Event-Driven Stabilization for Markov Jump Systems Based on Disturbance Observer

Haoyu Wang, Xinyu Hu, Zhilian Yan and Yebin Chen

***Abstract*—This paper is concerned with the event-driven stabilization of Markov jump systems with disturbances based on disturbance observer (DO). First, a DO is employed to estimate the disturbance generated by an exogenous system. A composite control scheme is then developed by integrating DO-based compensation with a switched event-driven control law, which ensures the stability of the closed-loop Markov jump system while optimizing network resource usage. Using a time-dependent and piecewise-defined Lyapunov function, we establish sufficient conditions for the existence of the desired observer and event-driven controller. Finally, a numerical example is presented to demonstrate the effectiveness of the proposed composite control scheme.**

***Index Terms*—Switched event-driven control, disturbance observer, Markov jump system, stabilization**

I. Introduction

SINCE the introduction of Markov jump systems (MJS) in 1961 [1], research in this field has gained significant momentum, owing to their remarkable effectiveness in modeling a variety of real-world systems characterized by variable structures and random changes [2], [3]. Today, MJSs have potential applications in various fields, including network control systems [4], economics [5], manufacturing applications [6], power systems [7]. It is well established that the stability of a dynamical system is a fundamental prerequisite for its practical applications. However, MJSs are essentially a special class of parameter-switching systems, where the switching (or jump) signals are governed by a Markov chain. This switching feature plays a critical role in the stability of MJSs. Consequently, the stabilization problem for MJSs has remained a key topic of research, as addressed in [8], [9]. A novel Lyapunov-Krasovskii function was proposed in [10] for the stability analysis and stabilization of MJSs with time-varying delays and an indeterminate transition rate matrix. In addition, a static output-feedback controller was designed to ensure the exponential stabilization of discrete-time MJSs within a hybrid design framework in [11].

In contemporary engineering control systems, disturbances and uncertainties are prevalent and can significantly degrade system performance. As a result, disturbance attenuation has become one of the primary objectives in controller design. To achieve this, a variety of advanced control strategies with disturbance attenuation capabilities have been developed, including model adaptive control [12], sliding mode control [13], and disturbance-observer-based control (DOBC) [14]. Among these methods, DOBC is particularly favored due to its ability to actively and effectively address disturbances [15]. The fundamental framework of DOBC involves constructing a DO to estimate disturbances, followed by the design of a composite controller consisting of a DO-based feedforward control term and a feedback control term [16]. The feedforward term compensates for the disturbances, while the feedback term is generally used to stabilize or track the nominal dynamics of the controlled system. Currently, DO has found widespread application in MJSs. In [17], a composite control scheme leveraging the DO was proposed to address control challenges in MJSs with time-varying delays. Additionally, the composite disturbance-resistant resilient control of nonlinear MJSs, subject to numerous disturbances and partially unknown transition probabilities, was also investigated in [18]. A composite disturbance-resistant control method, which ensures the stochastic stability of the closed-loop system, was developed by combining disturbance estimates with the conventional L_2-L_∞ resilient control law.

In the control community, time-driven mechanism (TDM), where sampling instants are initiated by a timer or clock, is widely used due to their simplicity in implementation, which facilitates the analysis and design of control systems [19], [20]. However, TDM may not be suitable for resource-constrained systems, as the choice of sampling instants does not account for the system state evolution or current computation or communication resource status [21]. To address this issue, a continuous event-triggered mechanism was proposed, where measurements are transmitted only when the relative change in the output exceeds a specified threshold [22]. Nevertheless, within a finite time frame, this mechanism can lead to an infinite number of triggering events, resulting in the Zeno phenomenon. In response, the periodic event-triggered mechanism was introduced, which checks the event-triggered condition at discrete time instants [23], [24]. This method prevents the Zeno phenomenon by ensuring a positive lower bound for the event interval. To further reduce the number of trigger events, a switched event-triggered mechanism was developed, which switches between periodic sampling and continuous event-triggered [25]. The principle behind this mechanism is that once a measurement is transmitted, the sensor must wait for a minimum period before continuously monitoring the trigger condition to determine the next triggering moment. Subsequent studies have demonstrated that this mechanism is both effective and technically viable [26], [27]. However, to the best of our knowledge, the switched event-driven control (SEDC) problem based on DO has not been thoroughly investigated, which serves as the main

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motivation for this work.

Building upon the previous discussions, this paper investigates the stabilization of MJSs with disturbance based on DO via SEDC. The remainder of the paper is organized as follows: Section II introduces MJSs, exogenous disturbances, DO, SED mechanism, and control law. In section III, we provide sufficient conditions for the closed-loop system to be asymptotically stable using linear matrix inequalities. Section IV validates the effectiveness of the proposed method through a simulation example. Section V concludes the paper.

II. Preliminaries

A. System description

Standard notations consistent with those outlined in [28] are used throughout the study. Consider the following continuous-time MJS

$$\dot{x}(t) = A(r(t))x(t) + B(r(t))(u(t) + d(t)), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the state vector and control input vector, respectively. $A(r(t))$ and $B(r(t))$ are known system matrices with approximate dimensions. $d(t) \in \mathbb{R}^m$ is external disturbance that satisfies the following Assumption 1. Let $r(t)$ be a right-continuous Markov process with values in the finite space $S = \{1, 2, ..., N\}$, having the following transition property:

$$Pr\{r(t+\delta) = j | r(t) = i\} = \begin{cases} \pi_{ij}\delta + o(\delta), & i \neq j \\ 1 + \pi_{ii}\delta + o(\delta), & i = j \end{cases}$$

where $\delta > 0$, and $\lim_{\delta\to 0}(o(\delta)/\delta) = 0$. The transition rate $\pi_{ij} \geq 0$ respresents the rate of transitioning from state i at time t to state j at time $t + \delta$, and the self-transition rate satisfies $\pi_{ii} = -\Sigma_{j=1,\, j\neq i}^{N}\pi_{ij}$ [29]. For convenience, let $r(t) = i$, where $i \in S$, and the system matrices associated with the ith mode are represented by A_i, B_i, and so on.

Assumption 1. The disturbance $d(t)$ is described by the subsequent exogenous system

$$\dot{\zeta}(t) = D_i\zeta(t), \quad d(t) = E_i\zeta(t), \tag{2}$$

where $D_i \in \mathbb{R}^{r\times r}$, $E_i \in \mathbb{R}^{m\times r}$ are known matrices.

B. Composite control law consisting of SEDC approach and DO-based compensation

To estimate the disturbance $d(t)$, the following DO is employed, given by:

$$\begin{aligned} \hat{d}(t) =& E_i\hat{\zeta}(t), \\ \hat{\zeta}(t) =& z(t) - L_i x(t), \\ \dot{z}(t) =& (D_i + L_iB_iE_i)(z(t) - L_ix(t)) \\ &+ L_i(A_ix(t) + B_iu(t)). \end{aligned} \tag{3}$$

Here $\hat{d}(t) \in \mathbb{R}^r$ represents the estimation of $d(t)$, and L_i denotes the DO gain matrix. The estimation error is expressed as

$$e_\zeta(t) = \zeta(t) - \hat{\zeta}(t). \tag{4}$$

Based on equations (2) and (3), we obtain the following estimation error dynamic equation:

$$\dot{e}_\zeta(t) = (D_i + L_iB_iE_i)e_\zeta(t). \tag{5}$$

Remark 1. An intermediate variable $z(t)$ is introduced in the design of the DO (3) to handle the derivative of the system variable $x(t)$, which may result in an inability to effectively track the disturbance $d(t)$.

The sensor waits for at least h seconds after sending the measurement data, then checks the event-triggered condition and sends the measurement value to the event-driven controller upon violation of the condition. This leads to the selection of sampling instants as follows:

$$t_{k+1} = min\left\{t \geq t_k + h | e_k^T(t)\Omega_ie_k(t) \geq \beta x^T(t)\Omega_ix(t)\right\}, \tag{6}$$

where $e_k(t) = x(t) - x(t_k)$, matrix $\Omega_i \geq 0$. $h > 0$ is a scalar, and β is the known threshold parameter. The triggering time between two adjacent events is at least h. Then, using the disturbance estimate $\hat{d}(t)$ from the DO (3) and the event-driven mechanism (6), the following control law is designed:

$$u(t) = -\hat{d}(t) + K_ix(t_k),\; t_k \leq t < t_{k+1}, \tag{7}$$

where K_i denotes control gain matrices. Substituting the controller (7) into system (1) yields that

$$\dot{x}(t) = \begin{cases} A_ix(t) + B_iK_ix(t_k) + B_iE_ie_\zeta(t), \\ \qquad\qquad t \in [t_k,\, t_k + h), \\ (A_i + B_iK_i)x(t) - B_iK_ie_k(t) + B_iE_ie_\zeta(t), \\ \qquad\qquad t \in [t_k + h,\, t_{k+1}). \end{cases} \tag{8}$$

The aim of this paper is to stabilize system (1) under controller (7). The following lemma, assumption and definition are demanded.

Lemma 1. (Jensen's Inequality) [30] For any positive definite matrix $N \in \mathbb{R}^{n\times n}$, and scalars p and q, consider a vector function $\varepsilon : [p,\, q] \to \mathbb{R}^n$. The following inequality holds:

$$F^TNF \leq (q-p)\int_p^q \varepsilon^T(\sigma)N\varepsilon(\sigma)d\sigma,$$

where

$$F = \int_p^q \varepsilon(\sigma)d\sigma.$$

Assumption 2. The pair A_i and B_i is controllable, while the pair D_i and B_iE_i is observable.

Definition 1. [31] (Infinitesimal operator) The notation $R(\mathbb{R}^n \times S; \mathbb{R}_+)$ is used to represent the family of all non-negative functions $V(x(t), r(t))$ defined on $\mathbb{R}^n \times S$, which are twice continuously differentiable in $x(t)$ and once continuously differentiable with respect to t. For $V(x(t), r(t)) \in R(\mathbb{R}^n \times S;\ \mathbb{R}_+)$, define the infinitesimal operator by

$$\begin{aligned} &\mathcal{A}V(x(t), r(t)) \\ &\quad= \lim_{\delta\to 0}\frac{1}{\delta}[EV(x(t+\delta), r(t+\delta)) \mid x(t), r(t) = i \\ &\qquad - V(x(t), r(t))]. \end{aligned}$$

Remark 2. Various event-triggered mechanisms have been proposed in the existing literature. In [32], an event-triggered mechanism based on a switched threshold was considered. A mixed switched event-triggered transmission mechanism was constructed in [33] by interpreting the resulting closed-loop system as a switching between systems with a dynamic threshold, including time-trigger, self-trigger, and discrete event-trigger systems. This paper focuses on switched between periodic sampling and continuous event-driven mechanism. According to the event-triggered mechanism (6), a switched closed-loop system (8) is proposed.

III. Main results

In this section, we use linear matrix inequalities (LMIs) to provide sufficient conditions for which the closed-loop system is asymptotic stability. Additionally, the design of the controller gain K_i and the observer gain L_i is presented.

Theorem 1. Given matrices K_i, L_i, and given parameters $\beta > 0$, $h > 0$, system (8) with controller (7) is asymptotic stability, if exist matrices $P_i > 0$, $M_i > 0$, $\Omega_i \geq 0$, $U > 0$, X, X_1, Y_1, Y_2, Y_3, Q_1 and Q_2 satisfying

$$\Xi_i > 0, \Phi_{1i} < 0, \Phi_{2i} < 0, \Phi_{3i} < 0, \tag{9}$$

where

$$\Xi_i = \begin{bmatrix} P_i + hHe(\frac{X}{2}) & h(-X + X_1) \\ * & hHe(-X_1 + \frac{X}{2}) \end{bmatrix}, \tag{10}$$

$$\Phi_{1i} = \begin{bmatrix} W_{11i} & \Delta_{12i} & W_{13i} & \Delta_{14i} & hY_1^T \\ * & \Delta_{22i} & \Delta_{23i} & \Delta_{24i} & hY_2^T \\ * & * & W_{33i} & \Delta_{34i} & hY_3^T \\ * & * & * & W_{44i} & \Delta_{45i} \\ * & * & * & * & -hU \end{bmatrix}, \tag{11}$$

$$\Phi_{2i} = \begin{bmatrix} W_{11i} & \Delta_{12i} + G_{12i} & W_{13i} & \Delta_{14i} \\ * & \Delta_{22i} + G_{22i} & \Delta_{23i} + G_{23i} & \Delta_{24i} \\ * & * & W_{33i} & \Delta_{34i} \\ * & * & * & W_{44i} \end{bmatrix}, \tag{12}$$

$$\Phi_{3i} = \begin{bmatrix} \Psi_{11i} & \Psi_{12i} & H_{13i} & \Delta_{14i} \\ * & \Delta_{22i} & H_{23i} & \Delta_{24i} \\ * & * & -\Omega_i & \Delta_{34i} \\ * & * & * & W_{44i} \end{bmatrix}, \tag{13}$$

$$\begin{aligned}
W_{11i} &= \Delta_{11i} + I_{11i}, \\
W_{13i} &= \Delta_{13i} + I_{13i}, \\
W_{33i} &= \Delta_{33i} + I_{33i}, \\
W_{44i} &= \Delta_{44i} + I_{44i}, \\
\Delta_{11i} &= He(-Y_1 + A_i^T Q_1), \\
\Delta_{12i} &= -Y_2 - Q_1^T + A_i^T Q_2 + P_i, \\
\Delta_{13i} &= Y_1^T - Y_3 + Q_1^T B_i K_i, \\
\Delta_{34i} &= 0, \\
\Delta_{44i} &= He(M_i D_i + M_i L_i B_i E_i), \\
\Delta_{14i} &= Q_1^T B_i E_i, \\
\Delta_{22i} &= He(-Q_2), \\
\Delta_{23i} &= Y_2^T + Q_2^T B_i K_i, \\
\Delta_{24i} &= Q_2^T B_i E_i, \\
\Delta_{33i} &= Y_3^T + Y_3, \\
\Delta_{45i} &= 0, \\
I_{11i} &= -He(\frac{X}{2}) + \sum_{j=1}^{s} \pi_{ij} P_j, \\
I_{13i} &= -X + X_1, \\
I_{33i} &= He(-X_1 + \frac{X}{2}), \\
I_{44i} &= \sum_{j=1}^{s} \pi_{ij} M_j, \\
G_{12i} &= hHe(\frac{X}{2}), \\
G_{22i} &= hU, \\
G_{23i} &= h(-X + X_1), \\
\Psi_{11i} &= He(Q_1^T A_i + Q_1^T B_i K_i) + \beta\Omega_i + \sum_{j=1}^{s} \pi_{ij} P_j, \\
\Psi_{12i} &= P_i - Q_i^T + (A_i + B_i K_i)^T Q_2, \\
H_{13i} &= -Q_1^T B_i K_i, \\
H_{23i} &= -Q_2^T B_i K_i.
\end{aligned}$$

Proof: Consider the following piecewise Lyapunov function:

$$V(t) = \begin{cases} \bar{V}(t), & t \in [t_k, t_k + h) \\ \tilde{V}(t), & t \in [t_k + h, t_{k+1}) \end{cases}$$

where

$$\begin{aligned}
\bar{V}(t) &= V_1(t) + V_2(t) + V_3(t), \\
\tilde{V}(t) &= V_1(t), \\
V_1(t) &= x^T(t) P_i x(t) + e_\zeta^T(t) M_i e_\zeta(t), \\
V_2(t) &= (h - \tau(t)) \int_{t_k}^{t} \dot{x}^T(s) U \dot{x}(s) ds, \\
V_3(t) &= (h - \tau(t)) \, [x^T(t), \, x^T(t_k)] \\
&\quad \times \begin{bmatrix} He(\frac{X}{2}) & -X + X_1 \\ * & He(-X_1 + \frac{X}{2}) \end{bmatrix} \times \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix},
\end{aligned}$$

and $\tau(t) = t - t_k$, $x_t(\theta) = x(t + \theta)$ for $\theta \in [-h, 0]$.

Evidently, $V_1(t)$, $V_2(t)$ and $x^T(t) P_i x(t)$ are positive definite. From (9) and (10), it follows that $V_3(t) + x^T(t) P_i x(t)$ is positive definite, since

$$\begin{aligned}
& V_3(t) + x^T(t) P_i x(t) \\
&= \xi^T(t)(\mathcal{F} + (t_k + h - t)\mathcal{M})\xi(t) \\
&= \xi^T(t)\Big(\frac{t - t_k}{h}\mathcal{F} + \frac{t_k + h - t}{h}\mathcal{F} + \frac{t_k + h - t}{h} h\mathcal{M}\Big)\xi(t) \\
&= \frac{t - t_k}{h}\xi^T(t)\mathcal{F}\xi(t) + \frac{t_k + h - t}{h}\xi^T(t)(\mathcal{F} + h\mathcal{M})\xi(t),
\end{aligned} \tag{14}$$

where $\xi = col\,\{x(t), \, x(t_k)\}$,

$$\mathcal{F} = \begin{bmatrix} P_i & 0 \\ 0 & 0 \end{bmatrix}, \; \mathcal{M} = \begin{bmatrix} He(\frac{X}{2}) & -X + X_1 \\ * & He(-X_1 + \frac{X}{2}) \end{bmatrix}.$$

For $t \in [t_k, \, t_k + h)$, we consider the following Lyapunov function:

$$V(t) = \bar{V}(t).$$

Then, based on Definition 1, it yields that

$$\begin{aligned}
\mathcal{A}V(t) &= 2\dot{x}^T(t) P_i x(t) + x^T(t) \sum_{j=1}^{s} \pi_{ij} P_j x(t) \\
&\quad + 2e_\zeta^T(t) M_i \dot{e}_\zeta(t) + e_\zeta^T(t) \sum_{j=1}^{s} \pi_{ij} M_j e_\zeta(t) \\
&\quad - \int_{t_k}^{t} \dot{x}^T(s) U \dot{x}(s) ds
\end{aligned}$$

$$
\begin{aligned}
&+ (h-\tau(t))\dot{x}^T(t)U\dot{x}(t) - x^T(t)He(\frac{X}{2})x(t)\\
&- x^T(t)He(-X+X_1)x(t_k)\\
&- x^T(t_k)He(-X_1+\frac{X}{2})x(t_k)\\
&+ (h-\tau(t))\dot{x}^T(t)He(X)x(t)\\
&+ 2(h-\tau(t))\dot{x}^T(X+X_1)x(t_k).
\end{aligned}
$$

Denoting

$$v_1 = \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} \dot{x}(s)ds,$$

we understand by $v_{1_{\tau(t)=0}}$ the following: $\lim_{\tau(t)\to 0} v_1 = \dot{x}(t)$. From Lemma 1, we can conclude

$$\int_{t-\tau(t)}^{t} \dot{x}^T(s)U\dot{x}(s)ds \geq \tau(t)v_1^T U v_1. \tag{15}$$

According to system (8), for any matrices Y_1, Y_2, Y_3, Q_1 and Q_2, applying the free weight matrix method yields that

$$
\begin{aligned}
0 &= 2[x^T(t)Y_1^T + \dot{x}^T(t)Y_2^T + x^T(t_k)Y_3^T]\\
&\quad \times [\tau(t)v_1 + x(t_k) - x(t)], && (16)\\
0 &= 2[x^T(t)Q_1^T + \dot{x}^T(t)Q_2^T]\\
&\quad \times [A_i x(t) + B_i K_i x(t_k) + B_i E_i e_\varsigma(t) - \dot{x}(t)]. && (17)
\end{aligned}
$$

Then, incorporating equations (16) and (17) into $\mathcal{A}V(t)$, we can deduce that

$$\mathcal{A}V(t) \leq \eta_1^T(t)\Lambda_i\eta_1(t),$$

where

$$
\Lambda_i = \begin{bmatrix}
W_{11i} & W_{12i} & W_{13i} & W_{14i} & \Delta_{15i}\\
* & W_{22i} & W_{23i} & \Delta_{24i} & \Delta_{25i}\\
* & * & W_{33i} & \Delta_{34i} & \Delta_{35i}\\
* & * & * & W_{44i} & \Delta_{45i}\\
* & * & * & * & \Delta_{55i}
\end{bmatrix}, \tag{18}
$$

$$
\begin{aligned}
\eta_1(t) &= col\left\{x(t),\, \dot{x}(t),\, x(t_k),\, e_\varsigma(t),\, v_1(t)\right\},\\
I_{12i} &= (h-\tau(t))He(\frac{X}{2}),\\
I_{22i} &= (h-\tau(t))U,\\
I_{23i} &= (h-\tau(t))(-X+X_1),\\
\Delta_{15i} &= \tau(t)Y_1^T,\\
\Delta_{25i} &= \tau(t)Y_2^T,\\
\Delta_{35i} &= \tau(t)Y_3^T,\\
\Delta_{55i} &= -\tau(t)U,\\
W_{12i} &= \Delta_{12i} + I_{12i},\\
W_{14i} &= \Delta_{14i} + I_{14i},\\
W_{22i} &= \Delta_{22i} + I_{22i},\\
W_{23i} &= \Delta_{23i} + I_{23i}.
\end{aligned}
$$

Clearly, equations (11) and (12) imply that $\tau(t) \to h$ and $\tau(t) \to 0$ in equation (18), respectively. According to (9)–(12), we can conclude that

$$\mathcal{A}\bar{V}(t) \leq \eta_1^T(t)\Lambda_i\eta_1(t) < 0. \tag{19}$$

The value of $V(t)$ switches at the instants t_k and $t_k + h$. For $t \in [t_k + h,\, t_{k+1})$, we apply the following function:

$$V(t) = \tilde{V}(t).$$

Equation (6) signifies that

$$0 \leq \beta x^T(t)\Omega_i x(t) - e_k^T(t)\Omega_i e_k(t). \tag{20}$$

Similar to equation (16), we can obtain

$$
\begin{aligned}
0 =& 2[x^T(t)Q_1^T + \dot{x}^T(t)Q_2^T]\\
&\times [(A_i + B_iK_i)x(t) - B_iK_ie_k(t) + B_iE_ie_\varsigma(t) - \dot{x}(t)].
\end{aligned} \tag{21}
$$

By adding (20) and (21) to $\mathcal{A}V(t)$, we obtain from Definition 1:

$$
\begin{aligned}
\mathcal{A}V(t) =& 2\dot{x}^T(t)P_ix(t) + 2e_\varsigma^T(t)M_i\dot{e}_\varsigma(t)\\
&+ x^T(t)\sum_{j=1}^{s}\pi_{ij}P_jx(t) + e_t^T\sum_{j=1}^{s}\pi_{ij}M_je_\varsigma(t)\\
&+ 2x^T(t)Q_1^T(A_i + B_iK_i)x(t)\\
&- 2x^T(t)Q_1^TB_iK_ie_k(t) + 2x^T(t)Q_1^TB_iE_ie_\varsigma(t)\\
&- 2x^T(t)Q_1^T\dot{x}(t) + 2\dot{x}^T(t)Q_2^T(A_i + B_iK_i)x(t)\\
&- 2\dot{x}^T(t)Q_2^TB_iK_ie_k(t) + 2\dot{x}^T(t)Q_2^TB_iE_ie_\varsigma(t)\\
&- 2\dot{x}^T(t)Q_2^T\dot{x}(t) + \beta x^T(t)\Omega_ix(t)\\
&- e_k^T(t)\Omega_ie_k(t).
\end{aligned}
$$

Let $\eta_2(t) = col\left\{x(t),\, \dot{x}(t),\, e_k(t),\, e_\varsigma(t)\right\}$, and from (9) and (13), we obtain that

$$\mathcal{A}V(t) \leq \eta_2^T(t)\Phi_{3i}\eta_2(t) < 0. \tag{22}$$

Since $V(t)$ is a piecewise function, it is necessary to show that $V(t)$ is continuous for $t \in [t_k,\, t_{k+1})$ at the instants t_k and $t_k + h$. The proof is outlined as follows:

$$
\begin{aligned}
V_2(t_k) &= V_3(t_k) = 0,\\
\lim_{x\to(t_k+h)^-} V_2(t) &= V_3(t) = 0.
\end{aligned} \tag{23}
$$

Therefore, $V(t)$ is continuous at the instants t_k and $t_k + h$, and we conclude that $V(t)$ is continuous for all $t \in [0,\, \infty)$. Thus, based on inequations (19) and (22), it holds for any $t \in [t_k,\, t_{k+1})$ that

$$\mathcal{A}V(t) < 0,$$

which implies that the system is asymptotically stable. This completes the proof. ■

Remark 3. Equation (23) proves the continuity of $V_2(t)$ and $V_3(t)$, thereby ensuring the continuity of Lyapunov function $V(t)$, which is essential for demonstrating the stability of the system.

Theorem 2. Given matrices $\bar{K}_i$, $\bar{L}_i$, and given parameters α, $\beta > 0$, $h > 0$, system (8) with controller (7) is asymptotic stability, if exist matrices $\bar{P}_i > 0$, $M_i > 0$, $\bar{\Omega}_i \geq 0$, $\bar{U} > 0$, $\bar{X}$, $\bar{X}_1$, $\bar{Y}_1$, $\bar{Y}_2$, $\bar{Y}_3$, $\bar{Q}_1$, and $\bar{Q}_2$ satisfying

$$\bar{\Xi}_i > 0, \bar{\Phi}_{1i} < 0, \bar{\Phi}_{2i} < 0, \bar{\Phi}_{3i} < 0, \tag{24}$$

where

$$
\bar{\Xi} = \begin{bmatrix}
\bar{P}_i + hHe(\frac{\bar{X}}{2}) & h(-\bar{X} + \bar{X}_1)\\
* & hHe(-\bar{X}_1 + \frac{\bar{X}}{2})
\end{bmatrix},
$$

$$
\bar{\Phi}_{1i} = \begin{bmatrix}
\bar{W}_{11i} & \bar{\Delta}_{12i} & \bar{W}_{13i} & \bar{\Delta}_{14i} & h\bar{Y}_1^T\\
* & \bar{\Delta}_{22i} & \bar{\Delta}_{23i} & \bar{\Delta}_{24i} & h\bar{Y}_2^T\\
* & * & \bar{W}_{33i} & \bar{\Delta}_{34i} & h\bar{Y}_3^T\\
* & * & * & \bar{W}_{44i} & \bar{\Delta}_{45i}\\
* & * & * & * & -h\bar{U}
\end{bmatrix},
$$

$$\bar{\Phi}_{2i} = \begin{bmatrix} \bar{W}_{11i} & \bar{\Delta}_{12i} + \bar{G}_{12i} & \bar{W}_{13i} & \bar{\Delta}_{14i} \\ * & \bar{\Delta}_{22i} + \bar{G}_{22i} & \bar{\Delta}_{23i} + \bar{G}_{23i} & \bar{\Delta}_{24i} \\ * & * & \bar{W}_{33i} & \bar{\Delta}_{34i} \\ * & * & * & \bar{W}_{44i} \end{bmatrix},$$

$$\bar{\Phi}_{3i} = \begin{bmatrix} \bar{\Psi}_{11i} & \bar{\Psi}_{12i} & \bar{H}_{13i} & \bar{\Delta}_{14i} \\ * & \bar{\Delta}_{22i} & \bar{H}_{23i} & \bar{\Delta}_{24i} \\ * & * & -\bar{\Omega}_i & \bar{\Delta}_{34i} \\ * & * & * & \bar{W}_{44i} \end{bmatrix},$$

$$\begin{aligned}
\bar{X} &= (Q_1^T)^{-1} X Q_1^{-1}, \\
\bar{X}_1 &= (Q_1^T)^{-1} X_1 Q_1^{-1}, \\
\bar{Y}_1 &= (Q_1^T)^{-1} Y_1 Q_1^{-1}, \\
\bar{Y}_2 &= (Q_1^T)^{-1} Y_2 Q_1^{-1}, \\
\bar{Y}_3 &= (Q_1^T)^{-3} Y_1 Q_1^{-1}, \\
\bar{G}_{12i} &= hHe(\frac{\bar{X}}{2}), \\
\bar{G}_{22i} &= h\bar{U}, \\
\bar{G}_{23i} &= h(-\bar{X} + \bar{X}_1^T), \\
\bar{H}_{13i} &= -B_i \bar{K}_i, \\
\bar{H}_{23i} &= -\alpha B_i \bar{K}_i, \\
\bar{\Delta}_{12i} &= -\bar{Y}_2 - Q_1^{-1} + \alpha (Q_1^T)^{-1} A_i^T + \bar{P}_i, \\
\bar{\Delta}_{14i} &= B_i E_i, \\
\bar{\Delta}_{22i} &= -\alpha He(Q_1^{-1}), \\
\bar{\Delta}_{23i} &= \bar{Y}_2^T + \alpha B_i \bar{K}_i, \\
\bar{\Delta}_{24i} &= \alpha B_i E_i, \\
\bar{W}_{11i} &= He(-\bar{Y}_1 + A_i Q_1^{-1}) + \sum_{j=1}^{s} \pi_{ij} \bar{P}_j, \\
\bar{W}_{13i} &= \bar{Y}_1^T - \bar{Y}_3 + B_i \bar{K}_i - (\bar{X} + \bar{X}_1), \\
\bar{W}_{33i} &= He(\bar{Y}_3 + \bar{X}_1 - \frac{\bar{X}}{2}), \\
\bar{W}_{44i} &= He(M_i D_i + \bar{L}_i B_i E_i) + \sum_{j=1}^{s} \pi_{ij} M_j, \\
\bar{\Psi}_{11i} &= He(A_i Q_1^{-1} + B_i \bar{K}_i) + \beta \bar{\Omega}_i + \sum_{j=1}^{s} \pi_{ij} \bar{P}_j, \\
\bar{\Psi}_{12i} &= \bar{P}_i - Q_1^{-1} + \alpha Q_1^T A_i^T + \alpha \bar{K}_i^T B_i^T, \\
\bar{P}_i &= (Q_1^T)^{-1} P_i Q_1^{-1}, \\
\bar{\Omega}_i &= (Q_1^T)^{-1} \Omega_i Q_1^{-1}, \\
\bar{U} &= (Q_1^T)^{-1} U Q_1^{-1}, \\
Q_2 &= \alpha Q_1,
\end{aligned}$$

and $\bar{K}_i = K_i Q_1^{-1}$, $\bar{L}_i = M_i L_i$.

Proof: By multiplying the inequalities $\bar{\Xi}_i > 0$, $\bar{\Phi}_{1i} < 0$, $\bar{\Phi}_{2i} < 0$ and $\bar{\Phi}_{3i} < 0$ on the left by the following matrices:

$$\begin{aligned}
&diag\left\{Q_1^{-1}, Q_1^{-1}, Q_1^{-1}, I, Q_1^{-1}\right\}, \\
&diag\left\{Q_1^{-1}, Q_1^{-1}, Q_1^{-1}, I\right\}, \\
&diag\left\{Q_1^{-1}, Q_1^{-1}, Q_1^{-1}, I\right\}, \\
&diag\left\{Q_1^{-1}, Q_1^{-1}\right\},
\end{aligned}$$

and then multiplying the result by their transposes on the right, we derive the result in inequation (9). Thus, the proof is complete. ■

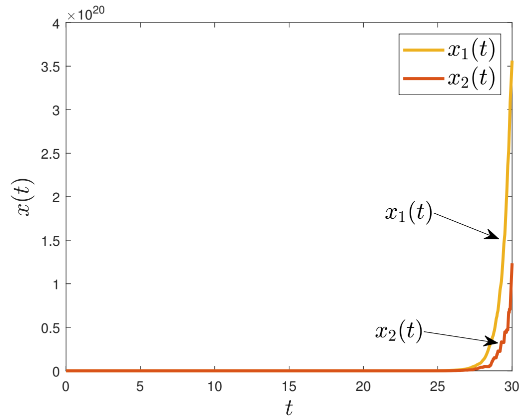

Fig. 1. State trajectories $x(t)$ without controller $u(t)$.

IV. Numerical Example

Consider a MJS (1) with parameters:

$$A_1 = \begin{bmatrix} 2.2 & -0.3 \\ 0.1 & -0.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.2 & -0.3 \\ 2.1 & -3 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 & 0.4 \\ -0.4 & 0 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The transition rate matrix is given by

$$\Pi = \begin{bmatrix} -1.4 & 1.4 \\ 1.3 & -1.3 \end{bmatrix}.$$

The initial conditions for the original system (1) and disturbance (2) are given by $x(0) = [0.1 \quad -0.1]^T$ and $\xi(0) = [1 \quad -2]^T$, respectively. The parameters are $h = 0.1$, $\alpha = 0.5$, $\beta = 0.5$. By solving the LMIs (24), the control gain K_i and observer gain L_i are obtained as:

$$\begin{aligned}
K_1 &= \begin{bmatrix} -7.9364 & 1.0581 \\ 0.0069 & -0.3738 \end{bmatrix}, \\
K_2 &= \begin{bmatrix} -5.4861 & 0.5856 \\ -1.7140 & 0.5869 \end{bmatrix}, \\
L_1 &= \begin{bmatrix} -2.3864 & 0.2678 \\ 0.4248 & -0.8387 \end{bmatrix}, \\
L_2 &= \begin{bmatrix} -2.2430 & 0.2517 \\ 0.3687 & -0.8274 \end{bmatrix}.
\end{aligned}$$

The simulation results are depicted in Figs. 1–6. Fig.1 shows that in the absence of the controller $u(t)$, the system state $x(t)$ diverges. In contrast, Fig. 2 shows that $x(t)$ ultimately converges to zero with the controller $u(t)$, indicating the effectiveness of the proposed controller. The controller $u(t)$ is illustrated in Fig. 3. The disturbance $d(t)$, its estimation $\hat{d}(t)$ and the estimation error $e_d(t)$ are presented in Fig. 4. Estimation error convergence to zero indicates that the DO can effectively identify and compensate for the disturbance $d(t)$. The event-driven release times and intervals are presented in Fig. 5. Finally, Fig. 6 illustrates the mode-switching process, where the system switched between Mode 1 and Mode 2.

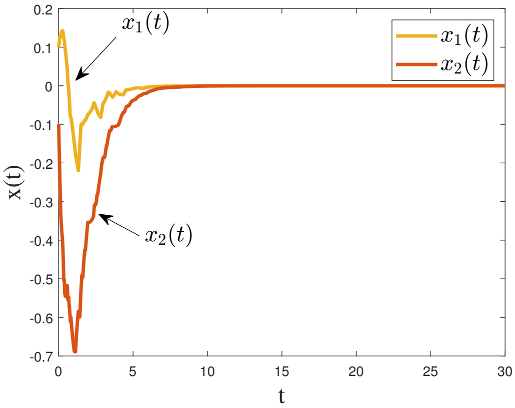

Fig. 2. State trajectories $x(t)$ by Theorem 2.

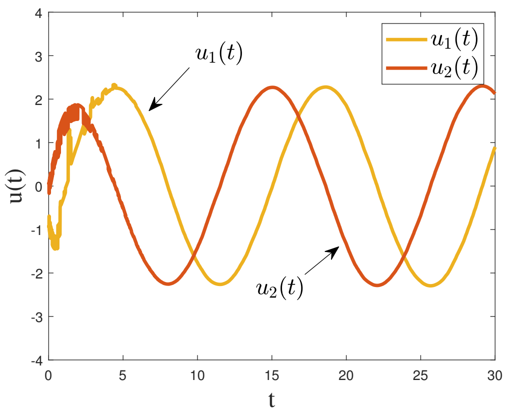

Fig. 3. Control input $u(t)$.

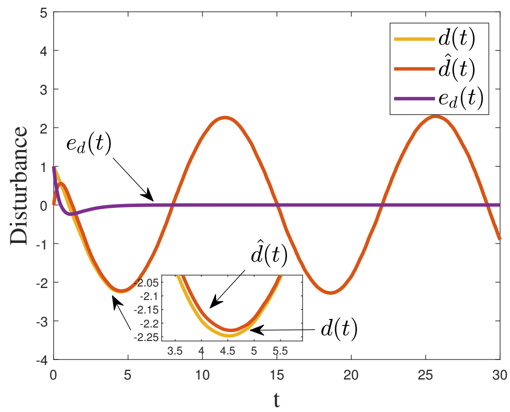

Fig. 4. Disturbance estimation error.

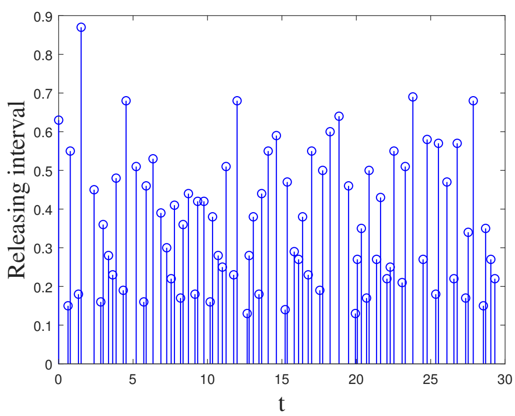

Fig. 5. The event-driven release instants and intervals.

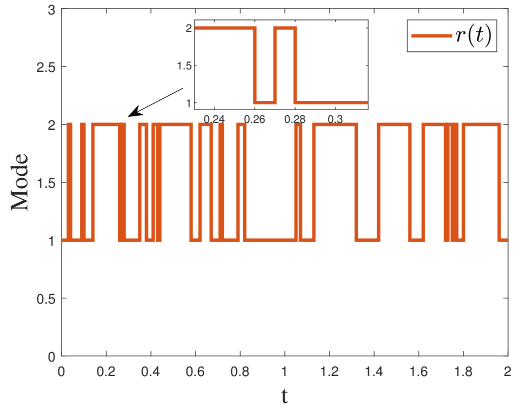

Fig. 6. Mode $r(t)$.

V. Conclusion

The stabilization problem of a class of MJSs with disturbance was investigated by synthesizing the SEDC approach and a DO-based compensation scheme in this paper. To begin with, a DO and an SED mechanism were adopted to estimate the disturbance and determine whether the measurement data should be sent, respectively. Then, a composite control scheme was constructed by combining the DO-based compensation with the SEDC. Using a time-dependent, piecewise-defined Lyapunov functional, a sufficient condition for ensuring the stability of the estimation error dynamics equation and the closed-loop Markov jump system was established in Theorem 1. On this basis, a co-design of the desired observer and event-driven controller was introduced in Theorem 2. Finally, a numerical example was provided to illustrate the effectiveness of the proposed composite control scheme.

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