A SEInIeR Cyber Public Opinion Propagation Prediction Model with Extreme Emotion Mechanism

Qiujuan Tong, Shengqi Yue, Jianke Zhang, Yihan Liu and Zheyu Han

Abstract—**As the landscape of social networks evolves, the dissemination of public opinion through online channels has become increasingly dominant. Consequently, researchers have turned their attention to the impact of emotional factors introduced by communicators when conveying public opinion. This paper proposes a novel SEInIeR public opinion communication prediction model, incorporating an extreme emotion mechanism, which acknowledges the inuential nature of speech under heightened emotional states. By integrating the traditional infectious disease SEIR model with fractional order differentiation, our model not only ensures the existence and stability of the disease-free equilibrium point E0 but also allows for more accurate prediction. Finally, the model has been validated and analyzed through simulation experiments and numerical simulations. Furthermore, example data were collected for analysis and tting. The experimental results provide substantial evidence for the accuracy of our theoretical analyses and the plausibility of the proposed model.**

Index Terms—**extreme emotions; SEInIeR model; stability analysis; public opinion dissemination; fractional differentiation**

I. INTRODUCTION

ONLINE social networks have emerged as the primary platform for the activities of netizens, exerting a significant inuence on the dissemination of public opinion. Platforms like Weibo, TikTok, and Twitter witness a substantial level of engagement from netizens. As of June 2022, the number of Internet users in China was 1.051 billion, with an internet penetration rate of 74.4 % [1]. Internet public opinion refers to the collective emotions, attitudes, and opinions expressed and disseminated through the Internet regarding various public affairs that concern individuals or are closely related to their interests [2]. It is crucial to acknowledge that netizens may express their personal opinions, which at times tend to be excessively one-sided and may even transform into baseless rumors as they spread. The proliferation of such remarks poses a threat to the establishment of a civilized online environment and has the potential to disrupt social order and stability.

The exploration of public opinion dissemination can be traced back to the 1920s. In 1927, Kermack proposed the classical SIR epidemic model after studying the epidemic pattern of bubonic plague [3]. Subsequently, scholars identified similarities between public opinion dissemination and disease communication and continued their research based on this premise. Daley et al. proposed the DK model to study the problem of information communication [4]. These two models are highly representative and have formed the theoretical foundation for the study of public opinion dissemination.

Subsequent to the initial models, scholars have made notable advancements in the field by incorporating various factors associated with the propagation of opinions. These factors encompass the hesitation mechanism [5], the forgetting mechanism [6], opinion leaders [7], and the discussion mechanism [8]. Subsequent research has investigated the impact of human behavior on opinion dissemination. Jiang proposed the 2I2SR model to explain the dissemination pattern of opinion rumors in a multilingual environment [9]. Yu developed the SIMR dissemination model for information transmission through two channels: friends and non-friends via marketing accounts [10]. Wang examined the situation of immunisers facing a new topic, taking into account that people's attention is drawn to new topics [11]. Di classified communicators into three categories: supporters, neutrals, and opponents, to investigate the impact of media on the development of public opinion [12].

In recent years, scholars have dedicated their research to understanding the inuence of netizens' emotional states on public opinion dissemination. Jin proposed that emotionality serves as a driving force for dissemination and suggested that emotional factors expedite online public opinion dissemination, thereby enhancing communication between information publishers and transmitters [13]. Tian added to the classical contagion model, highlighting the positive emotion's purication effect [14]. Zhang developed an emotional communication model among netizens that accounts for the cumulative effect of negative emotions. This model draws upon the theory of emotional infection and the classical epidemic dynamics model [15]. Geng, on the other hand, constructed a dual-intervention SEI2R1R2 model of network per medium based on the SEIR model. The model was explored under the condition of dual intervention by the government and the network media [16]. Zhao formulated the SIpInR model which takes into consideration the interactivity of emotions among internet users and encompasses dual

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emotional intersectionality [17].

This paper takes into account the emergence of extreme emotions during unexpected online public opinion dissemination. The model distinguishes between normal and extreme emotion disseminators and proposes a SEInIeR model for extreme emotions. Experimental validation is used to depict the evolution of major online public opinion based on this model.

This papers primary innovations are as follows:

1. A fractional-order differential has been introduced to the model due to the limitations in memorability observed with integer-order differential.

2. A single infection rate γ was proposed to represent the unidirectional connectivity between the two emotions, extreme and normal. The inflammatory nature of extreme emotions was then represented using an acceleration factor ε. The SEInIeR model was developed.

The remainder of this paper is structured as follows: The modeling procedure and fractional-order discretization are explained in Section II. The stability of the system is demonstrated and its equilibrium point is discussed in Section III. In Section IV, simulation experiments are conducted to verify the model. Section V validates the model using real-life examples. The final section VI provides a brief conclusion to the paper.

II. Establish of the model

A. Conformable Fractional Derivative

The differential order traditionally used is a local operator and an integer. However, the lack of memory effects makes utilizing integer-order differential to examine epidemic models limited. In contrast, a fractional-order differential system's next state depends not only on the current state but also on its historical state, which is the memory of the non-local operator [18]. The fractional-order differential has developed rapidly in recent decades. Classical denitions of fractional order differentiation include Riemann-Liouville [19] and Caputo [20]. The conformable fractional derivative (CFD) [21] is widely used due to its close relationship to the dening rst-order derivative.

Definition 1: Set a function $f(t)$: $[0,\infty) \to R$. For all $t>0$, $\alpha \in (0,1)$. the definition of CFD of $f(t)$ is:

$$D^\alpha f(t) = \lim_{\varepsilon \to \infty} \frac{f(t+\varepsilon t^{1-\alpha}) - f(t)}{\varepsilon} \tag{1}$$

when $t=0$, $D^\alpha f(0) = \lim_{t\to\infty} D^\alpha f(t)$

Theorem 1: The relationship between CFD [21] and the first derivative is as follows: $\alpha \in (0,1)$,$t>0$

$$D^\alpha f(t) = t^{1-\alpha} \frac{df(t)}{dt} \tag{2}$$

When $\alpha = 0$, CFD is the first derivative.

B. Initial SEIR Model

1) Model assumption

The traditional SEIR model divides netizen groups in the dissemination of online public opinion as follows: S (denoting the group that is not aware of the public opinion for the time being, called susceptible); E (denoting the group that has learned about the public opinion but has not yet disseminated it, called latent); I (denoting the group that has received the public opinion and has already begun to disseminate it, called infective); and R (denoting the group that has ceased to disseminate the public opinion, called removal).

2) Model state transfer rule

The densities of susceptible, latent, infected, and removed individuals are denoted by $S(t)$, $E(t)$, $I(t)$, and $R(t)$, respectively, at time t. The state transfer of the SEIR model is shown in Fig. 1.

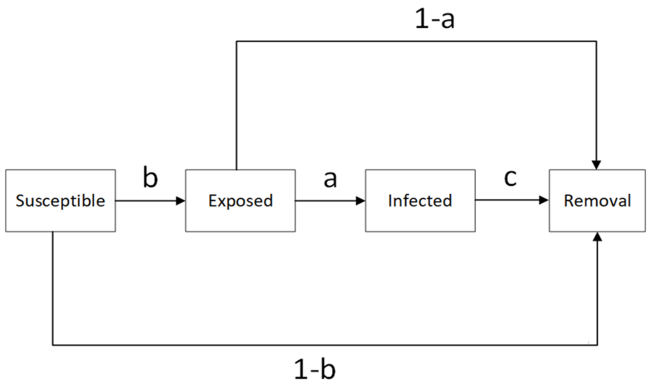


Fig. 1. The basic scheme of the initial SEIR model.

The specific characteristics of each node are described below:

For the S node: When susceptible individuals are exposed to infected individuals, they become aware of public opinion. A portion of the susceptible individuals are then converted to latent with a probability of b, while another fraction is directly converted to removers with a probability of $1-b$.

For the E node: Individuals in the latent group are infected with a probability of a and begin to spread public opinion. The remaining individuals become removers with a probability of $1-a$.

For the I node: During transmission, some infected individuals may become removers with a probability of c due to loss of interest or forgetfulness.

For the R node: The removers are no longer involved in the dissemination of public opinion and are not affected by it.

C. SEInIeR Model Based on Extreme Emotion Mechanism

1) Model assumption

Assumption 1: The definitions of the S, E, I, and R groups in this model are similar comparable to those in the conventional SEIR model. Depending on the infected person's emotional state, the I nodes are further divided into Ie (extreme emotion) and In (normal emotion). Ie denotes that the infected person is in an extreme emotional state and spreads public opinion by making irrational, radical, and extreme speeches, while In denotes that the infected person is in a normal emotional state and spreads public opinion by making normal speeches.

Assumption 2: The online social network platform experiences dynamic variations in its user base, represented by the parameters Λ and d, which stand for the number of people inflow and outflow per unit of time.

Assumption 3: Online public opinion can be influenced by emotional factors [13], and public communication behavior may also be impacted by the external utility of social group networks [22]. Extreme and normal emotions are two opposing types of emotions. Therefore, the emotional intensication degree q and the emotional divergence degree f are added here [17]. The former indicates the degree of communicator enhancement under the inuence of emotion, and the latter indicates the emotional network's external utility to the netizen group. In this model, f represents the normal emotional network external utility of the Internet user group, while $1f$ represents the extreme emotional network external utility of the Internet user group.

The densities of susceptible, latent, normal emotion transmitter, extreme emotion transmitter, and removal are denoted by $S(t)$, $E(t)$, $In(t)$, $Ie(t)$, and $R(t)$, respectively, at the moment of t. The state transfer of the SEInIeR model is shown in Fig. 2.

$$\begin{cases} \alpha_1 = (a+q)f \\ \alpha_2 = (a+q)(1-f) \\ \alpha_3 = (1-a)(1-q) \\ \beta_1 = cf \\ \beta_2 = c(1-f) \end{cases} \tag{3}$$

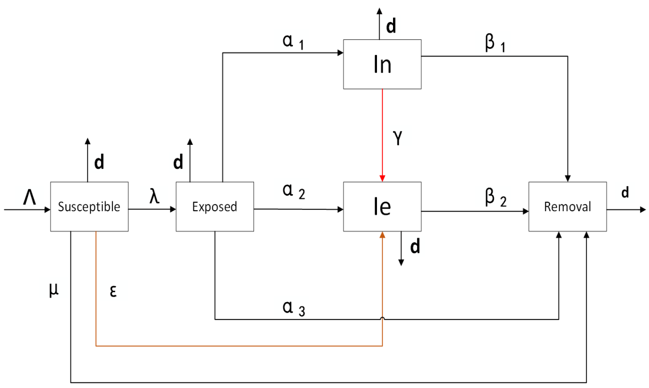


Fig. 2. The scheme of the SEInIeR model.

2) Model state transfer rule

Regarding the S node: At a specific transformation rate λ, members of the susceptible group who come into contact with infected persons may turn into lurkers. Some of them may not care about the public opinion or may become removers for other reasons and thus quit the propagation. The removal rate is set to μ. When communicating with individuals who have strong emotions in the Ie node, they may be influenced by public opinion that is also emotionally charged. Netizens in the S node skip the E node directly with an acceleration factor of ε, become members of the Ie node, and participate in public opinion dissemination.

Regarding the E node: The population of E node undergoes three dynamic change states. The first state involves expressing opinions and discussing public opinion through normal emotions, resulting in the individual in the E node transforming into a member of the In node with a probability of α_1. The second state involves expressing opinions and discussing public opinion through remarks with extreme emotions, resulting in the individual in the E node transforming into a member of the Ie node with a probability of α_2. The third state is when individuals decide not to participate in the discussion and dissemination and become removers with a probability of α_3, exiting the dissemination process.

Regarding the In node and the Ie node: During the dissemination process, it is essential to avoid extreme emotions, as they can be inflammatory. Internet users can become biased when exposed to statements with extreme emotions. Individuals in the In node may join the group of the Ie node with a probability of γ. However, it is difcult to convert extreme sentiment users into normal sentiment users, as they tend to scoff at or even speak ill of the normal sentiment users' remarks. Thus, there is only a one-way infection rate from In node to Ie node. Over time, individuals in both In and Ie nodes gradually lose interest or forget and become removers, with probabilities of β_1 and β_2, respectively.

Regarding the R node: The removers are no longer influenced by public opinion, are immune to the message, and are no longer involved in its dissemination.

3) Model building

After the analysis of the model laws of state transfer, the following differential equations of state transfer can be derived:

$$\begin{cases} \frac{dS(t)}{dt} = \Lambda - \lambda S(t)(In(t)+Ie(t)) - \varepsilon S(t)Ie(t) \\ \qquad - (\mu+d)S(t) \\ \frac{dE(t)}{dt} = \lambda S(t)(In(t)+Ie(t)) - (\alpha_1+\alpha_2+ \\ \qquad \alpha_3+d)E(t) \\ \frac{dIn(t)}{dt} = \alpha_1 E(t) - \gamma In(t)Ie(t) - (\beta_1+d)In(t) \\ \frac{dIe(t)}{dt} = \alpha_2 E(t) + \gamma In(t)Ie(t) + \varepsilon S(t)Ie(t) \\ \qquad - (\beta_2+d)Ie(t) \\ \frac{dR(t)}{dt} = \beta_1 In(t) + \beta_2 Ie(t) + \mu S(t) + \alpha_3 E(t) \\ \qquad - dR(t) \end{cases} \tag{4}$$

The fractional order state transfer differential equation, as defined by CFD, is:

$$\begin{cases} D^\alpha S(t) = t^{1-\alpha}\frac{dS(t)}{dt} = \Lambda - \lambda S(t)(In(t)+Ie(t)) \\ \qquad - \varepsilon S(t)Ie(t) - (\mu+d)S(t) \\ D^\alpha E(t) = t^{1-\alpha}\frac{dE(t)}{dt} = \lambda S(t)(In(t)+Ie(t)) \\ \qquad - (\alpha_1+\alpha_2+\alpha_3+d)E(t) \\ D^\alpha In(t) = t^{1-\alpha}\frac{dIn(t)}{dt} = \alpha_1 E(t) - \gamma In(t)Ie(t) \\ \qquad - (\beta_1+d)In(t) \\ D^\alpha Ie(t) = t^{1-\alpha}\frac{dIe(t)}{dt} = \alpha_2 E(t) + \gamma In(t)Ie(t) \\ \qquad + \varepsilon S(t)Ie(t) - (\beta_2+d)Ie(t) \\ D^\alpha R(t) = t^{1-\alpha}\frac{dR(t)}{dt} = \beta_1 In(t) + \beta_2 Ie(t) \\ \qquad + \mu S(t) + \alpha_3 E(t) - dR(t) \end{cases} \tag{5}$$

The overall population of the social networking site satisfies the equation $N(t) = S(t)+E(t)+I(t)+Ie(t)+R(t)$. $S(0), E(0), In(0), Ie(0), R(0) \geq 0$. Λ, d, λ, ε, α_1, α_2, α_3, β_1, β_2, $\gamma \in (0,1)$.

From (Eq. 5), we have:

$$\begin{cases} D^{\alpha}N(t) = \Lambda - dN(t) \\ N(t) = \dfrac{\Lambda}{d} + Ce^{-dt} \\ \lim\limits_{t\to\infty} N(t) = \dfrac{\Lambda}{d} \end{cases} \tag{6}$$

The system (5)'s positive invariant set is:

$$\Phi = \{(S, E, In, Ie, R) \in R_5^+ : S + E + In + Ie + R \leq \frac{\Lambda}{d}\}$$

As $R(t)$ does not affect the first four equations, the system (5) can be simplified in the following way:

$$\begin{cases} D^{\alpha}S(t) = t^{1-\alpha}\dfrac{dS(t)}{dt} = \Lambda - \lambda S(t)(In(t) + Ie(t)) \\ \qquad - \varepsilon S(t)Ie(t) - (\mu + d)S(t) \\ D^{\alpha}E(t) = t^{1-\alpha}\dfrac{dE(t)}{dt} = \lambda S(t)(In(t) + Ie(t)) \\ \qquad - (\alpha_1 + \alpha_2 + \alpha_3 + d)E(t) \\ D^{\alpha}In(t) = t^{1-\alpha}\dfrac{dIn(t)}{dt} = \alpha_1 E(t) - \gamma In(t)Ie(t) \\ \qquad - (\beta_1 + d)In(t) \\ D^{\alpha}Ie(t) = t^{1-\alpha}\dfrac{dIe(t)}{dt} = \alpha_2 E(t) + \gamma In(t)Ie(t) \\ \qquad + \varepsilon S(t)Ie(t) - (\beta_2 + d)Ie(t) \end{cases} \tag{7}$$

III. The Equilibrium point and stability analysis

A. Equilibrium point and Basic regeneration number

The online public opinion dissemination within the system eventually stabilizes, meaning that the values reach an equilibrium point. Let the right side of the (Eq. 7) be zero, and the disease-free equilibrium point (public opinion calming down, the number of both extreme and normal emotion transmitters are zero) E_0 and the public opinion prevailing equilibrium point E_1 can be obtained.

E_0 can be calculate at the disease-free equilibrium point:

$$E_0 = (S_0, E_0, In_0, Ie_0) = (\frac{\Lambda}{\mu + d}, 0, 0, 0)$$

The basic regeneration number in the infectious epidemic model [23] is used as a parameter to determine whether public opinion is still spreading. It indicates that the spread of public opinion is gradually subsiding when $R_0 < 1$, on the contrary, it is still spreading when $R_0 > 1$. This parameter can also be used for stability analysis. R_0 can be solved by the next generation matrix method [24].

Let $X(t) = (E(t), In(t), Ie(t), S(t))^{\top}$. As stated in system (7):

$$D^{\alpha}X(t) = F(x) - \Psi(x) \tag{8}$$

where,

$$F(x) = \begin{bmatrix} \lambda S(In + Ie) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Psi(x) =$$

$$\begin{bmatrix} (\alpha_1 + \alpha_2 + \alpha_3 + d)E(t) \\ \alpha_1 E(t) + \gamma In(t)Ie(t) + (\beta_1 + d)In(t) \\ -\alpha_2 E(t) - \gamma In(t)Ie(t) - \varepsilon S(t)Ie(t) + (\beta_2 + d)Ie(t) \\ -\Lambda + \lambda S(t)(In(t) + Ie(t)) + \varepsilon S(t)Ie(t) + (\mu + d)S(t) \end{bmatrix}$$

Do the Jacobi matrix calculation with $F(x)$, $\Psi(x)$, substitute E_0 respectively. We get:

$$J(F|E_0) = \begin{bmatrix} 0 & \lambda S_0 & \lambda S_0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} F_0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J(\Psi|E_0) =$$

$$\begin{bmatrix} (\sum_{i=1}^{3} \alpha_i) + d & 0 & 0 & 0 \\ -\alpha_1 & \beta_1 + d & 0 & 0 \\ -\alpha_2 & 0 & \beta_1 + d - \varepsilon S_0 & 0 \\ 0 & \lambda S_0 & \varepsilon S_0 & \mu + d \end{bmatrix}$$

$$= \begin{bmatrix} V_0 & 0 \\ C & C_1 \end{bmatrix}$$

$$F_0 = \begin{bmatrix} 0 & \lambda S_0 & \lambda S_0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V_0 = \begin{bmatrix} (\sum_{i=1}^{3} \alpha_i) + d & 0 & 0 \\ -\alpha_1 & \beta_1 + d & 0 \\ -\alpha_2 & 0 & \beta_1 + d - \varepsilon S_0 \end{bmatrix}$$

The next generation matrix FV^{-1} can be calculated as follows:

$$F_0 V_0^{-1} = \begin{bmatrix} A_1 & A_2 & A_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_1 = \frac{\lambda S_0[\alpha_2(\beta_1 + d) + \alpha_1(\beta_2 + d - \varepsilon S_0)]}{((\sum_{i=1}^{3} \alpha_i) + d)(\beta_1 + d)(\beta_2 + d - \varepsilon S_0)}$$

$$A_2 = \frac{\lambda S_0}{\beta_1 + d}$$

$$A_3 = \frac{\lambda S_0}{\beta_2 + d - \varepsilon S_0}$$

The spectral radius of the next generation matrix FV^{-1} is the basic regeneration number of system (7).

$$R_0 = \rho(F_0 V_0^{-1}) = A_1 \tag{9}$$

B. Equilibrium Stability Analysis

Theorem 2: For system (7), if $R_0 < 1$ and $\beta_2 + d - \varepsilon S_0 > 0$, then the disease-free equilibrium point is local asymptotical stable.

Proof 1: The Jacobi matrix of system (7) at E_0 is as follows:

$$J(E_0) =$$

$$\begin{bmatrix} -(\mu + d) & 0 & -\lambda_1 S_0 & -(\lambda_1 + \varepsilon)S_0 \\ 0 & (\sum_{i=1}^{3} \alpha_i) + d & \lambda_1 S_0 & \lambda_1 S_0 \\ 0 & \alpha_1 & -(\beta_1 + d) & 0 \\ 0 & \alpha_2 & 0 & -(\beta_2 + d) \end{bmatrix}$$

The characteristic polynomial of this matrix is given (In order to distinguish λ, the parameter λ in the matrix are denoted λ_1):

$$|\lambda E - J(E_0)| = (\lambda + \alpha_1 + \alpha_2 + \alpha_3 + d)(B_1\lambda^3 + B_2\lambda^2 + B_3\lambda + B_4) = 0$$

One of the eigenvalues is known to be significantly less than 0, then $f(\lambda) = B_1\lambda^3 + B_2\lambda^2 + B_3\lambda + B_4 = 0$

$$B_1 = 1$$
$$B_2 = \alpha_1 + \alpha_2 + \alpha_3 + \beta_1 + \beta_2 + 3d - \varepsilon S_0$$
$$B_3 = (\beta_1 + d)(\beta_2 + d - \varepsilon S_0) + (\alpha_1 + \alpha_2 + \alpha_3 + d)(\beta_1 + \beta_2 + 2d - \varepsilon S_0) - (\alpha_1 + \alpha_2)\lambda_1 S_0$$
$$B_4 = (\alpha_1 + \alpha_2 + \alpha_3 + d)(\beta_1 + d)(\beta_2 + d - \varepsilon S_0) - \lambda S_0[\alpha_2(\beta_1 + d) + \alpha_1(\beta_2 + d - \varepsilon S_0)]$$

If $R_0 < 1$ and $\beta_2 + d - \varepsilon S_0 > 0$, we obtain $B_2, B_3, B_4 > 0$. $B_1, B_2, B_3, B_4 > 0$, and they all have negative real parts. According to the Routh-Hurwitz criterion [25]. The disease-free equilibrium point is locally asymptotically stable.Conversely, if $R_0 > 1$ or $\beta_2 + d - \varepsilon S_0 < 0$, E_0 is unstable.

Theorem 3: For system (7), if $R_0 < 1$, $0 < \beta_2 + d - \varepsilon S_0 < 1$ and $0 < \beta_1 + d < 1$, then the disease-free equilibrium point is global asymptotic stable.

Proof 2: It can be obtained from the first formula in system (7): $\frac{dS}{dt} \le \Lambda - (\mu + d)S$. Through the constant variation method,we get $S \le \frac{\Lambda}{\mu+d} + Ce^{-(\mu+d)t}$, C is an arbitrary constant. A sufficiently small integer ζ can satisfy:

$$\begin{aligned} S \le& \Lambda/(\mu + d) + [\zeta(\mu + d)(\alpha_1 + \alpha_2 + \alpha_3 + d)(\beta_1 + d)(\beta_2 \\ &+ d - \varepsilon S_0)]/[\alpha_2(\beta_1 + d) + \alpha_1(\beta_2 + d - \varepsilon S_0)] \end{aligned} \tag{10}$$

The lyapunov function $L(t)$ is constructed:

$$\begin{aligned} L(t) =& (\alpha_1 + \alpha_2)E + (\alpha_1 + \alpha_2 + \alpha_3 + d)In \\ &+ (\alpha_1 + \alpha_2 + \alpha_3 + d)Ie \\ D^\alpha L(t) =& (\alpha_1 + \alpha_2)D^\alpha E(t) + (\alpha_1 + \alpha_2 + \alpha_3 + d)D^\alpha In(t) \\ &+ (\alpha_1 + \alpha_2 + \alpha_3 + d)D^\alpha Ie(t) \end{aligned}$$

Plug in the equations of system (7)

$$\begin{aligned} D^\alpha L(t) =& [\lambda S(\alpha_1 + \alpha_2) - (\alpha_1 + \alpha_2 + \alpha_3 + d)(\beta_1 + d)]In \\ &+ [\lambda S(\alpha_1 + \alpha_2) - (\alpha_1 + \alpha_2 + \alpha_3 + d)(\beta_2 + d - \varepsilon S_0]Ie \end{aligned}$$
$$A = \lambda S(\alpha_1 + \alpha_2) - (\alpha_1 + \alpha_2 + \alpha_3 + d)(\beta_1 + d)$$
$$B = \lambda S(\alpha_1 + \alpha_2) - (\alpha_1 + \alpha_2 + \alpha_3 + d)(\beta_2 + d - \varepsilon S_0)$$

From formula (10), we also get:

$$\begin{aligned} A \le& \lambda(\alpha_1 + \alpha_2)[(\Lambda/\mu + d) + \zeta(\mu + d)(\alpha_1 + \alpha_2 + \alpha_3 + d) \\ &(\beta_1 + d)(\beta_2 + d - \varepsilon S_0)]/[\alpha_2(\beta_1 + d) + \alpha_1(\beta_2 + d - \\ &\varepsilon S_0)] - (\alpha_1 + \alpha_2 + \alpha_3 + d)(\beta_1 + d) \\ &\le \lambda[\alpha_2(\beta_1 + d) + \alpha_1(\beta_2 + d - \varepsilon S_0)]\frac{\Lambda}{\mu + d} + \zeta\lambda(\alpha_1 + \\ &\alpha_2 + \alpha_3 + d)(\beta_1 + d)(\beta_2 + d - \varepsilon S_0)(\mu + d) - (\alpha_1 + \\ &\alpha_2 + \alpha_3 + d)(\beta_1 + d) \\ &= (\alpha_1 + \alpha_2 + \alpha_3 + d)(\beta_1 + d)(\beta_2 + d - \varepsilon S_0)[R_0 + \\ &\zeta\lambda(\mu + d) - \frac{1}{\beta_2 + d - \varepsilon S_0}] \end{aligned}$$

It is also can be obtained in the similar way:

$$\begin{aligned} B \le& (\alpha_1 + \alpha_2 + \alpha_3 + d)(\beta_1 + d)(\beta_2 + d - \varepsilon S_0) \\ &[R_0 + \zeta\lambda(\mu + d) - \frac{1}{\beta_1 + d}] \end{aligned}$$

when $R_0 < 1$, $0 < \beta_2 + d - \varepsilon S_0 < 1$ and $0 < \beta_1 + d < 1$,there is $\frac{dL}{dt} \le 0$.

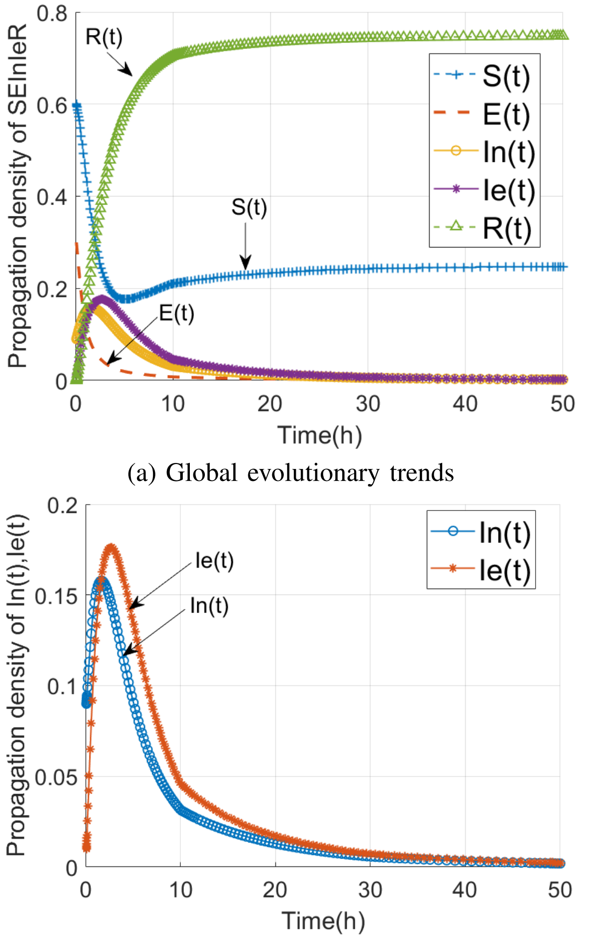
The system (7) is globally asymptotic stable, in accordance with the Lyapunov stability theorem [26].

IV. Simulation Results and Analysis

Numerical simulation experiments were conducted using Matlab in this section. The disease-free equilibrium point was veried, and the impact of each parameter on the process of public opinion dissemination was researched and discussed using the method of controlling variables.

A. Stability Simulation and Analysis of E_0

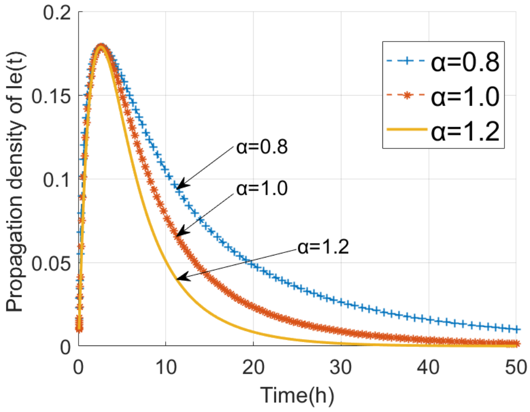
When a significant online public opinion arises, internet users frequently pay close attention to it. People rarely stay focused on a single piece of information for a prolonged amount of time because of information fragmentation. Therefore, let $a = 0.8$ and $c = 0.3$. In this era of information explosion, it is assumed that a major online opinion has not yet been disseminated. The majority of netizens on that social platform are in a susceptible state. Let $S(0) = 0.6$ and $E(0) = 0.3$. In the initial stage, the proportion of netizens with normal emotions is higher than that with extreme emotions, regardless of whether the emotions are normal or extreme. Specifically, $In(0) = 0.09$ and $Ie(0) = 0.01$. The external effect of extreme emotions is slightly stronger than that of normal emotions, with $f = 0.4$. At this point, netizens' emotions are intense, which may lead to susceptible individuals being driven towards communicators, so $q = 0.6$. The relevant parameters are set to: $\Lambda = 0.1$,$d = 0.1$, $\mu = 0.3$, $\lambda = 0.7$, $a = 0.8$, $c = 0.3$, $q = 0.6$, $f = 0.4$, $\gamma = 0.3$, $\varepsilon = 0.1$. The simulation results are as follows:



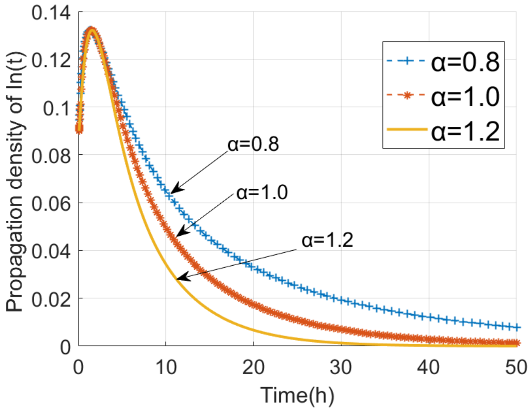
(a) Global evolutionary trends

(b) Magnified view of In(t),Ie(t)

Fig. 3. Global stablility of the disease-free equilibrium point $E_0 = (S_0, E_0, In_0, Ie_0) = (0.25, 0, 0, 0)$ for the system (7) .



(a) The effect of different differential orders with Ie



(b) The effect of different differential orders with In

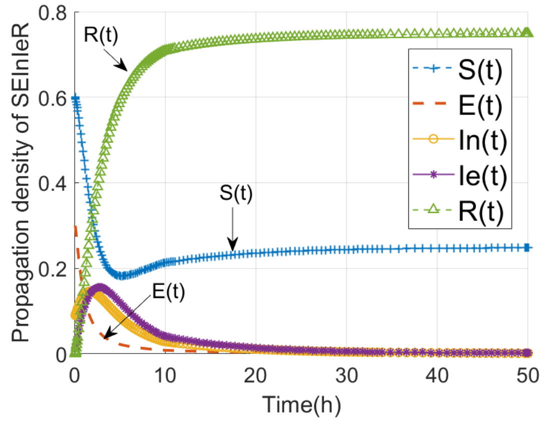
Fig. 4. Different differential orders of $\alpha \in 0.8, 1.0, 1.2$ on the dissemination of public opinion with extreme and normal emotions.

It can be obtained through calculation, $R_0 = 0.647 < 1$, $0 < \beta_2 + d - \varepsilon S_0 = 0.255 < 1$ and $0 < \beta_1 + d = 0.22 < 1$. From Fig. 3a, the system (7) will eventually arrive at $E_0 = (0.25, 0, 0, 0)$ as time goes on. This is in agreement with Theorem (3).

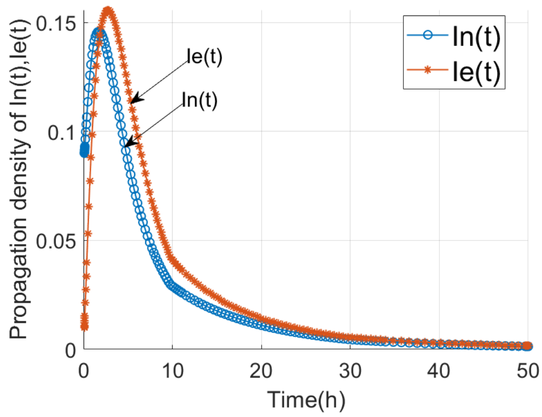
Fig. 3 illustrates a rapid increase in the number of emotion-driven communicators at the beginning. The number of extreme emotion communicators also increased sharply and surpassed that of normal emotion communicators. This suggests that extreme emotion communicators have a strong influence on public opinion at the start. Throughout the process of reaching the maximum value and the beginning of the decline, the density of extreme emotion communicators remained higher than that of emotion-normal communicators, indicating the persistence of extreme emotions' influence. S gradually moves near 0.25 over time, whereas E, In, and Ie go toward 0, signifying a slowdown in the spread of public opinion.

As the value of α decreases in Fig. 4, the curve converges at a slower rate, implying that online public opinion dissemination will take an extended time to end. Controlling and guiding the number of disseminators is the key to the governance of public opinion dissemination. The aim is to regulate and direct the quantity of highly emotive spreaders within the system (7). To determine the impact of modifications to each parameter on the transmitter count, the initial values for each group of internet users are maintained, remaining at $S(0) = 0.6$, $E(0) = 0.3$, $In(0) = 0.09$, $Ie(0) = 0.01$. Relevant simulation experiments are conducted.

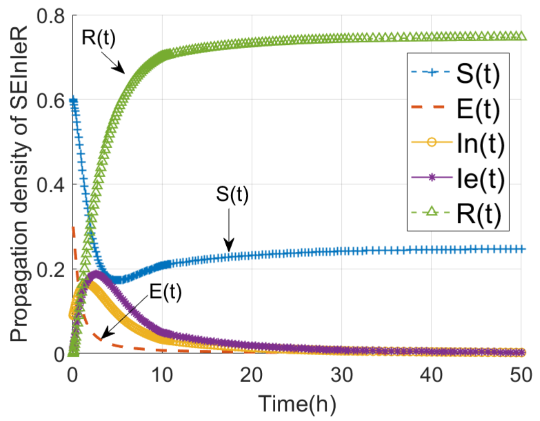
B. The Effect of Intensification degree and Divergence degree on Online Public Opinion Dissemination



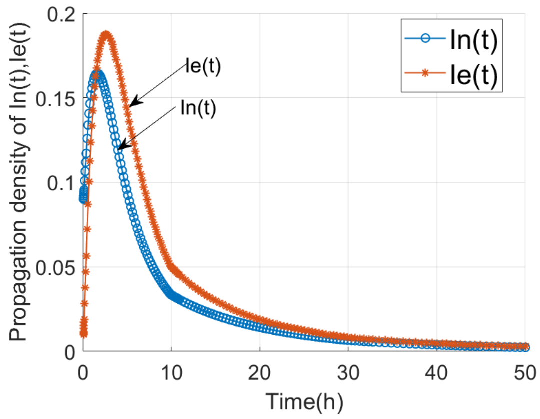
(a) Global evolutionary trends($q = 0.3, f = 0.4$)



(b) Magnified view of In(t),Ie(t)($q = 0.3, f = 0.4$)

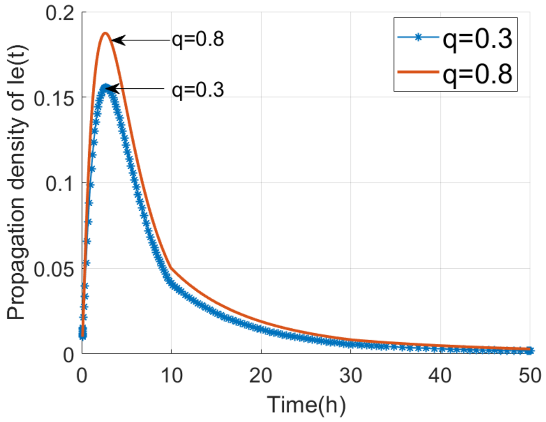


(c) Global evolutionary trends($q = 0.8, f = 0.4$)

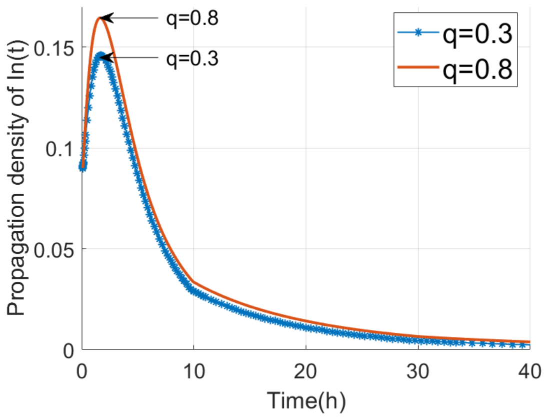


(d) Magnified view of In(t),Ie(t)($q = 0.8, f = 0.4$)

Fig. 5. The effect of intensification degree on online public opinion dissemination.

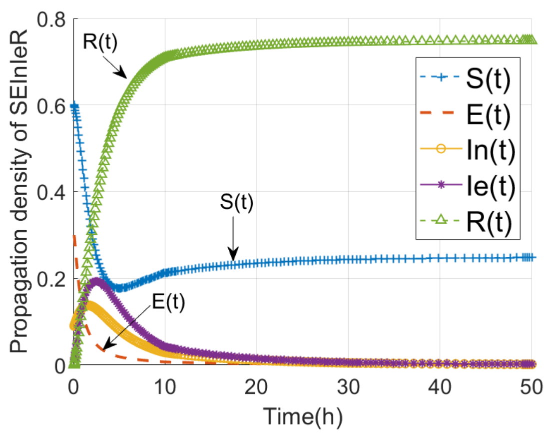


(a) The effect of different intensification degree with Ie

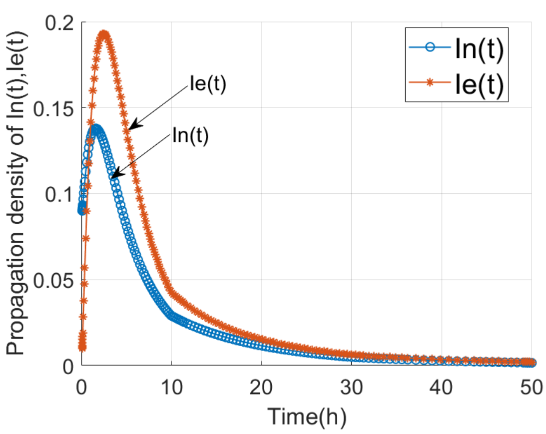


(b) The effect of different intensification degree with In

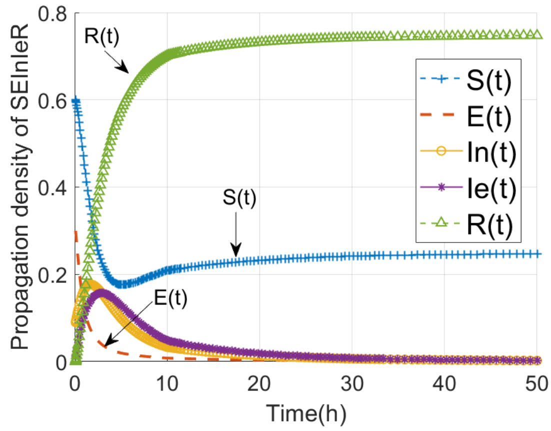
Fig. 6. The effect of intensification degree on the dissemination of extreme and normal emotions.



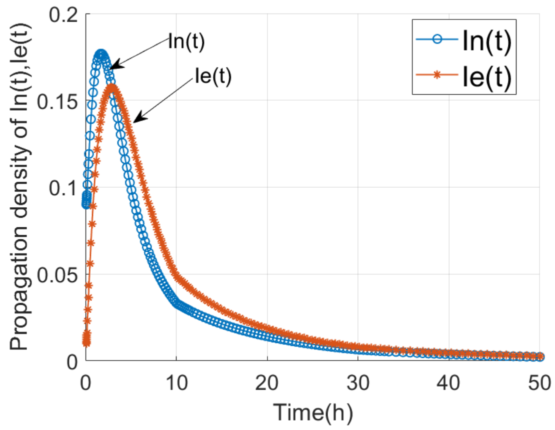
(a) Global evolutionary trends($f = 0.3, q = 0.6$)



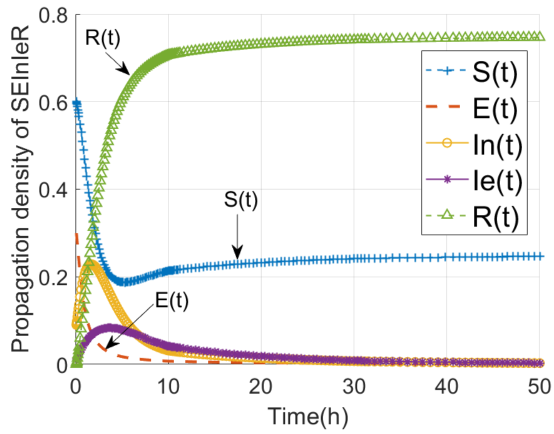
(b) Magnified view of In(t),Ie(t)($q = 0.3, f = 0.6$)



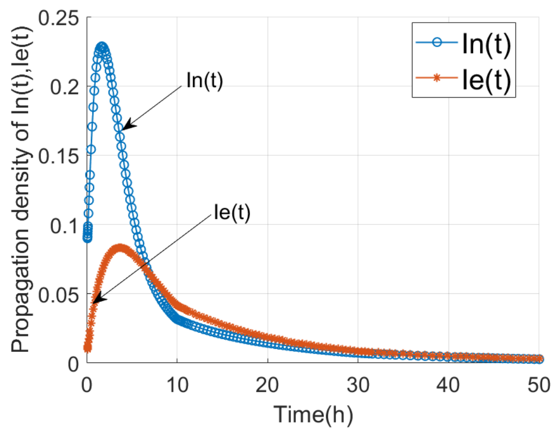
(c) Global evolutionary trends($f = 0.5, q = 0.6$)



(d) Magnified view of In(t),Ie(t)($f = 0.5, q = 0.6$)

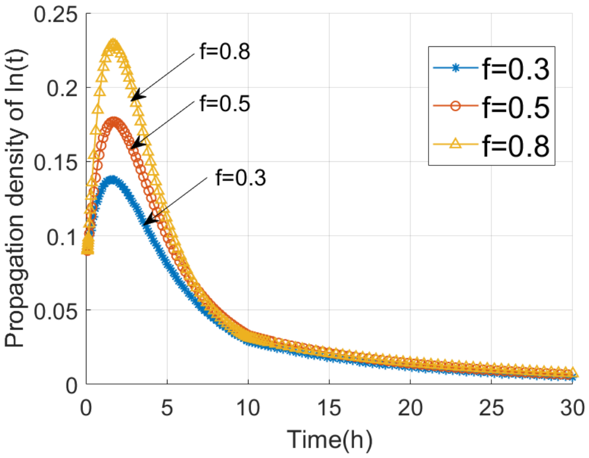


(e) Global evolutionary trends($f = 0.8, q = 0.6$)

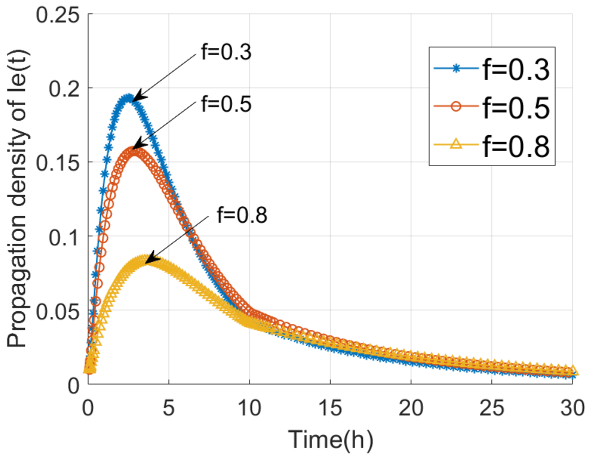


(f) Magnified view of In(t),Ie(t)($f = 0.8, q = 0.6$)

Fig. 7. The effect of divergence degree on online public opinion dissemination.



(a) The effect of different divergence degree with In



(b) The effect of different divergence degree with Ie

Fig. 8. The effect of divergence degree on the dissemination of extreme and normal emotions.

Both the number of emotionally normal communicators and the number of emotionally intense communicators rise with emotional intensication. Fig. 6 and Fig. 7 demonstrate that the degree of reinforcement has a greater impact on the degree of extreme emotional communication. During the dissemination of public opinion, it is crucial to avoid irrational and radical remarks, as they may incite unnecessary participation and comments. The benefits of social media as a type of self-media include quick distribution, great timeliness, and a variety of communication channels. The more self-media is involved, the greater the participation of the public. The degree of participation in self-media is directly proportional to the exposure of netizens to public opinion events and the intensity of their emotions [27]. Therefore, extreme emotions are more likely to be intensied.

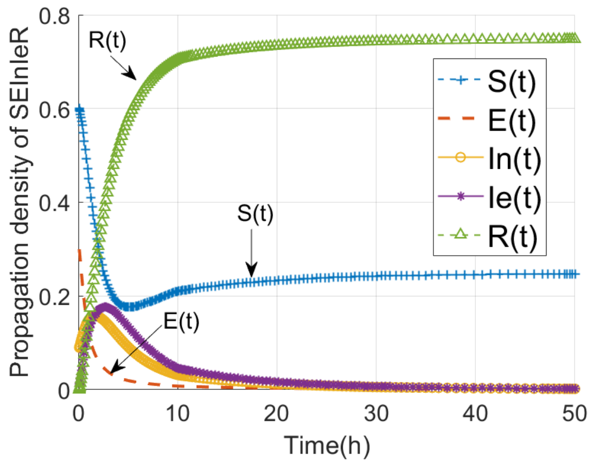
The impact of the divergence degree on the density of two types of emotion communicators is visible in Fig. 8. The three values represent different situations: $f = 0.3$ indicates that the external utility of the extreme emotion network is greater than that of the normal emotion network; $f = 0.5$ indicates that the external utility of the extreme emotion network is the same as that of the normal emotion network; and $f = 0.8$ indicates that the external utility of the extreme emotion network is less than that of the normal emotion network.

When $f = 0.3$, the density of extreme emotion communicators increases sharply and is significantly higher than that of normal emotion communicators. It remains higher than the latter even as it decreases. This suggests that when the public opinion atmosphere is dominated by extreme emotions, extreme emotion communicators begin to make reckless and extreme remarks early on in the process of public opinion dissemination. This causes pandemonium on online social networking platforms and has a more severe impact.

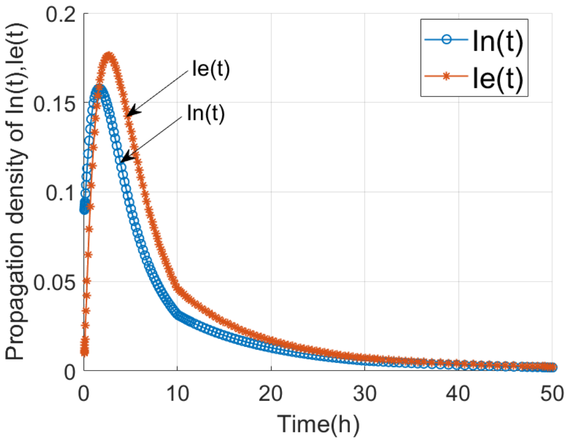
When $f = 0.5$, the density of communicators expressing normal emotions peaks shortly after the beginning of public opinion dissemination. During this period, the growth rate of normal emotion communicators exceeded that of extreme emotion communicators. However, as the density of normal emotion communicators begins to decline, the number of emotional extremists begins to exceed that of normal emotion communicators again. This indicates that the opinion climate in the online social network is in a phase of stalemate between the two emotions. Following the stalemate, the number of individuals expressing normal emotions decreased more rapidly than those expressing extreme emotions. This was due to the provocative and alluring nature of the extreme emotion communicators' statements, which caused some of the normal emotion communicators to engage in public opinion discussions and become one of them.

When $f = 0.8$, the density of normal emotion communicators greatly exceeds that of extreme emotion communicators, indicating that the public opinion atmosphere is dominated by normal emotions at this time. Early in the propagation process, the online social network environment is signicantly puried, and the percentage of extreme speech declines. After approximately 7 or 8 hours of public opinion spreading, the number of extreme emotion spreaders surpasses that of normal emotion spreaders. This indicates that as public opinion fervor subsides, extreme emotions dissipate at a significantly slower rate than normal emotions. Therefore, it is important to continue monitoring public opinion with extreme care and on time. When the public opinion atmosphere is dominated by normal emotions, the inuence of some extreme emotion communicators can be greatly reduced. This is the kind of public opinion atmosphere that is required on online social networks.

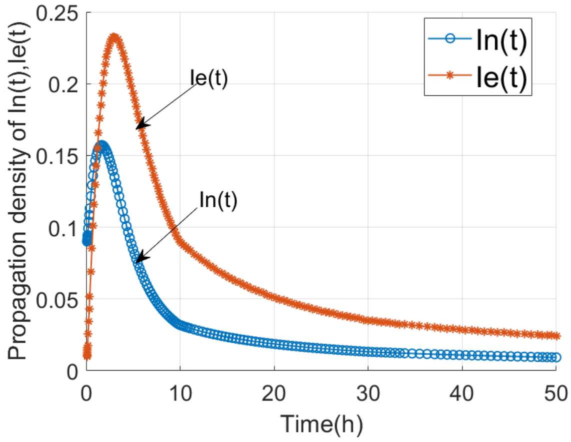
C. The Effect of Acceleration factor and one-way infection rate on Online Public Opinion Dissemination



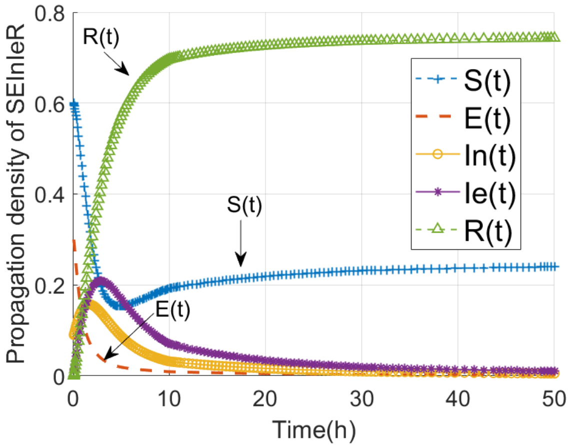
(a) Global evolutionary trends($\varepsilon = 0.1, \gamma = 0.3$)



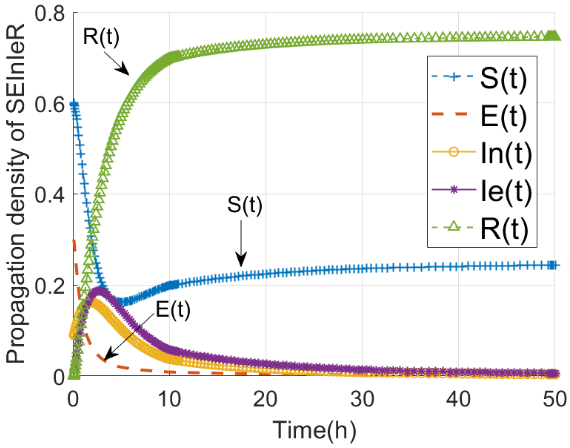
(b) Magnified view of In(t),Ie(t)($\varepsilon = 0.1, \gamma = 0.3$)



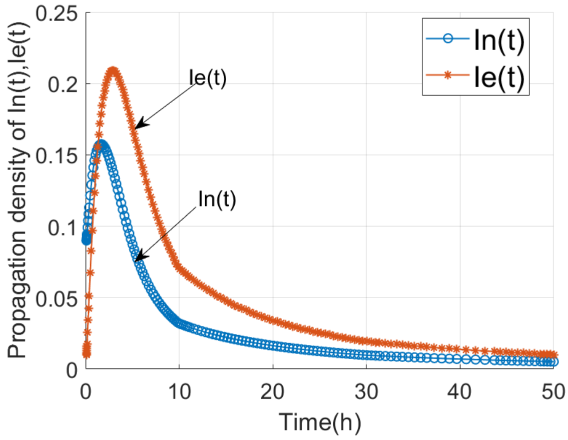
(f) Magnified view of In(t),Ie(t)($\varepsilon = 0.6, \gamma = 0.3$)



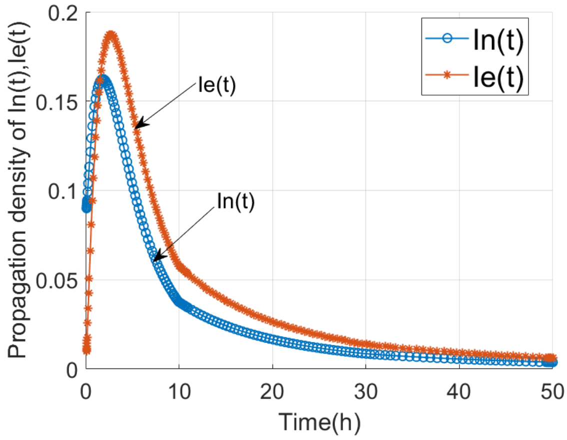
(c) Global evolutionary trends($\varepsilon = 0.4, \gamma = 0.3$)



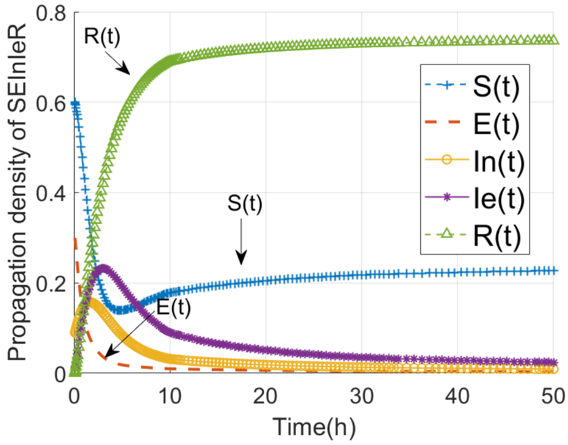
(g) Global evolutionary trends($\varepsilon = 0.3, \gamma = 0.1$)



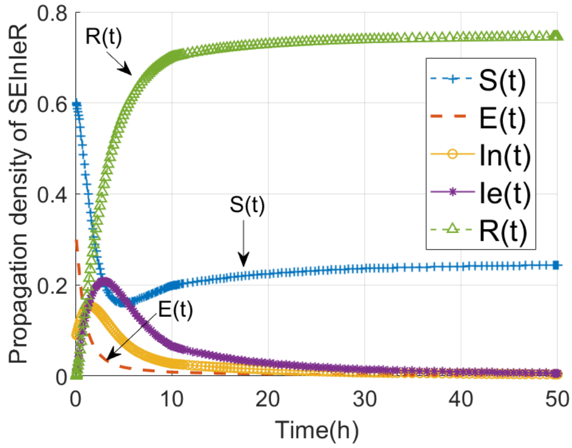
(d) Magnified view of In(t),Ie(t)($\varepsilon = 0.4, \gamma = 0.3$)



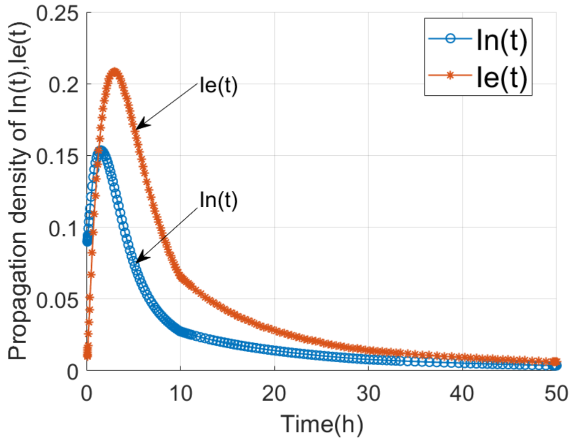
(h) Magnified view of In(t),Ie(t)($\varepsilon = 0.3, \gamma = 0.1$)



(e) Global evolutionary trends($\varepsilon = 0.6, \gamma = 0.3$)



(i) Global evolutionary trends($\varepsilon = 0.3, \gamma = 0.5$)



(j) Magnified view of In(t),Ie(t)($\varepsilon = 0.3, \gamma = 0.5$)

Fig. 9. The effect of acceleration factor and one-way infection rate on online public opinion dissemination.

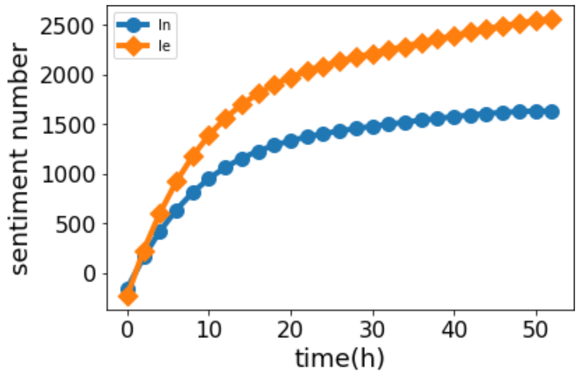
Fig. 9 illustrates how the density of two kinds of emotion spreaders rises in tandem with increases in the one-way infection rate and acceleration factor. The acceleration factor further increases the density difference between the two types of emotions. In the propagation of public opinion, individuals who are less informed about the topic are more susceptible to being inuenced by extreme emotions. This can cause them to skip the evaluation stage and move directly to the adoption stage. As the number of individuals in this group increases, the spread of extreme emotions becomes dominant and slows down the convergence of the curve, perpetuating the dissemination of public opinion.

V. Experimental Simulation of Actual Public Opinion Dissemination

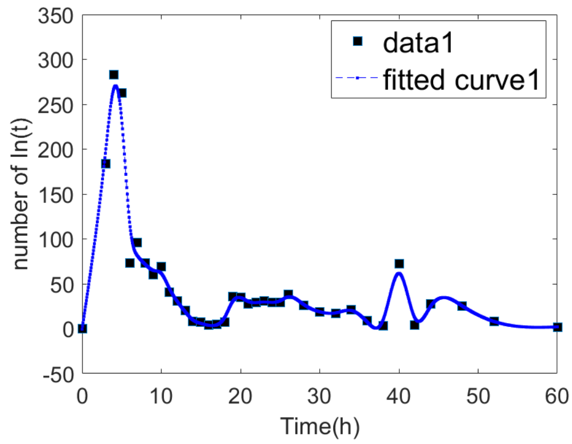
To show the logic of the SEInIeR model, this component of the study examines blog posts and comments associated with the microblog search query, "What does it mean when the birth population falls below 8 million?" Microblogs are highly interactive and open, making them a suitable source of data. The blog posts and comments related to this hot search were collected using an Octopus collector and Python crawler code. The time frame for this event is from 12:00 on May 28th to 0:00. on June 3rd, 2023. The number of blog posts and comments obtained after the deweighting process is 4356.

The text data were first subjected to lexical processing using the Jieba library and a deactivation word list to lter out common deactivated words. The training set was then chosen from the 60,000 positive and negative sentiment texts found in the Weibo sentiment analysis dataset. Finally, the sentiment tendency was labeled using the SnowNLP library. Each text was scored, with a score closer to 0 indicating a more negative tone and a score closer to 1 indicating a more positive tone. A rating of 0.3 was used as a cut-off point to distinguish between extreme emotional speech (scores less than 0.3) and normal emotional speech (scores greater than 0.3).

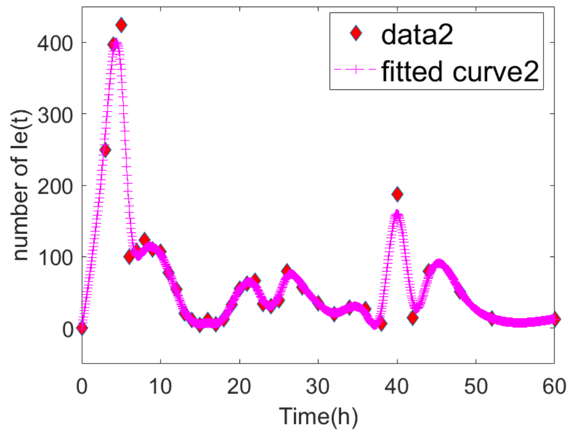
Fig. 10a illustrates the growth trend of the two types of emotions, The number of comments on the two types of emotions in each time interval was obtained by dividing time periods, and the results were plotted on a graph in Fig. 10b and Fig. 10c



(a) The growth trend of the two types of emotions

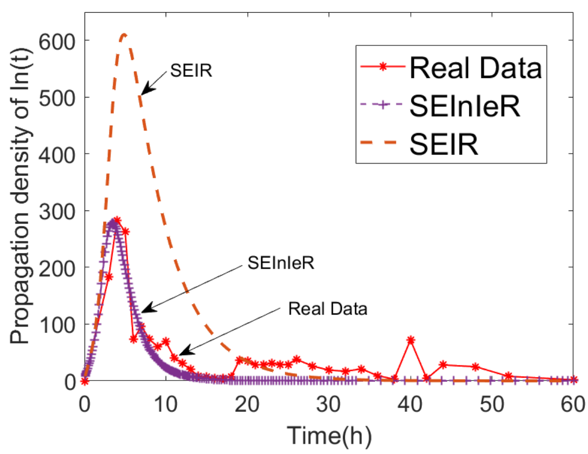


(b) The evolution trend of normal Emotions

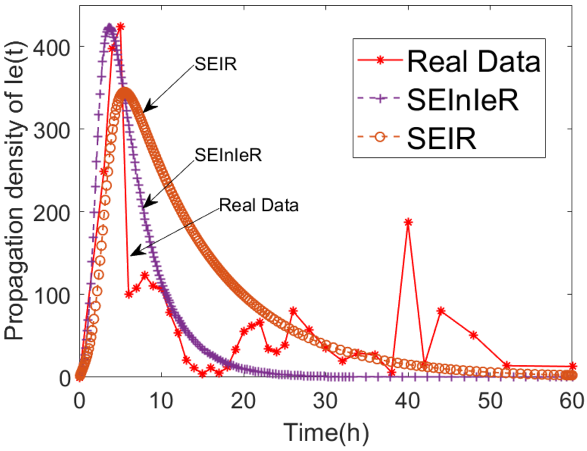


(c) The evolution trend of extreme Emotions

Fig. 10. Data acquisition and processing results graph.



(a) Error analysis of In



(b) Error analysis of Ie

Fig. 11. Error analysis and comparison

The comparison between the SEIR model without interaction between emotions and the SEInIeR model proposed in this paper are shown in Fig. 11.

TABLE I
MODEL COMPARING ERROR RESULTS.

Model	SEIR	SEInIeR
MAE	136.12	25.40
RMSE(In)	194.15	32.78
RMSE(Ie)	121.30	54.96

After comparing the results of the two models and the real data, it was found that the curve of the SEInIeR model is closer to the real data than that of the SEIR model. The SEInIeR model performs better at forecasting how the two types of emotions will grow. By choosing the data points, the two models' mean absolute error (MAE) and root mean square error (RMSE) were determined, as indicated in Tab. I. The SEInIeR model has a smaller error than the original SEIR model. Thus, the SEInIeR model based on extreme emotions demonstrates the superior predictive ability for the trend of public opinion communication during emergencies and more accurately depicts the process of online public opinion dissemination.

VI. CONCLUSIONS

In this paper, the standard SEIR model and emotional state are integrated, and fractional-order differential equations are introduced to create the SEInIeR model based on extreme emotions for the phenomenon of opinion dissemination in online social networks. Analysis is done on the equilibrium point's stability, the evolution of public opinion, and the method by which extreme and normal emotions change. Sentiment analyses were also conducted on actual cases of online public opinion communication to demonstrate the feasibility of the SEInIeR model. By comparing it with the original SEIR model, the SEInIeR model proposed in this paper can better predict the propagation trend of emotions in public opinion.

Through the simulation experiment with each parameter, the following conclusions are drawn:

1) Controlling the magnitude of emotional reinforcement can help regulate the frequency of extreme emotional communication. Self-media practitioners should refrain from using excessive subjective language when inuencing public opinion. Instead, they should strive to uncover the truth of the incident to prevent emotional exaggeration and the spread of extreme emotions through their statements.

2) Administrators of social networking platforms should strengthen their audit and inspection of speech to timely detect and delete irrational, radical, and other extreme speech. Users should be barred for persistent noncompliance in order to lessen the impact of extreme emotions on the outside world.

3) The incomplete understanding of public opinion among susceptible groups makes them vulnerable to extreme remarks. To control the one-way infection rate, the government and related organizations should issue early warnings and improve intervention mechanisms. This will enable more netizens to learn the facts and think for themselves. It is recommended that authorities communicate authoritative information through ofcial media channels to reduce the number of netizens who have been diverted by excessive statements.

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