\left(\alpha_1 M_{A_1}^U(x)+\alpha_1 N_{A_1}^U(x)\right)+\left(\alpha_2 M_{A_2}^U(x)+\alpha_2 N_{A_2}^U(x)\right)+\\
&\qquad\cdots+\left(\alpha_n M_{A_n}^U(x)+\alpha_n N_{A_n}^U(x)\right)\\
&=\alpha_1\left(M_{A_1}^U(x)+N_{A_1}^U(x)\right)+\alpha_2\left(M_{A_2}^U(x)+N_{A_2}^U(x)\right)+\\
&\qquad\cdots+\alpha_n\left(M_{A_n}^U(x)+N_{A_n}^U(x)\right)
\end{aligned}$$

TABLE I
THE IVIFN OF EACH EDGE IN MULTIGRAPH

Edge	Parameter 1	Parameter 2	Parameter 3	Combine parameter
e1(1,2)	(0.778,0.878), (0.022,0.122)	(0.769,0.869), (0.030,0.130)	(0.8,0.9), (0,0.1)	(0.782,0.882), (0.018,0.118)
e2(1,2)	(0.8,0.9), (0,0.1)	(0.8,0.9), (0,0.1)	(0.604,0.704), (0.196,0.296)	(0.735,0.835), (0.065,0.165)
e1(1,3)	(0.425,0.525), (0.375,0.452)	(0.567,0.667), (0.233,0.333)	(0.465,0.565), (0.335,0.435)	(0.486,0.586), (0.314,0.406)
e2(1,3)	(0.630,0.730), (0.169,0.269)	(0.527,0.627), (0.273,0.373)	(0.588,0.688), (0.212,0.312)	(0.582,0.682), (0.218,0.318)
e(1,4)	(0.114,0.214), (0.686,0.786)	(0,0.1), (0.8,0.9)	(0,0.1), (0.8,0.9)	(0.038,0.138), (0.762,0.862)
e(3,4)	(0.468,0.568), (0.332,0.432)	(0.365,0.465), (0.435,0.535)	(0.4,0.5), (0.4,0.5)	(0.411,0.511), (0.389,0.489)
e1(2,5)	(0.575,0.675), (0.225,0.325)	(0.653,0.753), (0.147,0.247)	(0.604,0.704), (0.196,0.296)	(0.611,0.711), (0.189,0.289)
e2(2,5)	(0.722,0.822), (0.078,0.178)	(0.759,0.859), (0.041,0.141)	(0.465,0.565), (0.335,0.435)	(0.649,0.749), (0.151,0.251)
e1(5,6)	(0.772,0.872), (0.028,0.128)	(0.769,0.869), (0.030,0.130)	(0.792,0.892), (0.008,0.108)	(0.778,0.878), (0.022,0.122)
e2(5,6)	(0.657,0.757), (0.143,0.243)	(0.724,0.824), (0.076,0.176)	(0.653,0.753), (0.147,0.247)	(0.678,0.778), (0.122,0.222)
e1(6,4)	(0.495,0.595), (0.305,0.405)	(0.608,0.708), (0.192,0.292)	(0.531,0.631), (0.269,0.369)	(0.544,0.644), (0.256,0.356)
e2(6,4)	(0.604,0.704), (0.196,0.296)	(0.698,0.798), (0.101,0.201)	(0.408,0.508), (0.392,0.492)	(0.570,0.670), (0.230,0.330)
e1(6,7)	(0.674,0.774), (0.126,0.226)	(0.709,0.809), (0.091,0.191)	(0.702,0.802), (0.098,0.198)	(0.695,0.795), (0.105,0.205)
e2(6,7)	(0.738,0.837), (0.062,0.162)	(0.765,0.865), (0.035,0.135)	(0.604,0.704), (0.196,0.296)	(0.702,0.802), (0.098,0.198)
e(4,7)	(0.663,0.763), (0.137,0.237)	(0.562,0.662), (0.238,0.338)	(0.620,0.720), (0.179,0.279)	(0.615,0.715), (0.185,0.285)
e1(7,8)	(0.632,0.732), (0.168,0.268)	(0.683,0.783), (0.116,0.216)	(0.661,0.761), (0.139,0.239)	(0.659,0.759), (0.141,0.241)
e2(7,8)	(0.666,0.766), (0.134,0.234)	(0.729,0.829), (0.071,0.171)	(0.490,0.590), (0.310,0.410)	(0.628,0.728), (0.172,0.272)
e(8,9)	(0.568,0.668), (0.232,0.332)	(0.648,0.748), (0.152,0.252)	(0.596,0.696), (0.204,0.304)	(0.604,0.704), (0.196,0.296)
e(4,9)	(0,0.1), (0.8,0.9)	(0.318,0.418), (0.481,0.581)	(0.073,0.173), (0.727,0.827)	(0.131,0.231), (0.669,0.769)

Since $M_{A_1}^U(x)+N_{A_1}^U(x)\le 1$ then $\alpha_1+\alpha_2+\cdots+\alpha_n\le 1$.

2. $\alpha_1+\alpha_2+\cdots+\alpha_n\le 1\Rightarrow f(A_i(x))$ is an IVIFN.

Based on definition 5.4, $f(A_i(x))$ can be write as

$$\begin{aligned}
&f(A_i(x))=\sum_{i\in\square}G_{\alpha_i}A_i(x)\\
&=G_{\alpha_1}A_1(x)+G_{\alpha_2}A_2(x)+\cdots+G_{\alpha_{n-1}}A_{n-1}(x)+G_{\alpha_n}A_n(x).\\
&=\left\langle\left(\alpha_1\mu_{A_1}^L(x),\alpha_1\mu_{A_1}^U(x)\right),\left(\alpha_1\eta_{A_1}^L(x),\alpha_1\eta_{A_1}^U(x)\right)\right\rangle+\\
&\qquad\left\langle\left(\alpha_2\mu_{A_2}^L(x),\alpha_2\mu_{A_2}^U(x)\right),\left(\alpha_2\eta_{A_2}^L(x),\alpha_2\eta_{A_2}^U(x)\right)\right\rangle+\cdots+\\
&\qquad\left\langle\left(\alpha_{n-1}\mu_{A_{n-1}}^L(x),\alpha_{n-1}\mu_{A_{n-1}}^U(x)\right),\left(\alpha_{n-1}\eta_{A_{n-1}}^L(x),\alpha_{n-1}\eta_{A_{n-1}}^U(x)\right)\right\rangle+\\
&\qquad\left\langle\left(\alpha_n\mu_{A_n}^L(x),\alpha_n\mu_{A_n}^U(x)\right),\left(\alpha_n\eta_{A_n}^L(x),\alpha_n\eta_{A_n}^U(x)\right)\right\rangle.
\end{aligned}$$

$$\begin{aligned}
&f(A_i(x))\\
&=\Big\langle\left(\alpha_1\mu_{A_1}^L(x)+\alpha_2\mu_{A_2}^L(x)+\cdots+\alpha_{n-1}\mu_{A_{n-1}}^L(x)+\alpha_n\mu_{A_n}^L(x)\right),\\
&\qquad\left(\alpha_1\mu_{A_1}^U(x)+\alpha_2\mu_{A_2}^U(x)+\cdots+\alpha_{n-1}\mu_{A_{n-1}}^U(x)+\alpha_n\mu_{A_n}^U(x)\right),\\
&\qquad\left(\alpha_1\eta_{A_1}^L(x)+\alpha_2\eta_{A_2}^L(x)+\cdots+\alpha_{n-1}\eta_{A_{n-1}}^L(x)+\alpha_n\eta_{A_n}^L(x)\right),\\
&\qquad\left(\alpha_1\eta_{A_1}^U(x)+\alpha_2\eta_{A_2}^U(x)+\cdots+\alpha_{n-1}\eta_{A_{n-1}}^U(x)+\alpha_n\eta_{A_n}^U(x)\right)\Big\rangle.
\end{aligned}$$

Theorem 2.8 proved.

TABLE II
THE VALUE FUNCTION OF EACH EDGE

Edge	Value Function
e1(1,2)	0.910614
e2(1,2)	0.858163
e1(1,3)	0.586586
e2(1,3)	0.689755
e(1,4)	0.091674
e(3,4)	0.502074
e1(2,5)	0.721743
e2(2,5)	0.76374
e1(5,6)	0.905566
e2(5,6)	0.795668
e1(6,4)	0.648833
e2(6,4)	0.677246
e1(6,7)	0.814387
e2(6,7)	0.822289
e(4,7)	0.726615
e1(7,8)	0.774685
e2(7,8)	0.741223
e(8,9)	0.714345
e(4,9)	0.193901

C. Modified Floyd-warshall Algorithms

Floyd-warshall is a dynamic programming algorithm used to determine the shortest path for all paths in a graph. This algorithm's weakness is that it is slower than other algorithms. The primary advantage, however, is that this algorithm can determine the shortest path among all existing paths in a single calculation. This advantage can provide decision-makers with more options.

This study modifies Floyd-warshall algorithm (FWA) by employing the IFN operator and side weights in the form of IFN and norms, which serve as decision-making indicators for determining the optimal path. The steps for FWA modification are as follows:

1. The initial step in iteration 1 should be an evaluation of the edge with the greater weight. As a result, iteration 1 will concentrate on eliminating multiple edges with provisions based on the goal of establishing the shortest path. As a result, the FWA must have explicit edge removal criteria or indicators.
2. Because the Floyd-Warshall method has an advantage, namely the evaluation of a path that connects the same two sites, but also because it prolongs the execution time, the evaluation of the path, known as the triangle operator, must be performed in the second iteration. This procedure is employed to save time during execution and to avoid edges with greater weights. If iteration 1 provides a simple graph from a multigraph, iteration 2 provides an optimal simple graph.
3. The results of identifying paths that connect the same a pair in the second iteration may be repeated for multiple iterations after that, such that the path assessment continues until the last iteration. The distinction is that there is no edge elimination after the third iteration since the optimal simple graph produced in the second iteration.
4. The FWA set up for the weight and path matrices can be altered to calculate the path length. The third iteration will begin by collecting all optimal pathways with three points, the fourth iteration with four points, and so on until the destination point is reached. By analysing the path in iteration 2, the path produced in the following iteration leads to the most optimal path indirectly.
5. The iteration comes to an end when the destination point is reached.

Furthermore, prior to the completion of IVIFM, the following actions are taken:

1. The function value is used to evaluate the multiple edges and path. The shortest path chooses the path with the highest value function. Because the value function is related to the degree of membership, the larger the value function, more probable it is that that path will be chosen as the shortest path.
2. Intuitionistic fuzzy numbers are formed with interval values as edges weights $\langle(a,b),(c,d)\rangle$ in such a way that the value function is
$$value\ function = \frac{a + a(1-a-c) + b + b(1-b-d)}{2}$$
3. The path weights are calculated by adding at least two edges weights
$$i - j = e(i,k) + e(k,j)$$
for $e(i,k) = \langle(a_1,b_1),(c_1,d_1)\rangle$ and
$e(k,j) = \langle(a_2,b_2),(c_2,d_2)\rangle$ then
$$e(i-j) = \left\langle\left(\frac{(a_1+a_2)}{2},\frac{(b_1+b_2)}{2}\right),\left(\frac{(c_1+c_2)}{2},\frac{(d_1+d_2)}{2}\right)\right\rangle,$$
4. Iteration calculations are performed in the same as in a normal graph, ensuring that each path weight obtained meets the requirements.

This modification is more effective since it has a smaller iteration then the Floyd-warshall its self. Another benefit from this modification is that from second iteration the simple graph become optimal simple graph.

III. RESULT AND DISCUSSION

The multigraph in figure 1 show that there are 9 vertices and 19 edges in the graph. Before we find the optimal path, first we will have to convert the real value to be IFN. The membership and non-membership function construct from half-trapezoidal approach. The formula that is used as follow:

$$\mu_i(x) = \begin{cases} \mu_a, & x < a \\ \dfrac{(a-x) + \mu_a(b-a)}{b-a}, & a \le x \le b \\ 0, & x > b \end{cases}$$

TABLE III
THE OPTIMAL PATH IN MULTIGRAPH

Edge	Value Function
1 – 2 – 5	0.83718
1 – 3 – 4	0.59591
2 – 5 – 6	0.83465
3 – 4 – 7	0.61434
3 – 4 – 9	0.34799
4 – 7 – 8	0.75065
5 – 6 – 4	0.79141
5 – 6 – 7	0.86393
6 – 7 – 8	0.79849
7 – 8 – 9	0.74452
1 – 2 – 5 – 6	0.87137
1 – 3 – 4 – 7	0.66127
2 – 5 – 6 – 4	0.75595
2 – 5 – 6 – 7	0.82847
3 – 4 – 7 – 8	0.69452
4 – 7 – 8 – 9	0.73250
5 – 6 – 7 – 8	0.81931
6 – 7 – 8 – 9	0.75642
1 – 2 – 5 – 6 – 7	0.84638
1 – 3 – 4 – 7 – 8	0.71798
2 – 5 – 6 – 7 – 8	0.80158
3 – 4 – 7 – 8 – 9	0.70443
5 – 6 – 7 – 8 – 9	0.76683
1 – 2 – 5 – 6 – 7 – 8	0.81076
2 – 5 – 6 – 7 – 8 – 9	0.75796
1 – 2 – 5 – 6 – 7 – 8 – 9	0.76255

$$\nu_i(x) = \begin{cases} 0, \ x < a \\ \dfrac{(b-x) + \nu_a(a-b)}{a-b}, \ a \le x \le b \\ \nu_a, \ x > b \end{cases}$$

In this mapping we use interval value by choosing the (μ_a, ν_a) so that we can have IVIFS as the weight of edges in figure 1.

The constant values $a_1, a_2, \ldots, a_n$ indicate the strength of the parameters' influence on the decision to be made. This study's constant value applies the same percentage to all parameters. In other words, the effect of all parameters is equal. The parameter combination provides a single value for using in IFM. Now we have IFM that consist combination parameter as edges weight.

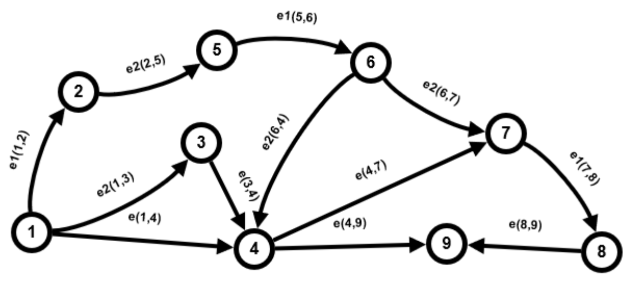

Fig. 2. Simple Graph of Transportation Network.

Now we can use the FWA modification to determine the optimal path in multigraphs (Figure 1). After identifying the vertices and edges in a multigraph, we will iterate. As shown in Figure 1, the multigraph has 9 vertices, 19 edges, and 7 edges that are multiple. After identifying the edges and weight in Table I, column combine parameter, we can begin the iteration to find the optimal path in multigraph.

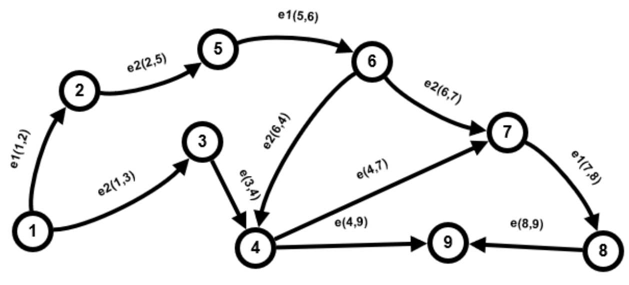

Fig. 3. Optimal Simple Graph of Transportation Network.

Iteration 1. The evaluation multiple edges that we use is the elimination of the smallest value function. Form Table I the red colors show that the edge was eliminated. The multigraph changes to be simple graph as Figure 2.

Iteration 2. Evaluation in second iteration in Figure 2 show that there are 2 triangle operators. The first is $e(1,4)$ and $e_2(1,3) + e(3,4)$, the second is $e_2(6,7)$ and $e_2(6,4) + e(4,7)$. By using value function we eliminate $e(1,4)$ since $e_2(1,3) + e(3,4)$ has the biggest, and we choose to use $e_2(6,7)$ instead of $e_2(6,4) + e(4,7)$ because value function of $e_2(6,7)$ bigger then $e_2(6,4) + e(4,7)$. From this iteration simple graph become optimal simple graph as Figure 3.

Iteration 3. Based on Figure 3, now we will find a path with length 2. There are 10 paths with 3 paths was reach the end point (3 – 4 – 9, 6 – 4 – 9, and 7 – 8 – 9). There is no evaluation in this iteration.

Iteration 4. Based on Figure 3, now we will find a path with length 3. There are 10 paths and 4 paths reach the end point (1 – 3 – 4 – 9, 4 – 7 – 8 – 9, 5 – 6 – 4 – 9 and 6 – 7 – 8). There are some evaluations in this iteration. We have to find which path is optimal between $e(4,9)$ and path 4 – 7 – 8 – 9 based on value function. Since value function of path 4 – 7 – 8 – 9 bigger than $e(4,9)$, $0.7325 > 0.1939$. We also have to choose which path is optimal between 6 – 4 – 9 or 6 – 7 – 8 – 9. Evaluation from value function show that value function of path 6 – 7 – 8 – 9, 0.75642, is bigger than value function of path 6 – 4 – 9, 0.43557.
For the rest iteration we will use path 4 – 7 – 8 – 9 instead of $e(4,9)$ and path 6 – 7 – 8 – 9 instead of path 6 – 4 – 9.

Iteration 5. There are 6 paths in this iteration and 2 paths has reach end point (3 – 4 – 7 – 8 – 9 and 5 – 6 – 7 – 8 –

9). There is no evaluation in this iteration.

Iteration 6. There are 3 paths in this iteration and 2 paths has reach end point (1 – 3 – 4 – 7 – 8 – 9 and 2 – 5 – 6 – 7 – 8 – 9). We have to choose which path is optimal between path 1 – 3 – 4 – 9 and path 1 – 3 – 4 – 7 – 8 – 9. Evaluation based on value function, show that path 1 – 3 – 4 – 7 – 8 – 9 is optimal.

Iteration 7. As the last iteration, we reach start point to end point that is path 1 – 2 – 5 – 6 – 7 – 8 – 9 with value function 0.76255. Compare to path 1 – 3 – 4 – 7 – 8 – 9, path 1 – 2 – 5 – 6 – 7 – 8 – 9 has the biggest value function. The optimal path from 1 to 9 should be path 1 – 2 – 5 – 6 – 7 – 8 – 9.

All the optimal path between any start point and end point in Figure 3 as shown in Table III. With total 7 iteration, the optimal solution for multigraph transportation network was solved. Table III contain all possible path from the problem statement.

IV. CONCLUSION

The modifications made to the Floyd-warshall algorithm permit optimal completion of paths in multigraphs. Even though the dual edges were ultimately eliminated, all sides had a chance of being selected because their side weights were based on a combination of multiple parameters. The optimal path of transportation network is 1 – 2 – 5 – 6 – 7 – 8 – 9 which reach from 7 iteration, faster compare to regular algorithm. Although the case study is a directed multigraph, multigraph problems can also be solved on undirected graphs. Then, a computer program must be developed to facilitate multigraph calculations with more vertices.

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