A Novel Whale Optimization Algorithm Based on Population Diversity Strategy

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***Abstract*—The whale optimization algorithm (WOA) is a meta-heuristic optimization algorithm inspired by the hunting behavior of humpback whales. Due to its simplicity and ease of implementation, WOA has become a popular algorithm for solving global optimization problems. However, like other meta-heuristic optimization algorithms, WOA also has some deficiencies. In order to overcome these shortcomings, a novel whale optimization algorithm based on population diversity strategy (referred to as $NWOA$) is proposed in this paper. This population diversity strategy aims to enhance the exploration property of $NWOA$ and avoid getting trapped in local optima. Numerical experiments demonstrate that the proposed method is capable of producing higher quality solutions.**

***Index Terms*—whale optimization algorithm; population diversity strategy; dynamic selective rule; swarm intelligence**

I. Introduction

ANY animal in nature possesses two fundamental instincts: reproduction and foraging. Reproduction ensures genetic continuity, foraging is essential for the continuation of their existence. Therefore, whether as individual animals or entire populations, having an effective foraging strategy can significantly increase their chances of survival. Different animals employ various foraging methods. By simulating foraging strategy, many meta-heuristic optimization algorithms [1-4] have been proposed.

Humpback whales primarily hunt small fish and shrimp. When they encounter a shoal of fish, humpback whales simply rush into the swarm and swallow the fish and shrimp together with water. However, this hunting method is ineffective when the fish swarm is dispersed. To address this dilemma, humpback whales have developed an effective foraging method known as bubble net feeding. Bubble net feeding can be divided into two categories based on the number of participating whales. The first category is humpback whale gathers in small groups to forage by the aid of "$bubble\ curtain\ wall$". The procedure is that whales continuously release bubbles around the prey. The bubbles form a precise curtain wall without any loopholes. Subsequently, a whale swims out from below of the fish swarm and gobble them down. The whale that is already full will take over his teammate to release bubbles, and teammates will take turns to prey. The other category is that a single whale utilizes a "$spiral\ bubble\ net$" to capture small fish and shrimp.

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By simulating the foraging behavior of humpback whales, Mirjalili et.al proposed a whale optimization algorithm[5] (WOA). During the past years, WOA has demonstrated significant success in solving various types of optimization problems characterized by non-convex, discontinuous and so on. However, similar to other meta-heuristic optimization algorithms, WOA also encounters several challenging issues. For instance, the lack of the balance between the exploration and the exploitation, they tend to struggle with poor convergence when solving complex multimodal function problems. These limitations have restricted the practical applications of WOA. A number of variant WOA algorithms have, hence, been proposed to solve these questions. These improvements can be categorized into two categories: 1) Introduce new solution updating strategy [6-9]. For example, Liu [6] introduced an improved WOA known as $IWOA$, incorporating an equiangular spiral updating strategy and a sound wave attenuation steering law. In the exploitation phase, the equiangular spiral updating equation is used to increase the convergence speed and exploitation ability of the search agent. In the exploration phase, the sound wave attenuation steering law (randomly swim model) is selected to update the position of whale. Chen et.al [8] gives a balanced whale optimization algorithm ($BWOA$) by integrating chaotic local search and l$\tilde{e}$vy flight. Ling [9] presented a comprehensive learning WOA which uses all other particles historical best information to update a whales position. 2) Hybridizaton of $WOAs$ with other swarm intelligence algorithms. For example, Mafarja et.al proposed two hybrid whale optimization algorithms called $WOASA-1$ and $WOASA-2$ in [10]. The objective of $WOASA$ is to effectively integrate SA without compromising their unique characteristics. Numerical experiments have shown that these hybrid wrapper methods can enhance the performance of WOA. These two category variants have clearly boosted the effectiveness of WOA. Nevertheless, there is still room for improvement the exploration capability of the search agent and reducing the risk of getting trapped in local optima. To address these drawbacks, this paper proposes a novel whale optimization algorithm based on population diversity strategy ($NWOA$).

The goal of this paper is to introduce a novel whale optimization algorithm. The key contributions of this algorithm are as follows: 1) Based on the diversity of population, the $NWOA$ algorithm can select a more effective strategy from the randomly search model and the actively model. This enables $NWOA$ algorithm to maintain a balance between exploration and exploitation abilities, thus avoiding search agent getting trapped in local optima. 2) Depending on the number of iterations, $NWOA$ has the capability to dynamically choose between exploitation or exploration equations. This approach allows search agent to explore new territories extensively during the early stages of iteration, while tending to exploit potential locations around g_{best} during later stages. To validate the performance of $NWOA$ algorithm, four engineering design optimization problems were conducted. The corresponding test results were compared with different WOA algorithms under fair and reasonable conditions.

The rest of this paper is organized as follows. Section 2 summarizes WOA algorithm. In Section 3, the $NWOA$ based on population diversity strategy is presented. Section 4 presents and discusses the experimental results. Finally, the conclusion is drawn in Section 5.

II. STANDARD WHALE OPTIMIZATION ALGORITHM

The entire process of WOA consists of three stages: the search for prey (exploration period), the shrinking encircling location update stage, and the logarithmic spiral updating position stage (exploitation period). The updating mechanisms of these three stages are independent, and controlled separately by the random number P and the coefficients vector A. The three updating mechanisms are modeled as follows:

A. Randomly search for prey (exploration period)

Humpback whales use ultrasound to transmit information about fish swarms. By assessing the intensity of ultrasound, whales can estimate the distance to their prey. When ultrasound is received from a distant location ($\vec{A} \geq 1$), the information about fish swarms may be distorted after traveling a long distance, so whales may passively swim towards their prey to begin an exploration period. Initially, whales select a random position from their current population as their target prey and then proceed to encircle this virtual prey in order to update their positions. This behavior can be represented by the following equations:

$$\vec{D} = \left|\vec{C}\vec{X}_{rand} - \vec{X}(t)\right|, \quad \vec{X}(t+1) = \vec{X}_{rand} - \vec{A}\vec{D} \quad (1)$$

Where $\vec{X}_{rand}$ is a random position vector chosen from the current population, $\vec{X}(t)$ is the current position vector, t indicates the current iteration. $\vec{A}$ and $\vec{C}$ are coefficient vectors, and calculated as follows:

$$\vec{A} = 2\vec{a} \cdot \vec{r} - \vec{a}, \quad \vec{C} = 2\vec{r} \quad (2)$$

Where $\vec{a}$ is linearly decreased from 2 to 0, and $\vec{r}$ is a random vector in [0,1].

B. Shrinking encircling location update mechanism

When the ultrasound originates from a nearby location ($|\vec{A}| < 1$), the intensity of the ultrasound may be stronger. So whale will actively swim towards their prey (X_{best}) and begin exploitation period. Initially, whales encircle this optimal location, and then attempt to update their positions towards the X_{best}. The update formula be expressed as follow:

$$\vec{D} = \left|\vec{C}\vec{X}_{best} - \vec{X}(t)\right|, \quad \vec{X}(t+1) = \vec{X}_{best} - \vec{A}\vec{D} \quad (3)$$

Eq.(3) allows search agent to update their position in the neighborhood of the current best solution and simulates encircling the prey.

C. Logarithmic Spiral updating mechanism

In order to mimic the spiral bubble net behavior of whales, the other position updating formula (logarithmic spiral updating mechanism) in exploitation period are designed as follows:

$$\vec{X}(t+1) = \left|\vec{X}_{best}(t) - \vec{X}(t)\right| e^{bl}\cos(2\pi l) + \vec{X}_{best}(t) \quad (4)$$

where $|\vec{X}_{best}(t) - \vec{X}(t)|$ is the distance of the whale to the prey; b is a constant for defining the shape of the logarithmic spiral; l is a random number in [-1,1].

"Shrinking encircling location update mechanism" and "Logarithmic Spiral updating mechanism" both belong to the actively model. Humpback whales use a random number p to control the choice between these two mechanisms during the exploitation period. These steps are repeated until termination criteria satisfy ending of WOA algorithm.

III. NOVEL WOA BASED ON POPULATION DIVERSITY STRAGETY

The random number p is utilized to control the selection update behavior of whale in WOA. When the value of random number p exceeds 0.5, the "Logarithmic spiral updating strategy" is employed to exploit a promising candidate solution. On the other hand, when the value of p is less than 0.5, the "Shrinking encircling location update mechanism" or the "Randomly search for prey rule" is utilized to search unexplored areas. However, it is worth noting that the design of parameter p does not take into account the influence of iteration number and population diversity. Therefore, introducing a quantity to describe their impact would be beneficial in achieving a better balance between the exploitation and exploration within WOA. In contrast, in the $NWOA$ algorithm, parameter p has been designed based on a population diversity index and iteration number. This approach allows for adaptive adjustment of local and global search capabilities to prevent whales from being trapped in local optima. Additionally, it enables whales to explore new virgin territory during early stages of the iteration and exploit potential locations around g_{best} in the later stages. The mathematical model is presented as follows:

A. Population diversity index

In order to adaptively adjust the exploration period (randomly search model) and exploitation period (actively model) of WOA, the diversity of the whole population will be introduced into the WOA algorithm. This incorporation allows the WOA algorithm to maintain a balance between its exploration and exploitation abilities. When the diversity of the whole population is large, humpback whales need to actively exploit potential candidate locations in order to improve convergence speed; on the contrary, they need to opt for the randomly search model in order to explore new territory. This way can increase the diversity of the population and avoid WOA algorithm trapping in local optima. The population diversity index σ is defined as follow:

$$\sigma = \frac{1}{NP \times \|U - L\|} \sum_{i=1}^{NP} \sqrt{\frac{1}{n} \sum_{j=1}^{n} (x_{ij} - \overline{x}_j)^2}, \qquad (5)$$

Where NP is the population size; n is the dimension of the search space; U and L are the upper and lower bound of the search space; $\overline{x}_j$ is the $j-th$ dimension of the mean value of the whole population. Next, Index σ was normalized by the following formula.

$$p = \frac{1}{e^{\sigma}}, \qquad (6)$$

From formula (6), it can be observed that normalized parameter p is distributed within the interval [0,1], with a larger diversity index σ resulting in a smaller normalized parameter p.

B. Novel selection update control mechanism

To compel humpback whales to explore new, untouched territory during the early stages of iteration or in the case of a small population diversity value, and to exploit potential location in later stages of the iteration or in the case of a large population diversity value, an adaptive threshold for selecting control parameter p_o has been designed based on the iteration number as follows:

$$p_o = \frac{it}{Max\ it}, \qquad (7)$$

Adaptive variation of the selecting control parameter p_o and the search vector $|\vec{A}|$ allow humpback whales to smoothly transit between exploration and exploitation. By modifying p_o and $|\vec{A}|$, some iterations are devoted to exploration and the the rest is dedicated to exploitation. In brief, if the normalized parameter p exceeds the threshold of the selected control parameter p_o, the randomly search model may be chosen to update the position of the humpback whales. If $p \le p_o$, and $|\vec{A}| < 1$, the shrinking encircling location update mechanism may be selected to update position of the humpback whales; If $p \le p_o$, and $|\vec{A}| \ge 1$, the logarithmic spiral updating mechanism may be selected to update position of the humpback whales. The selection update control rule is outlined as follows:

$$\vec{X}(t+1) = \begin{cases} \vec{X}_{rand} - \vec{A}\vec{D}, & if\ p > p_o \\ \vec{X}_{best} - \vec{A}\vec{D}, & if\ p \le p_o, |\vec{A}| < 1 \\ \left|\vec{X}_{best} - \vec{X}(t)\right| e^{bl} \cos(2\pi l) + \vec{X}_{best}, & otherwise \end{cases} \qquad (8)$$

Depending on the values of p and $|\vec{A}|$, whales have the ability to switch among a randomly search model, a shrinking encircling location update mechanism, and a logarithmic Spiral updating mechanism. The $NWOA$ algorithm is terminated when a termination criterion is satisfied. After discussing all the components of the $NWOA$ algorithm, the Pseudo-code of the $NWOA$ is shown as follows.

Algorithom 1. NWOA

```
Generate the initial population x_i (i=1,2,...,n).
  Calculate the fitness of each search agent.
  Determine the best solution X_best.
When the stopping criterion is not met do
Update a, A, C, l, p and p_o.
For i = 1 to n do
  if p ≤ p_o
    If |A| < 1
      Generate a new candidate according to (3).
    else if |A| ≥ 1
      Generate a new candidate according to (8).
    End if
  else if p > p_o
    Select a random search agent X_rand,
    Generate a new candidate according to (1).
  End if
End for
Check if any search agent goes beyond the search
  space and amend it.
Calculate the fitness of each search agent and
  update X_best if there is a better solution.
Set iter = iter + 1.
End when
```

IV. EXPERIMENTAL VALIDATION

A. Test Problems and Parameter Settings

To test the performance of the $NWOA$ algorithm, we conducted a series of experiments using four engineering design problems, the pressure vessel design problem, the cantilever beam design problem, the three-bar truss design problem and the speed reducer gearing system design problem. The performance of $NWOA$ algorithm was compared to that of the standard WOA[5], $IWOA$[6], $HMNWOA$[7], PSO[11], DE[12], GA[13], IHS[14] and ES[15] in terms of finding the best solution. In order to ensure a fair comparison, each engineering design problem was tested 30 runs, and the best solution from the statistical experimental data was reported. Additionally, all algorithms were implemented in Matlab 7.0 and simulations were run on Windows 10 with Intel (R) Core i7-4790 CPU @3.6GHz with 8GB memory capacity.

B. Performance of IWOA on engineering design problems

The first test function is a pressure vessel design problem which is characterized by non-linear and complex constraints. This problem serves as a well-established benchmark for validating meta-heuristic optimization algorithms. It involves four design variables: shell thickness (T_s), heads thickness (T_h), inner radius (R) and cylindrical section length (L). The primary objective is to minimize overall costs encompassing material expenses, manufacturing outlays, and welding expenditures, while adhering to nonlinear of stress and yield criteria constraints. This problem can be modeled as follows:

$$\min f = 0.6224 T_s R L + 1.7781 R^2 T_h + 3.1661 T_s^2 L + 19.84 T_s^2 R$$

subject to

$$-T_s + 0.0193R \leq 0,$$
$$-T_h + 0.00954R \leq 0,$$
$$-\pi LR^2 - \frac{4}{3}\pi R^3 + 1296000 \leq 0,$$
$$-240 + L \leq 0,$$

where

$$0.0625 \leq T_s, T_h \leq 99 \times 0.0625,\ 10 \leq R, L \leq 200,$$

The optimum solution obtained by $NWOA$ algorithm is compared with design results derived from PSO, GA, DE, WOA, IHS, ES, $IWOA$, $HMNWOA$ in Table 1. The comparative analysis reveals that $NWOA$ surpassed all other methods. The result of comparison demonstrates that population diversity strategy enables humpback whales to generate superior diversity, avoid search agent being trapped in local optimal positions, and increase exploration ability of $NWOA$ to an extent. Consequently, $NWOA$ can enhances search precision, reduces the number of failed search procedures and accelerates the discovery of improved solutions.

The second test function pertains to a cantilever beam design problem, with the objective to minimize the weight of cantilever beams. A cantilever beam is composed of five hollow blocks, the design variable x_1, x_2, x_3, x_4 and x_5 represent the length of each hollow block respectively. The mathematical expression governing this problem and its vertical displacement constraint are described by the following equation:

$$\min f = 0.6224(x_1 + x_2 + x_3 + x_4 + x_5)$$

subject to

$$\frac{61}{x_1^3} + \frac{27}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \leq 0,$$

where

$$0.01 \leq x_1, x_2, x_3, x_4, x_5 \leq 100,$$

This problem has been optimized by various methods such as PSO, GA, DE, WOA, $IWOA$, $HMNWOA$. The optimum solutions obtained by different algorithms are presented in Table 2. It can be observed from Table 2 that the solution obtained by $NWOA$ yields the best result among all these algorithms, with a value of 13.032661.

The third test function pertains to the three-bar truss design problem. This particular problem is formulated to determine the minimum volume of a three-bar truss through adjustments in the cross-sectional areas. It involves a nonlinear objective function and three inequality constraints aimed at limiting stress, deflection, and buckling, respectively. The problem can be represented as follows:

$$\min f = (2\sqrt{2x_1} + x_2)l$$

subject to

$$\frac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1x_2}p - \sigma \leq 0,$$
$$\frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2}p - \sigma \leq 0,$$
$$\frac{1}{\sqrt{2}x_2 + x_1}p - \sigma \leq 0,$$

where

$$0 \leq x_1, x_2 \leq 1, l = 100cm, p = 2kN, \sigma = 2kN/cm^2$$

where variables x_1, x_2 consisting of the cross-sectional areas of the two members. This problem has been optimized utilizing various methods such as PSO, GA, DE, WOA, $IWOA$, $HMNWOA$. The optimum solutions obtained by these algorithms are listed in Table 3. It is evident from Table 3 that $NWOA$ yielded the best value of 263.89584 (corresponding to $x_1 = 0.7886765$ and $x_2 = 0.4082443$), which significantly outperformed the results obtained by other algorithms used in this study.

The final test function pertains to the design problem of the speed reducer gearing system. The main objective is to minimize the weight of the speed reducer, which involves seven variables: face width x_1, module of teeth x_2, number of teeth on pinion x_3, length of first shaft between bearings x_4, length of second shaft between bearings x_5, diameter of first shaft x_6, and diameter of second shaft x_7. Variable x_3 is an integer value while the remaining variables are continuous. There are 11 constraint conditions in this problem primarily based on bending stress of the gear teeth, transverse deflections of the shafts, surface stress and stresses in the shafts. This problem can be formulated as follows:

$$\min f = 0.7854x_1x_2^2(3.33332x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2)$$

subject to

$$\frac{27}{x_1x_2^2x_3} - 1 \leq 0; \quad \frac{397.5}{x_1x_2^2x_3^2} - 1 \leq 0; \quad \frac{1.93x_4^3}{x_2x_6^4x_3} - 1 \leq 0;$$
$$\frac{1.93x_5^3}{x_2x_7^4x_3} - 1 \leq 0; \quad \frac{\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 16.9 \times 10^6}}{110x_6^3} - 1 \leq 0,$$
$$\frac{\sqrt{\left(\frac{745x_5}{x_2x_3}\right)^2 + 157.5 \times 10^6}}{85x_7^3} - 1 \leq 0, \quad \frac{x_2x_3}{40} - 1 \leq 0;$$
$$\frac{5x_2}{x_1} - 1 \leq 0; \quad \frac{x_1}{12x_2} - 1 \leq 0; \quad \frac{1.5x_6 + 1.9}{x_4} - 1 \leq 0;$$
$$\frac{1.1x_7 + 1.9}{x_5} - 1 \leq 0;$$

where

$$2.6 \leq x_1 \leq 3.6, \quad 0.7 \leq x_2 \leq 0.8, \quad 17 \leq x_3 \leq 28,$$
$$7.3 \leq x_4, x_5 \leq 8.3, \quad 2.9 \leq x_6 \leq 3.9, \quad 5 \leq x_7 \leq 5.5,$$

Table 4 presents the performance comparison results of the $NWOA$ with other optimization algorithms for the speed reducer gearing system design problem. In terms of the minimum function value f(x), the $NWOA$ achieved an optimization result of f(x) = 2994.471066 (when x_1, x_2, x_3, x_4, x_5, x_6 and x_7 are 3.5, 0.7, 17, 7.3, 7.715319, 3.350214 and 5.286654, respectively). This result is significantly superior to those obtained by the WOA, $IWOA$ and $HMNWOA$ algorithms.

In conclusion, it can be inferred that the $NWOA$ algorithm demonstrates superior performance in nearly all cases, yielding highly accurate solutions that closely approximate the optimal values across four engineering design problems. These results indicate that the innovative select control mechanism enables the $NWOA$ algorithm to escape local optima and ultimately converge on global optimum values. This is attributed to the incorporation of iteration

TABLE I: Optimum designs obtained by different algorithms for PV design problem.

Algorithm	Optimum Variables				Optimum	Feasible
	T_s	T_h	R	L	cost	solution
PSO	0.812500	0.437500	42.091266	176.746500	6061.0777	Y
GA	0.812500	0.437500	40.323900	200.000000	6288.7445	Y
DE	0.812500	0.437500	42.098411	176.637690	6059.7340	Y
ES	0.812500	0.437500	42.098370	176.637146	6059.7144	N
IHS	1.125000	0.625000	58.290150	43.6926800	7197.7300	N
WOA	0.823751	0.411093	42.611010	170.458911	5991.0648	Y
$NWOA$	0.812351	0.401546	42.090739	176.732115	5946.3657	Y
$BWOA$	1.258663	0.621865	65.179120	10.1987370	7318.0940	Y
$IWOA$	0.812361	0.401551	42.091257	176.725695	5946.3845	Y

TABLE II: Optimum designs obtained by different algorithms for cantilever beam design problem.

Algorithm	Optimum Variables					Optimum
	x_1	x_2	x_3	x_4	x_5	cost
PSO	5.935478	4.897605	4.495612	3.509344	2.103976	13.034310
GA	6.005523	5.305914	4.494750	3.513362	2.154234	13.365283
DE	6.027437	5.338575	4.490487	3.483437	2.134591	13.365745
WOA	6.042106	5.337772	4.472002	3.481961	2.140922	13.365892
$NWOA$	5.970552	4.872109	4.480494	3.486662	2.129548	13.032661
$BWOA$	5.951330	4.921423	4.558141	3.426154	2.090774	13.037924
$IWOA$	5.952851	4.902563	4.468554	3.461453	2.154621	13.033082

TABLE III: Optimum designs obtained by different algorithms for three-bar truss design problem.

Optimum variables	PSO	GA	DE	WOA	NWOA	HMNWOA	IWOA
x_1	0.801109	0.765336	0.770865	0.761453	0.788676	0.787317	0.797544
x_2	0.374171	0.482961	0.461118	0.491257	0.408244	0.412104	0.383725
$f(x)$	264.00494	264.76810	264.14535	264.49713	263.89584	263.89728	263.95201

TABLE IV: Optimum designs obtained by different algorithms for speed reducer gearing system design problem.

Optimum variables	PSO	GA	DE	WOA	NWOA	HMNWOA	IWOA
x_1	3.500000	3.500310	3.500022	3.500000	3.500000	3.500000	3.500000
x_2	0.700000	0.700045	0.700000	0.700000	0.700000	0.700000	0.700000
x_3	17.000000	17.000000	17.000012	17.000000	17.000000	17.000000	17.000000
x_4	8.211665	8.100158	7.300387	7.594086	7.300000	7.350422	7.300041
x_5	7.940123	7.815244	7.758575	7.913427	7.715319	7.865377	7.794689
x_6	3.361541	3.696135	3.713765	3.367444	3.350214	3.360230	3.350814
x_7	5.288635	5.374563	5.367734	5.347213	5.286654	5.326663	5.292783
$f(x)$	3011.655103	3159.928998	3153.879597	3044.841142	2994.471066	3026.428648	3000.268034

number and population diversity within the design process of parameter p. Specifically, this mechanism can enforce humpback whales to explore different regions and generate a range of high-quality solutions during the initial stages of iteration, thereby avoiding search agent being trapped in local optimal positions. In later stages of iteration, this mechanism guides humpback whales to exploit potential locations surrounding superior individuals produced in various generations, thereby enhancing exploitation and convergence capabilities to some extent. Ultimately, it can be concluded that the novel $NWOA$ algorithm can achieve an appropriate balance between exploration and exploitation, and find a better solution at a higher speed.

V. CONCLUSIONS

In this paper, a novel whale optimization algorithm based on population diversity strategy ($NWOA$) is proposed. The parameter p, which controls the selection update behavior of humpback whale, has been designed using a population diversity strategy and iteration number. This strategy allows for adaptive adjustment of the local and global search capabilities of humpback whales, encouraging exploration of new territory when the diversity index is small, and exploitation of potential locations around superior individuals when the diversity index is large. The performance of the $NWOA$ algorithm was compared with that of other evolutionary algorithms, using various engineering design problems. Numerical experiments demonstrate that the proposed method is capable of producing higher quality solutions.

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