The Metro Operation Plan with Overtaking and Passenger Travel Options

Yi Xie, Yan Liu, Ru Niu, Lanxue Dang, Peng Cao, Shuanglei Yang, Yan Zou, Jiping Zheng, Baojun Qiao

***Abstract*—Metro operation planning stands a pivotal concern in urban rail transit operation, significantly influencing service quality and operational costs. The complexity of planning is further amplified in urban rail transit lines that bridge cities with their suburbs and cater to long-distance passengers. To optimize metro operation planning, this paper introduces a hybrid algorithm that integrates the improved simulated annealing algorithm with the MSA algorithm, and it constructs a comprehensive model that optimizes the combined operation of express and local trains, alongside full-length and short-turn routing. This model closely mirrors real-world scenarios by considering passengers' optimal route choices and the overtaking of express trains. The algorithm's primary objective is to minimize both passengers' travel time and trains' running time, thereby reducing travel costs for suburban passengers and operational costs for metro companies. The efficacy of this algorithm has been validated through testing on both simulation data and the actual data from Zhengzhou Metro Line 2 in China.**

***Index Terms*—Urban rail transit, Express and local train, Full-length and short-turn routing, hybrid algorithm**

I. Introduction

WITH economic development and urban expansion, urban rail transit systems are continuously extending outward from city centers, creating a regional transit network that connects central and suburban areas. In the regional transit system, suburban passengers are dispersed and typically undertake longer journeys, whereas urban passengers are concentrated and usually have shorter trips. Given these characteristics, it is necessary to design an operation plan that balances safety, service quality, cost and other pertinent objectives.

The prevalence of extended operation distances and imbalanced passenger flow distribution in regional transit systems results in the inadequacy of the traditional full-length routing mode in solving the Metro Train Operation Problem (MTOP). Current research has proposed several methods to optimize regional transit operation plans, but there are still certain shortcomings. Firstly, existing research seldom takes into account passengers' travel choices comprehensively, rendering the optimization results difficult to implement in real-world scenarios. Secondly, operating single express-local train or employing full-length and short-turn routing may lead to increased passenger waiting times or transfer times. Thirdly, despite some urban rail lines incorporating skip-stop station to allow express trains to overtake local trains, there is scant research on overtaking stations. Therefore, devising methods to enhance the efficiency of train operation plans in regional transit represents both the focus and challenge of MTOP research.

To address the above problems, this paper proposes a hybrid optimization approach for urban rail transit operation planning that considers passenger travel preferences and overtaking mechanisms. It formulates an operation plan for the combined operation of express and local trains, alongside full-length and short-turn routing, while adhering to constraints such as travel intervals, turnaround times and full-load ratio. The primary objective of this optimization is to minimize both passenger travel costs and enterprise operational costs. A comparison with the real-world scenarios reveals that, during morning peak hours, train operation time and total passenger travel time can be reduced by 48.1% and 3.9% respectively, while during evening peak hours, these reductions are 43.2% and 3.55% respectively.

The paper is organized as follows. Section II reviews relevant literature on express-local train operations. Section III elaborates on the problem model. Section IV introduces the solution model, which integrates Simulated Annealing (SA) and Modified Simulated Annealing (MSA). Section V presents simulation tests and a real-case study to assess the effectiveness of the proposed approach. Finally, Section VI summarizes the key findings of this study and offers recommendations for future research endeavors.

II. Literature Review

The majority of current research on urban rail lines has centered on express-local train operation modes. Various

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TABLE I
COMPARISON OF RELEVANT STUDIES

References	PFD	D	Skip-stop plan	PT	TC	Objective	Algorithm
Wang et al.[1]	Off-peak	Bi	dynamic	No	No	PTT+TOC	Bilevel optimization
Niu et al.[2]	Any time	Un	static	No	Yes	SWT	GAMS
Zhang et al.[3]	Off-peak	Bi	FSSS	No	Yes	PTT	GA
Shang et al.[4]	oversaturation	Un	A/B	Yes	Yes	PC+TOC	LR
Sun et al.[5]	Off-peak	Un	A/B	Yes	Yes	PTT+TOC	PSO
Salama et al.[6]	static	Un	A/B	Yes	No	DAR	ENUM
Rajabighamchi et al.[7]	peak	Un	Robust stop-skip	Yes	No	PC	CPLEX
Li et al.[8]	Any time	Un	overtaking	Yes	Yes	PTT	BC
Shang et al.[9]	Morning peak	Un	time	NO	Yes	PTT	GA
Cao et al.[10]	peak	Bi	A/B	No	Yes	PTT+TC	GA+TA
Zhao et al.[11]	saturation	Bi	A/B	No	Yes	WT+TOT	ISA
This paper	any period	Un	A/B+overtake	Yes	Yes	WT+L	SA+MSA

PFD:passenger flow description, D:direction, PT:passenger transfer, TC:train capacity, SWT:sum waiting time, Bi:bi-directional, Un:unidirectional, TOT:train operation time, WT:wait time, PTT:passenger travel time, PC:passenger costs, TOC:train operation costs, DAR:direct arrival rate

scholars have extensively examined these modes in relation to different operating hours, environments and conditions.

Depending on different operation periods, express-local train plans can be categorized into peak and off-peak periods. Meanwhile, Zhang et al.[3] and Cao et al.[12] focused on the optimization method during off-peak periods and explored the application of Time Varying Model Predictive Control (MPC) in subway train scheduling and passenger flow management. Shang et al. [4], Rajabighamchi et al. [7] designed skip-stop plans to address high passengers demand during peak periods.

The two primary operation modes of express-local train operation plans are the A/B mode and the dynamic skip-stop mode. In the A/B mode, stations are classified as A and AB types, while trains are designated as A and B types. Specifically, Type A trains stop only at A and AB stations, and type B trains stop exclusively at AB stations. Niu et al.[2] optimized train timetables using a predetermined train skip-stop pattern, establishing a nonlinear integer programming model to minimize total passenger waiting time. Salama et al.[6] employed three scenarios—"pairing", "alternating", and "free assignment"—in their model to generate optimal stopping points for each scenario, ensuring the transfer did not increase travel costs while maintaining passenger travel time. Cao et al.[12] constructed a multi-objective optimization function focused on minimizing total passenger waiting time, travel time and train operation time. They designed an estimation method for train skip-stops that increased some passengers' waiting time to further decrease total travel time. Zheng et al.[13] formulated a 0-1 integer programming model aiming at minimizing passenger travel time, revealing that the A-AB-A station combination can significantly reduce total passenger travel time. Lee et al.[14] used a genetic algorithm to plan train skip-stops, reducing the total passenger travel time by 17-20%. Cao et al.[15] incorporated factors such as passenger flow congestion, train selection preference and train acceleration/deceleration processes into their model, adding train stopping operations to the passenger perception time model to better align with passenger needs.

In the dynamic skip-stop mode, trains' stopping stations are not fixed, allowing for greater adaptability to spatial and temporal changes in passenger flow. Wang et al.[1] developed a mixed integer linear programming model optimizing passenger travel time and energy consumption. They used a two-level optimization approach to obtain the optimal train skip-stop solution in a shorter time. Zhang et al.[3] designed a Dynamic Station Skipping (FSSS) model aimed at minimizing total passenger travel time while maintaining a stable number of skip-stops. This model accounts for scenarios where passengers board the wrong train, reflecting passenger flow patterns during off-peak periods. Rajabighamchi et al. [7] applied a simulated annealing algorithm to find the optimal skip-stop plan, minimizing total passenger travel time.

Some scholars have investigated the implementation of express-local train operation modes in unique circumstances. For example, during train operation disruptions, the temporary deployment of express-local trains during recovery phase accelerates passenger circulation and minimizes the accumulation of stranded passengers. Gao et al.[16] proposed a rescheduling optimization plan based on train skip stop strategy. Altazin et al.[17] generated express-local train operation plans aimed at minimizing both train recovery time and passenger waiting time.

Some researchers contend that introducing two or more routing modes on a single line can effectively address the imbalance in passenger flow distribution among different sections. Shi et al.[18] and Bai et al.[19] formulated optimization models for the operation plans of full-length and short-turn services. Tang et al.[20] established a two-tier planning framework for the operation of multi-routing express and local trains on urban rail lines.

Table I provides a comparative analysis of relevant studies, including the present one.

III. PROBLEM

A. Problem Description

In regional transit commutes, passengers often encounter extended travel distances and high passenger volumes along specific routes—challenges that a single train operating mode cannot effectively accommodate. Conversely, an express train skip-stop operation plan, tailored through passenger flow analysis, can significantly reduce passengers' travel time. In addition, when passenger flow is unevenly distributed across a section, incorporating additional short-turn routing operation modes can further reduce passenger waiting time and alleviate train congestion. Therefore, the service quality of urban rail transit systems can be enhanced by integrating full-length and short-turn routing with express-local trains when scheduling timetables.

When optimizing the train operation plan, this study takes into account both express-train overtaking and passenger flow assignment. When an express train overtakes a local train, the local train is required to halt at the auxiliary line of the station, while the express train proceeds via the main line. Regarding the passenger flow assignment problem, for a passenger flow $q_{i,j}^{t}$ originating and terminating at the same OD pair $c=\{i,j\}(i<j, \mathrm{i,j} \in S^{u} \mid \mathrm{i,j} \in S^{d})$ at time t, several travel options are available based on the type of OD pair, as shown in Fig. 1. Notably, station A serves as a halt for local trains, $\mathrm{S_A}$ is a collection of local stations; AB is a station accessible to both express and local trains; and $\mathrm{S_{AB}}$ is a collection of both express and local trains.

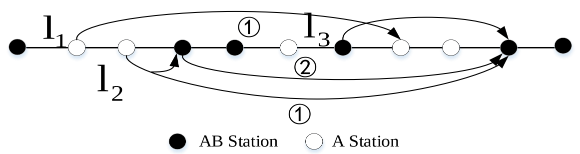


Fig. 1. Passenger Travel Behavior.

If a passenger's journey begins and ends at station A, i.e. $\mathrm{i,j} \in \mathrm{S_A}$, then $\mathrm{q_{i,j}^{t}}$ will take the slow train for the entire trip, as shown in scenario 1 on line $\mathrm{l_1}$. When a passenger departs from station A and is headed to station AB, i.e. $\mathrm{i} \in \mathrm{S_A}, \mathrm{j} \in \mathrm{S_{AB}}$ and $\mathrm{q_{i,j}^{t}}$ has the option to either take the local train throughout or transfer to an express train from the same platform as the local train, as shown in paths 1 and 2 on line $\mathrm{l_2}$. In this scenario, it is imperative to consider the influence of these two routing options on passenger travel choices. When a passenger's journey begins and ends at station AB, i.e $\mathrm{i,j} \in \mathrm{S_{AB}}$, and $\mathrm{q_{i,j}^{t}}$ can opt for either the local train or the express train for the entire journey, as shown in path $\mathrm{l_3}$.

B. Problem Modeling

In this section, we focus our research on single-directional train planning and develop an optimization model for the combined operation of full-length and short-turn routing, integrating express-local trains.

(1) Parameter table

Table II outlines the mathematical parameters used in this study.

(2) Constraints

The constraints involved in the model can be categorized into three types: route constraints, time constraints and passenger demand constraints.

Route constraints

This study employs the A/B method to designate stations with lower passenger demand along the route as overtaking stations, enhancing operational flexibility, denoted as $\forall \mathrm{i} \in \mathrm{S_A}$, effectively accommodating passenger flow requirements. The departure station $\mathrm{S_0}$ and terminal station $\mathrm{S_N}$ of the line are mandatory stops for local-express trains, following the AB type classification proposed by Vuchic et al.[21], where $\mathrm{S_0}, \mathrm{S_N} \in \mathrm{S_{A,B}}$.

Passengers waiting at stations $\mathrm{j}(\forall \mathrm{j} \in \mathrm{S_{A,B}})$ have the option to board any passing train type, while passengers waiting at stations $\mathrm{i}(\forall \mathrm{i} \in \mathrm{S_A})$ are restricted to boarding only local trains. To mitigate passengers' perceived waiting time, no two adjacent trains are allowed to skip the same station, i.e. $\forall \mathrm{i} \in \mathrm{S^k} \cup \mathrm{S^{k+1}}$.

Equation (1) means that a passenger disembarks at the nearest station AB for the express train.

$$h=\min(l \cap S_{AB}) \tag{1}$$

Train frequency affects the passing capacity of both express and local trains, which in turn impacts the number of stations skipped by express trains. Consequently, when the combined frequency of express and local trains reaches a critical threshold, the number of overstating stations for express trains diminishes to zero, eliminating the express-local train mode on the route, as shown in equation (2).

$$(f_0+f_1) \leq f_{\max} \tag{2}$$

Time constraints

Equation (3) is the train departure interval in the operation cycle.

$$t_{y,k}=\frac{T_i}{f_y} \tag{3}$$

where $\mathrm{t_{y,k}} \leq \mathrm{t_{max}}$. To satisfy passenger demand and ensure adequate line capacity, the train headway within the section should comply with the condition $\mathrm{t_{min}} \leq \mathrm{t_c} \leq \mathrm{t_{max}}, \forall \mathrm{c} \in \mathrm{C}$.

The congestion of trains in a particular section is correlated with the section's passenger flow. Based on the relationship between section passenger flow and train capacity, the degree of congestion can be categorized into three levels: no congestion, moderate congestion and severe congestion.

Passengers experience varying perceptions of time in these three congestion levels. Consequently, moderate and severe crowding coefficients are introduced to account for the additional passenger time perception coefficients caused by congestion, as shown in equation (4).

$$\alpha_c \begin{cases} 0, p \leq f \cdot M \\ \dfrac{p-f \cdot M}{f \cdot M}\beta_1, f \cdot M<p<f \cdot M' \\ \dfrac{M'-M}{M}\beta_1+\dfrac{p-f \cdot M'}{f \cdot M'}\beta_2, f \cdot M'<p \end{cases} \tag{4}$$

Passenger travel time includes train operation time and dwell time, so it is influenced by train congestion. The travel time for passengers on route l=i,j ($\forall \mathrm{i}<\forall \mathrm{j}, \mathrm{i,j} \in \mathrm{S}$) is shown in equation (5).

$$T_l=\sum_{c \in \{i,j\}} T_c^r\,(1+\alpha_c)+\sum_{i=i+1}^{j-1} T_i^w \tag{5}$$

Transfer time includes travel time for the transfer itself and waiting time on the platform. In this paper, transfers happen between express and local trains that share the same platform, rendering the travel time for the transfer negligible (i.e., 0). Hence, the transfer time equals the platform waiting time. Equation (6) represents the waiting time for a passenger who disembarks at station i during shift A and

TABLE II
NOTATIONS OF THE MODEL

Symbol	Description
Set	
S	Collection of all stations
C	Collection of all sections
S^O	Overtaking station collection
S_{AB}	Collection of type-AB stations
S_A	Collection of type-A stations
S^k	The shift k contains the collection of stations
S_B	Overtaking station collection of express trains
K	Collection of all shifts
T	Collection of a-whole-day's operations
Parameters	
p_c	Section passenger flow of section c$c \in [S_0, S_N]$
$p_{i,j}^k$	Passenger flow at station i to station j by shift k,$i,j \in S, k \in K$
$p_i^{d,k}$	Passenger flow of taking shift k to get off at station i
p_i^u	Passenger flow at station i,$i \in S$
p_i^d	Passenger flow of getting off at station i
y	Train type: 1 is local train, 0 is express train
f_y	Departure frequency of train type y
f_{max}	Maximum departure frequency
$T_{k,i}^w$	Dwell time of shift k at station i, $k \in K, i \in S$
$\frac{T_{0,c}^r}{T_{1,c}^r}$	Express/local train's running time in section c,$c \in [S_0, S_N]$
t_c	Headway of section c,$c \in [S_0, S_N]$
$t^{\min}$	Minimum headway of the line
$t^{\max}$	Maximum headway of the line
$\frac{t_{0,k}}{t_{1,k}}$	Headway of shift k,$k \in K$
t^t	Minimum turn back interval of line
T_k^s	Departure time of shift k,$k \in K$
$t_{k,i}^{arr}$	Time of arrival of shift k at station i
$t_{k,i}^{dep}$	Time of departure of shift k from station i
M	Train capacity
M'	Overload number of train
S_0	Starting station of the line
S'_0	Starting station of short-turn routing
S_N	Terminal station of the line
S'_N	Terminal station of short-turn routing
Variables	
γ	If the express train stops at station s, then $\gamma = 1$; otherwise, $\gamma = 0$
μ	Proportional factor of overtaking
H	Threshold of generalized travel cost
$\chi_{i,1}$	Probability of taking local train at station i, $i \in S$
$\chi'_{i,1}$	Probability of taking a local train at station i to transfer to an express train, $i \in S$
$\chi_{i,0}$	Probability of taking express train at station i, $i \in S$
ϕ	Full load factor parameter

waits for shift B to make a same-platform transfer.

$$T_i^w = \frac{1}{2} t_{k_2}, \quad (i \in S_{AB}) \tag{6}$$

When an express train overtakes a local train, the local train is required to enter the overtaking track and wait to avoid the express train, resulting in additional travel time for the passengers on the local train, as illustrated in equation (7).

$$T_l' = \begin{cases} T_l + T_i^w + t_{\min}, \gamma = 1 \\ T_l + t_{\min}, \gamma = 0 \end{cases} \tag{7}$$

The time consumption for passengers selecting various routes during their journey is analyzed as follows. Here, a passenger's departure station is denoted as i and the destination station as j (where i,j $\in$ S), with p_1, p_2, p_3 referring to passengers who choose routes l_1, l_2, l_3 respectively. The running time of trains of the same type within the same section remains consistent.

When a passenger opts to take the local train for the entirety of their journey, equation (8) represents the total waiting time experienced by the passenger, and equation (9) represents the passenger's time spent on board the train.

$$T_1 = \frac{p_1 + p_2 \cdot \chi_{i,1} + p_3 \cdot \chi_{i,1}}{2} f_1 \tag{8}$$

$$T_1^r = \sum_{c=i+1}^{j-1} T_{1,c}^r \tag{9}$$

When a passenger opts to take the express train for the entirety of their journey, equation (10) shows the total waiting time experienced by the passenger, and equation (11) shows the passenger's time on the train.

$$T_2 = \frac{p_3 \cdot \chi_{i,0}}{2} f_0 \tag{10}$$

$$T_2^r = \sum_{c=i+1}^{j-1} T_{0,c}^r \tag{11}$$

When a passenger opts to take the local train to station h and then transfer to an express train, equation (12) represents the passenger's total waiting time, and equation (13) represents the time of the passenger spent on board the train.

$$T_3 = \left(\frac{p_2 \cdot \chi'_{i,1}}{2} + \frac{p_3 \cdot \chi'_{i,1}}{2}\right)(f_0 + f_1) \quad (12)$$

$$T_3^r = \sum_{c=i+1}^{h-1} T_{1,c}^r + \sum_{c=h+1}^{j-1} T_{0,c}^r \quad (13)$$

Passenger demand constraints

The condition for operating express and local trains concurrently is met when the ratio of long-distance to short-distance passenger flow satisfies specific criteria; otherwise, it fails to align with the optimization objectives of the train operation plan. Equation (14) provides the number of passengers boarding and alighting at each station.

$$p_i = p_i^u + p_i^d \quad (14)$$

The model introduces an overtaking proportional factor A. In scenarios where the proportion of boarding and alighting passengers at a station is small enough, the express train opts to bypass the station, prioritizing the travel time of the majority of passengers over the waiting time of a small subset, as illustrated in equation (15).

$$p_i + H \le \mu, \quad i \notin S_{AB}, \forall c \in S \quad (15)$$

When planning express-local train operation, the fundamental premise of optimization is to fulfill passenger flow demand. Consequently, equation (16) imposes constraints on the frequency of trains traversing section c to ensure compliance with the maximum section passenger flow data.

$$p_c \le (f_0 + f_1) \cdot M \cdot \phi \quad (16)$$

This study takes into account passengers' transfer behavior between express and local trains and designs an optimization model based on the premise that passengers can transfer at most once and refrain from boarding trains heading in the opposite direction of their destination $(0 < \{k_1 \cap k_2 \cap ... \cap k_n\} \le 2)$, in order to simplify the complexity of operational organization.

(3) Optimization objectives

This study examines the combined operation of full-length and short-turn routing, as well as express and local trains, within the regional transit system, with the optimization objective of minimizing both passengers' total travel time and the overall train operation time.

Equation (17) delineates that the total train running time is contingent upon the frequency and operational sections of each train type.

$$T_R = f_1\left(\sum_{i=S'_0}^{S'_N-1} T_{k,i}^w + \sum_{c=C'_0}^{C'_N} T_{1,c}^r\right) + f_0\left(\sum_{i=S'_0}^{S'_N-1} T_{k,i}^w + \sum_{c=C'_0}^{C'_N} T_{0,c}^r\right) \quad (17)$$

The interval for shift k is denoted as $[C'_0, C'_N]$, and the corresponding stops are indicated as $[S'_0, S'_N]$. Based on the analysis of passengers' travel routes conducted in the previous section, the total travel time for passengers is presented in equation (18).

$$T_W = T_1^W + T_1^R + T_2^W + T_2^R + T_3^W + T_3^R \quad (18)$$

For the multi-objective comprehensive optimization, equation (19) uses the weighted value composition method to optimize the multi-objective function.

$$\min(\omega_1 T_R + \omega_2 T_W) \quad (19)$$

IV. SOLUTION ALGORITHM

Traditional heuristics improve the quality of the solution through neighborhood operators, and meta-heuristic algorithms can avoid falling into local optima and find near-optimal solutions under certain logic [22]. In this study, the problem is divided into three sub-problems: the optimization of full-length and short-turn routing + express-local train operations, the distribution of passenger flow, and the preparation of timetable. A hybrid algorithm integrating the simulated annealing algorithm (SA) and MSA algorithm is designed to solve these three sub-problems separately.

A. Overall Framework

The overall algorithm framework is described by pseudo code as Algorithm 1.

Algorithm 1 The Algorithm Framework for Train Operation Plan Optimization

Input: Passenger flow data, route data, decision variables
Output: Train operation plan
1 Sort out and analyze the passenger flow data.
2 The combination of full-length and short-turn routing + express-local train is determined according to the temporal and spatial distribution of passenger flow, as shown in Algorithms 2 and 3.
3 Search the neighborhood for the departure frequency of all types of trains, as shown in Algorithm 4.
4 Distribute the passenger flow through MSA, as shown in Algorithm 5.
5 Evaluate the solution quality of train operation plan.
6 Judge whether the algorithm meets the termination conditions. If not, return to step 3. If yes, proceed to the next step.
7 Prepare the train timetable and generate the train diagram based on the current optimal train operation plan in the previous step.

Due to the complexity of the problem's scale, this paper proposes a hybrid algorithm based on SA, which integrates MSA. The SA algorithm offers local search advantages, while the MSA algorithm determines passengers' route choices. This combination enhances the algorithm's effectiveness and accuracy.

B. Optimization of full-length and short-turn routing + express-local train operation plan

The SA algorithm is a global optimization search algorithm rooted in probability. It initiates from a relatively high initial temperature and conducts a neighborhood search within the solution space as the temperature parameter progressively decreases. Upon encountering a local optimal solution, it possesses the capability to probabilistically escape due to its probabilistic sudden jump characteristic, continuing until the global optimal solution for the objective function is identified. In this study, the SA algorithm, enhanced with an elite strategy, is utilized to solve for the optimal train operation plan.

In our algorithm, we scrutinize the number of boarding and alighting passengers at each station and introduce an overtaking proportional factor to address the "free assignment"-based express train operation mode. This approach takes into account the skip-stop issues associated with express trains from both the enterprise's and passengers' perspectives. The algorithm proceeds through the following steps:

Step 1. Initialization: Route, station and passenger flow data are analyzed.

Step 2. Determination of the skip-stop stations: By analyzing the proportion between the number of alighting passengers at each station and the number of overtaking passengers, we obtain a set of stations with high passenger flow as the candidate set for train stopping stations. This step helps to narrow down the initial solution space.

Step 3. Free assignment: The elements within the set S_B, acquired from Step 2, are distributed freely to generate multiple skip-stop station operation plans, thereby forming the solution space.

Step 4. Full-length express train operation program: When $S_B = \varphi$, the passenger flow assignment fails to meet the requirements for both long and short-distance travel characteristics. Consequently, at this point, $f_0 = 0$, only the full-length local train operation plan is considered. When $S_B \neq \varphi$, the temporal and spatial characteristics of passenger flow are analyzed, and an express train operation plan is devised. The pseudo-code for the algorithm is presented in Algorithm 2.

Algorithm 2 Express-local train operation formulate Algorithm

Input: OD matrix,μ

Output: Set of Express-local train operation plans
1 Based on the OD matrix and $p_i = p_i^u + p_i^d$,calculate the passenger flow $p_i, \forall i \in S$ for each station.
2 If p_i/ skip passengers p_{skip} at station i is greater than μ,then $S_s \leftarrow i$.
3 Arrange the full set $S_s$to generate multiple skip-stop train plan.
4 return full-length plan + skip-stop train operation plan.

Step 5. Short-turn routing local train operation program: The balance of a section is assessed based on its passenger flow. A section with a significant passenger flow is selected, taking into account the balance of the train's full-load rate, to formulate a short-turn routing operation plan. If the section passenger flow is balanced, a short-turn routing is deemed unnecessary. The pseudo-code for this algorithm is presented in Algorithm 3.

Algorithm 3 Short-turning train operation formulate Algorithm

Input: section passenger data P,ϕ

Output: Set of Short-turning train operation plans
1 for $i \leftarrow 0$ to N-1
do if $P[i] > f_{max} \bullet M \bullet \phi$
then $S_T \leftarrow i$
2 Arrange the full set $S_T$to generate multiple Short-turning train plan.
3 return short-turning train operation plan.

Step 6. Determination of train frequency: The operation plan generated through Steps 4 and 5 is fed into the simulated annealing algorithm, where a neighborhood search is conducted for train frequencies. This process ultimately outputs the targeted optimal express-local train operation plan. The pseudo-code for this algorithm is shown in Algorithm 4.

Algorithm 4 Improved Simulated Annealing Algorithm
Input: train operation plan
Output: train frequency
1 Set T_0andα.Let $n \leftarrow 0$.
2 Based on the amount of T,computer all protection function amounts for uncertain parameters.
3 Considering the amount of protection function value,compute the $MinR_i, \forall i \in K$,and find an initial solution r by algorithm 1. Let $R_{best} \leftarrow$ rand Go to 4.
4 Generate a neighborhood solution r′,using schedule r by simply changing one of the randomly selected binary values.
5 If $f(r') \leq f(r)$ or random $[0,1] < p$ then $r \leftarrow r'$,If $f(r) > f(E_{best})$,then $E_{best} \leftarrow f(r)$. If $f(r') \leq f(R_{best})$ then $R_{best} \leftarrow r$.
6 If the termination criterion is not satisfied then $T_n = \alpha \times T_{n-1}$,$n \leftarrow n+1$ and Go to 4; otherwise, stop and return $Max\{R_{best}, E_{best}\}$.

The aforementioned steps outline the simulated annealing algorithm based on the elite strategy for solving the train frequency problem. To guarantee the quality of the results, the elite strategy ensures that the best solution that is replaced is preserved within the elite group.

C. Solving passenger flow assignment

Given the variation in passenger comfort and time cost across different travel routes, passengers are inclined to select the most advantageous option. However, as the number of passengers on the most favorable route increases, passenger satisfaction tends to decline. Consequently, the optimal route shifts with changes in passenger assignment, rendering passenger flow assignment a dynamic and stochastic event. The MSA algorithm, a prevalent approach in addressing traffic flow assignment issues, represents a cyclical allocation technique that interweaves incremental and balanced allocation methods. It is also recognized as the quadratic weighted average or iterative weighting method. The underlying principle involves progressively adjusting the flow allocated to each segment until a balanced outcome is achieved. Therefore, this study employs the MSA algorithm to tackle the passenger flow route selection problem. The solution steps are outlined below:

Step 1 Initialization: Passengers are categorized based on a common origin-destination (OD) trip.

Step 2 Line impedance function: The travel impedance for the same route varies among different passengers. Therefore, the impedance function for passenger travel is characterized using random variables, as illustrated in equation (20).

$$c'^l_n = T_c^r + T_i^w, c_n^l = c'^l_n + \varepsilon_l \qquad (20)$$

where c_n^l is the estimated value of passengers' generalized travel cost; c'^l_n is the real value of passengers' generalized travel cost, and ε_l is a random term.

Step 3 Filter valid routes: The valid routes are filtered according to the threshold of the minimum generalized travel cost, as shown in equation (21).

$$c_i^l < (1+H)c_{min}^l \qquad (21)$$

Step 4 Passenger flow selection probability: It is assumed that the random error terms follow a Gumbel distribution and are mutually independent. The probability of passengers selecting a valid route n is presented in Equation (22).

$$P_n^l = \frac{\exp\left(-\frac{C_n^l}{\bar{C}}\right)}{\sum\exp\left(-\frac{C_n^l}{\bar{C}}\right)}, \quad \sum_{i=1}^{n} P_i^l = 1 \qquad (22)$$

Step 5 Circular assignment: Passengers are allocated to lines based on the probabilities generated by the line impedance and logit model function. The pseudo-code for this algorithm is displayed in Algorithm 5.

Algorithm 5 Passenger Flow Assignment Algorithm
Input: passenger flow M, Valid Path Collection$\{x_1...x_n\}$
Output: Number of people choosing each path
While
Calculate $I_1...I_n$
And $p_1...p_n \leftarrow$ Formula(22)
If(I_1==p_1 &&....&&I_n==p_n)
End
Else
$x_1 \leftarrow x_1$+(p_1*M-x_1)/count
Then count++

D. Train timetable preparation

The routes of trains can be arranged in accordance with the train operation plan. In practical operations, it is also crucial to determine the arrival and departure times of trains at stations, which is essential for planning train frequencies. By converting the train operation plan into a timetable, the operational status of trains during each time period can be visualized. This section delves into the coordination of conflicts among different types of trains on a line when preparing the train timetable, particularly focusing on the overtaking behavior of express and local trains to maximize the utilization of the line's carrying capacity.

During off-peak periods, the train departure interval is relatively wide, facilitating easier adherence to the departure interval without necessitating overtaking between express and local trains. The subsequent express trains maintain a safe tracking distance from the preceding local trains, ensuring independence between express and local trains on the line. This necessitates strict adherence to the departure interval for both express and local trains at the originating station, as illustrated in equation (23).

$$T_v^s - T_{v+1}^s = \sum_{i=1}^{n-1} T_{1,i}^r + T_{1,i}^w - T_{0,i}^r + T_{0,i}^w \qquad (23)$$

During peak periods, the frequency of train departures increases. When $t_{1,i}^{dep} < t_{0,i}^{dep}, t_{1,i+1}^{dep} > t_{0,i+1}^{dep}$, express trains overtake local trains in section [i,i+1]. In such a scenario, the departure interval between express and local trains at the originating station should be adjusted to avoid train conflicts. The overtaking of express trains at the overtaking station can be divided into two types: one where the express train stops at the overtaking station, and another where it does not, as depicted in Figure 2. Previous studies have shown that the non-stopping of express trains at overtaking station $h, h \in (S^0 \cap S_A)$, has minimal impact on line carrying capacity.

Taking into account passengers' perception of time on local trains, the avoidance time for local trains should be minimized, and express and local trains should arrive at the overtaking station with the smallest possible travel interval, as illustrated in Equation (24).

$$t_{\min} = \sum_{i=1+1}^{h-1} T_{1,i}^r - T_{0,i}^r, h \in S^O \qquad (24)$$

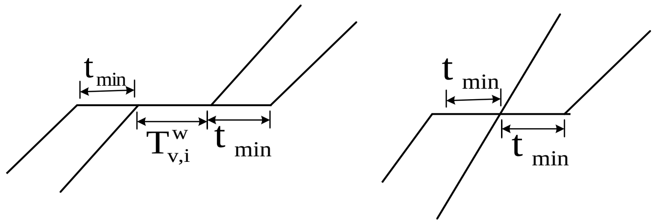
After analyzing the relationship between express trains and the overtaking station and computing the departure interval, we identify two scenarios.

1) Equation (25) indicates that at this point, the express train does not overtake the local train; instead, it lags behind, with its departure time being moved up.

$$T_v^s = T_v^s - \left(\sum_{j=i+1}^{h-1} T_{1,j}^r - T_{0,j}^r\right) - t_{\min} \qquad (25)$$

2) Equation (26) indicates that at this point, the express train overtakes the local train. Consequently, the express train is ahead of the local train, and its departure time is delayed.

$$T_v^s = T_v^s + \sum_{j=i+1}^{h-1} T_{1,j}^r - T_{0,j}^r + t_{min} \qquad (26)$$



(a) train stop (b) train do not stop

Fig. 2. Whether train stop at a overtaking station.

V. Case Study

This paper validates the design model and algorithm using both simulation data and actual data.

The experimental setup was conducted on a computer equipped with an Intel Core i5-7500 and 8G RAM, running the windows 10 (64-bit) operating system. The algorithm was implemented using the Java language. Passenger flow and route information were obtained through data collection efforts.

A. Simulation experiment

A small urban rail system has been simulated based on the actual line structure, comprising 15 stations and 14 sections along the railway.

(1) Set-up

The statistical OD data for passenger trips within 30-minute intervals are presented in Table III. The line base data are detailed in Tables IV and V, while the parameters used in the model are set as outlined in Table VI.

(2) Basic test results

With the objectives of minimizing both the total train running time and the total passenger travel time, we designed and obtained a train operation plan based on the model presented in Table VII. The resulting total train operation time is 18,302 seconds, and the total passenger travel time is 2,161,174,707 seconds.

The above represents the optimization result for the train operation plan within a 30-minute period. Given the high passenger flow in sections 6 to 10, short-turn routes have been introduced to alleviate transportation pressure. Conversely, due to the low number of boarding and alighting passengers at stations 5, 6, 8, and 9, express trains have been implemented that do not stop at these stations, thereby reducing the travel time for long-distance passengers.

Since the paper considers a train optimization plan that encompasses multiple issues, the impact of two specific sub-problems on the overall problem is analyzed below.

a) Stochastic user-equilibrium

To assess the influence of passenger selection outcomes on different routing within this algorithm, experiments are conducted by incorporating long-distance passenger flow data from simulations. Utilizing the variations in passenger choices across different routes, Table VIII analyzes the effect of passenger flow allocation on the optimized train operation plan's target value.

Table VIII demonstrates that the passenger flow distribution factor influences the train operation plan. By comparing the results, it is evident that the Stochastic User Equilibrium (SUE) approach can effectively decrease both train running time and passenger travel time. When passengers switch from local trains to express trains, their travel time is shortened, and their travel costs are optimized.

TABLE III
OD MATRIX (UNIT: PERSON)

Station	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	12	26	19	11	31	51	26	72	261	334	184	198	47
2		0	9	19	1	4	34	47	31	25	156	152	90	68	31
3			0	7	11	13	23	38	58	61	64	25	27	79	110
4				0	6	19	96	54	31	86	154	121	123	196	34
5					0	0	16	11	26	20	28	30	11	21	54
6						0	31	10	34	121	38	78	21	45	62
7							0	34	55	65	110	264	421	58	162
8								0	39	61	37	98	19	23	24
9									0	43	19	76	98	78	26
10										0	25	48	17	75	26
11											0	38	9	25	48
12												0	0	16	43
13													0	5	31
14														0	21

TABLE IV
TRAIN RUNNING TIME IN THE SECTION(UNIT:S)

Station	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Local train	124	94	122	105	131	160	200	131	608	617	540	755	545	728
Express train	107	81	105	90	123	125	181	120	543	552	485	674	488	651

TABLE V
STATION DWELL TIME (UNIT:S)

Station	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Dwell time	40	30	30	40	40	40	40	45	45	40	30	30	30	40

TABLE VI
EXPERIMENTAL PARAMETERS

Parameter	Value
Train capacity	1470 person
Overload number of the train	2205 person
Maximum headway	15min
Minimum headway	2min
Minimum turn back interval	2min
Overtaking proportional coefficient	4.0
Moderate crowding coefficient	1.86
Severe crowding coefficient	2.1
Transfer perception coefficient	1.2

TABLE VII
TEST RESULT

Type	Route	Skip station	Frequency	Operating interval
Local train one	1～15	0	2	15min
Local train two	6～10	0	2	15min
Express train	1～15	5, 6, 8, 9	1	30min

Additionally, the positioning of transfer stations impacts the planning of the train operation scheme. Therefore, rational allocation of passengers' travel routes can make the train operation scheme more realistic and practical.

b) Full-length and short-turn routing + Express-local train

When planning the train operation scheme, the express-local train strategy aims to address the travel time issues faced by long-distance passenger flow, while full-length and short-turn routing are employed to tackle the problem of uneven passenger flow distribution across different sections. This section tests the impact of both single routing mode and joint operation mode on passenger demand and enterprise costs, as shown in Table IX.

TABLE VIII
COMPARISON OF RESULTS CONSIDERING SUE

Contrasting factor	Train operation plan			Run time/s	Travel time/s
	Route	Skip station	Frequency		
Regardless of SUE	1-15	0	2	20680	216413444
	7-11	0	1		
	1-15	5,9	1		
Consider SUE	1-15	0	2	20140	2163952241
	7-11	0	1		
	1-15	3,5,6,8,9	1		

As Table IX illustrates, in the full-length and short-turn routing mode, the local trains on full-length routes operate frequently due to the dispersed passenger distribution and long travel distances, leading to an extended total train running time. In contrast, the express-local train mode effectively reduces passengers' total travel time through the skip-stop operation of express trains. When combining full-length and short-turn routing with the express-local train mode, short-turn routing enhances the attractiveness of short-distance travel, while express trains further decrease the travel time for long-distance passengers. Consequently, the joint implementation of full-length and short-turn routing alongside the express-local train mode helps minimize both enterprises' operating costs and passengers' travel expenses.

TABLE IX
COMPARISON OF THE RESULTS OF DIFFERENT OPERATING MODELS

Operation mode	Train operation plan			Run Time/s	Travel Time/s
	Route	Skip station	Frequency		
Full-length and short-turn routing	1-15 6-11	0 0	4 2	30172	21622808786
Express-local train	1-15 1-15	0 3,5,6,8,9	2 2	24366	2162957352
Full-length and short-turn routing + express-local train	1-15 7-11 1-15	0 0 3,5,6,8,9	2 1 1	20140	2163952241

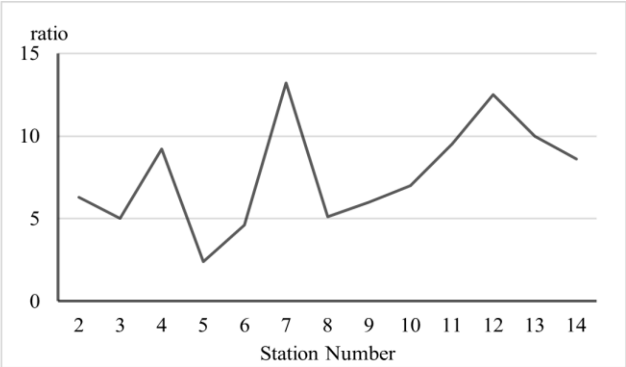


Fig. 3. Overtaking ratio of each station.

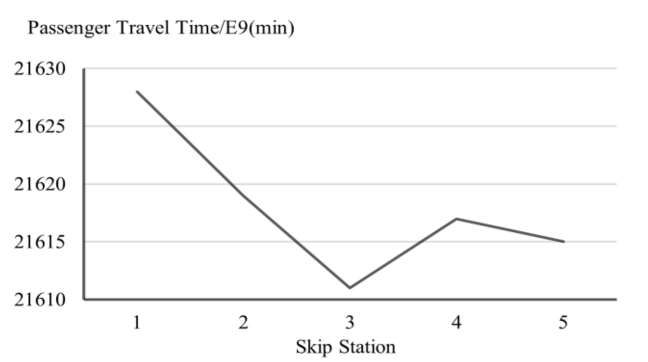


Fig. 4. Comparison results of the operation plan with different number of skip stations.

(3) Sensitivity Analysis

In this section, the train operation plan is analyzed with respect to two sensitive factors: the overtaking proportional factor and the number of skip stations.

a) Stochastic user-equilibrium

In this study, the decision to operate an express-local train is based on the ratio of boarding to alighting passengers, which establishes the critical condition for its operation. Keeping the passenger flow data constant, we adjust the overtaking coefficients to assess the express-local train operation plan under this critical condition. The overtaking proportion coefficients for each station are presented in Figure 3, while the operation plans corresponding to different ratios are detailed in Table X.

The results show that when the overtaking factor is set to 10, all stations on the line (excluding the departure and destination stations) fulfill the skip-stop condition. As a result, various suitable skip-stop combinations can be arranged across the entire line, leading to the most diverse express-local train plans. Conversely, when the overtaking coefficient is 5, only station 6 meets the skip-stop criteria, resulting in the least number of train plans. With 2 serving as the critical condition, as the overtaking ratio decreases, the passenger flow distribution on the line fails to meet the requirements for express-local train operation, resulting in only full-length routing local trains. The experimental findings suggest that an express-local train operation plan, tailored to passenger flow conditions, significantly reduces both the total train operation time and the overall passenger travel time compared to a local train operation plan. This better aligns with the needs of both enterprises and passengers.

b) Number of skip stations

Express and local trains are designed to cater to the needs of passengers traveling long distances. As the number of skip stations for express trains increases, their running cycle becomes shorter, thereby reducing the travel time for long-distance passengers. To analyze the impact on the total travel time of passengers along the route, we conducted tests by varying the number of skip stations, and the results are presented in Figure 4.

The experimental results reveal a non-linear negative correlation between the number of skip stations for express trains and the total travel time of passengers. When the waiting time of some passengers is compromised to optimize the overall travel time for passengers on the line, a critical point is reached. Beyond this point, a continuous increase in the number of overtaking stations leads to an increase in the total travel time of passengers.

B. Actual cases

In order to verify the effectiveness and applicability of the algorithm, this section selects the actual metro line case data to optimize the train operation plan.

(1) Case data

The experiment is conducted on Line 2 of the Zhengzhou subway's suburban network in China, as illustrated in Figure 5. This line spans approximately 42.1 kilometers and comprises 36 stations, with overtaking tracks established at the SWL and MZ stations. Passenger demand data is sourced from the Automatic Fare Collection (AFC) system. Figure 6 depicts the total passenger flow distribution from 6:00 am to 11:30 pm during a typical operational day

TABLE X
COMPARISON RESULTS OF OPERATION PLANS WITH DIFFERENT OVERTAKING RATIOS

Overtaking ratio	Overtaking station	Short-turn route	Train run time/s	Total passenger travel time/s
10	3, 5, 6, 9, 13	5-10	17804	2161546200
5	6	6-10	18692	2162557617
6.1	3, 5, 6, 8, 9	6-10	18126	2162432562
2	0	6-10	24278	2610628600

(passenger flow data recorded on December 9, 2019). During weekdays, the morning peak occurs between 7:30 am and 8:30 am, while the evening peak occurs between 6:00 pm and 7:00 pm. According to the actual timetable, the departure interval is 12.9 minutes during the morning peak and 16.1 minutes during the evening peak. The experimental parameters are set as follows: train capacity is 1470 passengers per train; maximum train full load ratio φ is 1; turnaround time for local train sets is 67.17 minutes; turnaround time for express train sets is 60.23 minutes; and the maximum and minimum departure intervals are 17 minutes and 2 minutes, respectively, across all operational periods of the day.

(2) Test results

The model developed in this study is capable of addressing issues pertaining to any time period. In this section, we focus on optimizing the train operation plan during the morning and evening peak hours, with an operating cycle of 30 minutes. The actual timetable for these peak hours is presented in Table XI, where FL denotes Full-length routing and ST denotes Short-turn routing.

In the actual timetable, the train operation plans for both the morning and evening peak hours are identical. However, as evident from Figure 5, the passenger flow characteristics of these two periods differ significantly, necessitating distinct treatment. Therefore, we have optimized the plans for each period based on their respective cycles, and the optimized train plan is presented in Table XII.

Since the central section of Line 2 traverses an urban area, passengers tend to flow towards the middle from both ends. Additionally, the presence of several transfer stations on the line contributes to a concentration of passenger flow. During the morning peak, stations 5, 19, 21, and 25 experience relatively low boarding and alighting passenger volumes, prompting the express train to skip these stations. Similarly, this skipping strategy is also applied during the evening peak period.

By analyzing the differences between the actual timetable and the optimized timetable through the use of objective functions, we can validate the efficacy of the algorithm. The comparison results are presented in Table XIII.

When compared to the actual timetable, during the morning peak hours, the average total train operation time and total passenger travel time decrease by 48.1% and 3.9% respectively. Similarly, during the evening peak period, these figures drop by 43.2% and 3.55% on average. The experiment reveals that the passenger flow is more evenly distributed over time during the morning peak compared to the evening peak, resulting in a higher quality of the train operation plan during the morning peak period. In the optimized plan, express trains are able to skip certain stops, leading to a significant reduction in total passenger travel time. Given that full-length routing local trains are a necessary operational mode for the line, the introduction of express trains increases the total train operation time.

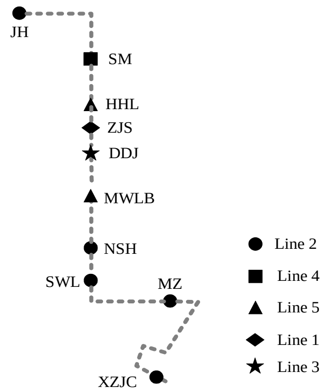


Fig. 5. Zhengzhou Line 2 Road map.

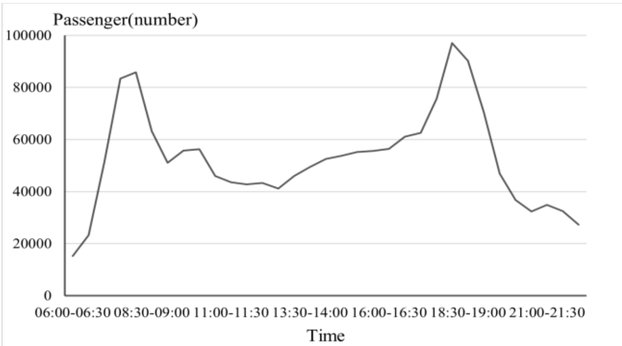


Fig. 6. Passenger flow time distribution.

TABLE XI
ACTUAL TIMETABLE OF ZHENGZHOU METRO LINE 2

Peak	Morning peak		Evening peak	
Time period	7:30-8:00	8:00-8:30	18:00-18:30	18:30-19:00
ST	1-22	1-22	1-22	1-22
Local train headway	FL:774s ST:258s	FL: 774s ST:258s	FL:774s ST:258s	FL:774s ST:258s

Therefore, the optimal solution strikes a balance between local and express trains.

The express train operation route map is generated based on the optimized express train operation plan, as shown in Figure 7. The procedures outlined in Section 4.4 are employed to determine the overtaking positions of express and local trains, compute the departure interval between them at the originating station, and compile the train operation map. The outcomes of these calculations are illustrated in Figures 8 and 9.

As shown in Figures 8 and 9, the departure interval between express and local trains during both the morning and evening peak hours is 616 seconds, while the departure interval between short-turn routing trains and express trains is 736 seconds. In the upward direction, the express train overtakes the local train at the SWL station. Given that the overtaking station is located towards the latter half of the line, the express train only needs to overtake once to meet the operational demand.

TABLE XII
OPTIMIZED TRAIN OPERATION PLAN

Planning Period	1800s			
	morning peak		evening peak	
	7:30-8:00	8:00-8:30	18:00-18:30	18:30-19:00
Skip stations	5, 16, 19, 21, 25, 31-35	2, 5, 16, 19, 21, 31-35	2, 5, 19, 21, 25, 31-35	2, 3, 5, 6, 25, 31-35
ST	0	0	0	12-23
Local train headway	FL:900s	FL:900s	FL:900s	FL:900s ST:900s
Express train headway	1800s	900s	900s	900s

TABLE XIII
COMPARISON OF THE ACTUAL OPERATION PLAN OF ZHENGZHOU LINE2 WITH THE OPTIMIZED TIMETABLE

	Total train operating time/min				Total passenger travel time/min			
	7:30-8:00	8:00-8:30	18:00-18:30	18:30-19:00	7:30-8:00	8:00-8:30	18:00-18:30	18:30-19:00
Original plan	517	517	517	517	435889	5342051	528402	560829
Optimized plan	239	299	302	287	419743	512129	507530	542817

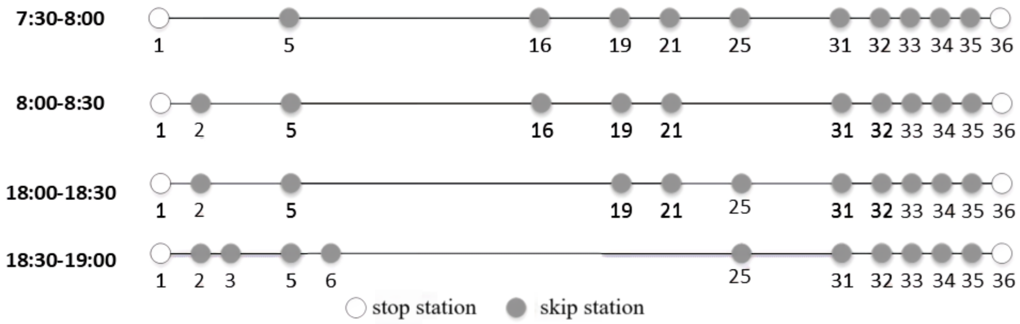


Fig. 7. Express train operation route.

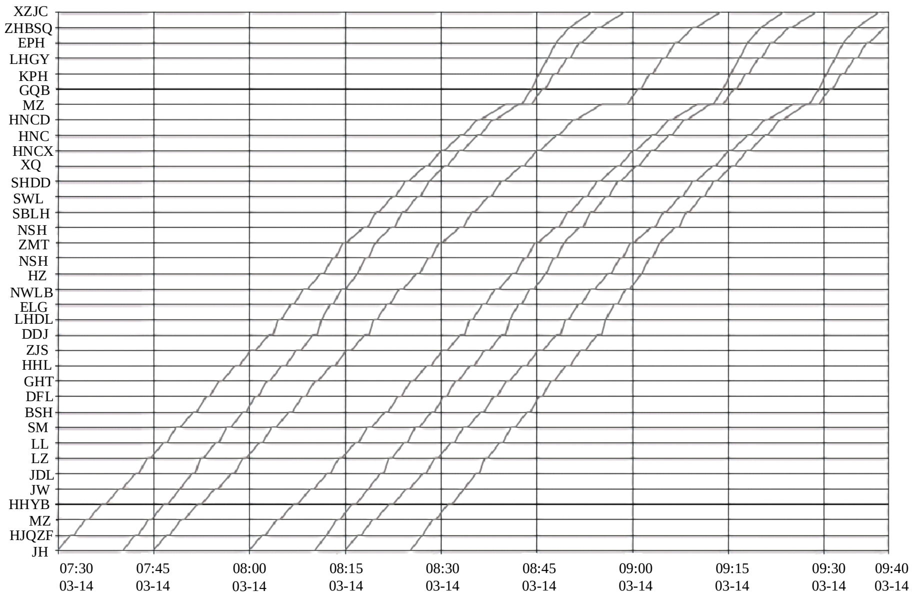


Fig. 8. Morning peak train operation chart.

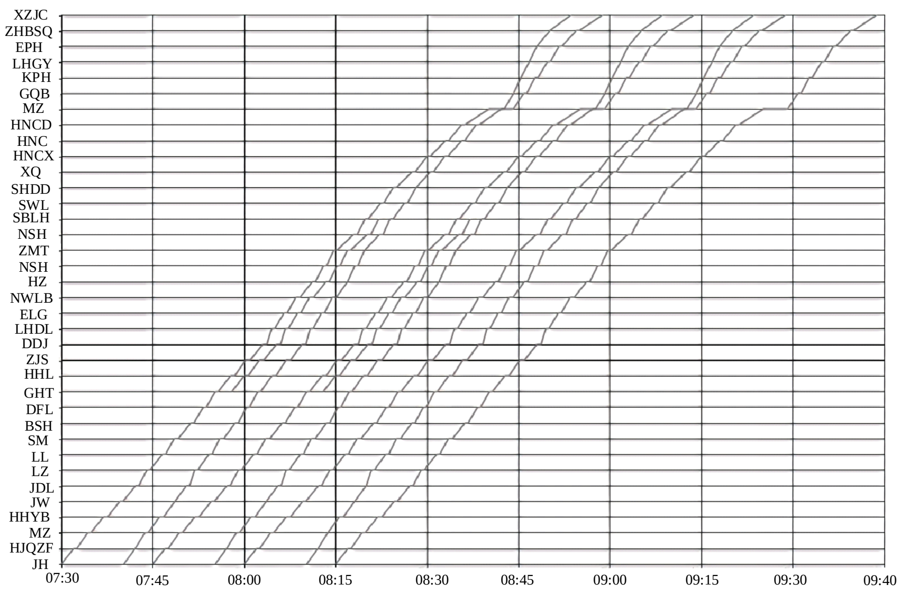


Fig. 9. Evening peak train operation chart.

VI. Conclusion

This paper delves into the optimization of train operation plans for regional transit systems. Taking into account passengers' travel preferences and express train overtaking behavior on the line, we devise a joint operation mode combining full-length and short-turn routing with express and local trains, tailored to the characteristics of passenger flow distribution. We then construct a dual-objective integer linear programming model and develop a hybrid algorithm combining MSA and SA to generate solutions. The efficacy of the algorithm is substantiated through simulation experiments, and a sensitivity analysis is conducted on pertinent factors to ascertain their respective impacts on the train operation plan. In a real-world application, a high-quality train timetable is derived from the operational data of Zhengzhou Metro Line 2 in China. Compared with the actual schedule, the method achieves an average reduction of 48.1% in train running time and 3.9% in passenger total travel time during the morning peak, and 43.2% and 3.55% respectively during the evening peak. These results have confirmed the practical viability of the proposed method.

The simulation experiment results demonstrate that, in comparison to traditional local train operation, the combined operation of express and local trains can significantly reduce both passenger travel time and train running time. Generally, as the number of stations skipped by express trains increases, the train cycle time and the travel time for long-distance passengers decrease. However, once the number of skipped stations exceeds a certain threshold, the demand of short-distance passengers will outweigh that of long-distance passengers, leading to potential negative impacts on the fast train operation scheme. Therefore, selecting appropriate parameters during the experimental process is crucial to avoid converging to a locally optimal operation plan.

In future research, it is necessary to take into account the influence of various time periods on objective functions and transform these functions into a single, multi-priority objective. For instance, during peak hours, express trains primarily cater to commuting passengers who are less concerned with train occupancy rates, hence the objective function should prioritize minimizing total passenger travel time. Conversely, during off-peak hours, with non-urgent, non-commuter traffic, the objective function should shift focus to minimizing enterprise costs. Furthermore, the majority of existing studies are tailored to specific application scenarios rather than offering a generalized solution method. Consequently, future research must enhance the algorithm's generality to facilitate its application across multiple lines.

References

[1] Wang Y, De Schutter B, van den Boom J, Ning B, Tang T, Efficient bilevel approach for urban rail transit operation with stop-skipping, IEEE Transactions on Intelligent Transportation Systems, vol.15, pp.2658-2670, 2014.

[2] Niu H, Zhou X, Gao R, Train scheduling for minimizing passenger waiting time with time-dependent demand and skip-stop patterns: Nonlinear integer programming models with linear constraints, Transportation Research Part B: Methodological, vol.76, pp.117-135, 2015.

[3] Zhang P, Liu X, Chen M, Optimal train scheduling under a flexible skip-stop scheme for urban rail transit based on smartcard data, 2016 IEEE International Conference on Intelligent Rail Transportation (ICIRT), IEEE. pp.13-22. 2016.

[4] Shang P, Li R, Liu Z, Yang L, Wang Y, Equity-oriented skipstopping schedule optimization in an oversaturated urban rail transit network, Transportation Research Part C: Emerging Technologies, vol.89, pp.321-343, 2018.

[5] Sun Y, Shi H, Wang Y, Chen S, Optimization on local/express train operation plan of urban rail transit by bi-level programming, J. Transp. Syst. Eng. Inf. Technol, vol.83, pp.160-167, 2018.

[6] Salama M, Abdelhafiez E, Shalaby M, Maximizing number of direct trips for a skip-stop policy in metro systems, Computers & Industrial Engineering, vol.137, pp.106091-106098, 2019.

[7] Rajabighamchi F, Hosein Hajlou, Hassannayebi E. A multi-objective optimization model for robust skip-stop scheduling with earliness and tardiness penalties, Urban Rail Transit, vol.5, pp.172-185, 2019.

[8] Li Z, Mao B, Bai Y, Chen Y, Integrated optimization of train stop planning and scheduling on metro lines with express/local mode, IEEE Access,vol.7, pp.88534-88546, 2019.

[9] Shang P, Li R, Yang L, Demand-driven timetable and stop pattern cooperative optimization on an urban rail transit line, Transportation Planning and Technology,vol.43, pp.78-100, 2020.

[10] Cao Z, Ceder A, et al, Robust and optimized urban rail timetabling using a marshaling plan and skip-stop operation, Transportmetrica A: Transport Science, vol.16, pp.1217-1249, 2020.

[11] Zhao S, Yang H, Wu Y, An integrated approach of train scheduling and rolling stock circulation with skip-stopping pattern for urban rail transit lines, Transportation Research Part C: Emerging Technologies, vol.128, pp.103170, 2021.

[12] Cao Z, Yuan Z, Li D, Estimation method for a skip-stop operation strategy for urban rail transit in china, Journal of Modern Transportation, vol.22, pp.174-182, 2014.

[13] Zheng L, Song R, He S, Li H, Optimization model and algorithm of skip-stop strategy for urban rail transit. Journal of the China Railway Society, vol.31, 1-8, 2009.

[14] Lee Y J, Shariat S, Choi K, Optimizing skip-stop rail transit stopping strategy using a genetic algorithm, Journal of Public Transportation, vol.17, pp.135-164, 2014.

[15] Cao Z, Yuan Z, Zhang S, Performance analysis of stop-skipping scheduling plans in rail transit under time-dependent demand, International Journal of Environmental Research and Public Health, vol.13, pp.707-712, 2016.

[16] Gao Y, Kroon L, Schmidt M, Yang L, Rescheduling a metro line in an over-crowded situation after disruptions, Transportation Research Part B: Methodological, vol.93, pp.425-449, 2016.

[17] Altazin E, Dauzere-Peres S, Rescheduling through stop-skipping in dense railway systems, Transportation Research Part C: Emerging Technologies, vol.79, pp.73-84, 2017.

[18] Shi F, Zhao S, Zhou Z, Wang P, Bell G, Optimizing train operational plan in an urban rail corridor based on the maximum headway function, Transportation Research Part C: Emerging Technologies, vol.74, pp.51-80, 2017.

[19] Guangzheng B , Jin G , Yang Y , On operation scheme of urban rail transit in long/short routing mode, Urban Mass Transit, 2015.

[20] Tang L, Xu X, Optimizing train plan with multi-routing and express-local modes for suburban rail transit based on bi-level programming, Transportation Systems Engineering and Information Technology,vol.18, pp.152-159, 2018.

[21] Vuchic V, Urban transit: operations, planning, and economics, John Wiley & Sons. 2017.

[22] Hou Y, Gu W, Wang C, Yang K, Wang Y, A Selection Hyper-heuristic based on Q-learning for School Bus Routing Problem, IAENG International Journal of Applied Mathematics, vol.52, no.4, pp.817-825, 2022.