Calculating Truth Degrees of Propositions on Quantum Computers

Songsong Dai

***Abstract*—Truth degree of a formula in classical propositional logic refers to a numerical or qualitative measure that indicates how true a proposition is. It's not just a binary true or false, but a spectrum that can include intermediate values. This manuscript presents a quantum approach for calculating the truth degree of formulas within the realm of classical two-valued propositional logic. We further present a quantum approach for calculating the similarity degree between formulas. Finally, several examples for calculating the truth degree and similarity degree on the HiQ quantum computing cloud platform are demonstrated.**

***Index Terms*—Classical propositional logic, truth degree, proposition, quantum computer.**

I. Introduction

IN two-valued logic (also call classical propositional logic), formulas (also call propositions) are interpreted as having exactly one truth value, either true or false. A formula A is a tautology (totally true) if $v(A) = 1$ for every valuation v. A formula A is a contradiction (totally false) if $v(A) = 0$ for every valuation v. The majoring of formulas are not the tautologies nor the contradictions. This naturally raises the question: how true are these formulas? In other words, is there a numerical or qualitative measure that can gauge the truthfulness of a proposition.

Wang et al. [1] (also see [2], [3]) introduced the truth degrees of propositions in two-valued logic. The theory of truth degrees is a measure of the the reliability of propositions. A tautology is more reliable than a contradiction. A proposition with a higher degree of truth is more reliable than a proposition with a smaller degree of truth. As all known, probabilistic logic introduced by Adams [4] is another successful realization of the idea of uncertainty of propositions. Our method is different from probabilistic logic. In probabilistic logic, the truth values of propositions are probability values (between 0 and 1), which is arbitrarily given. And in many probabilistic logics, truth value of a proposition is not determined by the truth values of its atom propositions (see [3], [5], [6]).

Based on the theory of truth degrees, the concepts of similarity degrees and distance between propositions have been proposed [1], [3], [7], [8]. Consequently, many researchers have improved the theory and extend it to various many-valued logical systems and fuzzy logical systems [9], [10], [11]. Furthermore, they have applied the theory of truth degrees to approximate reasoning [1], [12], predicate logic [13], modal logic [14] and rough logic [15], [16].

Manuscript received May 10, 2025; revised Aug. 28, 2025. This work was supported in part by the Zhejiang Provincial Natural Science Foundation of China under Grant No. LMS25A010011.

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The term "quantum" is increasingly prevalent in interdisciplinary scientific literature, as exploring quantum counterparts of classical models inspires technological advancements [17], [18], [19], [20], [21]. In this paper, quantum computing provides a new perspective on calculating the truth degrees of propositions. In quantum computing, a proposition's truth value can exist as a superposition of true and false, enabling parallel processing and faster results.

The structure of this article is organized as follows: Section II provides a review of the two-valued propositional logic system, including the truth degrees of propositions and the role of quantum gates. In Section III, we apply quantum circuits to propositions. Sections IV and V present quantum circuits designed to calculate the truth degree and the randomized truth degree of propositions, respectively. Subsequently, Sections VI and VII focus on quantum circuits for determining the similarity degree and the randomized similarity degree between propositions, respectively. Finally, Section VIII concludes our research findings.

II. Preliminaries

A. Two-valued propositional logic system

First let us recall the some notions of two-valued propositional logic system L. For more details, see [22], [23].

Suppose that $S = \{p_1, p_2, ...\}$ is a countable set and $\mathbf{F}(S)$ is the set of all propositions, which is free algebra of type $(\neg, \to)$ generated by S, where $\neg$ and $\to$ are unary and binary operators, respectively.

The axiom schemes of two-valued propositional logic system L given in [22], [23] are as follows:

(L1) $A \to (B \to A)$,
(L2) $A \to (B \to C) \to ((A \to B) \to (A \to C))$,
(L3) $(\neg A \to \neg B) \to (B \to A)$.

The deduction rule of L is Modus Ponens (briefly, MP): from A and $A \to B$ infer B.

Let $\Sigma = \{0, 1\}$, a valuation of $\mathbf{F}(S)$ is a homomorphism $v : \mathbf{F}(S) \to \Sigma$, where $v(\neg A) = 1 - v(A)$ and $v(A \to B) = v(A) \to v(B) = \max\{1 - v(A), v(B)\}$. Since $A \vee B = (A \to B) \to B$, $A \wedge B = \neg(\neg A \vee \neg B)$ and $A \leftrightarrow B = (A \to B) \wedge (B \to A)$. Hence, we have $v(A \vee B) = \max\{v(A), v(B)\}$, $v(A \wedge B) = \min\{v(A), v(B)\}$ and $v(A \leftrightarrow B) = \max\big(\min\{1 - v(A), 1 - v(B)\}, \min\{v(A), v(B)\}\big)$.

Denote Ω the set of all valuations. A proposition A is a tautology, denoted by $\vDash A$. if $v(A) = 1$ for every $v \in \Omega$, and A is a contradiction if $v(A) = 0$ for every $v \in \Omega$.

B. Truth degrees of propositions

In this paper, $\Sigma = \{0, 1\}$ and $\Sigma^m = \{0, 1\}^m$, $|S|$ represents the cardinality of a set S.

Assume that $A = A(p_1, p_2, ..., p_m)$ is a proposition built up from m atomic propositions $p_1, p_2, ..., p_m$ by using the connectives $\neg$ and $\rightarrow$. Then we can obtain a mapping $\bar{A}(x_1, x_2, ..., x_n) : \Sigma^m \rightarrow \Sigma$, called a m-ary truth function (or Boolean function) induced by A.

Definition 1. *[1] Let $A = A(p_1, p_2, ..., p_m)$ be a proposition containing m atomic proposition $p_1, p_2, ..., p_m$, $\bar{A}$ be the truth function induced by A. Define*

$$\tau(A) = \frac{|\bar{A}^{-1}(1)|}{2^m} \tag{1}$$

the truth degree of proposition A in L.

It is noteworthy that all atomic formulas are assigned a uniform truth degree of $\frac{1}{2}$. From the point of view of probability, the probability of each atomic formula is

$$\Pr(p_i = 1) = \Pr(p_i = 0) = \frac{1}{2},\ \forall i \in \{1, 2, \cdots, n\}, \tag{2}$$

and atomic formulas are mutually independent.

Then the truth degree of a proposition $A = A(p_1, p_2, ..., p_m)$ containing m atomic proposition $p_1, p_2, ..., p_m$ is the probability of A being true, i.e.,

$$\tau(A) = \Pr(A(p_1, p_2, ..., p_m) = 1). \tag{3}$$

For example, the truth degree of the conjunction of any two atomic formulas is precisely the product of the individual truth degrees of those formulas, i.e.,

$$\begin{aligned} &\tau(p \wedge q) = \Pr(p \wedge q = 1) \\ &= \Pr(p = 1) \cdot \Pr(q = 1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}. \end{aligned} \tag{4}$$

Let A, B be two propositions, the similarity degree between A and B is defined as

$$S(A, B) = \tau\big((A \rightarrow B) \wedge (B \rightarrow A)\big). \tag{5}$$

Wang et al. [11], [24] introduced the concept of the randomized truth degree of a formula, where the truth degree of each atomic formula is assigned a value randomly selected from the interval $(0, 1)$. Specifically, for each atomic formula p_i, the probability that $p_i = 1$ is given by d_i, where $d_i \in (0, 1)$ for all $i \in \{1, 2, \ldots, m\}$, i.e.,

$$\Pr(p_i = 1) = d_i \in (0, 1),\ \forall i \in \{1, 2, \cdots, n\}. \tag{6}$$

Consequently, $D = (d_1, d_2, \ldots, d_m)$ is defined as a random sequence. The randomized truth degree of a formula A, denoted as $\tau_D(A)$, is then the probability that the formula $A(p_1, p_2, \ldots, p_m) = 1$ under the random sequence D, i.e., the condition specified by Equation 6.

Let A, B be two propositions and D be a random sequence, the randomized similarity degree between A and B under random sequence D is defined as

$$S(A, B) = \tau_D\big((A \rightarrow B) \wedge (B \rightarrow A)\big). \tag{7}$$

C. Quantum gates

Some notions of of quantum computation are recalled below, whereas others refer to reports from [20].

The fundamental concept of quantum computation is the qubit, which is a pure state in the Hilbert space $\mathbb{C}^2$. The standard orthonormal basis of $\mathbb{C}^2$ is $\{|0\rangle, |1\rangle\}$, where

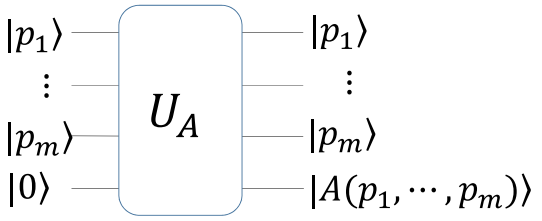


Fig. 1: Quantum circuit of the oracle function U_A for the proposition $A(p_1, p_2, ..., p_m)$.

$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. In some contexts, the state $|1\rangle$ is associated with truth, while $|0\rangle$ is associated with falsity. A pure state $|\xi\rangle$ is expressed as a coherent superposition of $|0\rangle$ and $|1\rangle$ with complex coefficients, given by the equation:

$$|\xi\rangle = \beta_0 |0\rangle + \beta_1 |1\rangle, \tag{8}$$

where $\beta_0, \beta_1 \in \mathbb{C}$ and satisfy the normalization condition:

$$|\beta_0|^2 + |\beta_1|^2 = 1. \tag{9}$$

An n-qubit quantum register is represented by a unit vector in the n-dimensional Hilbert space $\mathbb{H}^{(n)} = \underbrace{\mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2}_{n-\text{times}}$, where $\otimes$ denotes the tensor product of spaces. Any factorized unit vector $|\xi_1\rangle \cdots |\xi_n\rangle$ can also be expressed as $|\xi_1 \cdots \xi_n\rangle$.

The main objective of this paper is to design quantum circuits for calculating the truth degrees of propositions using quantum gates. As outlined in Table 1, six types of quantum gates will be employed: the Hadamard gate, the R_Y gate, the I gate, the Pauli-X (NOT) gate, the CNOT gate, and the Toffoli gate.

III. Quantum circuits of propositions

Indeed, a proposition $A = A(p_1, p_2, ..., p_m)$ containing m atomic proposition $p_1, p_2, ..., p_m$ is a m-bit input, 1-bit output Boolean function $A : \{0, 1\}^m \rightarrow \{0, 1\}$.

Note that any quantum gate is reversible, all the gates in a quantum circuit are required to be reversible. To overcome this, a useful concept is that of an oracle function U_A. Given a function $A : \{0, 1\}^m \rightarrow \{0, 1\}$, the function U_A is a (m+1)-input (m+1)-output Boolean function that is defined as follows:

$$U_A(p_1, p_2, ..., p_m, 0) = U_A\big(p_1, p_2, ..., p_m, A(p_1, p_2, ..., p_m)\big). \tag{10}$$

Figure 1 illustrates the quantum circuit for the oracle function U_A corresponding to the proposition $A(p_1, p_2, \ldots, p_m)$. The oracle functions of five connectives are given in Table 2. Additionally, Figure 2 presents the quantum circuits for the oracle functions of these connectives.

IV. Quantum circuit for calculating the truth degree

A. Approach

Let $A = A(p_1, p_2, ..., p_m)$ be a proposition containing m atomic proposition $p_1, p_2, ..., p_m$. For convenience, let $p = (p_1, p_2, ..., p_m)$. In this paper, we assume each proposition A is provided as an oracle U_A. We examine

TABLE I: QUANTUM GATES.

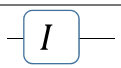
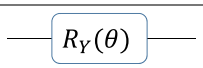
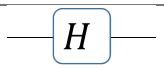
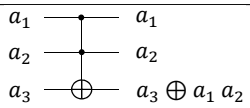
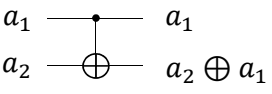
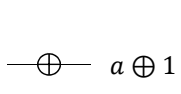
Gate	I	R_Y	Hadamard
Diagram			
Matrix	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.	$\begin{bmatrix} \cos(\frac{\theta}{2}) & -\sin(\frac{\theta}{2}) \\ \sin(\frac{\theta}{2}) & \cos(\frac{\theta}{2}) \end{bmatrix}$.	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.
Gate	Toffoli	CNOT	Pauli-X (NOT)
Diagram			
Matrix	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$.	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

TABLE II: Truth-table of the oracle functions U_{A_1}, U_{A_2},U_{A_3},U_{A_4} and U_{A_5} of $A_1(p) = \neg p$, $A_2(p,q) = p \wedge q$, $A_3(p,q) = p \vee q$, $A_4(p,q) = p \rightarrow q$ and $A_5(p,q) = p \leftrightarrow q$, respectively.

p	q	$U_{A_1}(p,0)$	$U_{A_2}(p,q,0)$	$U_{A_3}(p,q,0)$	$U_{A_4}(p,q,0)$	$U_{A_5}(p,q,0)$
0	0	01	000	000	001	001
0	1	01	010	011	011	010
1	0	10	100	101	100	100
1	1	10	111	111	111	111

the quantum algorithm for computing the truth degree of propositions. The quantum circuit of our method is shown in Figure 3 and the steps of the method are as follows.

(1). The initial state $|\alpha_0\rangle$ is $|0\rangle^{\otimes(m+1)}$.
(2). Apply the $H^{\otimes m} \otimes I$ on the initial state, producing

$$|\alpha_1\rangle = \sum_{p\in\{0,1\}^m} \frac{|p\rangle}{\sqrt{2^m}} \otimes |0\rangle . \tag{11}$$

(3). Perform the U_A on $|\alpha_1\rangle$, producing

$$|\alpha_2\rangle = U_A(|\alpha_1\rangle) = \sum_{p\in\{0,1\}^m} \frac{|p, A(p)\rangle}{\sqrt{2^m}}. \tag{12}$$

(4). Perform a single-qubit measurement on the bottom qubit of $|\alpha_2\rangle$ and denote the output $|\alpha_3\rangle$.

B. Analysis

Let

$$Q = \{p = (p_1, p_2, ..., p_m) \in \{0,1\}^m | A(p_1, p_2, ..., p_m) = 1\}, \tag{13}$$

$$S = \{p = (p_1, p_2, ..., p_m) \in \{0,1\}^m | A(p_1, p_2, ..., p_m) = 0\}. \tag{14}$$

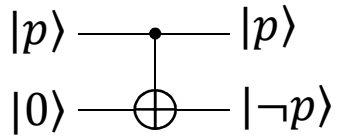
Then the truth degree of proposition A is $\tau(A) = \frac{|Q|}{2^m}$.

In step 3, the state of the qubit $|\alpha_2\rangle$ can be described as follows:

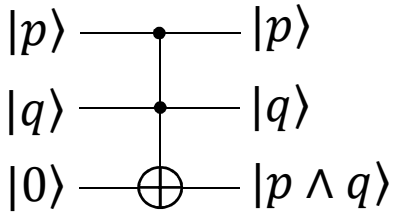
$$\begin{aligned} |\alpha_2\rangle &= \sum_{p\in\{0,1\}^m} \frac{|p, A(p)\rangle}{\sqrt{2^m}} \\ &= \sum_{p\in S} \frac{|p,0\rangle}{\sqrt{2^n}} + \sum_{p\in Q} \frac{|p,1\rangle}{\sqrt{2^m}}. \end{aligned} \tag{15}$$

When measuring the bottom qubit of $|\alpha_2\rangle$, the state is revealed to be $|0\rangle$ with probability $\frac{|S|}{2^m}$ and $|1\rangle$ with probability $\frac{|Q|}{2^m}$.

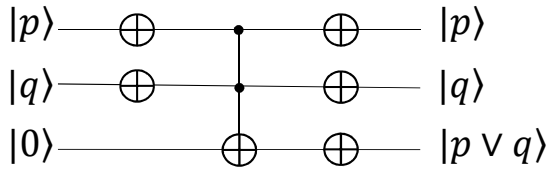
Therefore, all we need to do is repeat steps 1-4 a number of times to estimate the truth degree of the proposition.



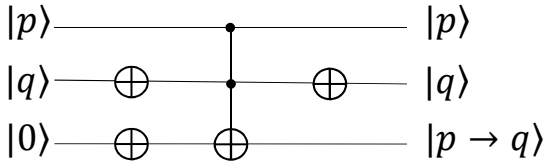
(a) Quantum circuit of the oracle function for $A(p) = \neg p$



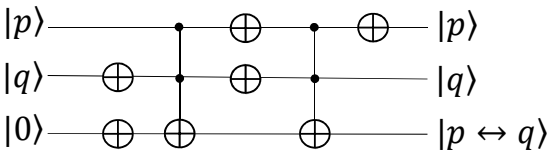
(b) Quantum circuit of the oracle function for $A(p, q) = p \wedge q$



(c) Quantum circuit of the oracle function for $A(p, q) = p \vee q$



(d) Quantum circuit of the oracle function for $A(p, q) = p \rightarrow q$



(e) Quantum circuit of the oracle function for $A(p, q) = p \leftrightarrow q$

Fig. 2: Quantum circuits of the oracle functions for connectives

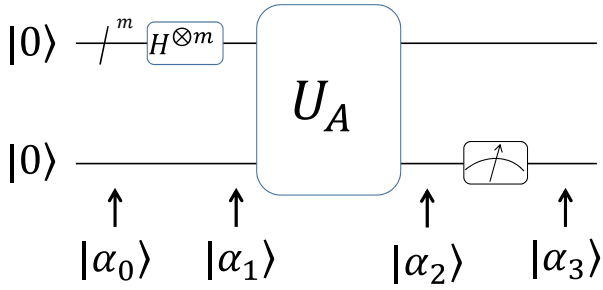


Fig. 3: Quantum circuit for calculating truth degree of the proposition A.

Classical algorithms, in the worst case, require evaluating $A(p)$ for all possible values, which involves 2^m computations. In contrast, our method leverages quantum parallelism to obtain the truth degree of the proposition in one time, storing the result as a quantum state in the last qubit of Fig. 3. This method can serve as a subroutine in Section VI for calculating the similarity between propositions, as this quantum state can be directly used as input for that computation. The primary drawback of our approach is that its output is a quantum state rather than a classical, crisp value. Therefore, multiple measurements are necessary to obtain an estimated value.

C. A sample example

Let us consider the proposition $A = p \rightarrow q$. It is easy to know that the truth degree of proposition A is $\tau(A) = 0.75$.

The proposed quantum implementation for calculating the truth degree of A is executed using Huawei's quantum computing cloud platform HiQ (https://hiq.huaweicloud.com/home). The quantum circuit utilized for this example is illustrated in Fig. 4(a), where the initial state is $|0\rangle\,|0\rangle\,|0\rangle$. Upon executing the program depicted in Fig. 4(a) on the HiQ quantum platform, the results are presented in Fig. 4(b). Notably, out of 10,000 repeated tests, the state was measured as 0 a total of 2,469 times, while the probability of the state being 1 was recorded as 7,531 times. Consequently, the truth degree of A on the HiQ quantum platform is calculated to be 0.7531, which closely approximates the ideal result of 0.75.

V. Quantum circuit for calculating the randomized truth degree

A. Approach

Let $D = (d_1, d_2, \ldots, d_m)$ be a random sequence in $(0, 1)$. Let $A = A(p_1, p_2, ..., p_m)$ be a proposition containing m atomic proposition $p_1, p_2, ..., p_m$. Here we examine the quantum algorithm for computing the randomized truth degree $\tau_D(A)$ of proposition A under the random sequence D. The quantum circuit of our method is shown in Figure 4 and the steps of the method are as follows.

(1). The initial state $|\alpha_0\rangle$ is $|0\rangle^{\otimes(m+1)}$.

(2). Apply the $R_Y(\theta_1) \otimes R_Y(\theta_2) \otimes \cdots \otimes R_Y(\theta_m)$ on the initial state where

$$\theta_i = 2\arcsin(\sqrt{d_i}), \tag{16}$$

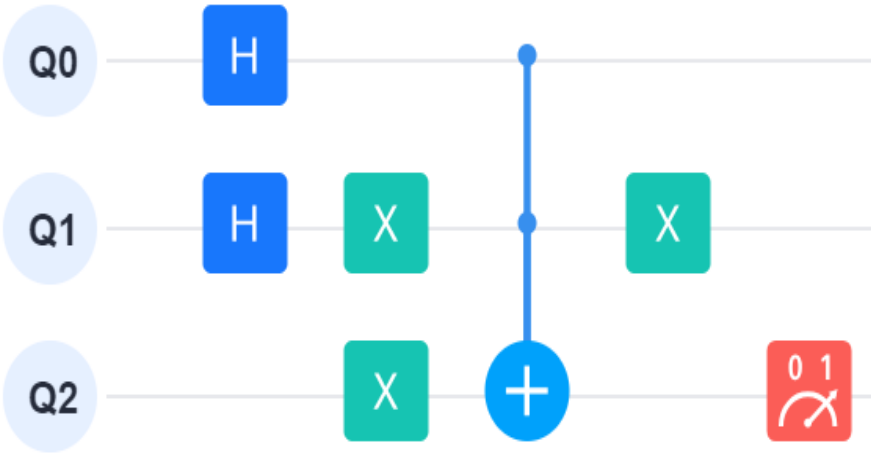
producing

$$|\alpha_1\rangle = R_Y(\theta_1)\,|0\rangle \otimes R_Y(\theta_2)\,|0\rangle \otimes \cdots \otimes R_Y(\theta_m)\,|0\rangle\,. \tag{17}$$

(3). Perform the U_A on $|\alpha_1\rangle$, producing

$$|\alpha_2\rangle = U_A(|\alpha_1\rangle). \tag{18}$$

(4). Perform a single-qubit measurement on the bottom qubit of $|\alpha_2\rangle$ and denote the output $|\alpha_3\rangle$.



(a)

(b)

Fig. 4: Quantum implementation for calculating truth degree of $A = p \to q$.

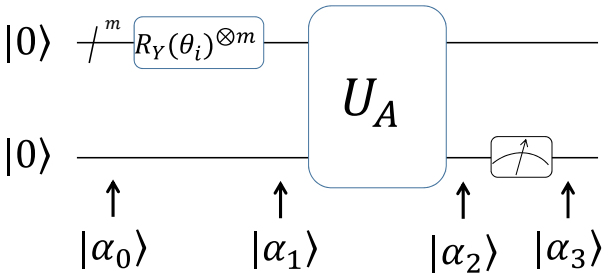


Fig. 5: Quantum circuit for calculating the randomized truth degree of proposition A.

B. Analysis

In step 2, for each i, we have $\theta_i = 2\arcsin(\sqrt{d_i})$, then

$$\begin{aligned} R_Y(\theta_i)|0\rangle &= \begin{bmatrix} \cos(\frac{\theta_i}{2}) & -\sin(\frac{\theta_i}{2}) \\ \sin(\frac{\theta_i}{2}) & \cos(\frac{\theta_i}{2}) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= \cos(\frac{\theta_i}{2})|0\rangle + \sin(\frac{\theta_i}{2})|1\rangle \\ &= \sqrt{1-d_i}\,|0\rangle + \sqrt{d_i}\,|1\rangle\,. \end{aligned} \tag{19}$$

The state $R_Y(\theta_i)|0\rangle$ is revealed to $|1\rangle$ with probability d_i. Consequently, when measuring the bottom qubit of $|\alpha_2\rangle$, the probability of it being $|1\rangle$ is equivalent to the probability that $A(p_1, p_2, \ldots, p_m) = 1$, given that $\Pr(p_i = 1) = d_i$ for all i.

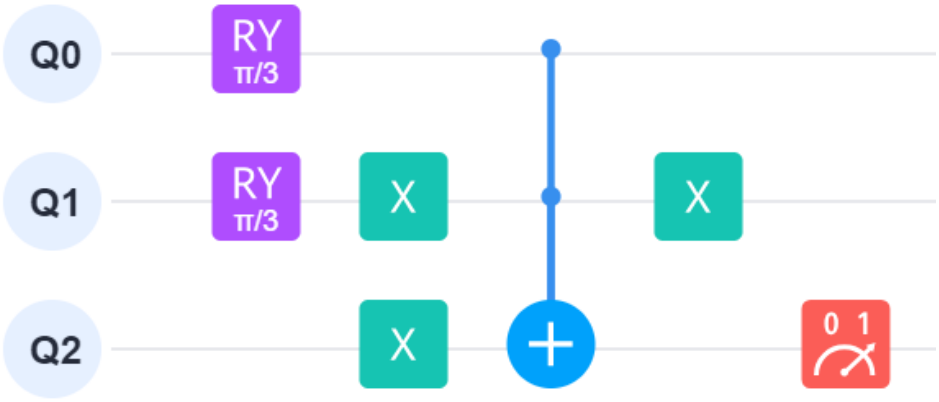
Thus, repeating above steps 1-4 a number of times allows us to estimate the proposition's randomized truth degree. Clearly, when the proposition is a tautology, the measurement result is always $|1\rangle$; when it is a contradiction, the result is always $|0\rangle$.

To compute the truth degree of a proposition, we utilize the Hadamard gate, which ensures that each atomic proposition has an equal probability of taking the value 0 or 1. In contrast, when calculating the randomized truth degree, we replace the H gate with the $R_Y(\theta)$ rotation gate. The parameter θ allows us to adjust the probability distribution of each atomic proposition taking the value 0 or 1. Consistent with our previous approach in Section IV, this method capitalizes on quantum parallelism: a single application of the U_A suffices to encode the randomized truth degree of proposition A into a quantum state, held in the final qubit depicted in Fig. 5. This method can serve as a subroutine in Section VII for calculating the randomized similarity between proposition, meaning this quantum state can then be used as input for the randomized similarity computation. A key limitation, however, remains: the algorithm's output is a quantum state rather than a classical, crisp value. Consequently, repeated measurements are necessary to yield an estimated value.

C. A sample example

Let us consider the proposition $A = p \to q$. If $\Pr(p = 1) = \Pr(q = 1) = 0.25$, then the randomized truth degree of proposition A under random sequence $D = (0.25, 0.25)$ is $\tau(A) = 0.8125$.

The proposed quantum implementation for calculating the randomized truth degree of A under random sequence



(a)

(b)

Fig. 6: Quantum implementation for calculating randomized truth degree of $A = p \to q$.

D is executed using Huawei's quantum computing cloud platform HiQ. The quantum circuit utilized for this example is illustrated in Fig. 6(a), where the initial state is $|0\rangle\,|0\rangle\,|0\rangle$. Upon executing the program depicted in Fig. 6(a) on the HiQ quantum platform, the results are presented in Fig. 6(b). Notably, out of 10,000 repeated tests, the state was measured as 0 a total of 1,925 times, while the probability of the state being 1 was recorded as 8,075 times. Consequently, the truth degree of A on the HiQ quantum platform is calculated to be 0.8075, which closely approximates the ideal result of 0.8125.

VI. Quantum Circuit for Calculating the Similarly Degree Between Propositions

A. Approach

Let $A = A(p_1, p_2, ..., p_m)$ be a proposition containing m atomic proposition $p_1, p_2, ..., p_m$ and $B = B(q_1, q_2, ..., q_n)$ be a proposition containing n atomic proposition $q_1, q_2, ..., q_n$. For convenience, let $p = (p_1, p_2, ..., p_m)$ and $q = (q_1, q_2, ..., q_n)$. Here we assume both propositions A and B are provided as two oracles U_A and U_B, respectively. We examine the quantum algorithm for computing the similarly degree between propositions A and B. The quantum circuit of our method is shown in Figure 5 and the steps of the method are as follows.

(1). The initial state $|\alpha_0\rangle$ is $|0\rangle^{\otimes(m+n+3)}$.
(2). Apply the $H^{\otimes m} \otimes I \otimes H^{\otimes n} \otimes I^2$ on the initial state,

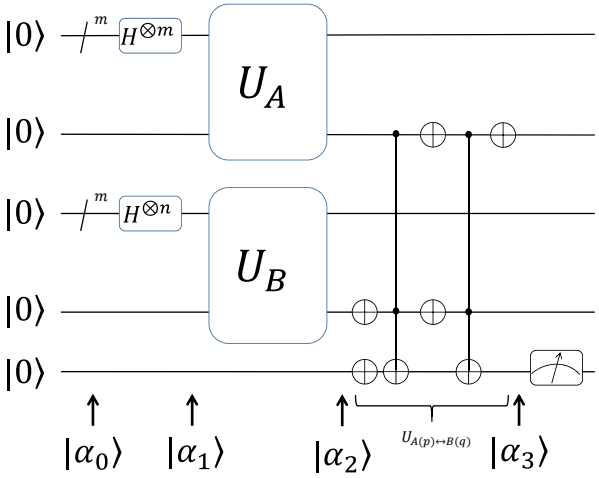
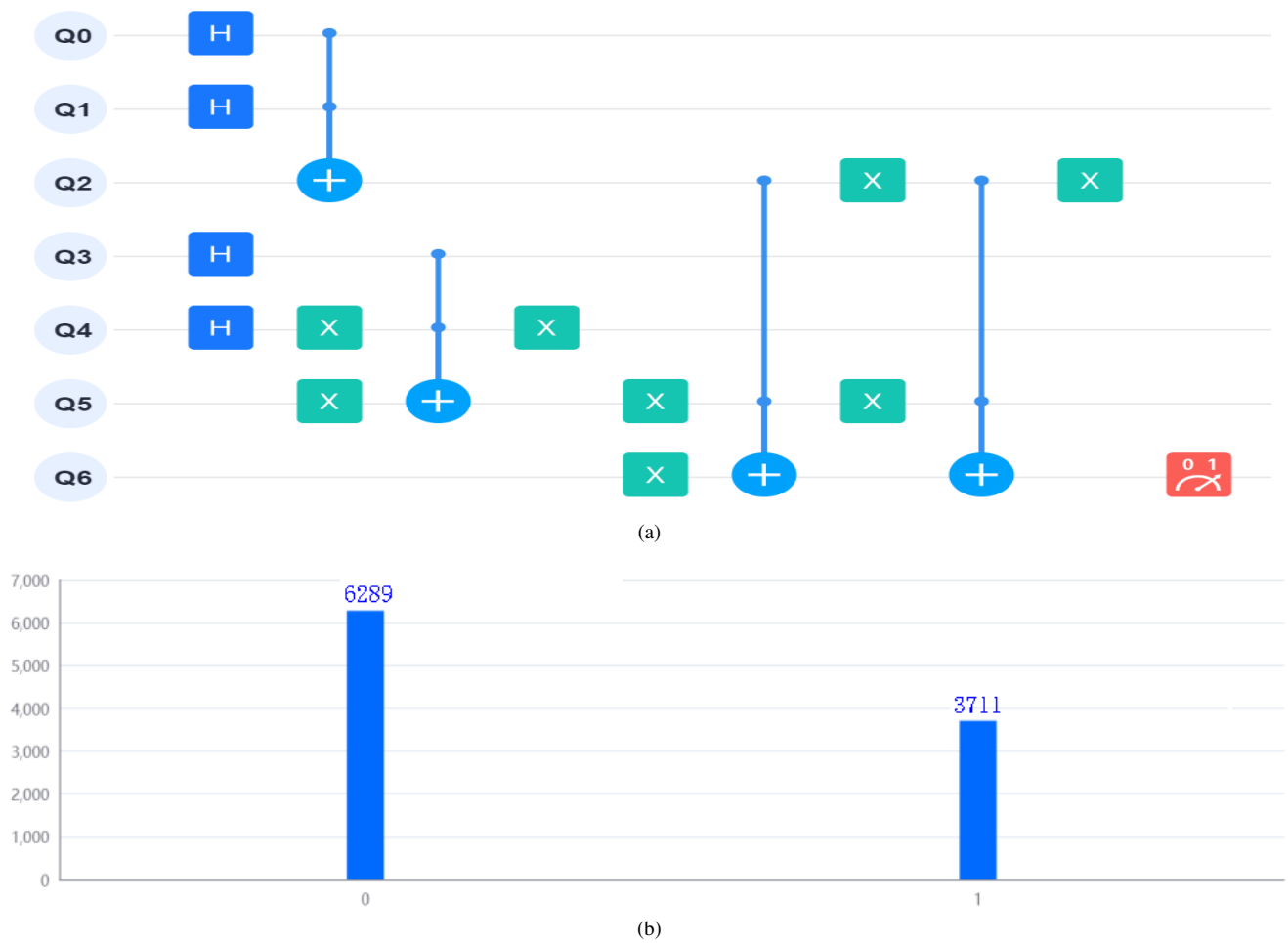


Fig. 7: Quantum circuit for calculating the similarly degree between propositions A and B.

producing

$$|\alpha_1\rangle = \sum_{p\in\{0,1\}^m} \frac{|p\rangle}{\sqrt{2^m}} \otimes |0\rangle \otimes \sum_{q\in\{0,1\}^n} \frac{|p\rangle}{\sqrt{2^n}} \otimes |0\rangle \otimes |0\rangle\,. \tag{20}$$



(a)

(b)

Fig. 8: Quantum implementation for calculating similarly degree between propositions A and B.

(3). Perform the $U_A \otimes U_B \otimes I$ on $|\alpha_1\rangle$, producing

$$\begin{aligned} &|\alpha_2\rangle \\ =&U_A(|\alpha_1\rangle) \\ =&\sum_{p\in\{0,1\}^m} \frac{|p, A(p)\rangle}{\sqrt{2^m}} \otimes \sum_{q\in\{0,1\}^n} \frac{|q, B(q)\rangle}{\sqrt{2^n}} \otimes |0\rangle . \end{aligned} \tag{21}$$

(4). Perform the $U_{A(p)\leftrightarrow B(q)}$ on the $n+1$th, $m+n+2$th, and $m+n+3$th qubits of $|\alpha_2\rangle$, as shown in Figure 5, and obtain $|\alpha_3\rangle$

(5). Perform a single-qubit measurement on the bottom qubit of $|\alpha_3\rangle$.

B. Analysis

From section 4, we can know that, in step 3, the $n+1$th and $m+n+2$th qubits of the state $|\alpha_2\rangle$ can be described as follows:

$$|\xi_1\rangle = \sqrt{1-\tau(A)}\,|0\rangle + \sqrt{\tau(A)}\,|1\rangle , \tag{22}$$

$$|\xi_2\rangle = \sqrt{1-\tau(B)}\,|0\rangle + \sqrt{\tau(B)}\,|1\rangle . \tag{23}$$

In step 4, we use $U_{A(p)\leftrightarrow B(q)}$ on $|\xi_1\rangle\,|\xi_2\rangle\,|0\rangle$ and obtain $|\xi_1\rangle\,|\xi_2\rangle\,|\xi_3\rangle$ where

$$\begin{aligned} |\xi_3\rangle = &\sqrt{1-\tau(A)\tau(B)-(1-\tau(A))(1-\tau(B))}\,|0\rangle \\ &+\sqrt{\tau(A)\tau(B)+(1-\tau(A))(1-\tau(B))}\,|1\rangle \end{aligned} \tag{24}$$

The state $|\xi_3\rangle$ is revealed to $|1\rangle$ with probability $\tau(A)\tau(B)+(1-\tau(A))(1-\tau(B))$ which is the similarly degree between propositions A and B. Thus, repeating above steps 1-5 a number of times allows us to estimate the similarly degree between propositions A and B.

C. A sample example

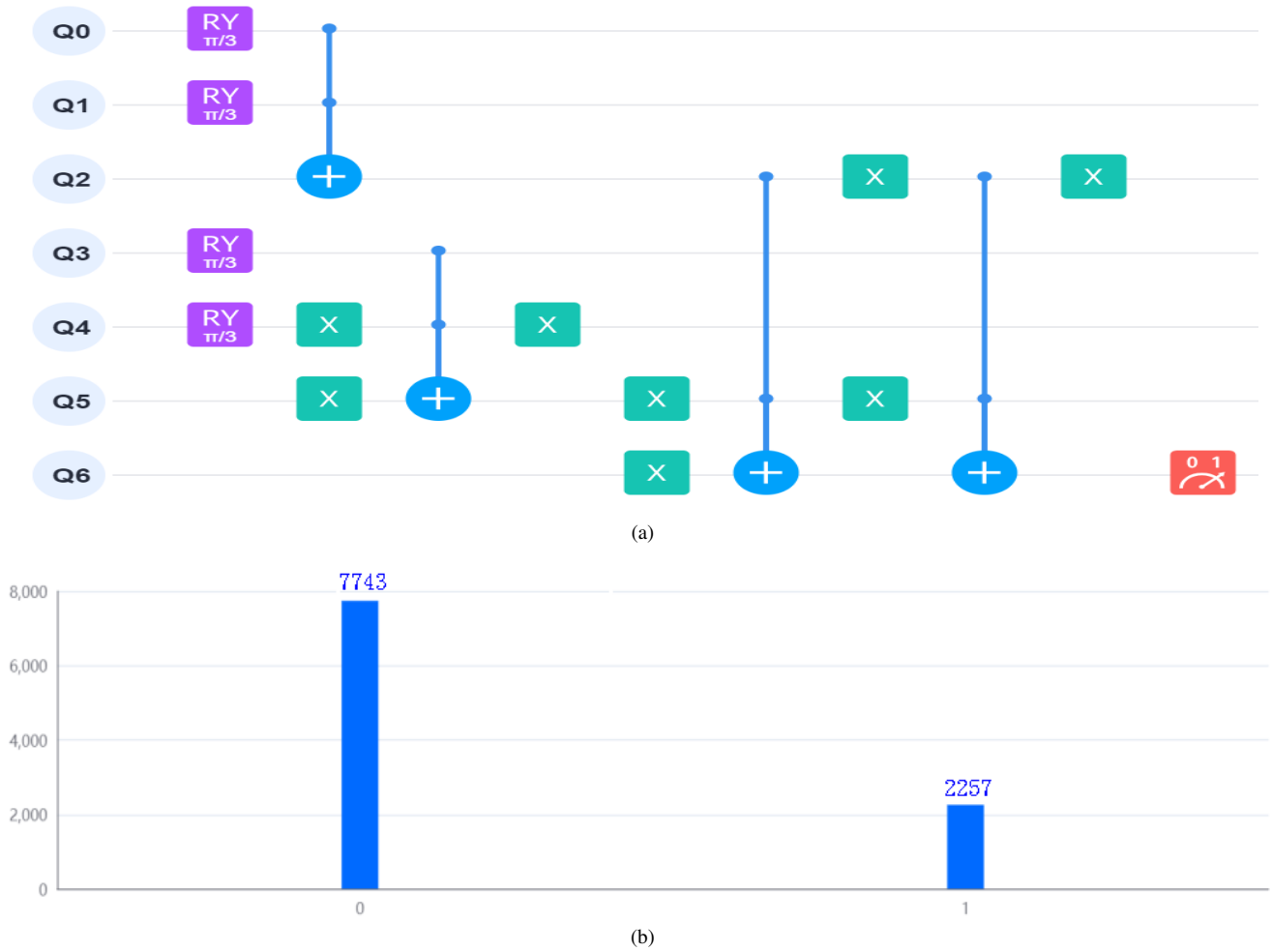
Let us consider the proposition $A = p \wedge q$ and $B = r \rightarrow s$. It is easy to know that the similarly degree between propositions A and B is $S(A,B) = 0.375$.

The proposed quantum implementation for calculating similarly truth degree between propositions A and B is implemented using Huawei's quantum computing cloud platform HiQ. The quantum circuit for this example is depicted in Fig. 8(a). Note that the initial state is $|0\rangle\,|0\rangle\,|0\rangle\,|0\rangle\,|0\rangle\,|0\rangle\,|0\rangle$. The result of executing the program of Fig. 8(a) on HiQ quantum platform is shown in Fig. 8(b). As it can be seen, the times of the state being 0 after the measurement is 6289, while the probability of the state being 1 is 3711, note that the number of repeated tests is 10000. The truth degree of A on HiQ quantum platform is 0.3711, which is very close to the ideal result 0.3711.

VII. Quantum Circuit for Calculating the Randomized Similarly Degree between Propositions

A. Approach

Let $A = A(p_1, p_2, ..., p_m)$ be a proposition containing m atomic proposition $p_1, p_2, ..., p_m$ and



(a)

(b)

Fig. 9: Quantum implementation for calculating randomized similarity degree between A and B.

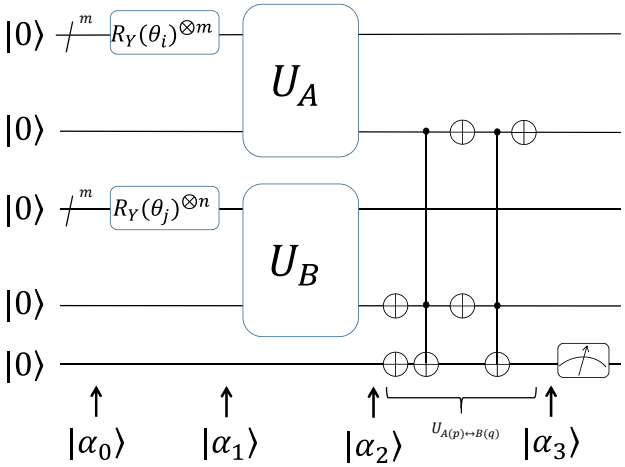


Fig. 10: Quantum circuit for calculating the randomized similarly degree between propositions A and B.

$B = B(q_1, q_2, ..., q_n)$ be a proposition containing n atomic proposition $q_1, q_2, ..., q_n$. Let $D_1 = (d_1, d_2, \ldots, d_m)$ and $D_2 = (e_1, e_2, \ldots, e_n)$ be two random sequences. Denote $D = (d_1, d_2, \ldots, d_m, e_1, e_2, \ldots, e_n)$. Let $\tau_{D_1}(A)$ and $\tau_{D_2}(B)$ be the randomized truth degrees of proposition A under the random sequence D_1 and proposition B under the random sequence D_2, respectively. The randomized similarly degree between propositions A and B under the random sequence D is $\tau_D(A \leftrightarrow B)$.

We examine the quantum algorithm for computing the randomized similarly degree between propositions A and B under the random sequence D. The quantum circuit of our method is shown in Figure 6 and the steps of the method are as follows.

(1). The initial state $|\alpha_0\rangle$ is $|\alpha_0\rangle$ is $|0\rangle^{\otimes(m+n+3)}$.

(2). Apply the $R_Y(\theta_1) \otimes \cdots \otimes R_Y(\theta_m) \otimes R_Y(\theta_{1'}) \otimes \cdots \otimes R_Y(\theta_n) \otimes I$ on the initial state where

$$\theta_i = 2\arcsin(\sqrt{d_i}), \quad i = 1, 2, ..., m, \tag{25}$$

$$\theta_{j'} = 2\arcsin(\sqrt{e_j}), \quad j = 1, 2, ..., n, \tag{26}$$

producing

$$\begin{aligned} &|\alpha_1\rangle \\ =&R_Y(\theta_1)|0\rangle \otimes \cdots \otimes R_Y(\theta_m)R_Y(\theta_{1'})|0\rangle \\ &\otimes \cdots \otimes R_Y(\theta_{n'})|0\rangle \otimes |0\rangle. \end{aligned} \tag{27}$$

(3). Perform the $U_A \otimes U_B \otimes I$ on $|\alpha_1\rangle$ and obtain $|\alpha_2\rangle$.

(4). Perform the $U_{A(p)\leftrightarrow B(q)}$ on the $n+1$th, $m+n+2$th, and $m+n+3$th qubits of $|\alpha_2\rangle$, as shown in Figure 5, and obtain $|\alpha_3\rangle$

(5). Perform a single-qubit measurement on the bottom qubit of $|\alpha_3\rangle$.

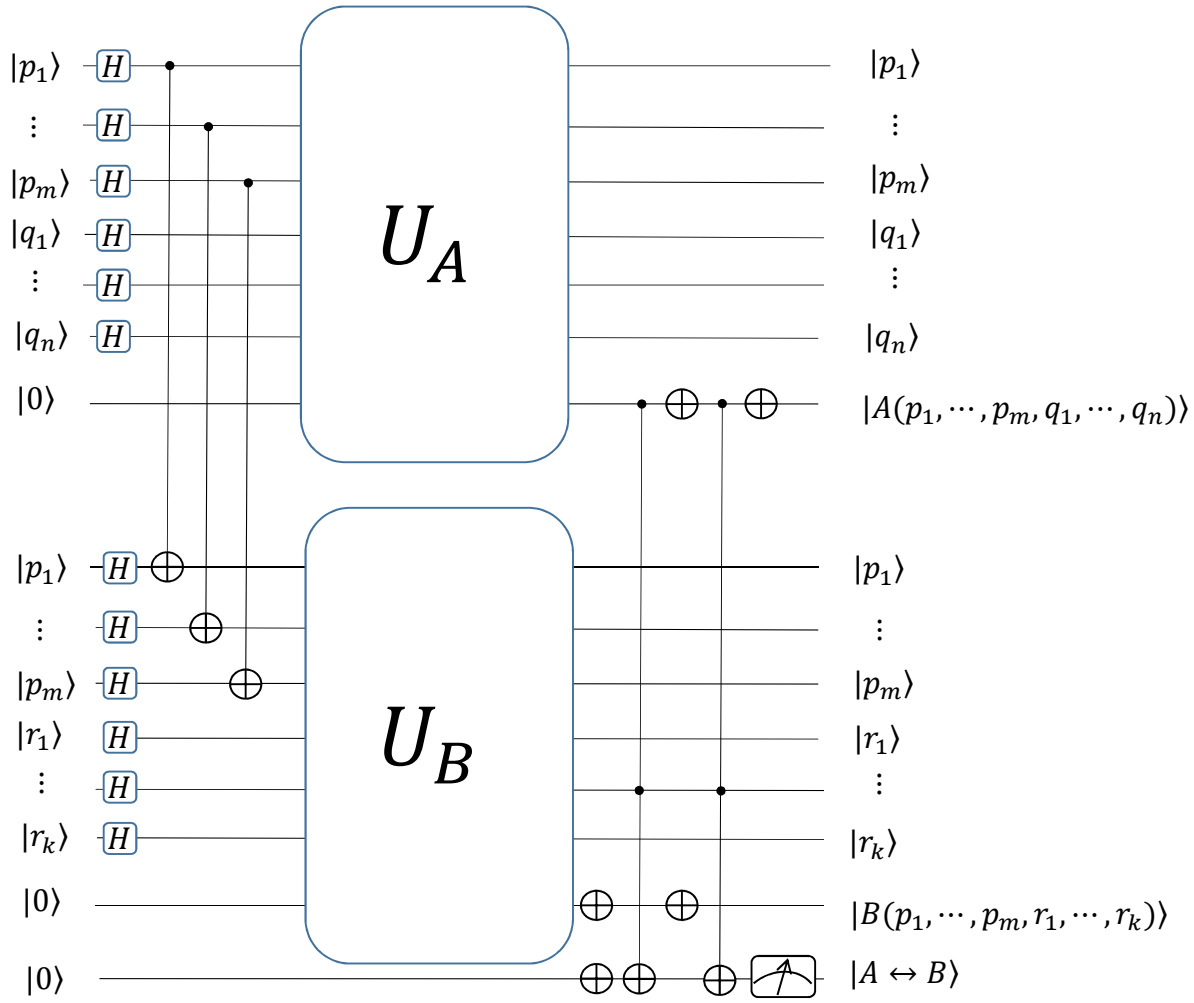


Fig. 11: Quantum circuit for calculating the similarly degree between propositions A and B with common atomic propositions.

B. Analysis

From section 5, we can know that, in step 3, the $n+1$th and $m+n+2$th qubits of the state $|\alpha_2\rangle$ can be described as follows:

$$|\xi_1\rangle = \sqrt{1-\tau_{D_1}(A)}\,|0\rangle + \sqrt{\tau_{D_1}(A)}\,|1\rangle\,, \tag{28}$$

$$|\xi_2\rangle = \sqrt{1-\tau_{D_2}(B)}\,|0\rangle + \sqrt{\tau_{D_2}(B)}\,|1\rangle\,. \tag{29}$$

In step 4, we use $U_{A(p)\leftrightarrow B(q)}$ on $|\xi_1\rangle\,|\xi_2\rangle\,|0\rangle$ and obtain $|\xi_1\rangle\,|\xi_2\rangle\,|\xi_3\rangle$ where

$$\begin{aligned}&|\xi_3\rangle\\ =&\sqrt{1-\tau_{D_1}(A)\tau_{D_2}(B)-(1-\tau_{D_1}(A))(1-\tau_{D_2}(B))}\,|0\rangle\\ &+\sqrt{\tau_{D_1}(A)\tau_{D_2}(B)+(1-\tau_{D_1}(A))(1-\tau_{D_2}(B))}\,|1\rangle\end{aligned} \tag{30}$$

The state $|\xi_3\rangle$ is revealed to $|1\rangle$ with probability $\tau_{D_1}(A)\tau_{D_2}(B)+(1-\tau_{D_1}(A))(1-\tau_{D_2}(B))$ which is the randomized similarly degree between propositions A and B under random sequence D. Thus, repeating above steps 1-5 a number of times allows us to estimate the randomized similarly degree between propositions A and B.

C. A sample example

Let us consider the proposition $A = p \wedge q$ and $B = r \rightarrow s$. If $\Pr(p = 1) = \Pr(q = 1) = \Pr(s = 1) = \Pr(t = 1) = 0.25$, then it is easy to know that the randomized similarly degree between propositions A and B is $S(A,B) = 0.2265625$ under random sequence $D = (0.25, 0.25, 0.25, 0.25)$.

The proposed quantum implementation for calculating similarly truth degree between propositions A and B is implemented using Huawei's quantum computing cloud platform HiQ. The quantum circuit for this example is depicted in Fig. 10(a). Note that the initial state is $|0\rangle\,|0\rangle\,|0\rangle\,|0\rangle\,|0\rangle\,|0\rangle\,|0\rangle$. The result of executing the program of Fig. 10(a) on HiQ quantum platform is shown in Fig. 10(b). As it can be seen, the times of the state being 0 after the measurement is 7743, while the probability of the state being 1 is 2257, note that the number of repeated tests is 10000. The truth degree of A on HiQ quantum platform is 0.2257, which is very close to the ideal result 0.2265625.

D. Discussion

In the previous sections, we presented quantum circuits for computing the similarity degree and randomized similarity degree between two propositions, A and B, under the assumption that they contain no common atomic propositions. We now extend our analysis to the more general case where A and B may contain overlapping atomic

propositions. Suppose $A = A(p_1, p_2, \ldots, p_m, q_1, q_2, \ldots, q_n)$ and $B = B(p_1, p_2, \ldots, p_m, r_1, r_2, \ldots, r_k)$, where $\{p_i\}_{i=1}^{m}$ are the common atomic propositions shared by both A and B. To handle this scenario, we propose a quantum circuit for evaluating their similarity degree. The quantum circuit is depicted in Figure 11. A key feature of this approach is that the first m qubits of A and the first m qubits of B are prepared in an entangled state. This ensures that their truth values are the same.

VIII. Conclusions

Quantum computers have demonstrated successful applications across various fields, and the quantum circuit model serves as a crucial tool in the theory of quantum computation. In this study, we employed quantum computers to estimate the truth degree of propositions and the similarity degree between propositions using quantum circuits. Our experimental evaluation, conducted on the HiQ quantum platform, confirmed the effectiveness of the proposed quantum circuits. This research aims to demonstrate the feasibility of quantum computers in computing truth degrees. Although truth degree theory is a valuable asset, it requires further exploration from the perspective of quantum computing. Future work should focus on investigating the implementation of quantum circuits for truth degree computation in many-valued logic systems.

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