A Tri-objective Collaborative Development Scheduling Model Based on Improved Fuzzy TOPSIS with Constraints

Junfeng Zhao, Nuo Guan, Xue Deng, Yan Xu, Jianxin Yang

***Abstract*—In the planning phase of collaborative development projects, a reasonable scheduling plan plays a pivotal role in shortening development cycle and reducing costs. However, in the traditional collaborative development scheduling models, the inherent fuzziness embedded in the scheduling process is ignored. Therefore, based on fuzzy TOPSIS, we construct a tri-objective (development time, development cost, and product quality) fuzzy collaborative development scheduling model with finite time and finite cost. The NSGA-II algorithm is used to solve the model, in which solutions meeting the constraints will be retained while those otherwise will be punished. Following the Pareto optimal solution set, the final solution is obtained using fuzzy TOPSIS CRITIC. Finally, empirical analysis verifies the effectiveness of the model and algorithm. The results show that in a fuzzy environment, collaborative development scheduling can be observed from a more practical perspective, and a more reasonable scheduling plan can be selected to reduce development time and costs. In empirical analysis, based on the proposed model and algorithm, scheduling schemes that satisfy constraints can be obtained with short time, low cost, and high quality.**

***Index Terms*— development scheduling, fuzzy multi-objective, triangular fuzzy number, ideal point method, NSGA-II algorithm**

I. Introduction

The burgeoning expansion of the global market has impelled organizations to augment their investments in product development as a means of sustaining their competitive edge [1]. Product development encompasses the competitive pursuits of minimizing risks through the acquisition of comprehensive market intelligence, cost reduction, and expediting time to market [2]. Consequently, product development assumes a pivotal position within the realm of business planning [3]. Nevertheless, the intricate nature and inherent uncertainties prevalent in the product development trajectory have compelled organizations to explore collaborative avenues for risk-sharing, cost minimization, accelerated time to market, quality enhancement, and capitalizing on complementary knowledge and competencies during the development phase. Hence, collaborative product development has emerged as a novel business paradigm, poised to augment the efficiency and efficacy of the development process [4].

In comparison with traditional development modalities, collaborative product development entails more malleable human and organizational behaviors, concomitantly giving rise to elevated levels of uncertainty [5]. The uncertainty endemic to collaborative product development has the potential to precipitate development outcomes that diverge substantially from anticipations. This admonishes enterprises to place greater emphasis on the uncertainties extant within the collaborative development process and to factor in diverse uncertainties during the planning stage of development projects. When formulating schedules for collaborative development, a more exhaustive analysis of the potential disparate outcomes under varying schedules ought to be carried out. Such systematic analysis can confer greater advantages upon the enterprise and equip it to brace for diverse contingencies. Owing to the equivocality of human judgment, evaluation outcomes exhibit fuzzy uncertainty when appraising the values of scheduling-related attributes. Should this uncertainty be disregarded, a comprehensive dissection of different schedules becomes infeasible, culminating in enterprises being unable to devise more satisfactory schedules for collaborative development, which might occasion losses for the enterprise. The fuzzy uncertainty permeating the scheduling process likewise imposes more exacting demands on collaborative development scheduling.

Hwang and Yoon [6] put forward a novel multi-attribute decision-making method known as the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). Since its introduction, TOPSIS has been extensively utilized to tackle a diverse array of multi-attribute decision-making conundrums in real-world scenarios. Owing to its succinct procedural steps and inherent logical rationality, it has maintained its standing as a leading multi-attribute decision-making paradigm. Behzadian et al. [7] comprehensively reviewed the application domains of the TOPSIS method, spanning supply chain management, production systems, market management, environmental management, human resources management, and numerous other sectors where the presence of TOPSIS is conspicuous. The fuzzy TOPSIS method has witnessed rapid proliferation, with a multitude of TOPSIS variants predicated on different fuzzy variables being proposed and deployed within assorted fuzzy decision-making contexts. Chen [8] devised a vertex approach to compute the distance between triangular fuzzy

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numbers and thereby extended the TOPSIS method to accommodate fuzzy environments. Zhang et al. [9] introduced the TOPSIS method under Pythagorean fuzzy sets, leveraging score functions and novel distance metrics. With the continuous evolution of TOPSIS methods in fuzzy environments, fuzzy TOPSIS methods have emerged as an efficacious and crucial means of resolving fuzzy multi-attribute decision-making issues.

In the planning stage of collaborative product development projects, a rational scheduling plan plays a significant role in curtailing project cycles and enhancing product quality [10]. Regarding the scheduling of collaborative development tasks, remarkable research achievements have been made both at home and abroad. Chen et al. [11] put forward a framework for task scheduling and change management in product collaborative development predicated on design structure matrices. Bao et al. [12] furnished the definitions of task fitness and task coordination efficiency, along with their respective calculation methods. On this foundation, a multi-objective optimization mathematical model for task allocation in product customization collaborative development was devised, and a dual-population adaptive genetic algorithm for task allocation in product customization collaborative development was introduced to tackle the model.

Zhang et al. [5] conducted an exploration of human behavior within collaborative development processes via agent-based simulation. Their focus primarily centered on the task planning behavior of designers and the resource conflict resolution behavior of managers. In this regard, they devised a collective utility function and a benefit strategy, aiming to curtail project development time and costs. Li et al. [10] incorporated the matching degree and resource category into the collaborative development project scheduling model. Subsequently, they formulated two static scheduling models to ascertain the scheduling scheme with the shortest development cycle. Moreover, they designed a simple genetic algorithm and a double-layer parthenogenetic algorithm respectively to address these models. Virtually all of the aforementioned collaborative development scheduling models were formulated within a deterministic environment. Through the solution of these models, a relatively optimal scheduling solution could be attained. Nevertheless, the fuzzy uncertainty inherent in the scheduling process was overlooked. They failed to recognize that when experts appraise the attribute values of diverse alternative solutions, the resultant evaluation values should be fuzzy rather than precise. This kind of fuzzy uncertainty constituted an important characteristic of scheduling itself, and the disregard of fuzzy uncertainty precluded enterprises from conducting a more comprehensive evaluation of alternative scheduling schemes. Sadeghi et al. [21] grasped the significance of expert judgment and applied the fuzzy sets theory to allocate four principal project objectives, namely time, cost, quality and resource-leveling.

In light of the foregoing discussion, it is necessary to construct a tri-objective collaborative development scheduling model by leveraging improved Fuzzy TOPSIS under constrained conditions. Our research has primarily focused on two key aspects. Firstly, we extend the collaborative development scheduling model to accommodate fuzzy environments. In contrast to precise numerical values, we employ language variables that align more closely with human judgment to assess the efficiency of enterprise task completion and the degree of enterprise collaboration. Triangular fuzzy numbers are then utilized to represent these language variables, thereby providing a more nuanced and realistic portrayal. Secondly, we establish a tri-objective fuzzy collaborative development scheduling model within a multi-constrained milieu. Specifically, we formulate a fuzzy collaborative development scheduling model subject to time and cost limitations, with development time, development cost, and product quality serving as the foundational pillars. In the context of the NSGA-II solving algorithm, with regard to constraints, a penalty mechanism is implemented to deal with individuals that fail to meet the stipulated constraints during the evolutionary process. This ensures that only those individuals conforming to the requirements are retained throughout the evolution. After deriving the Pareto optimal solution set, the Fuzzy TOPSIS CRITIC method is employed to rank and select the optimal subset. Finally, the efficacy of the model and algorithm was corroborated through case analysis, and sensitivity analysis was carried out on relevant parameters to further explore their impact and variability.

The rest of this paper is organized as follows. In Section II, the fundamental theories and relevant definitions are introduced. In Section III, a tri-objective collaborative development scheduling model is constructed by means of the improved Fuzzy TOPSIS. Section IV presents the Non-Dominated Sorting Genetic Algorithm II (NSGA-II) for solving the proposed model. In Section V, a numerical example is provided to validate the efficacy and practicality of the model. Concurrently, a detailed analysis of the specific parameters is conducted. Section VI serves as a summary of the work accomplished in this paper.

II. Preliminaries

A. Triangular Fuzzy Number

Definition 1 [13] If X is a domain, then the mapping

$$\mu_{\tilde{A}}(x): X \rightarrow [0,1], \tag{1}$$

defines a fuzzy set $\tilde{A}$ on X. The mapping $\mu_{\tilde{A}}$ is called the membership function of $\tilde{A}$.

Definition 2 [13] A fuzzy set $\tilde{A}$ in the domain $x_1, x_2 \in X$ is defined as convex if and only if for all $x_1, x_2 \in X$, we have

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \mathrm{Min}(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)). \tag{2}$$

where $\lambda \in [0,1]$.

Definition 3 [13] A fuzzy set $\tilde{A}$ in the domain $\exists x_i \in X,\ \mu_{\tilde{A}}(x_i) = 1$ is called normal, which means $\exists x_i \in X,\ \mu_{\tilde{A}}(x_i) = 1$.

Definition 4 [14] The α cut set of fuzzy number $\tilde{A}$ is defined as

$$\tilde{A}^{\alpha} = \{x_i : \mu_{\tilde{A}} \geq \alpha, x_i \in X\}. \tag{3}$$

Where $\alpha \in [0,1]$, $\tilde{A}^{\alpha}$ is a non-empty bounded closed interval on X, which can be denoted as $\tilde{A}^{\alpha} = \left[A_l^{\alpha}, A_u^{\alpha}\right]$.

Definition 5 [14] Triangular fuzzy number n can be defined as a triplet (n_1, n_2, n_3), with its membership function defined as

$$\mu_{\tilde{n}}(x)=\begin{cases} 0, & x<n_1; \\ \dfrac{x-n_1}{n_2-n_1}, & n_1 \le x \le n_2; \\ \dfrac{x-n_3}{n_2-n_3}, & n_2 \le x \le n_3; \\ 0, & x>n_3. \end{cases} \tag{4}$$

Definition 6 [15] If n is a fuzzy number and $\tilde{m}^{\alpha}>0, \alpha \in [0,1]$, then n is called a positive fuzzy number.

Definition 7 [14] Given two positive triangular fuzzy numbers $\tilde{m}=(m_1,m_2,m_3)$ and $\tilde{n}=(n_1,n_2,n_3)$, define the operation

(1) $\tilde{m}(+)\tilde{n}=(m_1+n_1, m_2+n_2, m_3+n_3)$; (5)

(2) $\tilde{m}(-)\tilde{n}=(m_1-n_1, m_2-n_2, m_3-n_3)$; (6)

(3) $\tilde{m}(\cdot)\tilde{n}=(m_1\cdot n_1, m_2\cdot n_2, m_3\cdot n_3)$; (7)

(4) $(\tilde{m})^{-1}=((m_3)^{-1},(m_2)^{-1},(m_1)^{-1})$; (8)

(5) $\tilde{m}(/)\tilde{n}=(m_1/n_3, m_2/n_2, m_3/n_1)$; (9)

(6) $k\tilde{m}=(km_1,km_2,km_3)$. (10)

Definition 8 [16] (Triangular Fuzzy Number Ranking) Given a normal triangular fuzzy number $\tilde{m}=(m_1,m_2,m_3)$, the abscissa of the centroid is $\bar{x}_0(\tilde{m})=\frac{1}{3}(m_1+m_2+m_3)$. For different normal triangular fuzzy numbers, they can be sorted according to $\bar{x}_0(\tilde{m})$, that is, the larger $\bar{x}_0(\tilde{m})$, the larger the corresponding triangular fuzzy number $\tilde{m}$. Furthermore, for a series of triangular fuzzy numbers $\tilde{m}_i=(m_{i1},m_{i2},m_{i3})$, $i=1,2,...,M$, define

$$\tilde{m}_{\max}=\max_i \tilde{m}_i, \tag{11}$$

where $\tilde{m}_{\max}\in\{\tilde{m}_i, i=1,2,...,M\}$, $\bar{x}_0(\tilde{m}_{\max})=\max_i \bar{x}_0(\tilde{m}_i)$.

Definition 9 [8] Given two triangular fuzzy numbers $\tilde{m}=(m_1,m_2,m_3)$ and $\tilde{n}=(n_1,n_2,n_3)$, the distance between them is defined as

$$d(\tilde{m},\tilde{n})=\sqrt{\frac{1}{3}[(m_1-n_1)^2+(m_2-n_2)^2+(m_3-n_3)^2]}. \tag{12}$$

Definition 10 [8] $\tilde{m}$ and $\tilde{n}$ are two triangular fuzzy numbers, then when $d(\tilde{m},\tilde{n})$ gradually approaches 0, the fuzzy number $\tilde{m}$ approaches $\tilde{n}$ closer and closer.

B. Fuzzy TOPSIS

Chen proposed the TOPSIS method in fuzzy environments in [8], and applied the TOPSIS method to fuzzy decision-making, providing a favorable tool for fuzzy multi-attribute decision-making problems. In this section, the specific steps of Chen's fuzzy TOPSIS method will be introduced.

First, we provide the distance measurement between fuzzy numbers. The specific steps of the fuzzy TOPSIS are as follows. A fuzzy multi-attribute decision-making problem is given, that is, the decision matrix $\tilde{D}$ and weight vector $\tilde{W}$ are given

$$\tilde{D}=\begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} \tag{13}$$

and

$$\tilde{W}= \tilde{w}_1 \quad \tilde{w}_2 \quad \cdots \quad \tilde{w}_n . \tag{14}$$

Where $\tilde{x}_{ij}$ $(\forall i, j)$ and $\tilde{w}_j$ ($i=1,2,...,\mathrm{m}, j=1,2,...,n$) are triangular fuzzy numbers, which can be represented as $\tilde{x}_{ij}=(a_{ij},b_{ij},c_{ij})$ and $\tilde{w}_j=(w_{j1},w_{j2},w_{j3})$.

In order to avoid the complex normalization formula in the traditional TOPSIS, a linear scale transformation method is used to convert attributes of different scales into comparable scales. Therefore, the normalized fuzzy decision matrix can be obtained, denoted as $\tilde{R}=[\tilde{r}_{ij}]_{m\times n}$. Using B and C to represent the benefit attribute set and cost attribute set, respectively, then we obtain

$$\tilde{r}_{ij}=(\frac{a_{ij}}{c_j^*},\frac{b_{ij}}{c_j^*},\frac{c_{ij}}{c_j^*}), \quad j\in B, \tag{15}$$

$$\tilde{r}_{ij}=(\frac{a_j^-}{c_{ij}},\frac{a_j^-}{b_{ij}},\frac{a_j^-}{a_{ij}}), \quad j\in C. \tag{16}$$

Where $c_j^*=\max_i c_{ij}, j\in B$ and $a_j^-=\max_i a_{ij}, j\in C$. The normalization method mentioned above limits the range of normalized triangular fuzzy numbers to [0,1]. Considering the different importance of different attributes, a weighted normalized fuzzy decision matrix is established

$$\tilde{V}=[\tilde{v}_{ij}]_{m\times n},\ \tilde{v}_{ij}=\tilde{r}_{ij}(\cdot)\tilde{w}_j,\ (i=1,2,...,m,\ j=1,2,...,n). \tag{17}$$

In the weighted normalized fuzzy matrix, each element is a normalized positive triangular fuzzy number, and their value range is within the closed interval [0,1]. Therefore, fuzzy positive ideal solution (FPIS) A^* and fuzzy negative ideal solution (FNIS) A^- can be defined as

$$A^*=(\tilde{v}_1^*,\tilde{v}_2^*,...,\tilde{v}_n^*) \tag{18}$$

and

$$A^-=(\tilde{v}_1^-,\tilde{v}_2^-,...,\tilde{v}_n^-). \tag{19}$$

Where $\tilde{v}_j^*=(1,1,1),\ \tilde{v}_j^-=(0,0,0)\ (j=1,2,...,n)$. The distance of positive and negative ideal solutions for each alternative is

$$d_i^*=\sum_{j=1}^{n} d(\tilde{v}_{ij},\tilde{v}_j^*),\ i=1,2,...,m \tag{20}$$

and

$$d_i^-=\sum_{j=1}^{n} d(\tilde{v}_{ij},\tilde{v}_j^-),\ i=1,2,...,m. \tag{21}$$

Where $d(\cdot,\cdot)$ represents the distance measurement of two fuzzy numbers.

After calculating d_i^* and d_i^- for each alternative $A_i (i=1,2,...,m)$, rank each alternative by defining a consistent coefficient by

$$CC_i=\frac{d_i^-}{d_i^*+d_i^-},\ i=1,2,...,m. \tag{22}$$

Obviously, as the consistent coefficient of the alternative tends to 1, it is closer to the positive ideal solution (A^*) and further away from the negative ideal solution (A^-). Furthermore, all alternative solutions are ranked by the consistent coefficient. The higher the consistent coefficient of the alternative solutions, the higher the ranking of the alternative solutions.

C. Fuzzy CRITIC

This section introduces the fuzzy CRITIC method put forth by Rostamzadeh et al. [17]. In the context of multi-attribute decision-making problems, attributes inherently can be regarded as a source of information, and objective weights are capable of reflecting the quantum of information contained within. CRITIC is a methodology employed for computing the objective weights of diverse attributes in multi-attribute decision-making scenarios [18]. The weights derived through this approach not only take into consideration the relative magnitude of each attribute but also factor in the conflicts among attributes. The magnitude of attribute comparison is gauged by means of the standard deviation, whereas the conflict between them is measured by the correlation coefficient. Rostamzadeh et al. [17] extended the CRITIC method to fuzzy environments and introduced the fuzzy CRITIC method. The specific computational steps of this method are delineated as follows.

In fuzzy multi-attribute decision-making problems, given the decision matrix $\tilde{D}$ and weight vector $\tilde{W}^o$, we have

$$\tilde{D}=\begin{bmatrix}\tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n}\\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn}\end{bmatrix} \tag{23}$$

and

$$\tilde{W}^o=\begin{bmatrix}\tilde{w}_1^o & \tilde{w}_2^o & \cdots & \tilde{w}_n^o\end{bmatrix}. \tag{24}$$

Where $\tilde{x}_{ij}\,(\forall i,j)$ and $\tilde{w}_j^o\ (j=1,2,...,n)$ are triangular fuzzy numbers, which can be represented as $\tilde{x}_{ij}=(x_{ij1},x_{ij2},x_{ij3})$ and $\tilde{w}_j^o=(w_{j1}^o,w_{j2}^o,w_{j3}^o)$. Use B to represent the benefit type attribute set and C to represent the cost type attribute set.

Step 1: Calculate each decision value after transformation and obtain the attribute vector

$$x_{ijk}^T=\begin{cases}\dfrac{x_{ijk}-x_{jk}^-}{x_{jk}^*-x_{jk}^-}, & \text{if } j\in B\\[2ex] \dfrac{x_{jk}^*-x_{ijk}}{x_{jk}^*-x_{jk}^-}, & \text{if } j\in C\end{cases} \tag{25}$$

and

$$\mathrm{x}_{jk}=(x_{1jk}^T,x_{2jk}^T,...,x_{njk}^T). \tag{26}$$

Where x_{ijk}^T is the change value of the k-th number $\tilde{x}_{ij}$ ($k=1,2,3$), x_{jk} represents the k-th vector of the j-th attribute, and $x_{jk}^*=\max_i x_{ijk}$, $x_{jk}^-=\min_i x_{ijk}$.

Step 2: Calculate the standard deviation σ_{jk} of each vector x_{jk} .

Step 3: Establish three symmetric matrices $[r_{jj'}^k]_{n\times n}$ $(k=1,2,3)$, where $r_{jj'}^k$ is the correlation coefficient between vector x_{jk} and vector $\mathrm{x}_{j'k}$.

Step 4: Calculate the information measurement for each attribute, i.e.

$$H_{jk}=\sigma_{jk}\sum_{j'=1}^{n}(1-r_{jj'}^k). \tag{27}$$

Step 5: Determine the unordered objective weight as

$$w'_{jk}=\frac{H_{jk}}{\sum_{j'=1}^{n}H_{j'k}}. \tag{28}$$

Step 6: Determine the fuzzy weights of each attribute by

$$\begin{aligned} &w_{jk}^o=w'_{jk'}\ ,\ k,k'\in\{1,2,3\}\\ &w_{j3}^o=\max_k w'_{jk},\ w_{j1}^o=\min_k w'_{jk}.\end{aligned} \tag{29}$$

The weight vector obtained from the fuzzy weight recombination of each attribute is $\tilde{W}^o=\begin{bmatrix}\tilde{w}_1^o & \tilde{w}_2^o & \cdots & \tilde{w}_n^o\end{bmatrix}$, which is the objective weight vector of the studied problem.

III. A Tri-objective Collaborative Development Scheduling Model Based on Improved Fuzzy TOPSIS with Constraints

A fuzzy collaborative development scheduling model was formulated with the objectives of minimizing development time and development cost, while maximizing product quality, under multiple constraints. This formulation took into account the finite development time and restricted development cost specific to the scenarios of product development. The model was addressed through the application of the NSGA-II and the enhanced fuzzy TOPSIS method. Enterprise A has a plan to develop a novel product, which comprises several subtasks. Enterprise A intends to jointly accomplish these development tasks with its suppliers. In order to economize on development time and costs, as well as enhance product quality, it is imperative to allocate the tasks in a rational manner among each participating enterprise.

Assuming that the development team has a total of m enterprises, $S=\{S_1,S_2,...,S_m\}$ represents the set of enterprises. The development project has n sub tasks, $P=\{P_1,P_2,...,P_n\}$ represents the set of tasks. Now it is necessary to assign each task in P to the enterprise in S, that is, to determine a mapping from P to S, denoted as

$$\mathrm{B}:P\rightarrow S. \tag{30}$$

Enterprise A does not merely regard project development time and costs as the reference factors for task scheduling. Instead, it also places significant emphasis on the differences in the expected product quality under varying scheduling scenarios. The enterprise harbors the aspiration of identifying superior scheduling plans to elevate the product quality. Additionally, for the timely launch of the product, the development duration of the new product ought not to be excessively protracted. To safeguard the enterprise's competitiveness, the development cost should be kept in check and not allowed to escalate to an exorbitant level.

Consequently, our objective is to point a scheduling scheme B that not only minimizes the project development time and development costs but also maximizes the product quality. This should be achieved while ensuring that both the development time and costs remain within the specified limits.

A. Objective Functions

1) Development Time

The time required to complete each task can be divided into two parts. For task *j*, the first part is the time spent on completing the task itself, which is denoted by $T_{j(Pdvp)}$, and the second part is the additional time spent on information exchange with other tasks, which is denoted by $T_{j(Pcol)}$. Then the total time spent on task *j* is

$$T_{j(P)} = T_{j(Pdvp)} + T_{j(Pcol)}. \tag{31}$$

Under scheduling B, the time spent by Enterprise *i* is

$$T_i = T_{i(dvp)} + T_{i(col)},\ i = 0,1,...,m. \tag{32}$$

Where $T_{i(dvp)}$ indicates the total time spent by Enterprise *i* on completing the tasks it is responsible for under scheduling B and $T_{i(col)}$ indicates the total additional time spent by Enterprise *i* on the interaction information between its responsible task and other tasks under scheduling B.

We use $[b_j]_{1\times n}$ to represent the task complexity vector and b_j is the complexity of task *j*; $[d_{j,l}]_{n\times n}$ represents the task dependency matrix and $d_{j,l}$ is the dependency between task *j* and task *l*; $[\tilde{\lambda}_{i,j}]_{m\times n}$ represents the efficiency matrix for completing enterprise tasks after conversion, and $[\tilde{\lambda}_{i,j}]_{m\times n}$ is a triangular fuzzy number; $[\tilde{\mu}_{i,l}]_{m\times m}$ represents the efficiency matrix of enterprise collaboration and $\tilde{\mu}_{i,l}$ is also a triangular fuzzy number.

After determining $[b_j]_{1\times n}$, $[d_{j,l}]_{n\times n}$, $[\tilde{\lambda}_{i,j}]_{m\times n}$ and $[\tilde{\mu}_{i,l}]_{m\times m}$, it is possible to calculate the time spent by each enterprise in the development process under scheduling B. The time spent on completing task *j* itself is the ratio of task *j*'s complexity to the completion efficiency of the corresponding enterprise, i.e. $b_j (/) \tilde{\lambda}_{ij}$. The time spent by Enterprise *i* on the task itself is

$$\tilde{T}_{i(dvp)} = \sum_{\mathrm{B}(P_j)=S_i} b_j (/) \tilde{\lambda}_{ij}. \tag{33}$$

The ratio of collaborative efficiency between two tasks corresponding to the enterprise is $d_{j,l} (/) \tilde{\mu}_{i,\mathrm{A}(P_l)}$, and then the time spent by Enterprise *i* on task collaboration is

$$\tilde{T}_{i(col)} = \sum_{\mathcal{B}(P_j)=S_i} \sum_{l=1}^{n} d_{j,l} (/) \tilde{\mu}_{i,\mathcal{A}(P_l)}. \tag{34}$$

Therefore, the time spent by Enterprise *i* throughout the entire development process is

$$\tilde{T}_i = \sum_{\mathrm{B}(P_j)=S_i} b_j (/) \tilde{\lambda}_{i,j} + \sum_{\mathrm{B}(P_j)=S_i} \sum_{l=1}^{n} d_{j,l} (/) \tilde{\mu}_{i,\mathrm{B}(P_l)}. \tag{35}$$

The total development time of the project can be obtained as

$$\tilde{T} = \max_i (\tilde{T}_i). \tag{36}$$

2) Development Cost

The following is the calculation of the development costs. Through effective communication with diverse suppliers and a comprehensive assessment of the enterprise's own capabilities, the cost that each enterprise needs to incur for the completion of each task can be ascertained. This gives rise to the task cost matrix $[c_{i,j}]_{m\times n}$, where $c_{i,j}$ denotes the cost requisite for Enterprise *i* to execute Task *j*. Consequently, under Scheduling B, the cumulative cost of the entire development process is

$$C = \sum_{j=1}^{n} c_{\mathcal{B}(P_j),P_j}. \tag{37}$$

3) Development Quality

To ensure the timely launch of new products, Enterprise A requires that the average expected development time $\tilde{T}$ of the product should not exceed T_0. As $\tilde{T}$ is a fuzzy number, the centroid mentioned in Definition 9 is used to describe its average level, which requires

$$\overline{x}_0(\tilde{T}) \le T_0. \tag{38}$$

It is worth noting that the calculation formula $\overline{x}_0(\tilde{T})$ for the center of mass, also known as the mean calculation formula for triangular fuzzy numbers under uniform distribution [19], $\overline{x}_0(\tilde{T})$ can also be considered as the mean of $\tilde{T}$. To ensure the competitiveness of new products in the market, the development cost C should not exceed C_0, i.e.

$$C \le C_0. \tag{39}$$

Subsequently, it is essential to compute the anticipated quality of the product under a particular scheduling scenario. Initially, given that the completion quality of diverse tasks exerts different degrees of influence on the quality of the eventual new product, experts are required to appraise the significance of the completion quality of each development task with respect to the quality of the final product. Scoring criteria can be established beforehand, as depicted in Table I.

TABLE I
EVALUATION CRITERIA FOR WEIGHTED LANGUAGE VARIABLES

Importance	Triangular fuzzy number
Very Low	(0.001, 0.001, 0.1)
Low	(0.001, 0.1, 0.3)
Above Low	(0.1, 0.3, 0.5)
Medium	(0.3, 0.5, 0.7)
Below High	(0.5, 0.7, 0.9)
High	(0.7, 0.9, 1.0)
Very High	(0.9, 1.0, 1.0)

TABLE II
EVALUATION CRITERIA FOR WEIGHTED LANGUAGE VARIABLES

Evaluation value	Triangular fuzzy number
Very Poor	(0.001, 0.001, 0.1)
Poor	(0.001, 0.1, 0.3)
Below Average	(0.1, 0.3, 0.5)
Average	(0.3, 0.5, 0.7)
Above Average	(0.5, 0.7, 0.9)
Good	(0.7, 0.9, 1.0)
Very Good	(0.9, 1.0, 1.0)

In accordance with Table I, experts employ the language variables presented in the table to assign ratings to the significance of each task. Upon the completion of the scoring process, we transform the scoring outcomes into triangular fuzzy numbers, thereby deriving the task importance vector $[\tilde{e}_j]_{1\times n}$. $\tilde{e}_j$ indicates the significance of the completion quality of the j-th task in relation to the quality of the final product. Subsequently, experts are required to assess and assign scores to the anticipated quality of completing each task, taking into account the actual circumstances of each enterprise. Scoring criteria can be established in advance, as illustrated in Table II.

In line with the scoring criteria, experts utilize language variables to appraise the quality of task completion for each enterprise. Subsequently, we transform each evaluation result into a fuzzy number according to the standard to obtain the enterprise task completion quality matrix $[\tilde{\nu}_{i,j}]_{m\times n}$, $\tilde{\nu}_{i,j}$ denotes the expected completion quality of the *i*-th enterprise for the *j*-th task. Leveraging the task importance vector $[\tilde{e}_j]_{1\times n}$ and the enterprise task completion quality matrix $[\tilde{\nu}_{i,j}]_{m\times n}$, it becomes feasible to compute the expected new product quality under specific scheduling B conditions

$$\tilde{Q}=\sum_{j=1}^{n}\tilde{e}_j(\cdot)\tilde{\nu}_{\mathrm{B}(P_j),j}. \tag{40}$$

4) Model Building

In summary, the established fuzzy collaborative development scheduling optimization model can be summarized as follows

$$\begin{cases} \min\ \tilde{T}=\max\limits_i\Big(\sum\limits_{\mathrm{B}(P_j)=S_i} b_j(/)\tilde{\lambda}_{i,j}+\sum\limits_{\mathrm{B}(P_j)=S_i}\sum\limits_{l=1}^{n} d_{j,l}(/)\tilde{\mu}_{i,\mathrm{A}(P_l)}\Big) \\ \min\ C=\sum\limits_{j=1}^{n} c_{\mathrm{B}(P_j),P_j} \\ \max\ \tilde{Q}=\sum\limits_{j=1}^{n}\tilde{e}_j(\cdot)\tilde{\nu}_{\mathrm{B}(P_j),j} \\ \text{s.t.}\quad 0\le \overline{x}_0(\tilde{T})\le T_0,\quad 0\le C\le C_0. \end{cases} \tag{41}$$

IV. Model Solution

A. NSGA-II Algorithm

In order to address the model, we initially employ NSGA-II to derive the Pareto optimal solution set. Subsequently, we proceed to select the optimal solution from within this set. The subsequent steps outline the specific procedures of the NSGA-II algorithm.

1) Calculate the target value of the population

There are tri-objective values to calculate here. For each individual in the population, the development time $\tilde{T}$, development cost C , and product quality $\tilde{Q}$ can be calculated according to Equations (38-40). To facilitate the sorting of target values and further facilitate the calculation of individual non-dominated levels, $\tilde{T}$ and $\tilde{Q}$ are transformed into their sorting indicators, namely centroid $\overline{x}_0(\tilde{T})$ and $\overline{x}_0(\tilde{Q})$. Then, $\overline{x}_0(\tilde{T})$, C and $\overline{x}_0(\tilde{Q})$ are used as the three objective values for individuals in the algorithm.

2) Punish individuals without meeting constraints

Due to conditional limitations on development time $\overline{x}_0(\tilde{T})$ and costs C , it is necessary to eliminate individuals who do not meet the conditions. In each iteration, the sum of target values $\overline{x}_0(\tilde{T})$ and C for each individual in the combined population is tested. If $\overline{x}_0(\tilde{T})>T_0$ or $C>C_0$, the individual will be punished, even if all tri-objective values become worse, making it difficult to be selected as the top level in the calculation of non-dominated levels, making it gradually eliminated during the evolution process. This penalty will increase as the number of iterations increases.

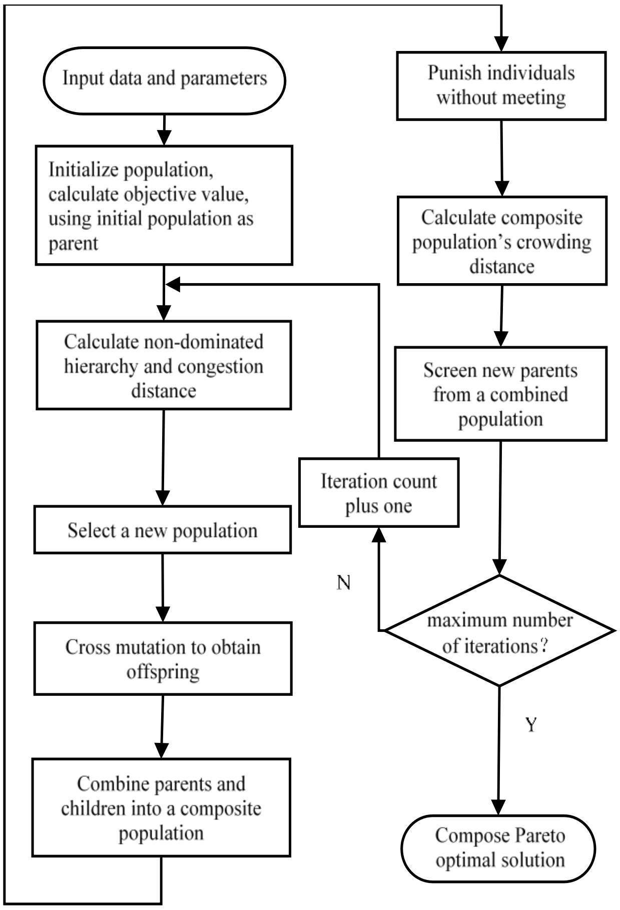

Fig. 1. The flowchart of the NSGA-II algorithm

3) Calculate crowding distance

Since there are currently three objectives and in order to estimate the degree of crowding for each individual at the objective value, the calculation of the individual crowding distance should also incorporate the individual's crowding distance with respect to the product quality objective. First, the individuals at the same non-dominated level within the population are extracted and ranked based on the three objectives. Once the ranking is completed, the calculation method for the crowding distance of individual i is as follows:

$$\begin{aligned} CD_i=&\frac{\overline{x}_0(\tilde{T})_{(i+1)}-\overline{x}_0(\tilde{T})_{(i-1)}}{\overline{x}_0(\tilde{T})_{\max}-\overline{x}_0(\tilde{T})_{\min}}+\frac{C_{(i+1)}-C_{(i-1)}}{C_{\max}-C_{\min}} \\ &+\frac{\overline{x}_0(\tilde{Q})_{(i+1)}-\overline{x}_0(\tilde{Q})_{(i-1)}}{\overline{x}_0(\tilde{Q})_{\max}-\overline{x}_0(\tilde{Q})_{\min}}. \end{aligned} \tag{42}$$

$\overline{x}_0(\tilde{T})_{\max}$ and $\overline{x}_0(\tilde{T})_{\min}$ represent the maximum and minimum values, respectively.

4) Algorithm process

Initially, input the data and set the parameters. Subsequently, initialize the population and compute the objective values of the initial population. Regard the initial population as the parent population, and calculate the non-dominated hierarchy and the crowding distance of the parent population. Thereafter, relying on the non-dominated hierarchy and the crowding distance, obtain a new population through the operations of selection, crossover, and mutation. Then, calculate the objective values for the individuals within the new population. Next, combine the new population and the parent population and record them as a composite population. Inspect the objective values of the composite population, and impose penalties on the individuals that fail to meet the constraints. Subsequently, calculate the non-dominated hierarchy and the crowding distance of the composite population. Finally, sort the combined population in ascending order according to the non-dominated hierarchy first, and then in descending order according to the crowding distance. Subsequently, select individuals from the top downwards, with the number of selected individuals being equal to the size of the parent population.

Subsequently, increment the iteration count by one and ascertain whether the maximum number of iterations has been attained. If it has not been reached, constitute a new parent population from the selected individuals and reiterate the aforementioned operations. Conversely, if the maximum number of iterations has been reached, then terminate the loop and compute the non-dominated level within the selected set of individuals. The individuals at level 1 will constitute the Pareto optimal solution set.

B. Selecting the Optimal Scheduling

We are required to select the optimal solution from the Pareto optimal solution set. Here, the enhanced fuzzy TOPSIS method is employed, specifically the fuzzy TOPSIS CRITIC method put forward by Rostamzadeh et al. [17]. This method enhances the attribute weighting approach in fuzzy multi - attribute decision - making problems. It takes into consideration the internal information of each attribute, presents this information in the form of objective weights, and ultimately combines these objective weights with subjective weights. In this paper, the fuzzy TOPSIS CRITIC method is utilized to optimize the Pareto optimal solution set. Firstly, the objective weights of each objective are computed using the fuzzy CRITIC method and subsequently integrated into the fuzzy TOPSIS method. Finally, the decision matrix, which is based on the diverse objective values of the Pareto optimal solutions, is derived as follows.

$$\tilde{D} = \begin{bmatrix} \tilde{T}_1 & C_1 & \tilde{Q}_1 \\ \tilde{T}_2 & C_2 & \tilde{Q}_2 \\ \vdots & \vdots & \vdots \\ \tilde{T}_m & C_m & \tilde{Q}_m \end{bmatrix}.$$

Where m is the number of Pareto optimal solutions, $\tilde{T}_i = (T_{i1}, T_{i2}, T_{i3})$ and $\tilde{Q}_i = (Q_{i1}, Q_{i2}, Q_{i3})$.

1) Fuzzy CRITIC [17]

First, each objective value is changed into

$$T_{ik}^T = \frac{T_k^* - T_{ik}}{T_k^* - T_k^-},\ C_i^T = \frac{C^* - C_i}{C^* - C^-},\ Q_{ik}^T = \frac{Q_k^* - Q_{ik}}{Q_k^* - Q_k^-}, \tag{43}$$

$$\mathrm{T}_k = (T_{1k}^T, T_{2k}^T, ..., T_{mk}^T), \tag{44}$$

$$\mathrm{C} = (C_1^T, C_2^T, ..., C_m^T), \tag{45}$$

$$\mathrm{Q}_k = (Q_{1k}^T, Q_{2k}^T, ..., Q_{mk}^T). \tag{46}$$

Where $T_k^* = \max_i(T_{ik})$, $T_k^- = \min_i(T_{ik})$, $C^* = \max_i(C_i)$, $C^- = \min_i(C_i)$, $Q_k^* = \max_i(Q_{ik})$ and $Q_k^- = \min_i(Q_{ik})$. Then calculate the standard deviations of T_k, C and Q_k respectively, and record them as σ_{1k}, σ_{2k} and σ_{3k}. Establish three symmetric matrices with a size of 3 * 3, as follows

$$[r_{jj'}^k] = \begin{bmatrix} r(\mathrm{T}_k, \mathrm{T}_k) & r(\mathrm{T}_k, \mathrm{C}) & r(\mathrm{T}_k, \mathrm{Q}_k) \\ & r(\mathrm{C}, \mathrm{C}) & r(\mathrm{C}, \mathrm{Q}_k) \\ & & r(\mathrm{Q}_k, \mathrm{Q}_k) \end{bmatrix}. \tag{47}$$

Where $r(\mathrm{T}_k, \mathrm{C})$ represents the linear correlation coefficient between vectors T_k and C.

Then calculate the information measurement for each attribute, i.e.

$$H_{jk} = \sigma_{jk} \sum_{j'=1}^{n} (1 - r_{jj'}^k). \tag{48}$$

Next, determine the size of the objective weights that have not yet been sorted, i.e.

$$w'_{jk} = \frac{H_{jk}}{\sum_{j'=1}^{n} H_{j'k}}. \tag{49}$$

Finally, calculate the fuzzy weights of each attribute:

$$\begin{gathered} w_{jk}^o = w'_{jk'},\ k, k' \in \{1,2,3\} \\ w_{j3}^o = \max_k w'_{jk},\ w_{j1}^o = \min_k w'_{jk}. \end{gathered} \tag{50}$$

The weight of development time is $\tilde{w}_1^o = (w_{11}^o, w_{12}^o, w_{13}^o)$, the weight of development cost is $\tilde{w}_2^o = (w_{21}^o, w_{22}^o, w_{23}^o)$, and the weight of product quality is $\tilde{w}_3^o = (w_{31}^o, w_{32}^o, w_{33}^o)$. The total objective weight is recorded as $\tilde{W}^o = \begin{bmatrix} \tilde{w}_1^o & \tilde{w}_2^o & \tilde{w}_3^o \end{bmatrix}$.

2) Fuzzy TOPSIS CRITIC [17]

Then, based on the objective weights $\tilde{W}^o = \begin{bmatrix} \tilde{w}_1^o & \tilde{w}_2^o & \tilde{w}_3^o \end{bmatrix}$ calculated above, the fuzzy TOPSIS method is used to sort and optimize the Pareto optimal solution set.

Step 1: Subjective weights $\tilde{W}^s = \begin{bmatrix} \tilde{w}_1^s & \tilde{w}_2^s & \tilde{w}_3^s \end{bmatrix}$ are given by experts, and their weighting criteria are referred to in Table I.

Step 2: Normalize subjective weights according to the following formula:

$$\tilde{w}_j^{sn} = \tilde{w}_j^s / [\overline{x}_0((\overset{3}{\underset{j=1}{(+)}}) \tilde{w}_j^s)]. \tag{51}$$

Step 3: The decision-maker provides the proportion ρ of subjective weight to the combined weight, and combines the subjective weight and objective weight according to this proportion to obtain the combined weight of each attribute

$$\tilde{w}_j = \rho \cdot \tilde{w}_j^{sn} (+) (1 - \rho) \cdot \tilde{w}_j^o. \tag{52}$$

Step 4: Normalize the decision matrix $\tilde{D}$, obtain

$$\tilde{T}n_i=(\frac{T_{\min,1}}{T_{i1}},\frac{T_{\min,1}}{T_{i2}},\frac{T_{\min,1}}{T_{i3}}). \tag{53}$$

$$Cn_i=\frac{C_{\min}}{C_i}. \tag{54}$$

$$\tilde{Q}n_i=(\frac{Q_{i1}}{Q_{\max,3}},\frac{Q_{i2}}{Q_{\max,3}},\frac{Q_{i3}}{Q_{\max,3}}). \tag{55}$$

Where $T_{\min,1}=\min_i T_{i,1}$, $C_{\min}=\min_i C_i$, $Q_{\max,3}=\max_i Q_{i3}$.

Step 5: Weighting the decision information after planning, obtain

$$\tilde{T}w_i=\tilde{T}n_i(\cdot)\tilde{w}_1, \tag{56}$$

$$\tilde{C}w_i=Cn_i(\cdot)\tilde{w}_2, \tag{57}$$

$$\tilde{Q}w_i=\tilde{Q}n_i(\cdot)\tilde{w}_3. \tag{58}$$

Step 6: Determine the positive and negative ideal solutions Z^+ and Z^-,

$$Z^+=(T^+,C^+,Q^+), \tag{59}$$

$$Z^-=(T^-,C^-,Q^-), \tag{60}$$

$$T^+=\max_i Tw_{i3}, C^+=\max_i Cw_{i3}, Q^+=\max_i Qw_{i3}, \tag{61}$$

$$T^+=\min_i Tw_{i1}, C^+\underset{i}{=}\min Cw_{i1}, Q^+=\min_i Qw_{i1}. \tag{62}$$

Where, Tw_{i3} represents the numerical value of the third term $\tilde{T}w_i$, Tw_{i1} represents the numerical value of the first term $\tilde{T}w_i$, and other symbols represent similar values.

Step 7: Calculate the distance from each solution to the positive and negative ideal solutions

$$d_i^+=d(\tilde{T}w_i,T^+)+d(\tilde{C}w_i,C^+)+d(\tilde{Q}w_i,Q^+), \tag{63}$$

$$d_i^-=d(\tilde{T}w_i,T^-)+d(\tilde{C}w_i,C^-)+d(\tilde{Q}w_i,Q^-). \tag{64}$$

Step 8: Calculate the consistent coefficient of each solution

$$CC_i=\frac{d_i^-}{d_i^++d_i^-},\ i=1,2,...,m. \tag{65}$$

Step 9: Rank each feasible solution within the Pareto solution set according to the consistency coefficient. The greater the consistency coefficient, the higher the ranking. Select the solution with the highest ranking as the optimal solution, which represents the optimal scheduling plan.

V. Numerical Analysis

The problem we are dealing with is an adaptation of the collaborative development scheduling problem proposed by Zhang [20]. In contrast to the original problem, our problem exhibits a higher degree of complexity. It encompasses an evaluation of the importance of each task and the anticipated completion quality of each task by the enterprises involved. Additionally, it incorporates constraints regarding development time and costs.

A. Problem Description

An existing enterprise, namely the core enterprise, formulates a plan to develop new products with the aim of further expanding its market share. The development of these new products encompasses a total of 11 subtasks. In order to enhance the development efficiency, the core enterprise has established a collaborative development team consisting of 6 enterprises, with the core enterprise itself being one of them. At present, the core enterprise is tasked with the rational allocation of these 11 tasks among these enterprises. Simultaneously, to guarantee the timely launch of the new products and considering the limited development funds available to the core enterprise, the core enterprise anticipates that the expected development time of the product should not exceed 180 days, and the development cost should not surpass 160,000 yuan.

Through the expert group, the task complexity and mutual task dependencies of these 11 development tasks can be determined, as shown below, where $[d_{j,l}]_{n\times n}$ is the symmetric matrix.

$$[b_j]=[10\quad 9\quad 9\quad 8\quad 8\quad 7\quad 7\quad 5\quad 5\quad 6\quad 6], \tag{66}$$

$$[d_{j,l}]=\begin{bmatrix}0&9&6&5&4&2&1&0&0&0&0\\ &0&8&6&5&5&1&0&0&0&0\\ &&0&2&8&0&0&0&0&0&0\\ &&&0&0&2&1&0&0&0&0\\ &&&&0&5&0&0&0&0&0\\ &&&&&0&0&0&5&4&2\\ &&&&&&0&5&5&2&0\\ &&&&&&&0&2&8&0\\ &&&&&&&&0&5&8\\ &&&&&&&&&0&5\\ &&&&&&&&&&0\end{bmatrix}_{11\times 11} \tag{67}$$

Furthermore, the expert group can rate the task completion efficiency of each enterprise and the collaborative efficiency between enterprises based on the scoring criteria $[b_j]_{1\times n}$ and $[d_{j,l}]_{n\times n}$ shown in Table II. The scoring results are as follows, where $[\tilde{\mu}_{i,l}]_{m\times m}$ is the symmetric matrix, refer to (68), (69).

Simultaneously, the expert group is able to analyze the significance of the task and derive the task importance vector. Through an analysis of the capabilities of each enterprise, an estimated evaluation of the task completion quality can be obtained for each enterprise when it comes to fulfilling different tasks, as in (70) and (71).

$$[b_j]=[\mathrm{H}\quad \mathrm{H}\quad \mathrm{H}\quad \mathrm{BH}\quad \mathrm{BH}\quad \mathrm{BH}\quad \mathrm{BH}\quad \mathrm{M}\quad \mathrm{M}\quad \mathrm{M}\quad \mathrm{M}]. \tag{70}$$

In the end, through the communication and negotiation with diverse supplier enterprises, along with the core enterprise's appraisal of its own task completion costs, the cost requirements of each enterprise for every single task can be determined, see (72).

B. Solution Results

To address the problem, initially, the enhanced NSGA-II is employed to identify the approximate Pareto optimal solution set from among all feasible solutions. Subsequently, the fuzzy Technique for Order Preference by fuzzy TOPSIS CRITIC method is utilized to optimize this Pareto optimal solution set. In this context, MATLAB is used for problem-solving. Likewise, the Pareto optimal solution set obtained by NSGA-II is only an approximation, and the outcomes from each run may vary. In practical applications, multiple runs can be carried out, and the results should be comprehensively taken into account.

1) Pareto optimal solution set

Size of population $NIND=100$, number of iterations

$MAXGEN = 400$, probability of crossover $Pc = 0.9$, and probability of mutation $Pm = 0.1$. Firstly, Figure 2 of the initial population can be obtained, with three coordinate axes representing the individual's three objective values $\overline{x}_0(\tilde{T})$, C and $\overline{x}_0(\tilde{Q})$. In the initial population, the minimum objective value $\overline{x}_0(\tilde{T})$ is 4249, the minimum objective value C is 336300, and the maximum objective value $\overline{x}_0(\tilde{Q})$ is 5.13.

The Pareto optimal solution set distribution obtained by running the program is shown in Figure 3. The Pareto optimal solution set contains a total of 49 Pareto optimal solutions. Among these solutions, the minimum value $\overline{x}_0(\tilde{T})$ is 101.98, the minimum value C is 148870, and the maximum value $\overline{x}_0(\tilde{Q})$ is 6.83, which is greatly improved compared to the initial population. Moreover, in the Pareto optimal solution set obtained, none of the individuals $\overline{x}_0(\tilde{T})$ exceeds 180 and none of the individuals C exceeds 160000.

2) Calculate the final solution

Among the 49 Pareto optimal solutions that have been derived, it is necessary to conduct a further identification of the optimal solution therefrom. Initially, taking into account the company's internal circumstances and its specific requirements for new products, the core enterprise determines the subjective weights for the three objectives of development time, development cost, and product quality. This determination is made by utilizing Table I as the evaluation criterion. The weights assigned to time, cost, and quality are obtained as follows:

$W = [\text{M} \;\; \text{M} \;\; \text{RH}]$.

And provide the proportion $\rho = 0.5$ of subjective weight to combined weight. Subsequently, leveraging the enhanced fuzzy TOPSIS method introduced in this chapter, specifically the fuzzy TOPSIS CRITIC method, the Pareto optimal solutions are ranked. Table III presents the three fuzzy weights corresponding to the three objectives; Table IV illustrates the scheduling arrangements with coefficients ranking among the top ten; Table V furnishes their corresponding development time, centroid values, and development costs; Table VI supplies the product quality, corresponding centroid values, as well as the values of the approximation coefficients. The last row of Tables V and VI shows the mean of each objective value in the Pareto optimal solution set.

Based on Tables V and VI, the highest approximation coefficient is 0.5182. In this instance, the estimated development time is (74.76, 98148.67), with the corresponding centroid of the triangular fuzzy numbers being 107.14, the estimated development cost is 159570 yuan, the expected product quality is (3.87,6.61,9.1), and the corresponding centroid of triangular fuzzy numbers is 6.53. It can be seen that the expected average development time of this schedule is less than 180 days, and the development cost is less than 160000 yuan. Then according to Table IV, the specific scheduling situation for this schedule is as follows: Enterprise 1 completes Task 1, Enterprise 2 completes Task 3 and Task 4, Enterprise 3 completes Task 2 and Task 5, Enterprise 4 completes Task 9 and Task 10, Enterprise 5 completes Task 6 and Task 7, and Enterprise 6 completes Task 8 and Task 11.

Based on Tables V and VI, it is evident that the centroid of the development time for the optimal scheduling is 107.14, which is significantly lower than the average value of 132.07 within the Pareto optimal solution set. The development cost of the optimal scheduling amounts to 159,570, which is marginally higher than the average of 155,430. Moreover, the product quality of the optimal scheduling stands at 6.53, being slightly higher than the average of 6.51. Through the computation utilizing the fuzzy TOPSIS CRITIC method, a scheduling plan was derived that exhibits superior development time and product quality compared to the mean values. However, this was accomplished at the expense of an increase in development costs. This outcome is still deemed acceptable, as when referring to the combined weights of various objectives in Table VI, the weight assigned to development costs is the lowest. This implies that the significance of development costs is the least among these objectives. In other words, enterprises are willing to tolerate a slightly higher expenditure on development costs in order to attain a shorter development time and higher product quality.

C. Sensitivity Analysis

In this section, sensitivity analysis will be conducted on parameters ρ , which refer to the proportion of subjective weights in calculating the combined weights of each objective. The corresponding proportion of objective weights obtained by fuzzy CRITIC is $1-\rho$. In the Pareto optimal solution set, the impact of scheduling ranking under different conditions ρ is analyzed, and the analysis results ρ can be used as further reference for selecting enterprise scheduling plans.

Table VII shows the sensitivity analysis results for ρ . Table VII shows the sorting changes of the top ten scheduling schemes with close coefficient sorting under different conditions ρ . The following sorting is used in the table to represent the schedule $\rho = 0.5$. For example, a value of 3 represents the schedule that ranks third ($\rho = 0.5$), sixth ($\rho = 0.1$), and second among other values ρ .

It can be discerned from Table VII that Schedules 1 and 3 rank in the top three on average except for time $\rho = 0.1$, but fall out of the top three at time $\rho = 0.1$. The underlying rationale for this phenomenon is that, according to the values of subjective and objective weights in Tables VI and VII, the larger the proportion of objective weights to the combined weight, i.e. the smaller the proportion of development time, the greater the proportion ρ . From Table V, it can be deduced that the development time of Schedules 1 and 3 ranks lower among the top ten schedules. When they reach a certain level, they are likely to be abandoned due to too long development time; Table VII illustrates that the ranking of Schedule 4 increases as it decreases. This is because the development time of Schedule 4 is very short, and the other two target values perform well. Consequently, when the weight assigned to the development time increases, the advantages of Schedule 4 become more pronounced. Moreover, it is evident that Schedule 2 demonstrates consistently good performance regardless of the specific values assumed by the relevant factors.

Based on the foregoing analysis, when the requirement for development time is not particularly stringent, either Schedule 1 or Schedule 3 would constitute a favorable option. In the event that the prompt development of the new product is a priority, Schedule 4 can be selected. Additionally, regardless of the circumstances, Schedule 2 remains a viable and commendable choice.

VI. Conclusion

We delved into the realm of fuzzy collaborative development scheduling under multiple constraints and objectives, and formulated a fuzzy collaborative development scheduling model. This model, with finite time and cost in consideration, aims to optimize development time, development cost, and product quality. During the solution process, the NSGA-II is once again employed to initially identify the Pareto optimal solution set. In order to select the optimal solution that adheres to the constraint conditions, a penalty mechanism is implemented during the evolutionary process for individuals that fail to meet the constraints. This mechanism causes the deterioration of their objective values, leading to their elimination from the evolutionary process. Finally, upon obtaining the Pareto optimal solution set, the most recent fuzzy TOPSIS CRITIC algorithm is utilized to derive the final solution. In the empirical analysis, leveraging the proposed model and algorithm, a scheduling scheme that satisfies the constraints while achieving short development time, low cost, and high quality can be obtained.

Due to the fact that intuitionistic fuzzy sets or hesitant fuzzy sets possess the capability to more accurately depict the judgments rendered by individuals in practical scenarios, they can be incorporated into collaborative development scheduling. By integrating these sets with the operational characteristics and ranking methodologies of the corresponding fuzzy sets, the realm of fuzzy collaborative development scheduling can be enhanced. Grounded in diverse practical application contexts, optimization objectives can be devised from the standpoints of boosting collaborative efficiency and elevating the strategic standing of enterprises. Simultaneously, distinct constraints can be established. It should be noted that this article has solely delved into the fuzzy uncertainty within collaborative development scheduling. In forthcoming research endeavors, we may attempt to explore the random uncertainty in scheduling or combine both types of uncertainties to formulate a model that is capable of yielding more rational scheduling outcomes.

$$[\lambda_{i,j}]=\begin{bmatrix} VG & A & A & BA & VP & VP & VP & VP & VP & VP & VP \\ VP & VG & G & AA & A & VP & VP & VP & VP & VP & VP \\ VP & G & VG & VG & AA & VP & VP & VP & VP & VP & VP \\ VP & VP & VP & VP & VP & VG & G & AA & AA & A & VG \\ VP & VP & VP & VP & VP & G & AA & VG & A & AA & G \\ VP & VP & VP & VP & VP & G & VG & G & VG & G & A \end{bmatrix}_{6\times 11}, \tag{68}$$

$$[\mu_{i,l}]=\begin{bmatrix} 1 & A & AA & A & G & AA \\ & 1 & AA & AA & A & AA \\ & & 1 & A & AA & A \\ & & & 1 & AA & A \\ & & & & 1 & A \\ & & & & & 1 \end{bmatrix}. \tag{69}$$

Remark: G=Good, AA=Above Average, A=Average, VP=Very Poor, VG=Very Good, BA=Below Average, H=High, BH=Below High, M=Medium.

$$[\nu_{i,j}]=\begin{bmatrix} G & AA & AA & A & VP & VP & VP & VP & VP & VP & VP \\ VP & G & VG & G & AA & VP & VP & VP & VP & VP & VP \\ VP & VG & AA & AA & AA & VP & VP & VP & VP & VP & VP \\ VP & VP & VP & VP & VP & G & A & VG & AA & AA & AA \\ VP & VP & VP & VP & VP & VG & G & AA & G & VG & A \\ VP & VP & VP & VP & VP & G & AA & AA & A & G & VG \end{bmatrix}. \tag{71}$$

$$[c_{i,j}]=\begin{bmatrix} 21000 & 13230 & 12300 & 9800 & 100000 & 100000 & 100000 & 100000 & 100000 & 100000 & 100000 \\ 100000 & 18500 & 15700 & 13000 & 12900 & 100000 & 100000 & 100000 & 100000 & 100000 & 100000 \\ 100000 & 16320 & 17700 & 17400 & 14500 & 100000 & 100000 & 100000 & 100000 & 100000 & 100000 \\ 100000 & 100000 & 100000 & 100000 & 100000 & 16950 & 14670 & 13000 & 13400 & 11980 & 16480 \\ 100000 & 100000 & 100000 & 100000 & 100000 & 14400 & 12800 & 15430 & 11900 & 13590 & 15000 \\ 100000 & 100000 & 100000 & 100000 & 100000 & 15000 & 16520 & 14000 & 15900 & 15200 & 12470 \end{bmatrix}. \tag{72}$$

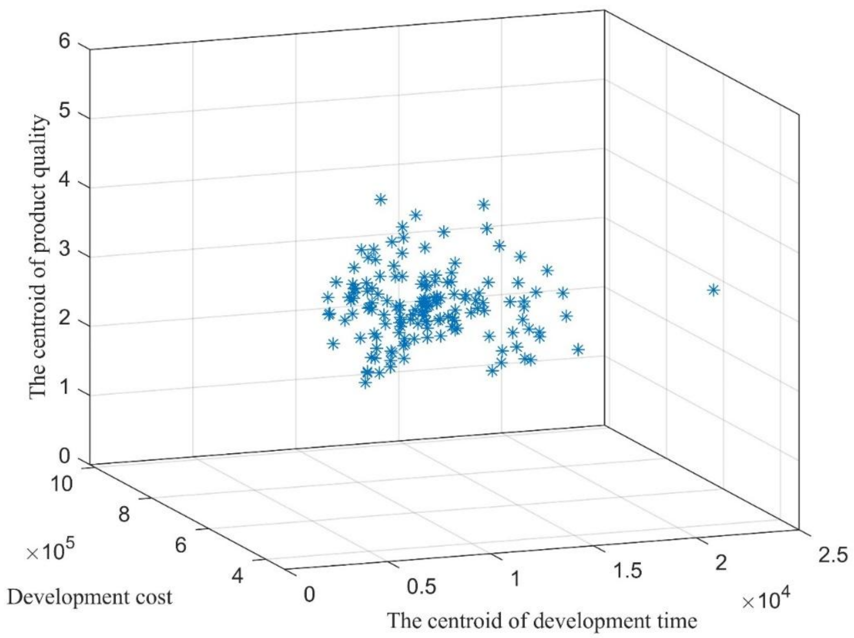

Fig. 2. Initial population distribution.

Remark: $NIND = 100$, $MAXGEN = 400$, $Pc = 0.9$, $Pm = 0.1$. The minimum objective value $\bar{x}_0(\tilde{T})$ is 4249, the minimum objective value C is 336300, and the maximum objective value $\bar{x}_0(\tilde{Q})$ is 5.13.

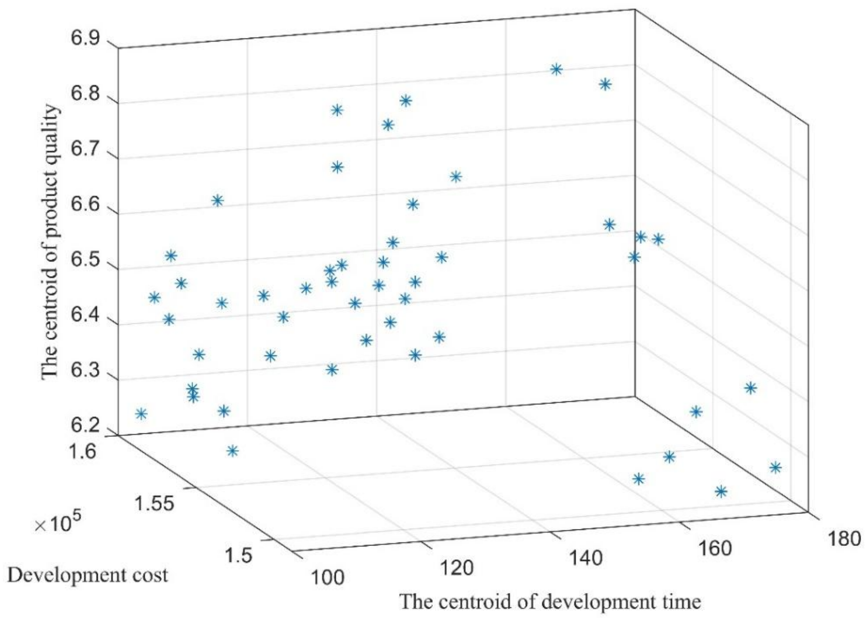

Fig. 3. Pareto optimal solution set distribution diagram.

Remark: The minimum value $\bar{x}_0(\tilde{T})$ is 101.98, the minimum value C is 148870, and the maximum value $\bar{x}_0(\tilde{Q})$ is 6.83, and none of the individuals $\bar{x}_0(\tilde{T})$ exceeds 180 and none of the individuals C exceeds 160000.

TABLE III
FUZZY WEIGHTS OF THREE OBJECTIVES

Weight type	Time weight	Cost weight	Quality weight
Subjective weight (Standardization)	(0.1765,0.2941,0.4118)	(0.1765,0.2941,0.4118)	(0.2941,0.4118,0.5294)
Objective weight (Fuzzy CRITIC)	(0.3393,0.3593,0.3613)	(0.3258,0.3418,0.3422)	(0.2985,0.3129,0.3189)
Combined weight ($\rho = 0.5$)	(0.2579,0.3267,0.3865)	(0.2511,0.318,0.377)	(0.2963,0.3623,0.4241)

TABLE IV
TOP 10 SCHEDULING CONDITIONS

Ranking \ Task	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11
1	1	3	2	2	3	5	5	6	4	4	6
2	1	3	3	2	2	5	4	4	5	4	6
3	1	3	1	2	3	5	5	6	4	4	6
4	1	3	2	2	3	5	4	4	5	4	6
5	1	3	1	2	3	5	5	4	4	6	6
6	1	3	3	2	2	6	5	4	5	4	6
7	1	3	2	2	3	6	5	4	5	4	6
8	1	3	1	2	3	6	5	4	5	4	6
9	1	3	1	2	3	5	6	4	5	4	6
10	1	3	1	2	3	5	5	4	6	4	6

Remark: Enterprise 1 completes Task 1, Enterprise 2 completes Task 3 and Task 4, Enterprise 3 completes Task 2 and Task 5, Enterprise 4 completes Task 9 and Task 10, Enterprise 5 completes Task 6 and Task 7, and Enterprise 6 completes Task 8 and Task 11.

TABLE V
DEVELOPMENT TIME AND DEVELOPMENT COST

Ranking	Development time ($\tilde{T}$)	$\bar{x}_0(\tilde{T})$	Development cost (C)
1	(74.76,98,148.67)	107.14	159570
2	(73.76,94,138.19)	101.98	159340
3	(74.76,98,148.67)	107.14	156170
4	(79.32,97.71,132.19)	103.07	158940
5	(80.08,98.76,138.73)	105.86	158390
6	(82.25,97.86,129.52)	103.21	158070
7	(82.17,102.86,144.19)	109.74	157670
8	(82.17,102.86,144.19)	109.74	154270
9	(80.19,99.08,139.3)	106.19	157390
10	(80.08,98.76,138.73)	105.86	157670
Mean value	—	132.07	155430

TABLE VI
PRODUCT QUALITY AND CONSISTENT COEFFICIENT

Ranking	Product quality ($\tilde{Q}$)	$\bar{x}_0(\tilde{Q})$	Consistent coefficient (CC)
1	(3.87,6.61,9.1)	6.53	0.5182
2	(3.57,6.31,8.87)	6.25	0.5169
3	(3.59,6.34,9)	6.31	0.5168
4	(3.85,6.58,8.97)	6.47	0.5166
5	(3.77,6.59,9.14)	6.5	0.5149
6	(3.67,6.52,9.14)	6.44	0.5149
7	(3.95,6.79,9.24)	6.66	0.5141
8	(3.67,6.52,9.14)	6.44	0.5129
9	(3.67,6.45,9.05)	6.39	0.5124
10	(3.65,6.39,8.93)	6.32	0.5101
Mean value	—	6.51	—

TABLE VII
SENSITIVITY ANALYSIS OF PARAMETER ρ

Ranking	$\rho=0.5$	$\rho=0.1$	$\rho=0.3$	$\rho=0.7$	$\rho=0.9$
1	1	4	1	1	1
2	2	2	3	3	3
3	3	5	4	2	2
4	4	1	2	4	4
5	5	6	6	5	6
6	6	3	5	7	7
7	7	8	7	6	5
8	8	7	8	8	8
9	9	9	9	9	9
10	10	10	10	10	13

Remark: A value of 3 represents the schedule that ranks third $\rho=0.5$, sixth $\rho=0.1$, and second among other values ρ.

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