Research on the Stochastic Evolutionary Game of Effective Supervision on Elderly Care Services under Internet Plus

Xiaoyan Cao, Xuelin Zhao, Jiading Wang

***Abstract*—In the context of the "Internet Plus" initiative, this study addresses the challenges faced in elderly care services and their supervision, with particular attention to the uncertain and stochastic elements inherent in the elderly care service market. A stochastic disturbance coefficient is introduced to construct a tripartite evolutionary game involving elderly care institutions, elderly residents, and government agencies. The behavioral stability conditions for each party are analyzed, and a MATLAB-based model is employed to simulate and evaluate the influence of key variables on the direction and magnitude of strategic evolution among the players. The findings reveal that both elderly care institutions and government agencies place significant emphasis on economic returns and social welfares, demonstrating high sensitivity to the influence of online public opinion. Furthermore, the introduction of appropriate incentives and necessary punitive measures not only reduces regulatory costs for the government agencies but also effectively encourages self-regulation among elderly care institutions and proactive oversight by the elderly themselves. This, in turn, facilitates the high-quality development of the elderly care industry. Moreover, governmental policy support or tax incentives aimed at guiding the investment behaviors of elderly care institutions can accelerate the adoption of self-discipline service strategies and improve the overall quality of elderly care services. Ultimately, by fulfilling its regulatory obligations and curbing opportunistic behaviors, the government can ensure the orderly operation of the elderly care services market and foster a sustainable environment for the industry's development.**

***Index Terms*—Internet Plus, Elderly care services, Effective supervision, Replicator dynamics, Stochastic evolutionary game.**

I. Introduction

POPULATION aging is a prominent and ongoing trend in China, where both the size and growth rate of the elderly population significantly surpass those of many other countries. According to data released by the National Bureau of Statistics of China in 2023, the number of individuals aged 60 and above reached 297 million, accounting for 21.1% of the total population. Among them, 217 million were aged 65 and above, representing 15.4% of the population [1]. Meanwhile, data from the Ministry of Civil Affairs indicate that by 2022, there were approximately 387,000 elderly care institutions of various types across the country, providing a total of 8.3 million beds. However, the number of certified elderly care workers nationwide remains critically low, at just around 40,000, highlighting a stark mismatch between the supply and demand for elderly care services. To address this imbalance, the National Health Commission has introduced the "9073 Plan", which proposes that approximately 90% of the elderly be cared for at home, 7% receive community-based care, and 3% reside in institutional elderly care facilities. Based on this strategy, the estimated total demand for elderly care beds across institutions and communities should be approximately 20 million, revealing a current shortfall of over 11 million beds [2]. Moreover, the diversity of elderly care services and the inconsistency in service quality further intensify the challenges faced by the elderly population.

Given these circumstances, considerable academic attention has been devoted to examining the quality of elderly care services and the effectiveness of regulatory frameworks. Chen et al. argued that the insufficient number of non-profit elderly care institutions nationwide leads to an inadequate supply of services to meet the basic care needs of the elderly. This issue is exacerbated by limited government oversight, which severely constrains the healthy development of the industry [3]. Wang noted that the roles and responsibilities of government agencies in elderly care supervision are often unclear, resulting in a fragmented regulatory system and suboptimal regulatory effectiveness [4]. Lei et al., through surveys conducted in several Chinese cities, identified complex administrative procedures and significant regulatory gaps in governmental supervision mechanisms. Both domestic and international scholars have increasingly employed game theory to analyze the regulatory challenges in elderly care services [5]. For instance, Yue et al. conducted a quantitative analysis of the impact of subsidies and penalties on elderly care regulation [6]. Mu et al. applied evolutionary game theory to explore the relationship between regulatory penalties and the self-regulatory behaviors of elderly care institutions, emphasizing that a well-balanced combination of incentives and regulatory pressure is essential for fostering institutional self-discipline [7]. Johanness found that increased participation by elderly individuals and external oversight significantly enhance the focus of elderly care institutions on service quality and safety [8].

In the era of information technology, the supervision of public opinion through various Internet media platforms serves as an effective supplement to governmental regulation and constitutes a vital channel through which the public can access information on elderly care services [9]. The Internet exhibits notable spillover effects, wherein the influence of online public opinion forms a key basis for strategic decision-

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Xiaoyan Cao is a lecturer at School of Logistics and E-commerce, Xinyang Agriculture and Forestry University, Xinyang 464000, China (E-mail: jsjlikui@xynu.edu.cn).

Xuelin Zhao is a lecturer at School of Computer and Information Technology, Xinyang Normal University, Xinyang 464000, China (E-mail: xlz_cit@xynu.edu.cn).

Jiading Wang is a lecturer at Department of Policy and Planning Sciences, University of Tsukuba, Ibaraki, 3058577, Japan (E-mail: wang.jiading.xg@alumni.tsukuba.ac.jp).

making by elderly care institutions, the elderly, and government agencies, directly impacting the costs and benefits faced by all parties involved [10]. Wang et al. argued that effective media supervision can reduce the regulatory burden on government agencies. However, enhancing the professional quality of media personnel remains essential [11]. Goodwin et al. emphasized the importance of standardizing supervisory mechanisms and guiding Internet platforms to plays a central role in the continuous oversight of the elderly care services market [12]. Fuentesm et al. contended that online supervision contributes to the refinement of regulatory policies and the enhancement of regulatory efficacy [13]. Wang introduced the concept of self-media into the regulatory framework for elderly care institutions and analyzed the impact and credibility of such media coverage on service supervision [14]. Hu et al. examined the dynamic evolution of elderly care supervision under the constraints of bounded rationality, with simulation results indicating the significant role of elderly individuals in the regulatory process [15].

In summary, evolutionary game theory has proven to be an effective analytical tool for studying the supervision mechanisms of elderly care services, with the Internet playing a crucial role in enhancing the overall regulatory system [16]. Nevertheless, existing research predominantly relies on qualitative approaches, lacking in-depth quantitative analysis and failing to incorporate real-world factors such as online public opinion. Furthermore, traditional evolutionary game models are typically confined to deterministic frameworks, overlooking stochastic disturbances that are often present in practice. This limitation restricts the applicability and generalizability of current research findings. Therefore, from the perspective of the "Internet Plus" strategy, this study comprehensively considers the influence of online public opinion and constructs an evolutionary game model involving elderly care institutions, elderly residents, and government agencies. A stochastic disturbance coefficient is introduced to account for uncertainties in the regulatory environment. The study investigates the stability of strategies adopted by the three players, explores the interdependence of their strategic choices, and analyzes the dynamic evolution of their decision-making processes through simulation [17].

II. Research Questions and Model Construction

A. Fundamental Assumptions of the Model

The regulation of elderly care services can be essentially characterized as a dynamic strategic game among three primary players: elderly care institutions, elderly residents, and government entities. Due to the presence of information asymmetry, the decision-making processes of these players are constrained by bounded rationality rather than full rationality. While elderly care institutions and elderly residents seek to optimize their individual payoffs, the government aims to maximize both its own interests and overall societal welfare. These strategic decisions are influenced by a combination of internal and external factors [18]. Based on the framework of bounded rationality, this study develops an evolutionary game model for supervising elderly care services, incorporating the following assumptions [19].

Hypothesis 1: Elderly care institutions charge fees in accordance with relevant regulatory standards and provide services to elderly residents, while bearing the operational costs related to facilities, infrastructure, and staff. Within the regulatory game, these institutions may adopt either a self-discipline or non-self-discipline strategy in service provision [20]. The self-discipline strategy, characterized by compliance with operational standards and delivery of high-quality care, results in stable returns, social recognition, and an expanded client base. It also qualifies the institution for tax incentives, service fee discounts, or other governmental rewards. In contrast, a non-self-discipline strategy, which involves circumventing regulations for speculative gains at the cost of service quality, poses substantial risks. Although it may yield short-term profits, it endangers the rights of elderly residents and may result in abusive practices. Once discovered, such misconduct can lead to official penalties, reputational damage, and client attrition.

Hypothesis 2: Elderly residents, as the direct recipients of care services, access these services by paying fees aligned with their individual and family needs. In response to the services received, residents can choose between active and passive supervision strategies [21]. An active supervision strategy entails a certain cost and requires the investment of time and effort. However, it contributes to improving service quality and maintaining social stability. To expand regulatory channels, mitigate extreme incidents, reduce regulatory costs, and enhance regulatory efficiency, the government agencies offer incentives to elderly individuals who actively participate in supervisory activities [22].

Hypothesis 3: As the guardian of public welfares, the government agencies are responsible for the daily oversight of elderly care services and bears the associated administrative costs, including staffing, equipment, and other supervisory resources. Within the regulatory game, the government may adopt either an active or passive supervision strategy [23]. Active supervision typically encourages elderly care institutions to maintain high service standards, thus enhancing public perceptions of the government's credibility and legitimacy. Conversely, a passive supervision strategy may reduce regulatory expenditures, but it increases the risk of undisciplined behavior by elderly care institutions. If poor service quality is exposed by the public or media, it can generate negative social impacts and subject the government to scrutiny or accountability from higher-level authorities.

Hypothesis 4: Amid the rapid acceleration of population aging, public concern regarding elderly care services for both current and future needs has intensified. Positive examples, quality services, or negative incidents in elderly care, when disseminated through platforms such as the Internet, can be rapidly amplified and give rise to widespread online public opinion. This, in turn, can have either positive or negative repercussions for the reputations of both elderly care institutions and the government-enhancing public trust and visibility in favorable cases or leading to reputational harm and financial losses in adverse situations [24].

B. Parameters Setting of the Model

To facilitate a comprehensive analysis of the costs, benefits, and potential losses experienced by each player under different strategic combinations, as well as to explore the dynamics and underlying motivations of strategic choices,

TABLE I
MAIN PARAMETERS AND THEIR MEANINGS

Parameters	Description
R_i	Daily operating payoffs of elderly care institutions, payoffs of elderly residents in these institutions, social welfares represented by government,when $i = 1, 2$, and $i = 3$, respectively
C_i	Operational costs of elderly care institutions, costs of elderly residents on supervision, costs of government agencies on supervision, when $i = 1, 2$,and$i = 3$, respectively
T_i	Coefficient of self-discipline services degree of the elderly care institutions, coefficient of supervision degree of the elderly, coefficient of supervision degree of the government, when $i = 1, 2$, and $i = 3$, respectively
x	Probability of self-discipline service of the elderly care institutions($x \in [0, 1]$)
y	Probability of active supervision of the elderly residents ($y \in [0, 1]$)
z	Probability of active supervision of the government ($z \in [0, 1]$)
g	Influence of online public opinion ($g > 1$)
V	Social acclaim resulting from self-discipline services in elderly care institutions, or the loss caused by non-self-discipline behaviors
M	Payoffs derived from the non-self-discipline behaviors of elderly care institutions
Q	Rewards from government for self-discipline or punishments for non-self-discipline of the elderly care institutions
S	Non-self-discipline services of elderly care institutions result in the loss of payoffs for the elderly residents
H	Rewards from government for elderly residents who actively supervise elderly care institutions
W	Social acclaim resulting from proactive supervision by government or reputation loss caused by passive supervision
N	When the lack of self-discipline by elderly care institutions is exposed, the higher-level government will penalize the government for their lax supervision

a set of model parameters is established, as summarized in Table I.

C. Payoff Matrix of the Model

Based on the aforementioned model assumptions and parameter settings, the strategic decision-making process in the evolutionary game is conceptualized as a dynamic, continuous interaction among elderly care institutions, elderly residents, and government agencies. These players make strategic choices while taking into account various factors such as participation costs, expected payoffs, penalty risks, and reputational outcomes [25]. According to the principles of dynamic modeling and variable interaction, the payoff matrix for this tripartite evolutionary game is constructed and presented in Table II.

III. Analysis of the Expected Functions for Replicated Dynamic Equations

A. Replicated Dynamic Equations

Based on the Table I and Table II, the expected payoffs for elderly care institutions adopting the self-discipline strategy is denoted as X_1, while that of the non-self-discipline strategy is denoted as X_2. The average expected payoff across both strategies is represented as $\overline{X}$, and the specific expressions are given as:

$$\begin{cases} X_1 = yz(R_1 + gV + Q - C_1) + y(1-z)(R_1 + gV + T_3Q - C_1) + z(1-y)(R_1 + gV + Q - C_1) + (1-y)(1-z)(R_1 + gV + T_3Q - C_1) \\ X_2 = yz(R_1 + M + gV - Q - T_1C_1) + y(1-z)(R_1 + M - gV - T_3Q - T_1C_1) + z(1-y)(R_1 + M - Q - gV - T_1C_1) + (1-y)(1-z)(R_1 + M - gV - T_3Q - T_1C_1) \\ \overline{X} = xX_1 + (1-x)X_2 \end{cases} \tag{1}$$

According to the Malthusian dynamics, the change in strategy adoption rates is proportional to differences in payoff [26]. The differential equation describing the evolutionary trajectory of elderly care institutions is:

$$F(X) = dx/dt = x(X_1 - \overline{X}) = x(1-x)(X_1 - X_2) \tag{2}$$

Similarly, for elderly residents, let Y_1 and Y_2 denote the expected payoffs of active and passive supervision strategies, respectively, and $\overline{Y}$ the average expected payoff. The expressions are:

$$\begin{cases} Y_1 = xz(R_2 + H - C_2) + x(1-z)(R_2 + T_3H - C_2) + z(1-x)(R_2 + H - S - C_2) + (1-x)(1-z)(R_2 + T_3H - S - C_2) \\ Y_2 = xz(R_2 - T_2C_2) + x(1-z)(R_2 - T_2C_2) + z(1-x)(R_2 - S - T_2C_2) + (1-x)(1-z)(R_2 - S - T_2C_2) \\ \overline{Y} = yY_1 + (1-y)Y_2 \end{cases} \tag{3}$$

The evolutionary dynamic equation for elderly residents is:

$$F(Y) = dy/dt = y(Y_1 - \overline{Y}) = y(1-y)(Y_1 - Y_2) \tag{4}$$

For government agencies, the expected payoffs of active and passive supervision strategies are denoted as Z_1 and Z_2, with the average expected payoff represented by $\overline{Z}$. The payoff functions are:

$$\begin{cases} Z_1 = xy(R_3 + gW - Q - H - C3) + x(1-y)(R_3 + gW - Q - C3) + y(1-x)(R_3 - H + gW + Q - C3) + (1-x)(1-y)(R_3 + gW + Q - C3) \\ Z_2 = xy(R_3 - gW - T_3Q - T_3H - T_3C3) + x(1-y)(R_3 - gW - T_3Q - T_3C3) + y(1-x)(R_3 - N - gW + T_3Q - T_3H - T_3C3) + (1-x)(1-y)(R_3 - N - gW + T_3Q - T_3C3) \\ \overline{Z} = zZ_1 + (1-z)Z_2 \end{cases} \tag{5}$$

TABLE II
PAYOFFS MATRIX OF THE MODEL

Players				Government departments	
				z	$1-z$
Elderly care institutions	x	Elderly	y	$R_1+gV+Q-C_1$ R_2+H-C_2 $R_3+gW-Q-H-C_3$	$R_1+gV+T_3Q-C_1$ $R_2+T_3H-C_2$ $R_3-gW-T_3H+Q+C_3$
			$1-y$	$R_1+gV+Q-C_1$ $R_2-T_2C_2$ $R_3+gW-Q-C_3$	$R_1+gV+T_3Q-C_1$ $R_2-T_2C_2$ $R_3-gW-T_3(Q+C_3)$
	$1-x$	Elderly	y	$R_1+M-gV-Q-T_1C_1$ $R_2+H-S-C_2$ $R_3-H+Q+gW-C_3$	$R_1+M-gV-T_3(Q+C_1)$ $R_2+T_3H-S-C_2$ $R_3-N-gW+T_3(Q-H-C_3)$
			$1-y$	$R_1+M-Q-gV-T_1C_1$ $R_2-S-T_2C_2$ $R_3+Q+gW-C_3$	$R_1+M-gV-T_3Q-T_1C_1$ $R_2-S-T_2C_2$ $R_3-N-gW+T_3(Q-C_3)$

Accordingly, the differential equation for the government's strategy evolution is:

$$F(Z) = dz/dt = z(Z_1 - \overline{Z}) = z(1-z)(Z_1 - Z_2) \quad (6)$$

Given that all variables $x, y, z \in [0,1]$, then $1-x, 1-y, 1-z \in [0,1]$, ensuring valid strategy proportions. After simplification, the final differential equations governing the dynamic evolution of each player's strategy are as follows:

$$\begin{cases} F(X) = x[2gV - M - C_1(1-T_1) + 2zQ + \\ \qquad 2T_3Q(1-z)] \\ F(Y) = y(-C_2 + T_3H + T_2C_2 + zH - zT_3H) \\ F(Z) = z[2gW + Q + N - C_3 - T_3Q + T_3C_3 - \\ \qquad x(2Q + N - 2T_3Q) - y(H - T_3H)] \end{cases} \quad (7)$$

B. Construction of the Stochastic Evolutionary Game Model

In the context of elderly care service supervision, strategic interactions among elderly care institutions, elderly residents, and government agencies are inherently uncertain. Players may flexibly alter their strategic choices based on individual payoffs, while emotional fluctuations and external risks may further influence decision-making. Furthermore, varying degrees of opportunistic behavior-aimed at maximizing individual gains are observed across participants. In the digital era, public engagement in supervision, including both social and online oversight, introduces sudden and unpredictable disturbances into the system. To reflect these real-world uncertainties, this study incorporates Gaussian white noise as a stochastic disturbance factor into the tripartite game model, resulting in a one-dimensional $It\hat{o}$ stochastic replicator dynamic equation [27].

$$\begin{cases} dx(t) = x(2gV - M - C_1(1-T_1) + 2zQ + \\ \qquad 2T_3Q(1-z))dt + \sigma x(t)d\omega(t) \\ dy(t) = y(-C_2 + T_3H + T_2C_2 + zH - \\ \qquad zT_3H)dt + \sigma y(t)d\omega(t) \\ dz(t) = z[2gW + Q + N - C_3 - T_3Q + T_3C_3 - \\ \qquad x(2Q + N - 2T_3Q) - y(H - T_3H)]dt + \\ \qquad \sigma z(t)d\omega(t) \end{cases} \quad (8)$$

In Equation(8), $\omega(t)$ represents a standard one-dimensional Brownian motion, capturing irregular fluctuations due to stochastic disturbances. The differential increment $d\omega(t) = \omega(t+h) - \omega(t)$ over an interval $h > 0$ follows a normal distribution $N(0, \sqrt{h})$, and σ denotes the intensity of the disturbance [15].

C. Analysis of the Existence and Stability of Equilibrium Solutions

At the initial stage of the tripartite evolutionary game, when $t = 0$, the system is defined by the initial state $x(0) = 0, y(0) = 0$, and $z(0) = 0$. Under these conditions, Equation (8) admits at least one trivial (zero) solution, indicating that in the absence of external stochastic disturbances, the system remains in a stable equilibrium. Therefore, the zero solution can be regarded as an equilibrium point of the corresponding deterministic differential equation. However, in real-world scenarios, the system is invariably subjected to external influences, which can disrupt the equilibrium and alter the stability of the game dynamics. The stability of the resulting stochastic differential equation is assessed using established stochastic stability criteria [28]. For the zero solution to retain its stability under stochastic perturbations, specific mathematical conditions must be fulfilled

$$\begin{cases} x(2gV - M - C_1(1-T_1) + 2zQ + 2T_3Q(1- \\ z)) \le -x \\ y(-C_2 + T_3H + T_2C_2 + zH - zT_3H) \le -y \\ z[2gW + Q + N - C_3 - T_3Q + T_3C_3 - x(2Q+ \\ N - 2T_3Q) - y(H - T_3H)] \le -z \end{cases} \quad (9)$$

By constraining the state variables $x, y, z \in [0,1]$, and appropriately simplifying Equation (8), we can derive the necessary conditions under which the zero solution remains stable in the stochastic framework:

$$\begin{cases} M - 2gV + C_1(1-T_1) - 2zQ - 2T_3Q(1- \\ z)) - 1 \ge 0 \\ C_2 - T_3H - T_2C_2 - zH + zT_3H) - 1 \ge 0 \\ C_3 - 2gW - Q - N + T_3Q - T_3C_3 + x(2Q+ \\ N - 2T_3Q) + y(H - T_3H)] - 1 \ge 0 \end{cases} \quad (10)$$

IV. NUMERICAL SIMULATION ANALYSIS

A. System Stability Analysis

Due to the nonlinearity of the $It\hat{o}$ stochastic differential equations (SDEs) presented in Equation (8), analytical solutions are generally infeasible [29]. However, the system

can be numerically solved using the Euler explicit forward method, leading to the discretized form:

$$\begin{cases} x_{n+1} = x_n + x(2gV - M - C_1(1-T_1) + 2zQ + \\ \qquad 2T_3Q(1-z))h + \Delta\omega(n)\sigma x(n) \\ y_{n+1} = y_n + y(T_3H - C_2 + T_2C_2 + zH - \\ \qquad zT_3H)h + \Delta\omega(n)\sigma y(n) \\ z_{n+1} = z_n + z(2gW + Q + N - C_3 - T_3(C_3 - \\ \qquad Q) - x(2Q + N - 2T_3Q) - y(H - \\ \qquad T_3H))h + \Delta\omega(n)\sigma z(n) \end{cases} \tag{11}$$

Based on standard modeling practices, the initial strategy adoption probability for all three players is set at 0.5, and the disturbance intensity is set to 0.8. The remaining parameters are selected randomly from parameter sets that satisfy the equilibrium stability conditions in Equation (10). Two example scenarios are considered:

Scenario 1: $T_1 = 0.4$, $T_2 = 0.8$, $T_3 = 0.5$, $g = 1.5$, $V = 4.5$, $M = 3.5$, $C_1 = 35$, C_2=24, $C_3 = 30$, $Q = 6$, $H = 5$, $W = 4.5$, $N = 3$;

Scenario 2: $T_1 = 0.3$, $T_2 = 0.6$, $T_3 = 0.2$, $g = 2$, $V = 1.5$, $M = 2.5$, $C_1 = 11$, $C_2 = 5$, $C_3 = 9$, $Q = 3$, $H = 1$, $W = 1.5$, $N = 2$.

Under a stochastic disturbance environment, Matlab software was utilized to simulate the dynamic evolution of the two scenarios. The simulation outcomes for the two scenarios are illustrated in Figures 1 and 2. These figures reveal that each player adjusted their strategy within the boundary range, with selection probabilities fluctuating around the initial value. This validates the effectiveness of the stability condition described in Equation (10).

Conversely, if the parameter settings fail to meet the conditions specified in Equation (10), the system will exhibit instability under random interference. In Scenario 1, by modifying only specific parameters, namely, $T_1 = 0.5$, $T_2 = 0.9$, and $T_3 = 0.6$, while keeping all other values constant, it becomes evident that Equation (10) is no longer satisfied. As a result, the evolution path of the system becomes unstable, as shown in Figure 3. A similar outcome is observed in Scenario 2 when $T_1 = 0.4$, $T_2 = 0.8$, and $T_3 = 0.6$, while maintaining all other parameters unchanged. This also violates the conditions of Equation (10), resulting in an unstable system state depicted in Figure 4. In summary, both positive and negative verifications of the stability condition in Equation (10) have been provided, thereby confirming its validity [30].

B. The Influence of Online Public Opinion on Strategy Selection

The Internet plays a pivotal role in amplifying public opinion, especially in the self-media era, where individuals act both as content creators and as disseminators of information. This transformation has reshaped the media ecosystem, leading to increased transparency and freedom in public opinion supervision. The emergence of the “Internet Plus” environment has substantially facilitated the high-quality development of elderly care services, while also introducing significant challenges in their supervision. To explore how

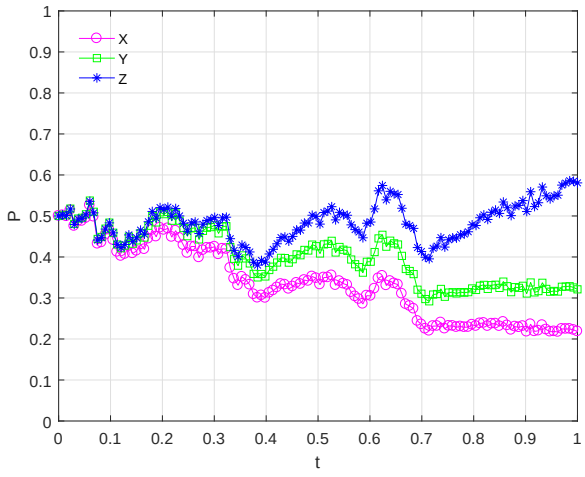

Fig. 1. Dynamic evolution path of stochastic disturbance (Scenario 1)

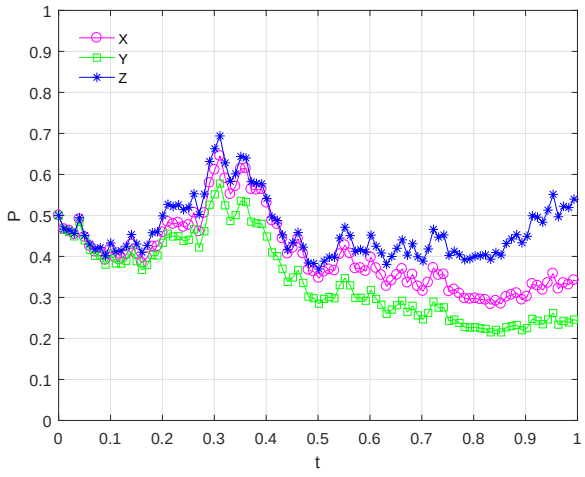

Fig. 2. Dynamic evolution path of stochastic disturbance (Scenario 2)

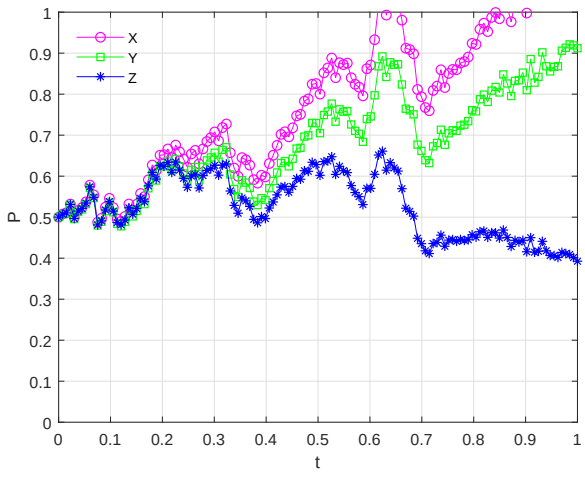

Fig. 3. Dynamic evolution path of stochastic disturbance (Scenario 3)

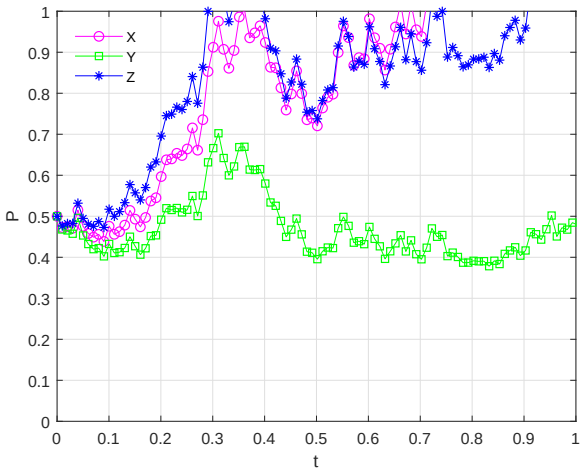

Fig. 4. Dynamic evolution path of stochastic disturbance (Scenario 4)

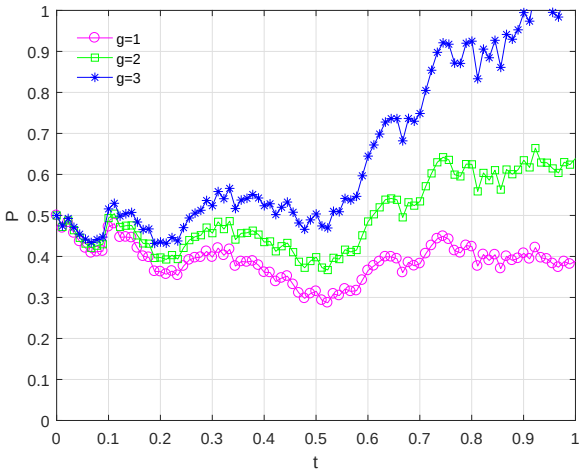

Fig. 5. The influence of online public opinion on elderly care institutions' strategies selection

online public opinion affects strategy choices among different players, a parameter g is introduced. The results, illustrated in Figures 5, 6, and 7, demonstrate how varying levels of online public opinion influence the strategic behavior of each entity. When $g = 1$, representing a moderate impact of public opinion, strategy fluctuations among the players are relatively minor, resembling traditional supervisory behavior. However, as g increases, noticeable shifts occur in the strategic choices of elderly care institutions and government agencies. These shifts are characterized by a growing preference for self-discipline and proactive supervision.

A comparison of Figures 5, 6, and 7 reveals that government agencies, representing public welfare and operating under stringent constraints such as scrutiny and evaluations from higher-level authorities are significantly more sensitive to online public opinion. This sensitivity is particularly pronounced in the context of elderly care services, where public discourse triggered by extreme events or representative cases imposes additional pressure on governmental bodies, prompting a heightened focus on routine supervision. In contrast, elderly residents are relatively less affected by online opinion dynamics. Due to lifestyle patterns or physical limitations, they often lack the necessary conditions, time, or energy to stay informed about online developments. Consequently, their primary concern lies in the direct quality and performance of the elderly care services they receive.

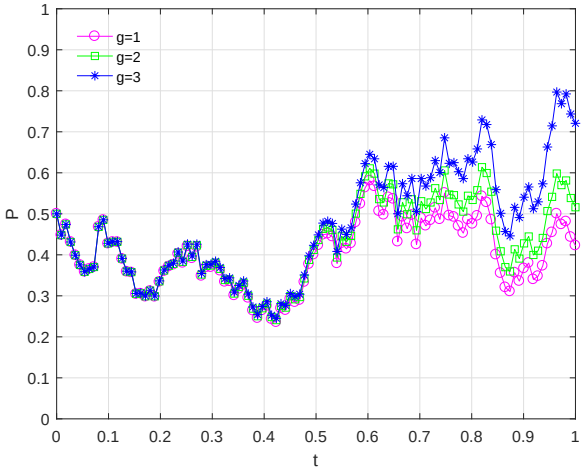

Fig. 6. The influence of online public opinion on elderly residents' strategies selection

C. The Influence of Social Reputation on Strategy Selection

Social reputation exerts a positive influence on both elderly care institutions and government agencies, though the magnitude of its impact varies, as demonstrated by the simulation results in Figures 8, 9, and 10. For elderly care institutions, social reputation serves as a critical indicator of service quality, directly affecting public perception and their capacity to attract elderly residents. As shown in Figure 8, a higher value of parameter V corresponds to a stronger inclination toward the self-discipline strategy. Institutions with favorable social reputations tend to achieve higher occupancy rates and enjoy a positive influx of residents, leading to enhanced social and economic outcomes and promoting sustainable development. Conversely, a poor reputation can significantly hinder an

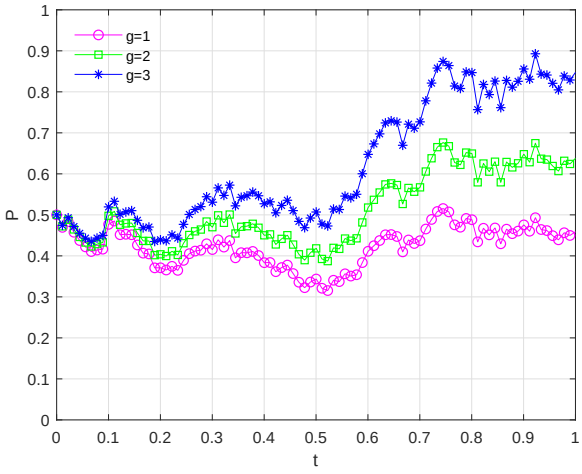

Fig. 7. The influence of online public opinion on governments' strategies selection

institution's operation, resulting in declining occupancy rates and diminished benefits. In extreme cases, adverse incidents such as abuse or neglect can lead to resident attrition and long-term reputational damage. As primary management entities, elderly care institutions are directly accountable for their profits and losses. While they benefit from the social acclaim associated with self-discipline strategies, they also bear the reputational costs of engaging in non-compliant behaviors. Figures 9 and 10 further indicate that variations in parameter V exert limited influence on the strategic decision-making of elderly residents and government agencies. The strategy evolution of these two groups remains relatively stable despite fluctuations in V. Nonetheless, as supervisory bodies, government agencies still exhibit moderate variation in response, reflecting their institutional role in regulation.

Similarly, Figure 11 shows that a higher value of parameter W corresponds to a stronger propensity for active supervision among government departments, resulting in accelerated strategy evolution. Governments prioritize the welfare of the elderly and the expectations of their families and society, maintaining strict oversight of elderly care services. This vigilance not only shields them from administrative penalties due to oversight failures but also helps uphold a positive public image and fosters credibility in governance. As shown in Figure 12, the degree of market standardization in elderly care services is closely tied to the regulatory effectiveness of government agencies. Elderly care institutions, driven by clear profit motives, demonstrate high risk-aversion and heightened sensitivity to changes in parameter W. Government agencies, motivated by societal recognition, are inclined to perform their supervisory roles diligently, thereby strengthening regulatory enforcement. Given the high speculative cost, elderly care institutions tend to favor self-discipline strategies. In cases where inadequate government oversight provokes public dissatisfaction or scrutiny from higher-level authorities, increased regulatory pressure ensues, including intensified inspections and sanctions targeting non-compliance. Anticipating potential income losses, institutions are compelled to adopt more disciplined operational behaviors. As illustrated in Figure 13, the strategic choices of elderly residents exhibit relatively low volatility. Over time, influenced by cultural norms and the maturation of democratic processes, Chinese citizens have developed a strong sense of reliance on government institutions. However, elderly residents tend to display limited initiative for self-supervision, particularly when such activities demand personal time or financial investment.

D. The Influence of Rewards and Punishments on Strategy Selection

As illustrated in Figure 14, simulation results reveal that elderly residents exhibit heightened sensitivity to government-provided incentives for participating in supervision. When the incentive parameter H increases from 1 to 5, the probability of active supervision by elderly residents rises markedly. As direct beneficiaries of elderly care services, residents are well-positioned to detect variations in service quality. Therefore, targeted incentives are critical for encouraging their involvement in supervising institutional self-discipline. Such incentives not only serve as a cost-effective

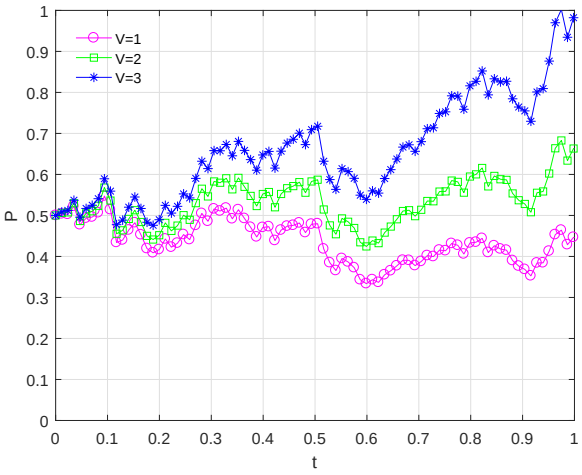

Fig. 8. The influence of social reputation on elderly care institutions' strategies selection

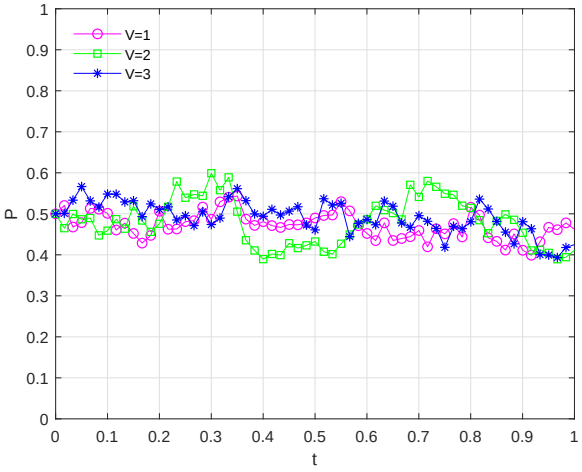

Fig. 9. The influence of social reputation on elderly residents' strategies selection

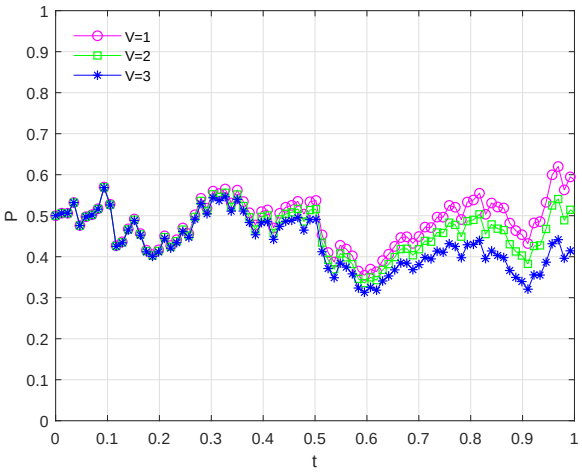

Fig. 10. The influence of social reputation on government agencies' strategies selection

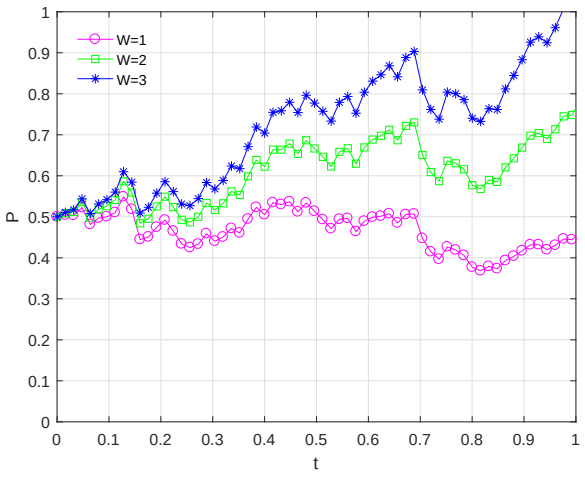

Fig. 11. The influence of online public opinion on government agencies' strategies selection

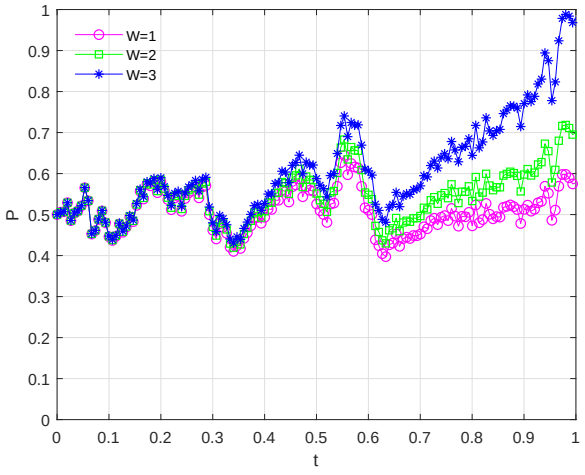

Fig. 12. The influence of online public opinion on elderly care institutions' strategies selection

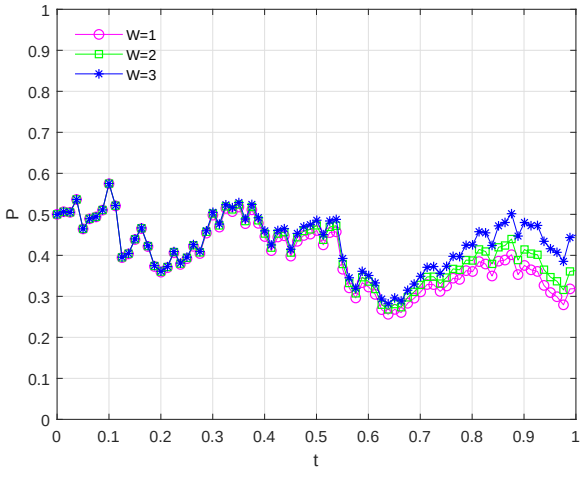

Fig. 13. The influence of online public opinion on elderly residents' strategies selection

strategy for the government to reduce the burden of routine supervision but also promote self-regulation among elderly care institutions. While governmental incentives effectively mobilize elderly residents and enhance the quality of care services, they inevitably lead to increased operational costs. This dynamic underscores the government's willingness to invest in incentivization as a mechanism to protect residents' rights and improve service standards. However, excessive incentive expenditures may conversely reduce the frequency of routine supervision, thereby undermining regulatory efficacy. Figure 15 highlights that the government's behavioral response is minimal at $H = 1$. When $H = 2$, signaling a modest rise in incentive costs, the government responds by intensifying its supervisory efforts and coordinating with elderly residents to raise supervision standards. This cooperation helps maintain order in the elderly care market and enhances the government's social reputation. However, as H continues to increase, escalating incentive costs begin to dampen regulatory enthusiasm, potentially leading to weaker oversight and governance lapses. Moreover, the supervisory performance of local government departments is regularly assessed by higher-level authorities, with implications for policy support, personnel advancement, and financial resource allocation. In a socially complex and sensitive environment, severe incidentsłsuch as elder abuse or bullyingłespecially those that generate online public backlash, are rigorously investigated by superior government bodies. As shown in Figure 16, elderly care institutions remain primarily driven by economic interests. These institutions tend to exhibit a weak sense of self-discipline and often disregard daily supervision by elderly residents. Even under low H values with limited government incentives, institutions are inclined toward speculative behavior. Nonetheless, increased government rewards and stronger supervision by elderly residents can suppress non-compliant behaviors to a certain extent. Despite this, institutions may still make minor strategic adjustments while retaining opportunistic tendencies.

Figure 17 indicates that government departments respond strongly to top-down punitive measures. Higher-level authorities wield significant influence over subordinate departments, especially through performance evaluations and promotion pathways. As the severity of punishment increases, competent local governments tend to adopt stricter regulatory approaches more rapidly, resulting in accelerated strategic evolution. Conversely, Figures 18 and 19 reveal that both elderly care institutions and residents exhibit limited sensitivity to changes in the punishment parameter N. Elderly care institutions, in particular, appear largely indifferent to punitive measures from higher-level government. Their strategic focus remains on short-term benefits, and their commitment to self-discipline is largely unaffected, regardless of punishment intensity. Although elderly residents show relatively greater sensitivity to N, their strategy choices remain constrained by factors such as limited supervisory capacity and weak enforcement mechanisms. As a result, their strategic behavior fluctuates within a narrow range, without significantly influencing the overall evolutionary trajectory.

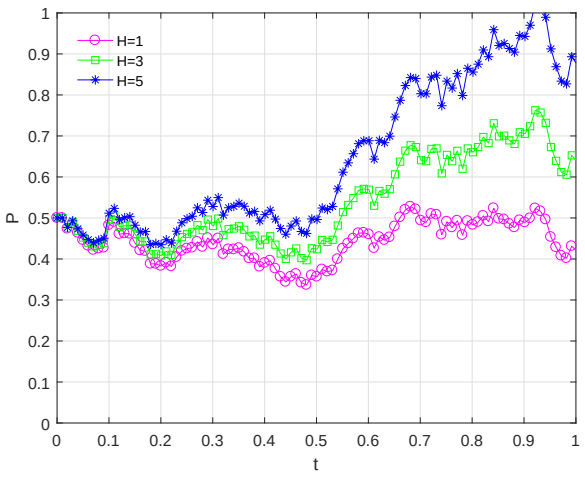

Fig. 14. The influence of rewards on elderly residents' strategies selection

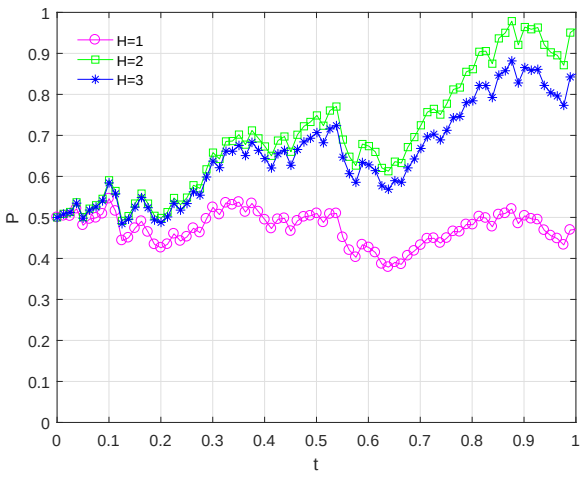

Fig. 15. The influence of rewards on governments' strategies selection

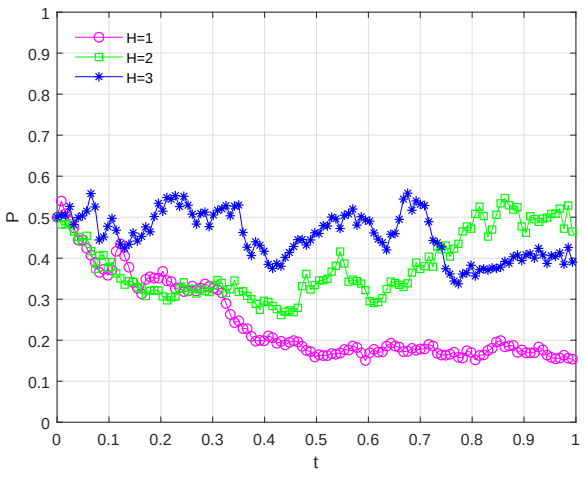

Fig. 16. The influence of rewards on elderly institutions' strategies selection

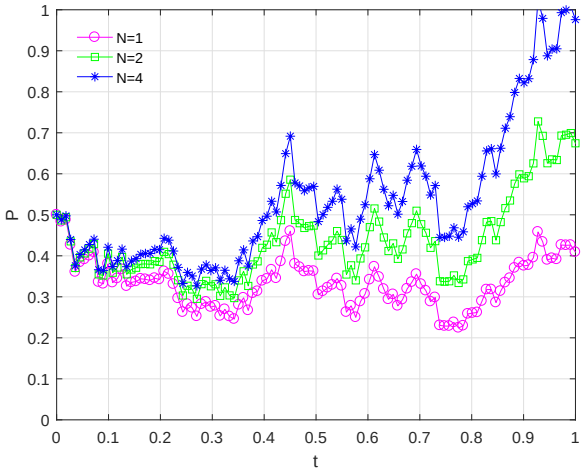

Fig. 17. The influence of rewards on governments' strategies selection from the higher-level government

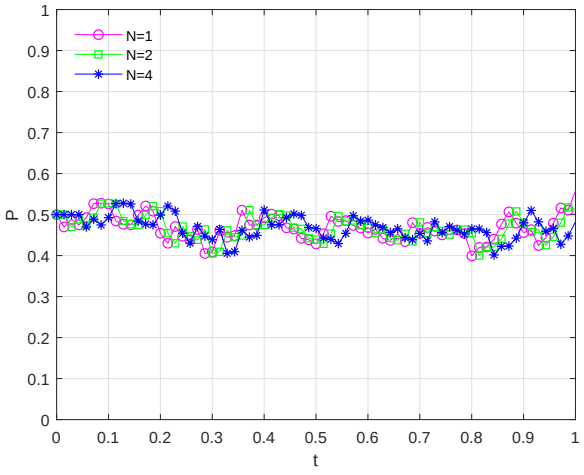

Fig. 18. The influence of rewards on elderly institutions' strategies selection from the higher-level government

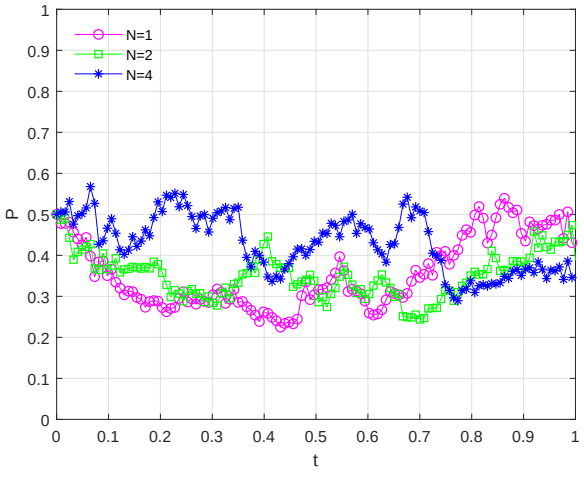

Fig. 19. The influence of rewards on elderly residents' strategies selection from the higher-level government

E. The Influence of Speculative Gains, Incentives, and Punishments on Strategy Selection

Elderly care institutions, as the principal market entities, are inherently profit-driven, seeking to maximize economic returns. Their standard income streams include service fees collected from elderly residents and government-provided subsidies. In addition to these legitimate earnings, institutions may also engage in speculative or illicit activities, such as unauthorized fundraising or investment schemes, including so-called "golden sunset" projects to obtain additional profits. As shown in Figure 20, a clear negative correlation exists between speculative gains and the inclination of institutions to adopt self-discipline strategies. When speculative returns rise, the incentive to maintain regulatory compliance diminishes. Conversely, as government rewards for self-regulating institutions increase, the level of institutional self-discipline improves correspondingly. Figure 21 further reveals that increased speculative behavior within elderly care institutions adversely affects the development of supervision by elderly residents. Currently, China's democratic framework for elderly care oversight faces limitations, including underdeveloped reporting channels and insufficient supervisory mechanisms. These deficiencies allow unchecked speculation in care institutions to undermine the well-being of elderly residents and suppress their capacity for effective oversight. Governmental supervisory behavior evolves distinctly from that of elderly residents, as illustrated in Figure 22. When speculative returns are low, government agencies often adopt a laissez-faire stance. However, as speculative behaviors intensify and their profits rise, government agencies respond with enhanced regulatory efforts. This escalation is typically driven by mounting public opinion pressure and evaluation criteria imposed by higher-level authorities, ultimately promoting the evolution of more stringent regulatory strategies over time.

Moreover, government-provided rewards for self-regulating institutions play a decisive role in fostering institutional compliance. As illustrated in Figure 23, policy incentivesłsuch as tax benefits or direct financial supportłsignificantly promote the adoption of high-quality service practices. In contrast, reductions in such benefits or increased penalties for non-compliant behavior lead to a resurgence of opportunistic and speculative actions. Given the rapidly aging population and the pace of economic and societal transformation, government agencies are required not only to fulfill their supervisory responsibilities but also to meet the rising expectations of the elderly and their families. Figure 24 highlights that competent government departments exhibit a strong willingness to expand incentive programs. By doing so, they aim to stimulate continuous quality improvements among elderly care institutions and drive high-quality development within the sector. These efforts are typically rewarded through favorable social evaluations and enhanced policy and financial support from superior governmental bodies. Such outcomes contribute to the coordinated development of regulatory institutions and the elderly care industry as a whole. In contrast, as shown in Figure 25, elderly residents remain largely indifferent to the variations in government-imposed rewards and punishments targeting elderly care institutions. Even with significant fluctuations in the Q parameter, residents' strategic preferences remain stable, indicating minimal responsiveness to external incentive structures. This observation underscores the limited effectiveness of such policy tools in mobilizing active engagement from elderly residents. Collectively, these findings suggest that while government incentives and punitive measures significantly influence the strategic decisions of elderly care institutions and supervisory departments, they exert minimal impact on the behavior of elderly residents. These insights underscore the need for more targeted and participatory supervisory mechanisms to enhance the overall effectiveness of governance in the elderly care sector.

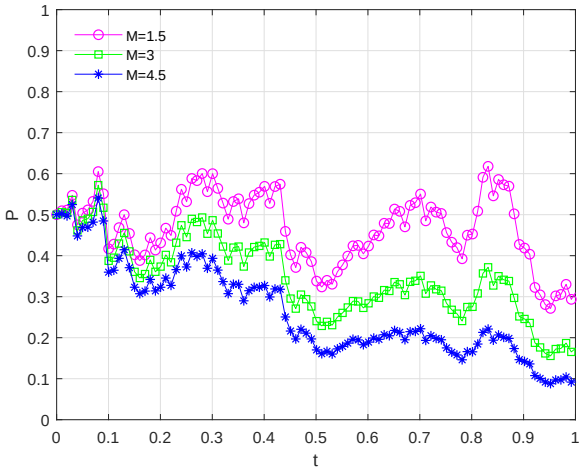

Fig. 20. The influence of speculative gains on elderly care institutions' strategies selection

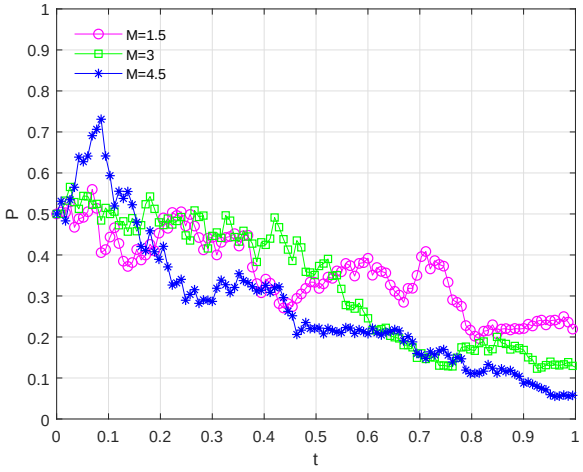

Fig. 21. The influence of speculative gains on elderly residents' strategies selection

F. The Influence of Elderly Residents' Payoff Loss on Strategy Selection

Within the framework of a market economy, the primary objective of elderly care institutions is to pursue and maximize their economic returns. While self-discipline services generate stable and legitimate benefits, non-compliant, speculative behaviors may offer increased short-term profits often

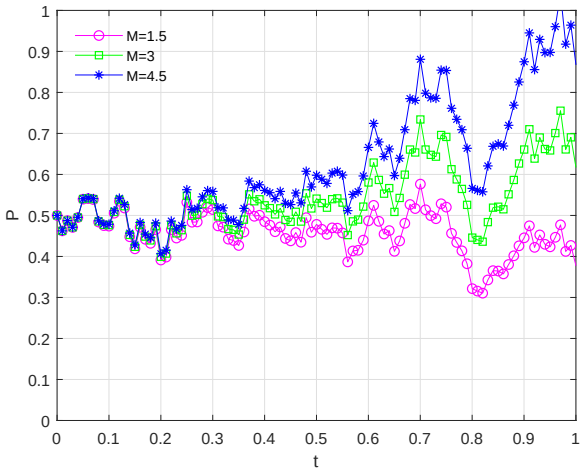

Fig. 22. The influence of speculative gains on governments' strategies selection

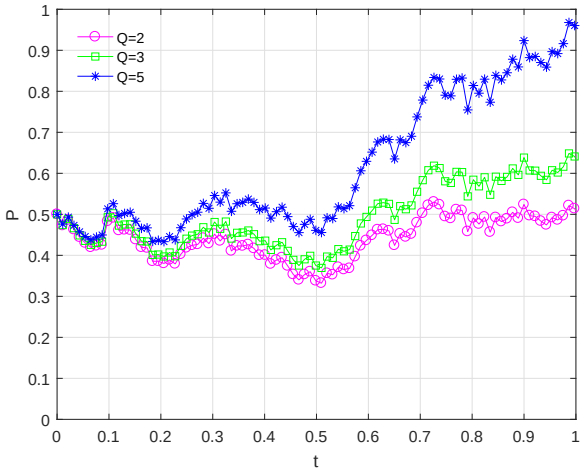

Fig. 23. The influence of incentives and punishments on elderly institutions' strategies selection

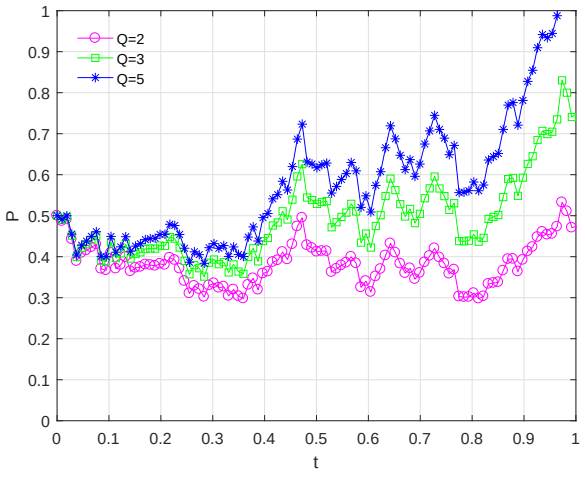

Fig. 24. The influence of incentives and punishments on governments' strategies selection

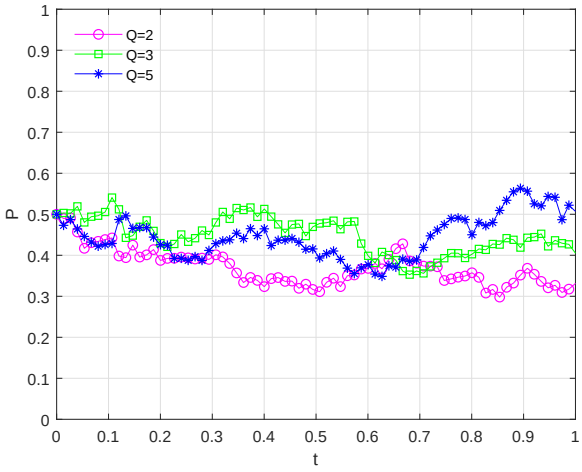

Fig. 25. The influence of incentives and punishments on elderly residents' strategies selection

at the expense of elderly residents' rights and well-being. As the direct recipients of care services, elderly residents are the most vulnerable to harm resulting from such misconduct. Accordingly, the infringement upon their interests serves as a direct and observable measure of institutional behavior. In China, the development of supervisory and management mechanisms within the elderly care sector currently lags behind broader economic development priorities. Given the substantial population of elderly residents and the proliferation of care institutions, speculative practices that compromise the rights of residents have become a prevalent issue. As a socialist country, China holds its government agencies accountable for ensuring social equity and justice. Therefore, any measurable loss experienced by elderly residents should theoretically influence the strategic behavior of all players. However, as demonstrated in Figure 26, even though elderly care institutions provide tangible services, the reduction in elderly residents' payoffs has limited impact on the institutions' strategic orientations. The decision to adopt self-discipline strategies appears to be only marginally affected, and observable changes are primarily attributed to external pressures—such as increased oversight from residents or shifts in governmental supervisory focus.

In contrast, Figure 27 illustrates a more pronounced behavioral response from elderly residents themselves. When the reduction in benefits is minor ($S = 1$ or $S = 2$), residents tend to adopt a tolerant approach, refraining from active resistance or escalation. However, substantial service degradation—resulting in significant personal loss—triggers a shift in strategic behavior. In such cases, elderly residents may engage in face-to-face complaints, formal petitions, or reports to protect their interests. If no corrective measures are taken, switching to alternative care institutions becomes a viable strategy to mitigate ongoing losses. Figure 28 highlights the dynamic response of government agencies to the erosion of elderly residents' welfares. When speculative behavior in elderly care institutions results in visible harm, government departments tend to adopt more proactive supervisory roles to fulfill their regulatory obligations. On the other hand, when the perceived impact on elderly residents' interests is minimal or when elderly care institutions exert undue

influence over regulatory personnel, government intervention may be delayed or weakened. Such regulatory lapses not only compromise social fairness and justice but also erode public confidence in government institutions, damaging their credibility and legitimacy. In summary, the strategic responses of the three main players, elderly care institutions, elderly residents, and government agencies, exhibit varying levels of sensitivity to the loss of resident payoffs. While elderly care institutions show only limited responsiveness, residents and government agencies demonstrate more adaptive behaviors in response to tangible welfare losses. These findings emphasize the importance of strengthening supervisory mechanisms and enhancing institutional accountability to safeguard the interests of the elderly.

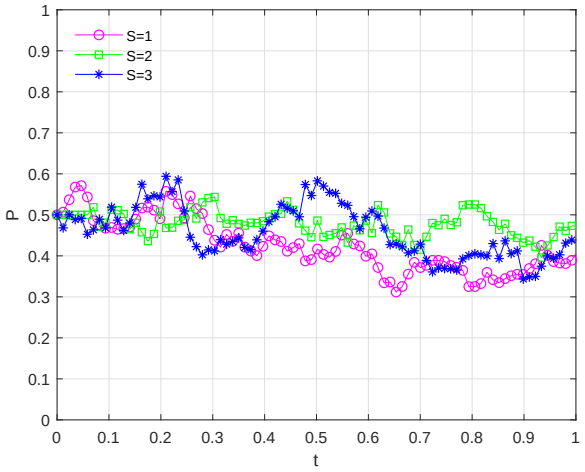

Fig. 26. The influence of elderly residents' payoffs loss on themselves strategies selection

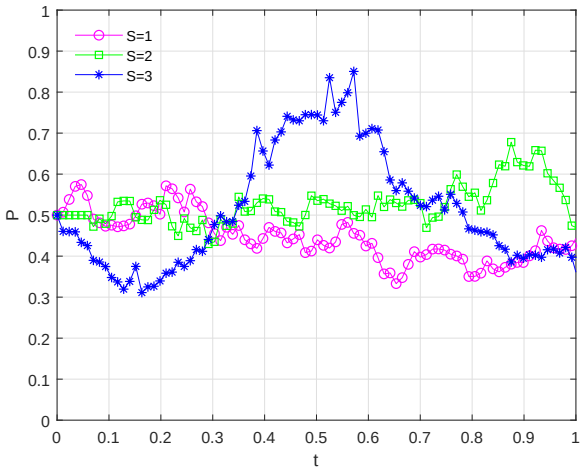

Fig. 27. The influence of elderly residents' payoffs loss on elderly institutions' strategies selection

V. Conclusions

In recent years, the intensification of population aging in China has brought elderly care services to the forefront of public discourse, making them not only a matter of societal concern but also a critical component of public welfare.

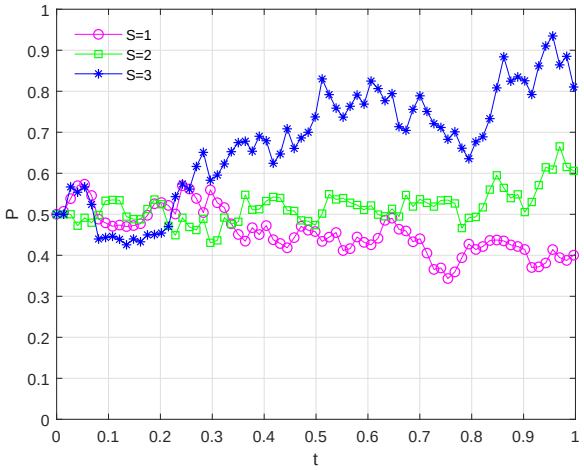

Fig. 28. The influence of elderly residents' payoffs loss on the governments' strategies selection

This study has demonstrated that the strategic choices of the three key actors in the regulatory frameworkłelderly care institutions, elderly residents, and government agencies are interdependent. Through the formulation and implementation of scientifically grounded strategies, the probabilities of institutional self-discipline, active resident supervision, and effective governmental regulation can be significantly enhanced. This, in turn, can elevate the overall quality of elderly care services and foster the high-quality development of the industry.

A. Scientific and Standardized Supervision of Online Platforms

In the digital era, online platforms possess substantial capacity for amplification and information dissemination, presenting both opportunities and challenges for elderly care governance. Elderly care institutions should actively enhance their digital capabilities, showcasing their service offerings and personalized care solutions via internet platforms to attract elderly residents, increase revenue, and build societal credibility. Institutions must also respond promptly to resident feedback to mitigate reputational risks and avoid online public opinion crises. Concurrently, government agencies should establish mechanisms for information screening and guide online platforms to disseminate content in a rational and truthful manner through legal frameworks, regulatory measures, and incentive-penalty systems. These measures help curb misinformation and speculative activities, fostering a positive and trustworthy information environment. Online media, while ensuring compliance with legal and ethical standards, can strengthen credibility by providing objective and professional supervision, thereby supporting the healthy development of the elderly care ecosystem.

B. Proactive Implementation of Self-Discipline by Elderly Care Institutions

To address the diversified needs of the aging population, elderly care institutions must proactively offer standardized, diversified, and specialized services. Enhancing intrinsic motivation for self-discipline and delivering high-quality care

can increase institutional competitiveness and social trust. Institutions should adhere to policy regulations regarding service fees and investment practices, accept oversight from civil affairs, financial, and market regulatory bodies, and avoid opportunistic behaviors that may damage social reputation or economic performance. Furthermore, embracing digital transformation is essential. By developing intelligent care solutions and leveraging information technologies, institutions can improve service quality and efficiency, aligning their operations with the evolving demands of the digital era.

C. Active Participation of Elderly Residents in Supervision

Due to information asymmetry in the elderly care market, external regulators often face high costs in acquiring accurate data. Therefore, encouraging elderly residents and their families to actively participate in the supervision process is critical. Providing appropriate incentives can extend the supervisory chain and enhance regulatory efficiency. Establishing robust channels for feedbackłsuch as online platforms, telephone hotlines, and formal petitionsłalongside effective problem-solving mechanisms, can help safeguard the legitimate rights and interests of the elderly. Optimizing supervision processes empowers residents and cultivates a supportive ecological environment for institutional growth.

D. Governmental Accountability and Regulatory Responsibilities

The government must take a proactive role in overseeing elderly care services by fostering inter-departmental collaboration and constructing a comprehensive regulatory framework. Enhancing coverage and inspection frequency, and offering incentives for institutions that demonstrate exemplary self-discipline, can promote a culture of healthy competition and compliance. Policy tools such as financial subsidies, tax incentives, and preferential loans should be leveraged to reduce operational burdens and stimulate institutional vitality. Additionally, public opinion can be harnessed through media platforms to create a synergistic regulatory effect, reduce enforcement costs, and boost regulatory efficiency. Strategies such as “online exposure” and “whistleblower rewards” should be utilized to motivate public, media, and welfare organizations to report violations, helping eliminate rent-seeking and speculative behaviors and ensuring the integrity of the elderly care market.

E. Coordinated Supervision and Governance of Elderly Care Services

Under the dual pressures of surging demand and limited payment capacities, many care institutions primarily fulfill basic custodial roles. As profit-driven entities, speculative behavior is difficult to eliminate. Elderly residentsłespecially those with cognitive or physical impairmentsłoften passively accept services due to limited alternatives and monitoring capacity. The study findings indicate that elderly residents exhibit low responsiveness to various institutional or policy parameters, placing strong reliance on government oversight. Therefore, government agencies must fulfill their supervisory roles in critical areas such as food safety, healthcare, service pricing, and recreational services. Through the engagement of third-party agencies and intelligent technologies, governments can enhance supervisory effectiveness and promote the sound development of the elderly care market. In doing so, they ensure that the elderly can truly enjoy the fruits of societal progress, live with dignity, and realize the broader social vision of aging in peace and happiness.

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