Optimization Model for Outpatient Services Capacity Management in Different Patient Conditions and Diseases

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Abstract—The high population in cities such as Medan, the capital of North Sumatra Province, Indonesia, is observed to have led to the need for more patient clinics with good health services. These clinics are required to have optimal capacity management to meet the health services expectation of patient. However, the main problem identified is uncertainty associated with the number of outpatients, including the first visit (FV) and revisit (RV). This research was conducted to propose optimization model to determine the minimal capacity necessary for a particular breach risk in order to achieve the intended lead time. The intention was to fix the constraints of the prior model by selecting the equivalent deterministic individual opportunity restrictions depending on the probability knowledge distribution. The purpose of model was to optimize the management of outpatient requests not fulfilled completely, the limited range of appointment times for returning patient, and the average appointment lead time for old patient. The results showed that the probability of the number of FV patient arriving at time unit i and provided with an appointment at time unit j being greater than or equal to their stochastic arrival in time unit i was 0.51. Each test was found to be different due to the application of random values generated from computer memory. It was observed that the probability value recorded was higher than previous model.

Index Terms—Optimization, Model, Outpatient Services, Uncertain Conditions, Patient Conditions.

I. Introduction

Universally, hospitals are built to provide health care services for people. These hospitals are required to provide a comprehensive range of medical services for inpatient and outpatient in the community. The inpatient facilities are normally equipped to handle critical medical situations and offer 24/7 care while outpatient services allow people to access medical care without needing prolonged hospitalization. This dual method helps ensure that individuals receive appropriate medical attention based on the severity and nature of their health conditions. The provision of timely access is essential in outpatient clinic that receives several appointment requests from different classes of patients every day. However, the needs of thess patients are sometimes not prioritized to prevent overbooking or idle slots in the calendar of the physicians.

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This shows the importance of having efficient capacity management and scheduling strategies due to the limited capacity of the clinic. Uncertainties have been associated with patient, for example, multiple classes of those in outpatient clinic studied in this research are observed to have several demands. These classes include internal patient referred by other departments within a clinic, new and external ones, established patients with previous visit records, and any of these classes engaged in subsequent visits to receive follow-up care.

Demand forecasting is a subject observed to have been studied extensively [1] with significant results in the retail industry as well as in managing inventory within companies such as Dell [2]. Several comparative analyses have been applied to this forecasting method with a focus on the accuracy of their results. For example, neural networks were compared with traditional econometric methods and generally found to have better outcomes [3]. Meanwhile, the summary of good cases conducted using traditional methods such as autoregression, moving average, and exponential smoothing was presented in [4] and they were reported to be simple.

Some studies have also been conducted on outpatient care model. For example, [5] focused on appointment scheduling at the department in West China Hospital (WCH) and the pilot data analysis showed that the use of the scheduling window improved the appointment system. The research analyzed two scenarios associated with the willingness of the patient to wait for their planned visits in order to have a comprehensive understanding of the process. This led to the development of a stylized single-server queue model to determine the ideal scheduling window. The results showed that the application of this scheduling window was not realistic to reduce the overall cost per day of the appointment system when patient was less sensitive to time delays or ready to wait for the planned services. However, model considered the time delay sensitivity of the patient (i.e., the possibility of seeking services elsewhere) and the potential cost of the physician's idle time.

Nguyen et al. [6] also used optimization methods to determine the optimal resource allocation solutions in outpatient clinics. The network flow method allowed the complex interactions between different resources within the clinical system to be modeled and optimized. The results showed that the proposed network flow method could assist in planning tactical resources for outpatient clinics. This was showed by the opportunity provided for the healthcare providers to optimize resource allocation, minimize patient waiting time, and improve overall services efficiency. In this research, an attempt was made to determine capacity required to accept the requests of all the patient. However, it was discovered that an excessive capacity level was needed to accommodate the unknown future demand of the FV patient. This required formulating the objective of the system in terms of the risk of violation (ε) which focuses on the probability that not all patient requests were met and observed to have exceeded ε. Therefore, the objective of model was to determine the minimum capacity needed to achieve the target grace period for a given breach risk. The main problem identified was non-optimal maximum capacity and this led to the inability to meet all outpatient requests, limited range of appointment times for returning patient, and the average appointment lead time for old patients due to emergencies which made it difficult to predict the time for new ones.

These problems led to the need to optimize model to determine capacity required to receive all patient requests. An over-capacity level was recommended due to the difficulty in predicting the demand from FV patient in the future. Therefore, the objective of the system was expressed in terms of the risk of violation (ε) again with the focus on the probability that not all patient demands would be honored and found to be higher than ε. Model was intended to identify the minimal capacity necessary to accomplish the specified grace time for a specific breach risk. Monte Carlo algorithm [7], [8], [9], [10], [11] was adopted to estimate the probability of model being developed. This algorithm has been previously used by [12] to construct model for the probability distribution of travel laws and charge features to forecast load demand as scale electric vehicles (EVs) were connected to the electric grid. A replica was also used by Hartland et al. [13] to obtain precise estimates of experimental and theoretical uncertainties. This was based on the opportunity provided to develop probability distributions in SMEFT's degrees of freedom space. Moreover, it has been extensively used to determine the uncertainty in life cycle assessments [14]. Monte Carlo simulation [15] has also been applied several times to assess the probabilities before and after an evaluation, and the results were compared with the action priority form to confirm their values as high, medium, or low. The research was also used to generate a new model to be used for demand forecasting and capacity management for bed usage during a pandemic.

II. LITERATURE REVIEW

Brown et al. [16] developed a network flow-based mathematical model to optimize resource allocation and schedule of patient examinations. Empirical data were used to test and validate model. The results showed that the network flow method improved capacity efficiency of outpatient clinics by reducing patient waiting times and optimizing resource use. This research contributed to the development of methods to enhance capacity management of outpatient services. The network flow method could be used by clinics and healthcare providers to make better decisions in optimizing services and increasing patient satisfaction.

White et al. [17] also developed a mathematical model based on integer programming using patient examination time, resource allocation, and patient preferences as variables. Moreover, existing constraints such as clinic capacity, services time, and physician preferences were also considered. Model was validated through simulations and experiments and its performance in improving scheduling efficiency and resource use in outpatient clinics was also evaluated. The results showed that the proposed integer programming model could improve operational efficiency and optimize patient examination scheduling, thereby reducing patient waiting time and improving clinical services.

Thompson et al. [18] introduced a mathematical model based on linear programming using resource allocation, services time, and patient needs as variables. Furthermore, existing constraints such as clinic capacity, examination time, and patient priority were also considered. Model was validated through simulations and experiments, and its performance in increasing the efficiency of resource allocation in outpatient clinics was also assessed. The results showed that the proposed linear programming model improved operational efficiency and optimized resource allocation. This subsequently led to a reduction in the patient waiting time, an increase in productivity, and the maximization of resources in outpatient clinics.

A different method was applied by Martinez et al. [19] through the suggestion of an agent-based simulation model where each patient was identified as an agent. The variables considered were the number of resources, patient examination time, and patient waiting time. Moreover, iterative simulation and optimization methods were applied to determine the optimal capacity configuration needed to achieve the best performance in efficiency and patient satisfaction. The results showed that the proposed simulation-based optimization model improved operational efficiency and resource use in outpatient clinics. This method allowed clinics to plan better capacity, minimize patient waiting time, and enhance the quality of services.

Lee et al. [20] developed model to achieve an optimal schedule for outpatient examinations in order to minimize patient waiting time. The main focus was on the improvement of services efficiency and time allocation of outpatient clinics. This led to the development of optimization-based mathematical model with constraints and desired goals including the maximum acceptable waiting time. Some optimization methods were applied to determine the best solution that satisfied these constraints. The results showed that the proposed optimization model was able to reduce waiting time for patient in outpatient clinics. The method was expected to provide an opportunity for healthcare providers to create more efficient schedules, minimize waiting times, and improve patient satisfaction.

Xu et al. [21] also proposed a fuzzy-based mathematical model with several objectives such as minimizing patient waiting time, maximizing resource use, and reducing operational costs. Optimization methods were applied to determine the optimal solutions and the results showed the ability of the proposed multi-objective fuzzy model to improve scheduling efficiency and resource use in outpatient clinics. It was expected to allow healthcare providers to make better decisions in the face of uncertain patient attendance, optimize patient appointment scheduling, and improve operational efficiency.

Zhou et al. [22] developed optimization model that considered the minimization of patient waiting times, maximization of resource use, and reduction of patient cancellations and no-shows. Optimization methods were also applied to determine the best solution to achieve these goals. The results showed the ability of the two-stage optimization model proposed to assist in improving the scheduling efficiency and reducing the impact of cancellations and absences in outpatient clinics. The method was expected to be useful for healthcare providers in making initial scheduling more efficient and rescheduling optimally after patient cancellations or no-shows.

Li et al. [23] proposed optimization model to improve patient satisfaction in outpatient clinics by considering the heterogeneous preferences of patient. The method allowed healthcare providers to make schedules that work with patient preferences, minimize patient waiting time, and improve services quality. This research was observed to have contributed to the development of optimization methods and model for scheduling patient examinations in outpatient services. Moreover, the consideration of the heterogeneous preferences of patient led to the improvement of patient satisfaction, operational efficiency, and provision of more personalized services to patient.

Chen et al. [24] were also observed to have used optimization methods to determine the optimal scheduling solutions with due consideration for the inaccuracy of patient arrival times. A mathematical model was developed to ensure more accurate and efficient timing of patient examinations, thereby reducing waiting times and increasing services efficiency. The results showed that the proposed optimization model improved the efficiency of patient examination scheduling in outpatient clinics by considering untimely patient arrivals. The method was expected to assist healthcare providers in optimizing scheduling, minimizing waiting time, and increasing patient satisfaction.

Suman et al. [25] also applied the DMAIC (Define-Measure-Analyze-Improve-Control) Six Sigma method to identify and reduce the total time patient spend in the surgical department of a hospital. The method increased process efficiency by identifying and overcoming variability and nonconformities. The mean time spent and standard deviation were calculated as 210.9 hours and 67.02 hours, respectively. Moreover, individual cause and effect analysis was also conducted on waiting time (WT) for surgery and length of stay (LOS) after surgery.

Mosca et al. [26] applied the "Engineering 4.0" concept in developing safe operating rooms for patient and medical staff. The term focused on a further industrial transformation of "Industry 4.0" with the focus on applying advanced technologies such as the Internet of Things (IoT), artificial intelligence (AI), big data processing, and automation to increase efficiency and safety in different sectors, including health care. The purpose was to preserve the integrity of tens of thousands of patients that lost their lives on the operating block due to external causes. It was discovered that the better use of an optimized management structure could provide significant benefits.

Cong et al. [27] used a sub-graph method to evaluate the impact of gene regulatory modules on renal clear cell carcinoma of the kidney (KIRC). The results showed that the combined effect of cancer-causing genes such as tumor suppressor, oncogenes, and DNA repair genes dramatically increased the probability of developing tumor metastasis. This led to the development of a new method to construct gene regulatory modules using a directed sub-graph method in order to reduce false positive results and identify highly relevant regulatory modules for tumor metastasis research.

Yanuar et al. [28] also identified the best model for LOS for inpatient COVID-19 patient in West Sumatra, Indonesia. The Los data were observed to have been skewed to the right or violated linear model assumptions, thereby leading to the preference for a quantile method. The asymptotic variance of the quantile regression was estimated by constructing confidence intervals for the parameters of interest. The results proved that wild bootstrap quantiles tended to produce the shortest confidence intervals while the diagnosis and outcome were found to have a statistically significant impact on the COVID-19 patient admitted to Los hospitals.

Lee et al. [29] used decision-making trials and laboratory evaluation methods to identify the critical dimensions of the Chinese version of the safety attitude questionnaire. The purpose was to improve the patient safety culture in Taiwan based on the perspective of experts. The causal dimensions were identified to be stress recognition, management perception, emotional exhaustion, and work-life balance while the affected variables were teamwork climate, safety climate, job satisfaction, and working conditions. The improvements in the dimensions were observed to have a little impact but a focus on the causal extent enhanced the measurement and performance of other directly affected variables. It was discovered that emotional exhaustion was the most critical dimension followed by management perceptions had a significant influence on other dimensions. This showed that hospital management needed to address emotional exhaustion and management perception to improve the patient safety culture.

Lin et al. [30] proposed a new framework called Patient Similarity Evaluation (PSE) to incorporate temporal information into embedded medical concepts for patient representational learning. The PSE combined Siamese Convolutional Neural Network (CNN) with Spatial Pyramid Pooling (SPP) to measure the similarity of all pairs of patient in order to predict the future health status of patient earlier and more precisely. The experimental results showed that this proposed framework performed better than all baseline methods.

Kusuma et al. [31] recommended a new coordinated ambulance routing model considered suitable for the COVID-19 pandemic. Model was designed based on three steps including the hospital patient allocation, ambulance-patient dispatch, and ambulance-pickup dispatch sequencing. The two objectives were to minimize the number of unserved patient and the total travel distance. Model was developed using cloud theory-based annealing simulations. The results showed its ability to outperform existing uncoordinated model in terms of number of unserved patient, total travel distance, and average travel distance. It was discovered that there were zero unserved patient when their number did not exceed the slots available in the entire hospital. Model also led to a 12 to 19 percent reduction in total trip distance and 27 to 29 percent in average trip distance.

III. Research Method

A. Research Framework

A research framework is a structured plan or outline guiding the research process. It normally provides a clear and systematic method to design, conduct, and analyze research projects. Research framework is essential because it allows proper organization, rigorous and systematic analysis, and effective communication of results. The components usually vary depending on the type of research and the specific questions being addressed. Therefore, the framework applied in this research is presented in the following Figure 1:

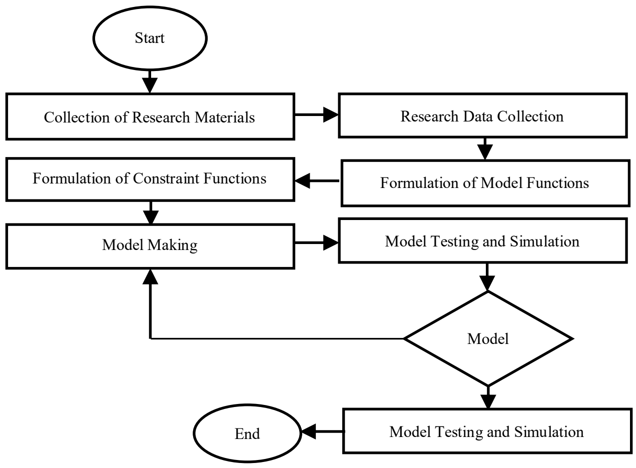

Fig 1. Research Framework

The nine steps in the framework are explained as follows:

1. Collection of Research Materials: Literature and research materials relevant to the research topic were retrieved. These were in the form of books, scientific journals, articles, or other sources of information to understand the issues being studied.
2. Collection of Research Data: Necessary data were gathered to answer the research questions through surveys, interviews, observation, or document analysis. The aim was to collect information considered relevant to the research topic.
3. Formulation of Model Functions: The data collection process was followed by the development of model functions to analyze data and answer research questions. These were in the form of theories, hypotheses, or conceptual frameworks developed based on data and research materials.
4. Formulation of Constraint Functions: Constraint functions to limit or control the variables in the research were developed to assist in designing an adequate research methodology and maintaining factors having a potential influence on the research results.
5. Modeling: Model was developed based on model and constraint functions previously developed. This could be mathematical, statistical, or contextual depending on the type of research being conducted.
6. Model Testing and Simulation: The development of model was followed by tests and simulation. The purpose of the test was to assess the accuracy and validity as well as identify the weaknesses and strengths of model.
7. Proposed Model: The testing and simulation process was followed by the improvement of model through revision or further development to enhance its quality and effectiveness.

B. Research Data

Daily data on new and old outpatient visits for 2016-2021 obtained from the Vita Insane Hospital Pematangsiantar City. It was discovered that the new outpatient was 2192 records and the geriatric were also 2192.

IV. Results and Discussion

This research was conducted based on optimization model developed by Nguyen et al. [6]. The model was to improve the non-optimal maximum capacity. This was considered important due to the inability to meet all outpatient requests, limited range of appointment times for returning patient, and average appointment lead time for old patients due to emergencies which made it difficult to predict the time for new ones. The results of previous relevant studies were discussed before presenting the ones observed from this current research. Moreover, the notations used in model are stated as follows:

u, v, w Target appointment grace period for the median, pth percentile, and 100th percentile of each (time unit).

p pth percentil ($50 < p < 100$).

$[a, b]$ Patient appointment time limits RV(Re-Visit)

$\bar{a}$ Average appointment time for RV patient ($a < \bar{a} < b$) (time units).

α, β Constant rate of discharge for FV and RV patient, where $0 < \alpha, \beta < 1$

τ^f, τ^r Consultation period for FV and RV patient

r_j^f, r_j^r Predefined total number of FV and RV patient in time units j

$\tilde{f}_i$ Stochastic arrival of FV patient in time units i.

d_j^r The total number of RV discharges after appointments in time units j.

x_{ij} Number of FV patient that showed up for appointments at the time unit-th, arrived at the appointment time j, and listed in the system as RV patient j after their visit at the time unit

y_{ij} Number of patient scheduled for a time unit i, had a different appointment at that time j, and listed as RV patient j after time unit.

z_{ij} Number of FV patient observed at the time i and assigned a time unit appointment j

c_i Total capacity needed at a given time unit i.

c_i^f, c_i^r Time units needed to accommodate FV and RV patient i

S The size of the arrival horizon in time units

T The time units included in the planning horizon, $T = \max(S + b, S + w)$

L^m, L^p The set of z_{ij} that satisfies the median grace period target (u) and the p-th percentile.

N_1, N_2, N Index sets, $N_1 = \{1,2, \ldots, S\}$, $N_2 = \{S + 1, S + 2, \ldots, T\}$, $N = N_1 \cup N_2$

ε Possible violation

$\Pr(A)$ The likelihood of an event A

$F_i(\,)$ The distribution function of cumulative data for $\tilde{f}_i$

μ_i, σ_i median and range of uncertain arrivals $\tilde{f}_i$.

The mixed integer model (P1) developed by earlier scholars to address capacity planning problem with demand uncertainty is presented as follows [6]:

[P1]:

Min q (1)

With constraints:

$q \geq c_j, \forall j \in N$ (2)

$Pr\left(\sum_{j=i}^{T} Z_{ij} \geq \tilde{f}_i;\ \forall i \in N_1\right) \geq 1-\varepsilon$ (3)

$\sum_{j=i}^{T} Z_{ij} = 0;\ \forall i \in N_2$ (4)

$x_{ij} - (1-\alpha) z_{ij} = 0, \forall i,j \in N$ (5)

$$\left(r_j^f + r_j^r + \sum_{i=1}^{j} x_{ij} + \sum_{i=1}^{j} y_{ij}\right) - \left(d_j^r + \sum_{i=j}^{T+1} y_{ji}\right) = 0, \forall j \in N \quad (6)$$

$$d_j^r - \beta\left(r_j^f + r_j^r + \sum_{i=1}^{j} y_{ij} + \sum_{i=1}^{j} x_{ij}\right) = 0, \forall j \in N \quad (7)$$

$y_{ij} = 0, \forall j - i < a, \forall i,j \in N$ (8)

$y_{ij} = 0, \forall j - i < b, \forall i,j \in N$ (9)

$\sum_{i=S+1}^{T} d_i^r = 0$ (10)

$\sum_{z_{ij} \in L^m} z_{ij} \geq \left(\frac{1}{2}\sum_{i=1}^{S}\sum_{j=i}^{T} z_{ij}\right) + 1$ (11)

$\sum_{z_{ij} \in L^p} z_{ij} \geq \frac{p}{100}\left(\sum_{i=1}^{S}\sum_{j=i}^{T} z_{ij}\right)$ (12)

$z_{ij} = 0, \forall j - i > w, \forall j - i < 0$ (13)

$\sum_{j=1}^{T}\sum_{i=i}^{j} (j-i)\, y_{ij} - \bar{a}\sum_{j=1}^{T}\sum_{i=i}^{j} y_{ij} \leq 0$ (14)

$c_j^f - \left(\frac{1}{1-\alpha}\tau^f r_j^f + \tau^f \sum_{i=i}^{j} z_{ij}\right) = 0, \forall j \in N$ (15)

$c_j^r - \left(\frac{1}{1-\beta}\tau^r r_j^r + \frac{1}{1-\beta}\tau^r \sum_{i=i}^{j} y_{ij}\right) = 0, \forall j \in N$ (16)

$c_j - \left(c_j^f + c_j^r\right) = 0, \forall j \in N$ (17)

$x_{ij}, y_{ij}, z_{ij}, c_i^f, c_j^r, d_i^f, d_j^r \geq 0, \forall i,j \in N$ (18)

$z_{ij} \in Z^+, \forall i,j \in N$ (19)

Objective (1) minimized the maximum capacity required per unit of time to obtain a predetermined target grace period. The needed maximum capacity was mentioned in Constraint (2) while Constraints (3), (4), and (5) showed that flow was conserved at the FV node (5). The constraint required a minimal probability that all patient requests could be scheduled (3). Moreover, the inequality on the left side should be used to design any patient requests inside the arrival horizon (3). The random nature of arrivals showed that conditions (3) was also a chance limitation. No demand was considered after the arrival horizon as showed in Constraint (4). Furthermore, the number of FV patient to remain RV was set by the cap based on the initial appointment (5), limits (6), and an illustration of the flow conservation in the RV node (7). A unit of time for all the appointments of the FV and RV patient, as well as their total number determined in advance are presented in the first set of brackets of Constraint (6). The second bracket represents the outflow of all appointments scheduled for a unit of time including the number of RV patient discharged following their visit and then the subsequent visits. Meanwhile, the number of patients released after a visit was constrained by constraint number seven. All RV patient was required to have a grace period within a limited range of appointments due to Constraints (8) and (9). No RV patient discharge was also permitted following the final limit of the arrival horizon in Constraint (10). The median, 100th percentile, and 100th percentile appointment deadlines were required to be reached according to constraints (11), (12), and (13). The limitation of the typical RV appointment grace period was presented in Constraint (14) while the overall capacity needed was calculated in units of time through (15) and (16). The power required for FV and RV patient was also determined in units of time using (17). The space needed was for both returning and new FV or RV patient. Meanwhile, Constraints (18) and (19) served as the prerequisites for integrality and non-negative variables.

The Ideas from earlier studies were used to create an equivalent capacity planning deterministic model. The process focused on approximating the joint opportunity constraint issue convexly in this section as a deterministic linear program. The issue associated with opportunity constraints (3) in the P1 joint was broken down into individual opportunity constraints to provide another approximation. The decomposition was calculated according to the inequality of Boole [32] $Pr(\cup_i A_i) \leq \sum_i \Pr(A_i)$. This showed that the probability of one event happening was not more than the total probability of all possible circumstances. The shared probability constraint limited the likelihood that all FV patient was scheduled for all available periods (3). Moreover, the application of the inequality intersection presented in conditions led to the reformulation of the equation as follows:

$Pr\left(\cap_{i \in N_1}\left\{\sum_{j=i}^{T} Z_{ij} - \tilde{f}_i \geq 0\right\}\right) \geq 1-\varepsilon$ (20)

This was rewritten using $Pr(\cap_i A_i) = 1 - Pr(\cup_i (A_i)^c)$, where $(A_i)^c$ was the complement of A_i: $1 - Pr\left(\cup_{i \in N_1}\left\{\sum_{j=i}^{T} Z_{ij} - \tilde{f}_i \geq 0\right\}^c\right) \geq 1-\varepsilon$. Therefore, Constraint (20) was observed to be equivalent to the following Constraint (21):

$Pr\left(\cup_{i \in N_1}\left\{\sum_{j=i}^{T} Z_{ij} - \tilde{f}_i < 0\right\}\right) \leq \varepsilon$ (21)

This created an upper bound on the likelihood that a violation would occur within one or more periods, with a violation occurring when an FV patient could not be scheduled. The limit on the left-hand side was obtained by applying Boole's inequality to Constraint (21) as follows:

$Pr\left(\cup_{i \in N_1}\left\{\sum_{j=i}^{T} Z_{ij} - \tilde{f}_i < 0\right\}\right) \leq \sum_{i \in N_1} Pr\left(\sum_{j=i}^{T} Z_{ij} - \tilde{f} < 0\right)$ (22)

This was based on the consideration that the $Pr\left(\sum_{j=i}^{T} Z_{ij} - \tilde{f}_i < 0\right) \leq \varepsilon_i$ and $\sum_{i \in N_1} \varepsilon_i \leq \varepsilon$ were proposed using the following individual opportunity constraints as a conservative approximation of Constraint (3).

$Pr\left(\sum_{j=i}^{T} Z_{ij} - \tilde{f}_i < 0\right) \leq \varepsilon_i, \forall i \in N_1$ (23)

The value on the right side of $\varepsilon_i \geq 0$ was selected and this showed that $\sum_{i \in N_1} \varepsilon_i = \varepsilon$. This showed the application of (22) and (23) ensured the fulfillment of (21). The Constraint (23) was rewritten as:

$Pr\left(\tilde{f}_i - \sum_{j=i}^{T} Z_{ij} \leq 0\right) \geq 1 - \varepsilon_i, \forall i \in N_1$ (24)

Constraint (24) was provided with a lower bound on the chance of scheduling FV patient during each period. The value of ε_i was recommended to be set to $\varepsilon_i = \frac{\varepsilon}{S}$ [33] to produce the following opportunity constraint:

$Pr\left(\tilde{f}_i - \sum_{j=i}^{T} Z_{ij} \leq 0\right) \geq 1 - \frac{\varepsilon}{S}, \forall i \in N_1$ (25)

The legal opportunity limitations (3) could be broken down into a denser set of individual opportunity constraints than (3) in problem (P1). Therefore, the deterministic equivalent of this particular probability restriction was established based on the information gathered about the probability distribution.

An equivalent deterministic model was also developed for the full distribution of information on demand arrival uncertainties. The probability distribution of the uncertainty of the arrival of requests was assumed to be known every time in this section $\tilde{f}_i$. For example, it was assumed that request arrivals had a known mean and variance and were normally distributed. Moreover, the cumulative distribution function was required to be stated as $\tilde{f}_i$ of $F_i()$. This led to the presentation of Constraint (25) as follows:

$$F_i\left(\sum_{j=i}^{T} Z_{ij}\right) \geq 1 - \frac{\varepsilon}{S}, \forall\, i \in N_1 \quad (26)$$

This (26) was further converted into a linear constraint using the inverse of the cumulative distribution function as follows:

$$\sum_{j=i}^{T} Z_{ij} \geq F_i^{-1}\left(1 - \frac{\varepsilon}{S}\right), \forall\, i \in N_1 \quad (27)$$

The probability constraint (3) in (P1) was replaced by (27) in order to obtain a tractable linear optimization problem in the following relationship.

[P1-a]:

Min q (1a)

With constraints:

$q \geq c_j, \forall\, j \in N$ (2a)

$\sum_{j=i}^{T} Z_{ij} \geq F_i^{-1}\left(1 - \frac{\varepsilon}{S}\right), \forall\, i \in N_1$ (3a)

Constraints (4) – (19) (4a)

Optimization model obtained was reduced to an algorithm using the Monte Carlo simulation based on the following steps:

1. Initialize the variables N, T, epsilon, and S.
   Initialize the vector F containing the value Fi for each i in N.
   Initialize the Z matrix with size N x T filled with normally distributed random numbers.
2. For each i in N, calculate the values $F_{inverse}$ = qnorm(1 - epsilon/S, mean=0, sd=1) / sqrt(T) and F_i inverse = $F_{inverse}$ + (mean(Z[i,j]) / sd (Z[i,j])).
3. For every i in N, calculate $S_i = 0$.
4. Repeat for $N_{iteration}$
5. For every i in N, do a Monte Carlo simulation by:
   a. Calculate the value (Z_i = sum (Z [i, j])).
   b. If Z_i >= $F_{i\ invers}$, add 1 to the value of S_i.
6. Calculate the probability value for each i in N using probabilities[i] = $S_i/N_{iteration}$.
7. Calculate the overall probability value using total $_{prob}$ = prob (1 - probabilities).
8. Print the overall probability values.

The analysis conducted on the previous model limits using the R language produced a probability value of 0 while the optimized new model had 0.51 which was considered a significant increase.

Program code Using the R Language in the Previous Model:

```
# Monte Carlo function to calculate probability
  monte_carlo <-function(Z, f_hat, n_iter, threshold)
  {
  S <- 0
  For (i in 1:n_iter) {
      Z_i <- sum(Z[i,])
      if (Z_i >= f_hat) {
        S <- S + 1
      }
      }
  return(S/n_iter)
  }
  # Initialize N and epsilon values
  N <- 100
  epsilon <- 0.01
  # Value initialization T, f_hat, and Z
  T <- 10
  f_hat <- c(5,6,7,8,9)
  Z <- matrix(rnorm(N*T), ncol=T)
  # Calculate probability using Monte Carlo
  probabilities <- numeric(length(f_hat))
  for (i in 1:length(f_hat)) {
  prob_i <- monte_carlo(Z[,i], f_hat[i], 10000,
  threshold=epsilon)
   probabilities [i] <- prob_i
  }
  # Calculate the overall probability using the multiplication
  method
  total_prob <- 1
  for (i in 1:length(probabilities)) {
   total_prob <- total_prob * probabilities[i]
  }
  total_prob <- 1 - total_prob
  # Print overall probability results
  cat("Overall probability:", total_prob, "\n")
```

Program code Using the R Language on an Optimized Model:

```
  # Determine the value T, Fi^(-1), and ε/S
  T <- 100
  Fi_inv <- 0.5
  epsilon_over_S <- 0.01
  # Create many random samples for the variable Zij
  Z <- matrix(rnorm(T * T), ncol = T)
  # Calculate the number of samples for each i in the set N1
  sum_Z <- apply(Z, 1, function(row) sum(row[1:T]))
  # Evaluate conditions: sum_Z >= Fi_inv *(1-
epsilon_over_S)
  for each i in N1
  pass_condition <- sum_Z >= Fi_inv * (1 - epsilon_over_S)
  # Calculates probability by calculating the average of
conditions that are met
  probability <- mean(pass_conditions)
  # Shows probability estimation results
  cat("Probability estimation:", probability, "\n").
```

The algorithm produced was tested using the R language and the results showed a probability value of 0.51. This showed the presence of 0.51 more FV patient scheduled for appointments at time unit j and arrived at time unit *i* compared to those that stochastically arrived at unit time j. Each test was different because random values were generated from the computer memory for each occasion.

V. Conclusion

In conclusion, model was able to optimize the problems associated with managing unfulfilled outpatient requests, a limited range of appointment times for returning patient, and the average appointment lead time for old patient due to emergencies. Model also fixed the constraints identified in the previous model by determining the schedule based on the probability distribution information and the deterministic equivalent of the individual opportunity limitations. Moreover, the probability value increased by 0.51 compared to the previous model.

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