Stochastic Nonlinear Systems Adaptive Neural Network Control Based on Observer with Incomplete Measurements

Jiaxin Yang, Chuang Gao*, and Zhongfeng Li*

Abstract—**An adaptive neural network control approach is proposed for a class of strict-feedback nonlinear stochastic systems with incomplete measurements. The systems under consideration experience data loss or transmission medium saturation, leading to incomplete measurements during data transmission. Two distinct control laws, based on state observers, are designed to handle different transmission scenarios using the backstepping method. Neural network approximation is employed to estimate the unknown functions. To address the complexity explosion in traditional control design, dynamic surface control (DSC) is utilized. The stability of the stochastic system is analyzed, and conditions for uniform ultimate boundedness in the mean square sense are derived. Experimental results confirm the reliability of the proposed controller.**

Index Terms—**adaptive neural control, stochastic nonlinear system, incomplete measurement, dynamic surface control.**

I. Introduction

In recent decades, advancements in the Internet of Things (IoT), communication technologies, and electronics have spurred significant growth in control systems. This evolution has led to both increased system complexity and the development of advanced control techniques. Control systems have transitioned from managing simple, isolated entities to handling complex, interconnected systems consisting of multiple subsystems. The emergence of Cyber-Physical Systems (CPSs) has garnered particular interest among researchers [1, 2]. CPSs are sophisticated systems that enable the interconnection of previously isolated environments, greatly improving system efficiency and data transmission capabilities. These systems find applications across various sectors, including remote patient monitoring, intelligent transportation, smart grids, smart factories, and connected vehicles [3, 4]. However, the interaction between subsystems brings critical challenges related to state estimation and stability, which are essential for the practical deployment of CPSs.

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In a CPS, subsystems communicate and exchange information via networks to enhance the overall system intelligence. As such, the quality of communication plays a pivotal role in the reliability and stability of the system. In particular, limitations in communication networks, particularly wireless channels, can result in issues such as state loss, quantization errors, and delays [5, 6]. These challenges can significantly affect state estimation, control, and overall system stability. Since state estimation relies on sensor data, much research has focused on developing robust state estimation techniques and control strategies to handle incomplete observations. Some studies have addressed controller design for CPSs under data loss conditions [7, 8]. Furthermore, Lu et al. [9] proposed a state estimation approach for incomplete state observations under network attacks, enhancing the overall estimation performance. Lu et al. [10] also addressed the event-triggered control problem in CPSs subjected to actuator attacks, incorporating a widening matrix of state and attack data with a Luenberger observer and controller[11]. Research by Han et al. [12] and Fei et al. [13] has focused on controlling systems under spoofing or model attacks.

Stochastic disturbances, which are inherent in practical systems, are often a source of instability. Consequently, controller design for stochastic systems has become an essential area of research. For instance, Han et al. [12] studied uncertain stochastic strict-feedback systems and proposed a fuzzy controller. Wang et al. [14] tackled nonlinear systems with input saturation, while the multi-objective H_2/H_∞ control problem for uncertain systems with state time lags was also explored.

Nonlinearity is a common characteristic in real-world systems, and designing controllers for nonlinear systems remains a challenging task. As a result, the control and state estimation of nonlinear CPSs have become important research topics. Li et al. [15] studied nonlinear CPSs under denial-of-service (DoS) attacks and proposed fuzzy systems for approximation, along with a periodic event-triggered control strategy to enhance communication efficiency. Zhang et al. [16] developed a state estimation filter for incomplete measurements, while Ma et al. [17] proposed a model-free control framework for systems under DoS attacks. Additionally, Liu et al. [18] designed two state estimators for nonlinear CPSs with incomplete measurements. Jiang et al. [19] focused on state estimation for stochastic systems under random packet loss, and An et al. [20] explored decentralized control for nonlinear CPSs subjected to intermittent DoS attacks, proposing a switch-type state estimator and convex design conditions for back-stepping controllers based on fuzzy logic approximation.

Large-scale systems often face additional challenges due to the time-sharing nature of communication channels, where nodes may not be able to access the network synchronously. In cognitive radio systems, for example, primary users occupy most of the communication bandwidth, with secondary users only able to access the network when the primary user releases the channel [21]. These communication constraints significantly impact control performance. Zhang et al. [22] employed fuzzy models to approximate nonlinear processes measured by wireless sensors, considering various communication constraints and providing sufficient conditions. However, system instability may occur if information collected prior to packet loss or network attacks is not appropriately managed. Thus, designing a state estimator capable of handling network failures is crucial.

This paper investigates the control of stochastic systems subject to deteriorated information transmission. Specifically, observers and controllers are designed to address both normal communication and blocked communication scenarios (i.e., data loss). The proposed design ensures that all system signals remain bounded in the mean square sense. In cases of data loss, the controller utilizes available system output information during normal communication to enhance system performance. The contributions of this work are summarized as follows:

- Incomplete measurements due to transmission issues can lead to instability in random CPSs. Observers are designed for two scenarios: the normal case where system output information is available, and the data loss case where transmission saturation and data loss occur. Output feedback controllers are constructed for both cases, differing from Long et al. [23], which only addresses normal data transmission. Probabilistic stability conditions are analyzed for both cases.
- Dynamic surface control methods are utilized in the controller design for stochastic nonlinear systems with incomplete measurements. These methods prevent computational issues due to differential explosion in the design process. This not only reduces computational complexity but also simplifies the design.
- The adaptive radial basis function neural network control method is adopted, requiring only one adaptive parameter, significantly reducing the computational workload.

The structure of the paper is organized as follows: Section 2 presents the system model and discusses transmission scenarios. Section 3 addresses the state observers, system controllers, and the associated stability analysis. Experimental results to demonstrate the effectiveness of the designed controllers are provided in Section 4. Section 5 summarizes the paper and discusses potential future work.

II. System Description and Preliminaries

A strict-feedback nonlinear stochastic CPS can be described as:

$$\begin{aligned} dx_i(t) &= \left(f_i\left(\bar{x}_i(t)\right) + x_{i+1}\right)dt + \varphi_i(x_1)dw(t), \\ dx_n(t) &= \left(f_n\left(\bar{x}_n(t)\right) + u\right)dt + \varphi_n(x_1)dw(t), \end{aligned} \quad (1)$$

where the system's output is given by:

$$y = \rho_1 x_1 + \rho_2 \sigma(x_1) + \rho_3 \varsigma(x_1). \quad (2)$$

where, $\bar{x}_i(t) = [x_1(t), \cdots, x_i(t)]^T \in \mathbb{R}^i$ for $i = 1, 2, \cdots, n$, and $u, y \in \mathbb{R}$ represent the state vector, control input, and output, respectively. The functions $f_i(\cdot) : \mathbb{R}^i \to \mathbb{R}$ and $\varphi_i(\cdot) : \mathbb{R}^i \to \mathbb{R}^{1\times r}$ are unknown smooth functions with $f_i(0) = 0$ and $\varphi_i(0) = 0$. Additionally, we assume that $\varphi_i(x_1) = x_1\psi_i(x_1)$, where $\psi_i(x_1)$ denotes a known smooth function, and

$$\begin{aligned} \varphi(x_1) &= \left[\varphi_1^T(x_1), \ldots, \varphi_n^T(x_1)\right]^T, \\ \psi(x_1) &= \left[\psi_1^T(x_1), \ldots, \psi_n^T(x_1)\right]^T. \end{aligned} \quad (3)$$

where, $w(t) \in \mathbb{R}^r$ represents random thermal motion. The functions $\sigma(\cdot)$ and $\varsigma(\cdot)$ correspond to saturation and data-loss scenarios, respectively. The parameters ρ_1, ρ_2, ρ_3 represent different system states, where each $\rho_i \in \{0, 1\}$ and $\sum_{i=1}^{3} \rho_i = 1$. Specifically, $\rho_1 = 1$ indicates normal measurement, $\rho_2 = 1$ denotes saturation, and $\rho_3 = 1$ corresponds to a data-loss scenario.

If the output measurement x_1 satisfies $|x_1| < x_{\max}$, where $x_{\max}$ is the saturation threshold, then $\rho_1 = 1$, and $\rho_2 = \rho_3 = 0$. If $|x_1| > x_{\max}$, we have $\rho_2 = 1$, with $\rho_1 = \rho_3 = 0$. The saturation behavior is described as:

$$\delta(x_1) = x_{\max} \cdot \mathrm{sign}(x_1).$$

In the event of data loss or sensor failure (e.g., during a system attack), $\rho_3 = 1$, with $\rho_1 = \rho_2 = 0$. The lost data is substituted by the most recent acquisition state. Therefore, the data-loss function is given by:

$$\varsigma(x_1) = x_1',$$

where x_1' represents the most recent observed system output. This work assumes that the duration of the data-loss situation is known, so x_1' remains available.

Remark 1: Packet loss can occur for various reasons, such as wireless network limitations or time-division multiplexing in communication channels. Additionally, communication networks may be susceptible to attacks (e.g., denial-of-service), resulting in damaged or unavailable data. These scenarios are categorized as data-loss cases. In such instances, some existing works (e.g., [20]) discard the data and assume zero observations, which can degrade state estimation performance. In contrast, we use the last valid observation to replace the lost data, mitigating the impact of data loss.

Next, we consider a general stochastic nonlinear system:

$$dX(t) = f(X)dt + \varphi(X)dw(t), \quad (4)$$

where $X \in \mathbb{R}^n$, and f and φ are Lipschitz continuous functions.

Definition 1: Given $V(X) \in C^2$, associated with the stochastic system described by (4), the differentiator L is defined as:

$$LV(X) = \frac{\partial V}{\partial x} f(X) + \frac{1}{2}\text{tr}\left(\varphi^T(X)\frac{\partial^2 V}{\partial x^2}\varphi(X)\right). \quad (5)$$

Remark 2 [24]: The term $\frac{1}{2}\text{tr}\left(h^T \frac{\partial^2 V}{\partial X^2} h\right)$ is referred to as the Ito correction term. The presence of the second-order differential $\frac{\partial^2 V}{\partial X^2}$ complicates controller design, making it more challenging compared to the deterministic case.

To design a stable controller for the system, we introduce the following assumptions:

Assumption 1: Given known constants m_i and μ, for all $X_1, X_2 \in \mathbb{R}^n$, the following inequalities hold:

$$\begin{aligned} \|f_i(X_1) - f_i(X_2)\| &\leq m_i \|X_1 - X_2\|, \\ \|\varphi(X_1) - \varphi(X_2)\| &\leq \mu \|X_1 - X_2\|. \end{aligned} \quad (6)$$

Lemma 1: For all $(x, y) \in \mathbb{R}^2$, the following inequality holds:

$$xy \leq \frac{\varepsilon^p}{p}|x|^p + \frac{1}{q\varepsilon^p}|y|^p,$$

where $\varepsilon > 0$, $p > 1$, $q > 1$, and $(p-1)(q-1) = 1$.

III. Controller Design and Stability Analysis

This section presents an estimator-based adaptive control scheme for system (1) using the backstepping method. Since only the system's output in its nominal state is measurable, a state observer is needed for controller design. Note that the system output can be affected by two phenomena: device saturation and data loss. Under saturation, the output is limited to the saturation value $\sigma(x_1)$, which represents the last reliable measurement before saturation. In the event of data loss, the most recent valid output is used as a replacement for the lost data.

Both saturation and packet loss can be considered instances of a "data-loss" scenario, where the current observation is replaced with the last valid one. From the controller design perspective, these two phenomena can be treated uniformly as the data-loss case.

A. State estimator and dynamic feedback design in normal case

In the normal case, where output data is available, a state estimator can be developed based on the output y. The state estimator for system (1) is given by:

$$\begin{aligned} \dot{\hat{x}}_{ci} &= \hat{x}_{c(i+1)} + k_{ci}(y - \hat{x}_{c1}), \\ \dot{\hat{x}}_{cn} &= u_c + k_{cn}(y - \hat{x}_{c1}), \end{aligned} \quad (7)$$

where $\hat{x}_{ci}$ denotes the estimate of the state x_i, and k_{ci} is a design parameter.

Define the estimation error in the normal case as $e_{ci} = x_i - \hat{x}_{ci}$ for $i = 1, \ldots, n$. The overall estimation error vector is:

$$e_c = \bar{x}_n - \overline{\hat{x}}_{cn}, \quad (8)$$

where $\bar{x}_n = [x_1, \ldots, x_n]^T$ and $\overline{\hat{x}}_{cn} = [\hat{x}_{c1}, \ldots, \hat{x}_{cn}]^T \in \mathbb{R}^n$, with $e_{c1} = x_1 - \hat{x}_{c1}$.

Using equations (1) and (7), the time derivative of the estimation error is:

$$\begin{aligned} de_c &= (Ae_c + F(\bar{x}(t)) - K_c e_{c1})dt + \varphi^T(x_1)dw(t), \\ &= ((A - K_c C)e_c + F(\bar{x}(t)))dt + \varphi^T(x_1)dw(t) \end{aligned} \quad (9)$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & & & \\ \vdots & & I_{n-1} & \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \\ F(\bar{x}(t)) &= [f_1(x_1), \ldots, f_n(x_n)]^T, \\ K_c &= [k_{c1}, \ldots, k_{cn}]^T, \\ C &= [1, 0, \ldots, 0]. \end{aligned}$$

Note: Ensure that the matrix $A - K_c C$ is Hurwitz by selecting appropriate gains k_{ci} to guarantee exponential stability of the estimation error in the mean-square sense.

To ensure system stability, $(A - K_c C)$ must be a strict Hurwitz matrix, which can be achieved by selecting appropriate gains K_c.

Define $\lambda = \lambda_{\min}(P)\lambda_{\min}(Q_c)$, where $Q_c = -(P(A - K_c C) + (A - K_c C)^T P)$. This leads to the inequality:

$$e_c^T P e_c \left[e_c^T \left(P(A - K_c C) + (A - K_c C)^T P\right) e_c\right] \leq -\lambda_c \|e_c\|^4 \quad (10)$$

where Q_c and P are symmetric positive definite matrices.

Finally, the dynamics of the entire system, incorporating both the estimation error and the state estimate, are described by:

$$\begin{aligned} de_c &= ((A - K_c C)e_c + F_c(\bar{x}(t)))dt + \varphi^T(x)dw(t), \\ d\hat{x}_{c1} &= (\hat{x}_{c2} + k_{c1}e_{c1})dt, \\ d\hat{x}_{c2} &= (\hat{x}_{c3} + k_{c2}e_{c1})dt, \\ &\vdots, \\ d\hat{x}_{cn} &= (u_c + k_{cn}e_{c1})dt. \end{aligned} \quad (11)$$

The backstepping procedure comprises n steps, with each step involving the design of a virtual control function $\hat{\alpha}_{ci}$ based on an appropriate Lyapunov function. The actual control law u_c is then formulated. To start the backstepping design, define the parameter θ_c as:

$$\theta_c = \max\left\{N_{ci}\|W_{ci}^*\|^2 : i = 0, 1, 2, \ldots, n\right\}.$$

Since $\|W_{ci}^*\|$ is unknown, θ_c is also unknown. Define the parameter error as $\widetilde{\theta}_c = \theta_c - \hat{\theta}_c$, where $\hat{\theta}_c$ is the estimate of θ_c. The virtual control signal is defined as:

$$\alpha_{ci}(X_{ci}) = -\frac{1}{2a_{ci}^2}Z_{ci}^3\hat{\theta}_c, \quad i = 1, \ldots, n-1, \quad (12)$$

where $Z_{c1} = x_1$, and $Z_{ci} = \hat{x}_{ci} - \alpha_{cif}$ for $i = 2, \ldots, n$. The design parameters satisfy $a_{ci} > 0$. The state vector is given by $X_{c1} = x_1$, and $X_{ci} = (e_{c1}, \overline{\hat{x}}_{ci}, \bar{\alpha}_{cif}, \dot{\alpha}_{cif})^T$ for $i = 2, \ldots, n-1$, with $\overline{\hat{x}}_{ci} = [\hat{x}_{c1}, \hat{x}_{c2}, \ldots, \hat{x}_{ci}]^T$.

Theorem 1: Consider system (1) with the state observer given by (7), the control signal:

$$u_c = -\frac{1}{2a_{cn}^2} Z_{cn}^3 \hat{\theta}_c,$$

virtual control laws α_{ci} as in (12), and the adaptive law:

$$\dot{\hat{\theta}}_c = \sum_{i=1}^{n} \frac{r_c}{2a_{ci}^2} Z_{ci}^6 - k_c \hat{\theta}_c, \tag{13}$$

where a_{ci} $(i = 1, 2, \ldots, n)$, r_c, and k_c are design parameters. Then, the stochastic system can achieve M-SGUUB behavior with probability $1 - \varepsilon$ in $\Omega(\varepsilon)$.

To analyze the stability condition of the estimation error in (8), consider the Lyapunov function candidate $V_{c0} = \frac{1}{2}(e_c^T P e_c)^2$. Its time derivative along with (11) is given by:

$$\begin{aligned} LV_{c0} &= e_c^T P e_c \left[2e_c^T P(Ae_c + F - K_c e_{c1})\right] \\ &+ \text{tr}\left\{\varphi^T(x_1)\left(2Pe_c e_c^T P\right)\varphi(x_1)\right\} \\ &+ \text{tr}\left\{\varphi^T(x_1)\left(e_c^T P e_c P\right)\varphi(x_1)\right\} \\ &= e_c^T P e_c \left[e_c^T (A - K_c C)^T P e_c\right] \\ &+ e_c^T P(A - K_c C)e_c + 2e_c^T PF \\ &+ \text{tr}\left\{\varphi^T(x_1)\left(2Pe_c e_c^T P\right)\varphi(x_1)\right\} \\ &+ \text{tr}\left\{\varphi^T(x_1)\left(e_c^T P e_c P\right)\varphi(x_1)\right\} \end{aligned} \tag{14}$$

By substituting (10) into (14), we have:

$$\begin{aligned} LV_{c0} &\le -\lambda \|e_c\|^4 \\ &+ 2e_c^T P e_c\, e_c^T PF \\ &+ \text{tr}\left\{\varphi^T(x_1)\left(2Pe_c e_c^T P\right)\varphi(x_1)\right\} \\ &+ \text{tr}\left\{\varphi^T(x_1)\left(e_c^T P e_c P\right)\varphi(x_1)\right\} \end{aligned} \tag{15}$$

Since $F = [f_1(x_1), \ldots, f_n(x_n)]^T$ is unknown, for any $\varepsilon_{i0} > 0$, there exists an RBF NN: $W_{i0}^{*T} S_{i0}(X_0)$ such that

$$f_i(X_0) = W_{i0}^{*T} S_{i0}(X_0) + \delta_{i0}(X_0), \quad \|\delta_{i0}(X_0)\| \le \varepsilon_{i0},$$

where $X_0 = x$, $X_0 \in \Omega_{X_0} = \{X_0 | x \in \Omega_x\}$, $\delta_{i0}(X_0)$ is the approximation error, and Ω_x is a compact set through which the state trajectories evolve. Then:

$$F = W_{c0}^{*T} S_0(X_{c0}) + \delta_{c0}(X_{c0}), \quad \|\delta_{c0}(X_{c0})\| \le \varepsilon_{c0} \tag{16}$$

Since $0 < S_{c0}^T S_{c0} \le N_{c0}$, and by the definition of θ_c with $\|W_{c0}^*\|^4 S_{c0}^4 \le \theta_c^2$, the following holds for $X_0 \in \Omega_{X_0}$:

$$\begin{aligned} 2e_c^T P e_c\, e_c^T PF &= 2e_c^T P e_c\, e_c^T P\left(W_{c0}^{*T} S_{c0}(X_{c0}) + \delta_{c0}(X_{c0})\right) \\ &\le \frac{3}{2}\|P\|^{\frac{8}{3}} \|e_c\|^4 + \frac{1}{2}\theta_c^2 + \frac{1}{2}\varepsilon_{c0}^4 \end{aligned} \tag{17}$$

We also have:

$$\begin{aligned} & \text{tr}\left\{\varphi^T(x_1)\left(2Pe_c e_c^T P + e_c^T P e_c P\right)\varphi(x_1)\right\} \\ \le\ & n\sqrt{n}\|\varphi(x_1)\|^2 \|P\|^2 \|e_c\|^2 \\ \le\ & \frac{3n\sqrt{n}}{2\eta_{c0}^2}\|\varphi(x_1)\|^4 + \frac{3n\sqrt{n}\eta_{c0}^2}{2}\|P\|^4 \|e_c\|^4 \\ =\ & \frac{3n\sqrt{n}}{2\eta_{c0}^2} x_1^4 \|\psi(x_1)\|^4 + \frac{3n\sqrt{n}\eta_{c0}^2}{2}\|P\|^4 \|e_c\|^4 \end{aligned} \tag{18}$$

where $\eta_{c0} > 0$ ensures $p_{c0} > 0$ with:

$$p_{c0} = \lambda_c - 3\|P\|^{\frac{8}{3}} - \frac{3n\sqrt{n}\eta_{c0}^2}{2}\|P\|^4 \tag{19}$$

Substituting (16), (17), and (18) into (15) yields:

$$LV_{c0} \le -p_{c0}\|e_c\|^4 + \frac{3n\sqrt{n}}{2\eta_{c0}^2} x_1^4 \|\psi(x_1)\|^4 + \frac{1}{2}\theta_c^2 + \frac{1}{2}\varepsilon_{c0}^4 \tag{20}$$

In this section, the DSC approach is employed to address the complexity explosion issue. Define the change of coordinates:

$$\begin{aligned} Z_{c1} &= y \\ Z_{ci} &= \hat{x}_{ci} - \alpha_{cif}, \quad i = 2, 3, \ldots, n \end{aligned} \tag{21}$$

where α_{cif} is the output of a filter, with $\alpha_{c(i-1)}$ serving as the input in the normal case. Applying Ito's differentiation rule yields:

$$\begin{aligned} dZ_{c1} &= \left(f_1(x_1) + e_{c2} + \hat{x}_{c2}\right)dt + \varphi^T(x_1)dw(t) \\ dZ_{ci} &= \left(\hat{x}_{c(i+1)} + k_{ci}e_{c1} - \dot{\alpha}_{cif}\right)dt, \\ & \quad i = 2, \ldots, n. \end{aligned} \tag{22}$$

Step 1: Consider the Lyapunov function candidate $V_{c1} = V_{c0} + \frac{1}{4}Z_{c1}^4 + \frac{1}{2r_c}\tilde{\theta}_c^2$. Differentiating V_{c1} with respect to time yields:

$$\begin{aligned} LV_{c1} &= LV_{c0} + Z_{c1}^3\left(f_1(x_1) + x_2\right) \\ &+ \frac{1}{2}\text{tr}\left(\varphi_1^T(x_1) 3Z_{c1}^2 \varphi_1(x_1)\right) - \frac{1}{r_c}\tilde{\theta}_c \dot{\hat{\theta}}_c \\ &= -p_{c0}\|e_c\|^4 + Z_{c1}^3\left(f_1 + e_{c2} + \hat{x}_{c2}\right) \\ &+ Z_{c1}^3\left(\frac{3n\sqrt{n}}{2\eta_{c0}^2} Z_{c1}\|\psi(x_1)\|^4\right) \\ &+ \frac{3}{2}Z_{c1}^2 \varphi_1^T(x_1)\varphi_1(x_1) \\ &- \frac{1}{r_c}\tilde{\theta}_c\dot{\hat{\theta}}_c + \frac{1}{2}\theta_c^2 + \frac{1}{2}\varepsilon_{c0}^4 \end{aligned} \tag{23}$$

Applying Young's inequality gives:

$$\begin{aligned} Z_{c1}^3 e_{c2} &\le \frac{3}{4}\eta_{c1}^{\frac{4}{3}} Z_{c1}^4 + \frac{1}{4\eta_{c1}^4} e_{c2}^4 \\ &\le \frac{3}{4}\eta_{c1}^{\frac{4}{3}} Z_{c1}^4 + \frac{1}{4\eta_{c1}^4}\|e_c\|^4 \end{aligned} \tag{24}$$

$$\begin{aligned} \frac{3}{2}Z_{c1}^2 \varphi_1^T(x_1)\varphi_1(x_1) &\le \frac{3}{4}\eta_{c2}^2 \|\varphi_1(x_1)\|^4 Z_{c1}^4 \\ &+ \frac{1}{4\eta_{c2}^2} \end{aligned} \tag{25}$$

where $\eta_{c1}, \eta_{c2} > 0$ are design parameters. Substituting (23) and (24) into (22), we get:

$$\begin{aligned} LV_{c1} \le &-p_{c1}\|e_c\|^4 + Z_{c1}^3(\hat{x}_{c2} + \bar{f}_{c1}) - \frac{1}{2}\tilde{\theta}_c\dot{\hat{\theta}}_c \\ &+ \frac{1}{2}\theta_c^2 + \frac{1}{2}\varepsilon_{c0}^4 - \frac{3}{4}Z_{c1}^4 + \frac{1}{4\eta_{c2}^2} \end{aligned} \tag{26}$$

where

$$\begin{aligned} p_{c1} &= p_{c0} - \frac{1}{4\eta_{c1}^4}, \\ \bar{f}_{c1} &= f_1 + \frac{3n\sqrt{n}}{2\eta_{c0}^2}\|\psi(x_1)\|^4 \\ &\quad + \left(\frac{3}{4}\eta_{c1}^{\frac{4}{3}} + \frac{3}{4}\eta_{c2}^2\|\psi_1(x_1)\|^2 + \frac{3}{4}\right)Z_{c1}. \end{aligned}$$

Now, consider the intermediate control signal:

$$\hat{\alpha}_{c1}(X_{c1}) = -(l_{c1}Z_{c1} + \bar{f}_{c1}),$$

where $l_{c1} > 0$ is a design parameter. Then (26) can be rewritten as:

$$\begin{aligned} LV_{c1} &\leq -p_{c1}\|e_c\|^4 + Z_{c1}^3(\hat{x}_{c2} - \hat{\alpha}_{c1}) \\ &- l_{c1}Z_{c1}^4 - \frac{3}{4}Z_{c1}^4 \\ &- \frac{1}{2}\tilde{\theta}_c\dot{\hat{\theta}}_c + \frac{1}{2}\theta_c^2 \\ &+ \frac{1}{2}\varepsilon_{c0}^4 + \frac{1}{4\eta_{c2}^2} \end{aligned} \tag{27}$$

However, $\hat{\alpha}_{c1}$ is an unknown function and cannot be implemented directly. For any constant $\varepsilon_{c1} > 0$, there exists an RBF NN: $W_{c1}^{*T}S_{c1}(X_{c1})$ that can approximate $\hat{\alpha}_{c1}(X_{c1})$ as:

$$\hat{\alpha}_{c1}(X_{c1}) = W_{c1}^{*T}S_{c1}(X_{c1}) + \delta_{c1}(X_{c1}), \quad \|\delta_{c1}(X_{c1})\| \leq \varepsilon_{c1},$$

where $\delta_{c1}(X_{c1})$ denotes the approximation error, and $X_{c1} \in \Omega_{X_{c1}} = \{X_{c1} \mid x \in \Omega_{X_{c1}}\}$. Based on the definition of θ_c and α_{c1}, we have:

$$\begin{aligned} -Z_{c1}^3\hat{\alpha}_{c1} &= -Z_{c1}^3\left(W_{c1}^{*T}S_{c1}(X_{c1}) + \delta_{c1}(X_{c1})\right) \\ &\leq \frac{N_{c1}}{2a_{c1}^2}Z_{c1}^6\|W_{c1}^*\|^2 + \frac{1}{2}a_{c1}^2 + \frac{3}{4}Z_{c1}^4 + \frac{1}{4}\varepsilon_{c1}^4 \\ &\leq \frac{1}{2a_{c1}^2}Z_{c1}^6\theta_c + \frac{1}{2}a_{c1}^2 + \frac{3}{4}Z_{c1}^4 + \frac{1}{4}\varepsilon_{c1}^4 \end{aligned} \tag{28}$$

$$Z_{c1}^3\alpha_{c1} = -\frac{1}{2a_{c1}^2}Z_{c1}^6\hat{\theta}_c \tag{29}$$

where $a_{c1} > 0$ is a design parameter. Then,

$$\begin{aligned} LV_{c1} &\leq -p_{c1}\|e_c\|^4 - l_{c1}Z_{c1}^4 \\ &+ Z_{c1}^3(\hat{x}_{c2} - \alpha_{c1}) \\ &+ \frac{1}{r_c}\tilde{\theta}_c\left(\frac{r_c}{2a_{c1}^2}Z_{c1}^6 - \dot{\hat{\theta}}_c\right) \\ &+ \Delta_{c1} \end{aligned} \tag{30}$$

where $\Delta_{c1} = \frac{1}{2}\theta_c^2 + \frac{1}{2}\varepsilon_{c0}^4 + \frac{1}{2}a_{c1}^2 + \frac{1}{4}\varepsilon_{c1}^4 + \frac{1}{4\eta_{c2}^2}$. Using $\hat{x}_{c2} = Z_{c2} + \alpha_{c2f}$, (30) becomes:

$$\begin{aligned} LV_{c1} &\leq -p_{c1}\|e_c\|^4 - l_{c1}Z_{c1}^4 \\ &+ Z_{c1}^3(Z_{c2} + \alpha_{c2f} - \alpha_{c1}) \\ &+ \frac{1}{r_c}\tilde{\theta}_c\left(\frac{r_c}{2a_{c1}^2}Z_{c1}^6 - \dot{\hat{\theta}}_c\right) \\ &+ \Delta_{c1} \end{aligned} \tag{31}$$

A first-order filter with time constant κ_{c2} is introduced as:

$$\kappa_{c2}\dot{\alpha}_{c2f} + \alpha_{c2f} = \alpha_{c1}, \quad \alpha_{c2f}(0) = \alpha_{c1}(0)$$

Let $\chi_{c2} = \alpha_{c2f} - \alpha_{c1}$, so $\dot{\alpha}_{c2f} = -\frac{\chi_{c2}}{\kappa_{c2}}$ and

$$\dot{\chi}_{c2} = \dot{\alpha}_{c2f} - \dot{\alpha}_{c1} = -\frac{\chi_{c2}}{\kappa_{c2}} + B_{c2}(X_1)$$

where

$$B_{c2}(X_1) = \frac{3}{2a_{c1}^2}Z_{c1}^2\dot{Z}_{c1}\hat{\theta}_c + \frac{1}{2a_{c1}^2}Z_{c1}^2\dot{\hat{\theta}}_c$$

The inequality in (31) can be expressed as:

$$\begin{aligned} LV_{c1} &\leq -p_{c1}\|e_c\|^4 - l_{c1}Z_{c1}^4 \\ &+ Z_{c1}^3Z_{c2} + Z_{c1}^3\alpha_{c2f} \\ &+ \frac{1}{r_c}\tilde{\theta}_c\left(\frac{r_c}{2a_{c1}^2}Z_{c1}^6 - \dot{\hat{\theta}}_c\right) \\ &+ \Delta_{c1} \end{aligned} \tag{32}$$

Step m: $(2 \leq m \leq n-1)$. The Lyapunov function is defined as:

$$V_{cm} = V_{c(m-1)} + \frac{1}{4}Z_{cm}^4 + \frac{1}{4}\chi_{cm}^4 \tag{33}$$

Differentiating yields:

$$\begin{aligned} LV_{cm} &\leq -p_{c1}\|e_c\|^4 - \sum_{i=1}^{m-1}l_{ci}Z_{ci}^4 + \sum_{i=1}^{m-1}Z_{ci}^3Z_{c(i+1)} \\ &+ \sum_{i=1}^{m-1}Z_{ci}^3\chi_{c(i+1)} + Z_{cm}^3(\hat{x}_{c(m+1)} + \overline{f}_{cm}) \\ &- \sum_{i=1}^{m-1}\left(\frac{\chi_{c(i+1)}^4}{\kappa_{c(i+1)}} - \chi_{c(i+1)}^3B_{c(i+1)}(X_i)\right) \\ &+ \frac{1}{r_c}\tilde{\theta}_c\left(\sum_{i=1}^{m-1}\frac{r_c}{2a_{ci}^2}Z_{ci}^6 - \dot{\hat{\theta}}_c\right) \\ &- \frac{3}{4}Z_{cm}^4 + \Delta_{c(m-1)} \end{aligned} \tag{34}$$

where $\overline{f}_{cm}(X_m) = k_{cm}e_{c1} - \dot{\alpha}_{cmf} + \frac{1}{4}Z_{cm} + \frac{3}{4}Z_{cm}$.

Define the intermediate control signal $\widehat{\alpha}_{cm}(X_{cm})$ as:

$$\widehat{\alpha}_{cm} = -(l_{cm}Z_{cm} + \overline{f}_{cm})$$

where $l_{cm} > 0$ is a design parameter. Adding and subtracting $\widehat{\alpha}_{cm}(X_{cm})$ in (34) yields:

$$\begin{aligned} LV_{cm} &\leq -p_{c1}\|e_c\|^4 - \sum_{i=1}^{m}l_{ci}Z_{ci}^4 + \sum_{i=1}^{m-1}Z_{ci}^3Z_{c(i+1)} \\ &+ \sum_{i=1}^{m-1}Z_{ci}^3\chi_{c(i+1)} + \frac{1}{r_c}\tilde{\theta}_c\left(\sum_{i=1}^{m-1}\frac{r_c}{2a_{ci}^2}Z_{ci}^6 - \dot{\hat{\theta}}_c\right) \\ &+ Z_{cm}^3(\hat{x}_{c(m+1)} - \widehat{\alpha}_{cm}) \\ &- \sum_{i=1}^{m-1}\left(\frac{\chi_{c(i+1)}^4}{\kappa_{c(i+1)}} - \chi_{c(i+1)}^3B_{c(i+1)}(X_i)\right) \\ &+ \Delta_{c(m-1)} - \frac{3}{4}Z_{cm}^4 \end{aligned} \tag{35}$$

Similarly, $\widehat{\alpha}_{cm}(X_{cm})$ can be approximated by the RBF NN: $W_{cm}^{*T}S_{cm}(X_{cm})$ as

$$\begin{aligned}\widehat{\alpha}_{cm}(X_{cm}) &= W_{cm}^{*T} S_{cm}(X_{cm}) + \delta_{cm}(X_{cm}),\\ |\delta_{cm}(X_{cm})| &\le \varepsilon_{cm},\end{aligned}$$

where $\delta_{cm}(X_{cm})$ denotes the approximation error. Combining the above yields:

$$\begin{aligned}-Z_{cm}^3\widehat{\alpha}_{cm} &\le \frac{1}{2a_{cm}^2}Z_{cm}^6\theta_c + \frac{1}{2}a_{cm}^2\\ &\quad + \frac{3}{4}Z_{cm}^4 + \frac{1}{4}\varepsilon_{cm}^4 \qquad (36)\\ Z_{cm}^3\alpha_{cm} &= -\frac{1}{2a_{cm}^2}Z_{cm}^6\hat{\theta}_c \qquad (37)\end{aligned}$$

where $a_{cm} > 0$ is a design parameter.

Given $\hat{x}_{c(m+1)} = Z_{c(m+1)} + \alpha_{c(m+1)f}$, and substituting (36), (37) into (35), it can be expressed as:

$$\begin{aligned}LV_{cm} &\le -p_{c1}\|e_c\|^4 - \sum_{i=1}^{m} l_{ci}Z_{ci}^4 + \sum_{i=1}^{m-1} Z_{ci}^3 Z_{c(i+1)}\\ &\quad + \sum_{i=1}^{m-1} Z_{ci}^3\chi_{c(i+1)}\\ &\quad + Z_{cm}^3(Z_{c(m+1)} + \alpha_{c(m+1)f} - \alpha_{cm})\\ &\quad - \sum_{i=1}^{m-1}\left(\frac{\chi_{c(i+1)}^4}{\kappa_{c(i+1)}} - \chi_{c(i+1)}^3 B_{c(i+1)}(X_i)\right)\\ &\quad + \frac{1}{r_c}\tilde{\theta}_c\left(\sum_{i=1}^{m}\frac{r_c}{2a_{ci}^2}Z_{ci}^6 - \dot{\hat{\theta}}_c\right) + \Delta_{cm} \qquad (38)\end{aligned}$$

where

$$\Delta_{cm} = \frac{1}{2}\theta_c^2 + \frac{1}{2}\varepsilon_{c0}^4 + \frac{1}{2}\sum_{i=1}^{m}\left(a_{ci}^2 + \frac{1}{2}\varepsilon_{ci}^4\right) + \frac{1}{4\eta_{c2}^2}.$$

Let α_{cm} pass through a filter with coefficient $\kappa_{c(m+1)}$, producing $\alpha_{c(m+1)f}$:

$$\begin{aligned}\kappa_{c(m+1)}\dot{\alpha}_{c(m+1)f} + \alpha_{c(m+1)f} &= \alpha_{cm},\\ \alpha_{c(m+1)f}(0) &= \alpha_{cm}(0).\end{aligned}$$

Define $\chi_{c(m+1)} = \alpha_{c(m+1)f} - \alpha_{cm}$ as the filtering error, then $\dot{\alpha}_{c(m+1)f} = -\frac{\chi_{c(m+1)}}{\kappa_{c(m+1)}}$, and

$$\dot{\chi}_{c(m+1)} = \dot{\alpha}_{c(m+1)f} - \dot{\alpha}_{cm} = -\frac{\chi_{c(m+1)}}{\kappa_{c(m+1)}} + B_{c(m+1)}(X_m),$$

where

$$B_{c(m+1)}(X_m) = \frac{3}{2a_{cm}^2}Z_{cm}^2\dot{Z}_{cm}\hat{\theta}_c + \frac{1}{2a_{cm}^2}Z_{cm}^2\dot{\hat{\theta}}_c.$$

The inequality (38) can then be expressed as:

$$\begin{aligned}LV_{cm} &\le -p_{c1}\|e_c\|^4 - \sum_{i=1}^{m} l_{ci}Z_{ci}^4 + \sum_{i=1}^{m} Z_{ci}^3 Z_{c(i+1)}\\ &\quad + \sum_{i=1}^{m} Z_{ci}^3\alpha_{c(i+1)f} + \Delta_{cm}\\ &\quad - \sum_{i=1}^{m-1}\left(\frac{\chi_{c(i+1)}^4}{\kappa_{c(i+1)}} - \chi_{c(i+1)}^3 B_{c(i+1)}(X_i)\right)\\ &\quad + \frac{1}{r_c}\tilde{\theta}_c\left(\sum_{i=1}^{m}\frac{r_c}{2a_{ci}^2}Z_{ci}^6 - \dot{\hat{\theta}}_c\right) \qquad (39)\end{aligned}$$

Step n: Consider the Lyapunov function:

$$V_{cn} = V_{c(n-1)} + \frac{1}{4}Z_{cn}^4 + \frac{1}{4}\chi_{cn}^4, \qquad (40)$$

and its derivative:

$$\begin{aligned}LV_{cn} &\le -p_{c1}\|e_c\|^4 - \sum_{i=1}^{n-1} l_{ci}Z_{ci}^4 + \sum_{i=1}^{n-1} Z_{ci}^3 Z_{c(i+1)}\\ &\quad + \sum_{i=1}^{n-1} Z_{ci}^3\chi_{c(i+1)} + Z_{cn}^3(u_c + \bar{f}_{cn})\\ &\quad + \frac{1}{r_c}\tilde{\theta}_c\left(\sum_{i=1}^{n-1}\frac{r_c}{2a_{ci}^2}Z_{ci}^6 - \dot{\hat{\theta}}_c\right)\\ &\quad + \Delta_{c(n-1)} - \frac{3}{4}Z_{cn}^4\\ &\quad - \sum_{i=1}^{n-1}\left(\frac{\chi_{c(i+1)}^4}{\kappa_{c(i+1)}} - \chi_{c(i+1)}^3 B_{c(i+1)}(X_i)\right) \qquad (41)\end{aligned}$$

where $\bar{f}_{cn} = k_{cn}e_{c1} - \dot{\alpha}_{cnf} + \frac{3}{4}Z_{cn}$.

The intermediate control signal is defined as $\widehat{\alpha}_{cn} = -(l_{cn}Z_{cn} + \bar{f}_{cn})$, with $l_{cn} > 0$ being a design parameter. Then we have:

$$\begin{aligned}LV_{cn} &\le -p_{c1}\|e_c\|^4 - \sum_{i=1}^{n} l_{ci}Z_{ci}^4 + \sum_{i=1}^{n-1} Z_{ci}^3 Z_{c(i+1)}\\ &\quad + \sum_{i=1}^{n-1} Z_{ci}^3\chi_{c(i+1)}\\ &\quad + Z_{cn}^3(u_c - \widehat{\alpha}_{cn}) - \frac{3}{4}Z_{cn}^4\\ &\quad - \sum_{i=1}^{n-1}\left(\frac{\chi_{c(i+1)}^4}{\kappa_{c(i+1)}} - \chi_{c(i+1)}^3 B_{c(i+1)}(X_i)\right)\\ &\quad + \frac{1}{r_c}\tilde{\theta}_c\left(\sum_{i=1}^{n-1}\frac{r_c}{2a_{ci}^2}Z_{ci}^6 - \dot{\hat{\theta}}_c\right) + \Delta_{c(n-1)} \qquad (42)\end{aligned}$$

Similarly, the RBF NN $W_{cn}^{*T}S_{cn}(X_{cn})$ is used to approximate $\widehat{\alpha}_{cn}(X_{cn})$. By definition of u_c:

$$\begin{aligned}-Z_{cn}^3\hat{\alpha}_{cn} &\le \frac{1}{2a_{cn}^2}Z_{cn}^6\theta_c + \frac{1}{2}a_{cn}^2\\ &\quad + \frac{3}{4}Z_{cn}^4 + \frac{1}{4}\varepsilon_{cn}^4 \qquad (43)\\ Z_{cn}^3 u_c &= -\frac{1}{2a_{cn}^2}Z_{cn}^6\hat{\theta}_c \qquad (44)\end{aligned}$$

where $a_{cn} > 0$ is a design parameter. Substituting (43) and (44) into (42) gives:

$$\begin{aligned}LV_{cn} &\le -p_{c1}\|e_c\|^4 - \sum_{i=1}^{n} l_{ci}Z_{ci}^4 + \sum_{i=1}^{n-1} Z_{ci}^3 Z_{c(i+1)}\\ &\quad + \sum_{i=1}^{n-1} Z_{ci}^3\chi_{c(i+1)}\\ &\quad - \sum_{i=1}^{n-1}\left(\frac{\chi_{c(i+1)}^4}{\kappa_{c(i+1)}} - \chi_{c(i+1)}^3 B_{c(i+1)}(X_i)\right)\\ &\quad + \frac{1}{r_c}\tilde{\theta}_c\left(\sum_{i=1}^{n}\frac{r_c}{2a_{ci}^2}Z_{ci}^6 - \dot{\hat{\theta}}_c\right) + \Delta_{cn} \qquad (45)\end{aligned}$$

where

$$\Delta_{cn} = \frac{1}{2}\theta_c^2 + \frac{1}{2}\varepsilon_{c0}^4 + \frac{1}{2}\sum_{i=1}^{n}\left(a_{ci}^2 + \frac{1}{2}\varepsilon_{ci}^4\right) + \frac{1}{4\eta_{c2}^2}.$$

From the definition of $\dot{\hat{\theta}}_c$:

$$\begin{aligned} LV_{cn} \leq\ & -p_{c1}\|e_c\|^4 - \sum_{i=1}^{n} l_{ci}Z_{ci}^4 \\ & + \sum_{i=1}^{n-1} Z_{ci}^3 Z_{c(i+1)} + \sum_{i=1}^{n-1} Z_{ci}^3 \chi_{c(i+1)} \\ & + \frac{k_c}{r_c}\tilde{\theta}_c\hat{\theta}_c \\ & - \sum_{i=1}^{n-1}\left(\frac{\chi_{c(i+1)}^4}{\kappa_{c(i+1)}} - \chi_{c(i+1)}^3 B_{c(i+1)}(X_i)\right) \\ & + \Delta_{cn} \end{aligned} \tag{46}$$

Also:

$$\begin{aligned} Z_{ci}^3 Z_{c(i+1)} &\leq \frac{3}{4}Z_{ci}^4 + \frac{1}{4}Z_{c(i+1)}^4 \\ Z_{ci}^3 \chi_{c(i+1)} &\leq \frac{3}{4}Z_{ci}^4 + \frac{1}{4}\chi_{c(i+1)}^4 \\ \tilde{\theta}_c\hat{\theta}_c &= \tilde{\theta}_c(\theta_c - \tilde{\theta}_c) \leq -\frac{1}{2}\tilde{\theta}_c^2 + \frac{1}{2}\theta_c^2 \\ \left|\chi_{c(i+1)}^3 B_{c(i+1)}\right| &\leq \frac{3}{4}\pi_c^{\frac{4}{3}} B_{c(i+1)}^{\frac{4}{3}}\chi_{c(i+1)}^4 + \frac{1}{4\pi_c^4} \end{aligned} \tag{47}$$

where $\pi_c > 0$ is a design constant. There exists $N_{c(i+1)} > 0$ such that $|B_{c(i+1)}| \leq N_{c(i+1)}$. Then:

$$\begin{aligned} LV_{cn} \leq\ & -p_{c1}\|e_c\|^4 - \sum_{i=1}^{n}\left(l_{ci} - \frac{7}{4}\right)Z_{ci}^4 \\ & - \sum_{i=1}^{n-1}\left(\frac{1}{\kappa_{c(i+1)}} - \frac{1}{4} - \frac{3}{4}\pi_c^{\frac{4}{3}}B_{c(i+1)}^{\frac{4}{3}}\right)\chi_{c(i+1)}^4 \\ & - \frac{k_c}{2r_c}\tilde{\theta}_c^2 + \overline{\Delta}_{cn} \end{aligned} \tag{48}$$

where $\overline{\Delta}_{cn} = \Delta_{cn} + \frac{1}{4\pi_c^4} + \frac{k_c}{2r_c}\theta_c^2$.

B. State estimator and dynamic feedback design in data-losing case

In the data-losing case, current measurements cannot be used due to transmission errors. Instead of discarding these data, the previous valid observation is employed as a substitute. In this scenario, it is assumed that $x_1 \neq y$. Define the estimation error as $e_{si} = x_i - \hat{x}_{si}$, with $e'_{s1} = x'_1 - \hat{x}_{s1}$, and the difference $\Delta e_{s1} = e_{s1} - e'_{s1} = x_1 - x'_1$, where $\hat{x}_{si}$ denotes the estimated state under data loss, and x'_1 represents the prior normal observation.

The estimation error in this case is expressed as:

$$e_s = \bar{x}_n - \overline{\hat{x}}_{sn}$$

where $\bar{x}_n = [x_1, \ldots, x_n]^T$ and $\overline{\hat{x}}_{sn} = [\hat{x}_{s1}, \ldots, \hat{x}_{sn}]^T$ represent the actual and estimated state vectors, respectively. The first element of the error is $e_{s1} = x_1 - \hat{x}_{s1}$.

The state estimator for this case is given by:

$$\begin{aligned} \dot{\hat{x}}_{si} &= \hat{x}_{s(i+1)} + k_{si}(x'_1 - \hat{x}_{s1}) \\ \dot{\hat{x}}_{sn} &= u_s + k_{sn}(x'_1 - \hat{x}_{s1}) \end{aligned} \tag{49}$$

where $\hat{x}_{si}$ for $i = 1, 2, \ldots, n$ are the state estimates under data loss, and k_{si} are design parameters.

Remark 3: The parameters k_{ci} and k_{si} significantly influence system stability and performance. If k_{ci} or k_{si} are too small, convergence will be slow; if too large, instability may result.

The time derivative of the estimation error is:

$$\begin{aligned} de_s &= (Ae_s + F - K_s e'_{s1})dt + \varphi^T(x_1)dw \\ &= (Ae_s + F - K_s e_{s1} + K_s\Delta e_{s1})dt + \varphi^T(x_1)dw \\ &= ((A - K_sC)e_s + K_s\Delta e_{s1} + F)dt \\ &\quad + \varphi^T(x_1)dw \end{aligned} \tag{50}$$

The virtual control function $\hat{\alpha}_{si}$ is developed using a Lyapunov approach, and the actual control input u_s is derived accordingly. To initiate the backstepping design, define the parameter θ_s as:

$$\theta_s = \max\left\{N_{si}\|W_{si}^*\|^2 : i = 0, 1, 2, \ldots, n\right\}.$$

where W_{si}^* is an unknown constant, making θ_s also unknown. Define the parameter estimation error as $\tilde{\theta}_s = \theta_s - \hat{\theta}_s$, where $\hat{\theta}_s$ is the estimate of θ_s. The virtual control signal is:

$$\alpha_{si}(X_{si}) = -\frac{1}{2a_{si}^2}Z_{si}^3\hat{\theta}_s, \quad i = 1, \ldots, n-1 \tag{51}$$

where $Z_{s1} = x_1$, $Z_{si} = \hat{x}_{si} - \alpha_{sif}$ for $i = 2, \ldots, n-1$, $a_{si} > 0$ are design parameters, $X_{s1} = x_1$, and $X_{si} = (e_{s1}, \overline{\hat{x}}_{si}, \bar{\alpha}_{sif}, \dot{\alpha}_{sif})^T$, with $\overline{\hat{x}}_{si} = [\hat{x}_{s1}, \hat{x}_{s2}, \ldots, \hat{x}_{si}]^T$.

Theorem 2: Consider system (1) with estimator (49), control law:

$$u_s = -\frac{1}{2a_{sn}^2}Z_{sn}^3\hat{\theta}_s,$$

virtual control signals α_{si} in (51) and the adaptive law:

$$\dot{\hat{\theta}}_s = \sum_{i=1}^{n}\frac{r_s}{2a_{si}^2}Z_{si}^6 - k_s\hat{\theta}_s \tag{52}$$

where a_{si} for $i = 1, \ldots, n$, r_s, and k_s are positive design parameters. Then the system is $M\text{-}SGUUB$ with probability $1 - \varepsilon$ in $\Omega(\varepsilon)$.

Assumption 2: There exists a known constant h such that:

$$\|K\Delta e_{s1}\| \leq h, \quad h \geq 0. \tag{53}$$

The Lyapunov function is chosen as $V_{s0} = \frac{1}{2}(e_s^T P e_s)^2$. Its derivative is:

$$\begin{aligned} LV_{s0} =\ & e_s^T P e_s\left(e_s^T\left(P(A - K_sC) + (A - K_sC)^T P\right)e_s\right) \\ & + 2e_s^T P K_s\Delta e_{s1} + 2e_s^T PF \\ & + \mathrm{tr}\left(\varphi^T(x_1)\left(2Pe_se_s^TP + e_s^TPe_sP\right)\varphi(x_1)\right) \end{aligned} \tag{54}$$

Similar to (16), we have:

$$F = W_{s0}^{*T} S_{s0}(X_{s0}) + \delta_{s0}(X_{s0}), \quad \|\delta_{s0}(X_{s0})\| \le \varepsilon_{s0}.$$

Since $0 < S_{s0}^T S_{s0} \le N_{s0}$ with N_{s0} being the dimension of S_{s0}, and from the definition of θ_s, it follows:

$$\|W_{s0}^*\|^4 S_{s0}^4 \le \theta_s^2.$$

Therefore, the following inequality holds for $X_{s0} \in \Omega_{X_{s0}}$:

$$\begin{aligned} 2e_s^T P e_s e_s^T P F &\le \frac{3}{2}\|P\|^{\frac{8}{3}}\|e_s\|^4 + \frac{1}{2}\|W_{s0}^*\|^4 S_{s0}^4 \\ &\quad + \frac{3}{2}\|P\|^{\frac{8}{3}}\|e_s\|^4 + \frac{1}{2}\delta_{s0}^4 \\ &\le 3\|P\|^{\frac{8}{3}}\|e_s\|^4 + \frac{1}{2}\theta_s^2 + \frac{1}{2}\varepsilon_{s0}^4. \end{aligned} \quad (55)$$

Additionally:

$$\begin{aligned} &\mathrm{tr}\{\varphi^T(x_1)(2Pe_se_s^TP + e_s^TPe_sP)\varphi(x_1)\} \\ &\le n\left\|\varphi^T(x_1)(2Pe_se_s^TP)\varphi(x_1)\right\|_F \\ &\quad + n\left\|\varphi^T(x_1)(e_s^TPe_sP)\varphi(x_1)\right\|_F \\ &\le n\sqrt{n}\left\|\varphi^T(x_1)(2Pe_se_s^TP)\varphi(x_1)\right\| \\ &\quad + n\sqrt{n}\left\|\varphi^T(x_1)(e_s^TPe_sP)\varphi(x_1)\right\| \\ &\le 3n\sqrt{n}\|\varphi(x_1)\|^2\|P\|^2\|e_s\|^2. \end{aligned} \quad (56)$$

Using **Assumption 1**, we have:

$$\begin{aligned} \|\varphi(x_1)\| &= \|\varphi(\hat{x}_{s1}) + \varphi(x_1) - \varphi(\hat{x}_{s1})\| \\ &\le \|\varphi(\hat{x}_{s1})\| + \mu\|e_s\| \end{aligned} \quad (57)$$

Thus, the following inequality holds:

$$\begin{aligned} &\mathrm{tr}\left\{\varphi^T(x_1)\left(2Pe_se_s^TP + e_s^TPe_sP\right)\varphi(x_1)\right\} \\ &\le 3n\sqrt{n}\left(\|\varphi(\hat{x}_{s1})\| + \mu\|e_s\|\right)^2 \\ &\quad \times \|P\|^2\|e_s\|^2 \\ &= 3n\sqrt{n}\|\varphi(\hat{x}_{s1})\|^2\|P\|^2\|e_s\|^2 \\ &\quad + 6n\sqrt{n}\mu\|\varphi(\hat{x}_{s1})\|\|P\|^2\|e_s\|^3 \\ &\quad + 3n\sqrt{n}\mu^2\|P\|^2\|e_s\|^4 \\ &\le \left(\frac{3n\sqrt{n}}{2\eta_{s0}}\|\varphi(\hat{x}_{s1})\|^4 + \frac{3n\sqrt{n}\eta_{s0}}{2}\|P\|^4\|e_s\|^4\right) \\ &\quad + 3n\sqrt{n}\mu^2\|P\|^2\|e_s\|^4 \\ &\quad + \left(\frac{3n\sqrt{n}\mu}{2\eta_{s1}^4}\|\varphi(\hat{x}_{s1})\|^4\right. \\ &\quad \left. + \frac{9n\sqrt{n}\mu\eta_{s1}^4}{2}\|P\|^4\|e_s\|^4\right) \end{aligned} \quad (58)$$

where $\eta_{s0}, \eta_{s1} > 0$ are design parameters.

According to **Assumption 2**, we obtain:

$$\begin{aligned} 2e_s^TPe_se_s^TPK\Delta e_{s1} &\le 2\|e_s\|^3\|P\|^2 h \\ &\le \frac{3}{2}\eta_{s2}^{\frac{4}{3}}\|P\|^{\frac{8}{3}}\|e_s\|^4 \\ &\quad + \frac{1}{2\eta_{s2}^4}h^4 \end{aligned} \quad (59)$$

where $\eta_{s2} > 0$. Define:

$$\begin{aligned} p_{s0} &= \lambda - 3\|P\|^{\frac{8}{3}} - \frac{3n\sqrt{n}\eta_{s0}}{2}\|P\|^4 \\ &\quad - \frac{9n\sqrt{n}\mu\eta_{s1}^{\frac{4}{3}}}{2}\|P\|^{\frac{8}{3}} \\ &\quad - 3n\sqrt{n}\mu^2\|P\|^2 - \frac{3}{2}\eta_{s2}^{\frac{4}{3}}\|P\|^{\frac{8}{3}} \end{aligned} \quad (60)$$

Substituting (45) and (58)-(60) into (49) yields:

$$\begin{aligned} LV_{s0} &\le -p_{s0}\|e_s\|^4 + \left(\frac{3n\sqrt{n}}{2\eta_{s0}} + \frac{3n\sqrt{n}\mu}{2\eta_{s1}^4}\right)\|\varphi(\hat{x}_{s1})\|^4 \\ &\quad + \frac{1}{2}\theta_s^2 + \frac{1}{2}\varepsilon_{s0}^4 + \frac{1}{2\eta_{s2}^4}h^4 \end{aligned} \quad (61)$$

To address complexity explosion, the DSC approach is applied. Backstepping design is implemented through a coordinate transformation:

$$\begin{aligned} Z_{s1} &= x_1, \\ Z_{si} &= \hat{x}_{si} - \alpha_{sif}, \quad i = 2, 3, \ldots, n. \end{aligned} \quad (62)$$

where α_{sif} is the filtered signal of $\alpha_{s(i-1)}$. Differentiation yields:

$$\begin{aligned} dZ_{s1} &= (f_1(x_1) + x_2)\,dt + \varphi_1(x_1)dw \\ &\quad + \frac{1}{2}\mathrm{tr}\left(\varphi_1^T(x_1)3Z_{s1}^2\varphi_1(x_1)\right) \\ dZ_{si} &= \left(\hat{x}_{s(i+1)} + k_{si}e_{s1} - \dot{\alpha}_{sif}\right)dt, \\ &\quad i = 2, \ldots, n. \end{aligned} \quad (63)$$

Step 1: Define the Lyapunov function $V_{s1} = V_{s0} + \frac{1}{4}Z_{s1}^4 + \frac{1}{2r_s}\tilde{\theta}_s^2$ with

$$\begin{aligned} LV_{s1} &= LV_{s0} + Z_{s1}^3(f_1(x_1) + x_2) \\ &\quad + \frac{1}{2}\mathrm{tr}\left(\varphi_1^T(x_1)3Z_{s1}^2\varphi_1(x_1)\right) - \frac{1}{r_s}\tilde{\theta}_s\dot{\hat{\theta}}_s \\ &= -p_{s0}\|e_s\|^4 + Z_{s1}^3\left(f_1(\widehat{x}_{s1}) + f_1(x_1) - f_1(\widehat{x}_{s1})\right. \\ &\quad \left. + e_{s2} + \widehat{x}_{s2}\right) + \frac{3}{2}Z_{s1}^2\left(\varphi_1^T(x_1)\varphi_1(x_1)\right) \\ &\quad + \left(\frac{3n\sqrt{n}}{2\eta_{s0}} + \frac{3n\sqrt{n}\mu}{2\eta_{s1}^4}\right)\|\varphi(\widehat{x}_{s1})\|^4 \\ &\quad + \frac{1}{2}\theta_s^2 + \frac{1}{2}\varepsilon_{s0}^4 + \frac{1}{2\eta_{s2}^4}h^4 - \frac{1}{r_s}\tilde{\theta}_s\dot{\hat{\theta}}_s \end{aligned} \quad (64)$$

One has:

$$\begin{aligned} Z_{s1}^3 e_{s2} &\le \frac{3}{4}\eta_{s3}^{\frac{4}{3}}Z_{s1}^4 + \frac{1}{4\eta_{s3}^4}e_{s2}^4 \\ &\le \frac{3}{4}\eta_{s3}^{\frac{4}{3}}Z_{s1}^4 + \frac{1}{4\eta_{s3}^4}\|e_s\|^4 \end{aligned} \quad (65)$$

$$\begin{aligned} Z_{s1}^3(f_1(x_1) - f_1(\widehat{x}_{s1})) &\le m_1|Z_{s1}|^3|e_{s1}| \\ &\le \frac{3}{4}m_1^{\frac{4}{3}}\eta_{s4}^{\frac{4}{3}}Z_{s1}^4 \\ &\quad + \frac{3}{4\eta_{s4}^4}\|e_s\|^4 \end{aligned} \quad (66)$$

Next, consider the trace term:

$$
\begin{aligned}
&\frac{3}{2}Z_{s1}^2\mathrm{tr}\big(\varphi_1^T(x_1)\varphi_1(x_1)\big)\\
&=\frac{3}{2}Z_{s1}^2\big(\varphi_1^T(\widehat{x}_{s1})+\varphi_1^T(x_1)-\varphi_1^T(\widehat{x}_{s1})\big)\\
&\times\big(\varphi_1(\widehat{x}_{s1})+\varphi_1(x_1)-\varphi_1(\widehat{x}_{s1})\big)\\
&\le\frac{3}{2}Z_{s1}^2\big(\|\varphi_1(\widehat{x}_{s1})\|^2+\mu e_{s1}^2+\mu\|\varphi_1(\widehat{x}_{s1})\|^2+\mu^2e_{s1}^2\big)\\
&=\frac{3(1+\mu)}{4}Z_{s1}^4\|\varphi_1(\widehat{x}_{s1})\|^2+\frac{3\mu(1+\mu)}{4}Z_{s1}^4e_{s1}^2\\
&\le\frac{3(1+\mu)\eta_{s5}^2}{4}Z_{s1}^4+\frac{3(1+\mu)}{4\eta_{s5}^2}\|\varphi_1(\widehat{x}_{s1})\|^2\\
&+\frac{3\mu(1+\mu)\eta_{s6}^2}{4}Z_{s1}^4+\frac{3\mu(1+\mu)}{4\eta_{s6}^2}\|e_s\|^4 \qquad (67)
\end{aligned}
$$

where η_{s3}, η_{s4}, η_{s5}, and $\eta_{s6}>0$ are design parameters. Substituting (65), (66), and (67) into (64) yields:

$$
\begin{aligned}
LV_{s1}\ \le\ & -\Big(p_{s0}-\frac{1}{4\eta_{s3}^4}-\frac{3}{4\eta_{s4}^4}-\frac{3\mu(1+\mu)}{4\eta_{s6}^2}\Big)\|e_s\|^4\\
&+Z_{s1}^3(\hat{x}_{s2}+\bar{f}_{s1})-\frac{1}{r_s}\tilde{\theta}_s\dot{\hat{\theta}}_s-\frac{3}{4}Z_{s1}^4\\
&+\left(\frac{3n\sqrt{n}}{2\eta_{s0}}+\frac{3n\sqrt{n}\mu}{2\eta_{s1}^4}+\frac{3(1+\mu)}{4\eta_{s5}^2}\right)\|\psi(\hat{x}_{s1})\|^4\\
&+\frac{1}{2}\theta_s^2+\frac{1}{2}\varepsilon_{s0}^4+\frac{1}{2\eta_{s2}^4}h^4 \qquad (68)
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{f}_{s1}\ =\ & f_1(\widehat{x}_{s1})+Z_{s1}\Big(\frac{3}{4}\eta_{s3}^{\frac{4}{3}}+\frac{3}{4}m_1^{\frac{4}{3}}\eta_{s4}^{\frac{4}{3}}\Big)\\
&+\frac{3(1+\mu)\eta_{s5}^2}{4}+\frac{3\mu(1+\mu)\eta_{s6}^2}{4}+\frac{3}{4}
\end{aligned}
$$

Now, take the intermediate control signal $\widehat{\alpha}_{s1}(X_{s1})$ as

$$\widehat{\alpha}_{s1}(X_{s1})=-\big(l_{s1}Z_{s1}+\bar{f}_{s1}\big)$$

where $l_{s1}>0$ is a design parameter. Then, one has:

$$
\begin{aligned}
LV_{s1}\ \le\ & -p_{s1}\|e_s\|^4-l_{s1}Z_{s1}^4+Z_{s1}^3(\hat{x}_{s2}-\widehat{\alpha}_{s1})\\
&-\frac{3}{4}Z_{s1}^4-\frac{1}{r_s}\tilde{\theta}_s\dot{\hat{\theta}}_s+\frac{1}{2}\theta_s^2+\frac{1}{2}\varepsilon_{s0}^4\\
&+\left(\frac{3n\sqrt{n}}{2\eta_{s0}}+\frac{3n\sqrt{n}\mu}{2\eta_{s1}^4}+\frac{3(1+\mu)}{4\eta_{s5}^2}\right)\|\psi(\hat{x}_{s1})\|^4\\
&+\frac{1}{2\eta_{s2}^4}h^4 \qquad (69)
\end{aligned}
$$

where $p_{s1}=p_{s0}-\frac{1}{4\eta_{s3}^4}-\frac{3}{4\eta_{s4}^4}-\frac{3\mu(1+\mu)}{4\eta_{s6}^2}$.

The RBF NN: $W_{s1}^{*T}S_{s1}(X_{s1})$ is adopted to approximate the unknown function $\widehat{\alpha}_{s1}(X_{s1})$, such that

$$
\begin{aligned}
\widehat{\alpha}_{s1}(X_{s1})\ &=\ W_{s1}^{*T}S_{s1}(X_{s1})+\delta_{s1}(X_{s1})\\
|\delta_{s1}(X_{s1})|\ &\le\ \varepsilon_{s1}
\end{aligned}
$$

where $\delta_{s1}(X_{s1})$ denotes the approximation error, $X_{s1}\in\Omega_{X_{s1}}=\{X_{s1}|x\in\Omega_{X_{s1}}\}$. From the definition of θ_s and α_{s1}, one has

$$
\begin{aligned}
-Z_{s1}^3\hat{\alpha}_{s1}\ &=\ -Z_{s1}^3\big(W_{s1}^{*T}S_{s1}(X_{s1})+\delta_{s1}(X_{s1})\big)\\
&\le\ \frac{N_{s1}}{2a_{s1}^2}Z_{s1}^6\|W_{s1}^*\|^2+\frac{1}{2}a_{s1}^2\\
&\quad+\frac{3}{4}Z_{s1}^4+\frac{1}{4}\varepsilon_{s1}^4\\
&\le\ \frac{1}{2a_{s1}^2}Z_{s1}^6\theta_s+\frac{1}{2}a_{s1}^2+\frac{3}{4}Z_{s1}^4+\frac{1}{4}\varepsilon_{s1}^4 \qquad (70)
\end{aligned}
$$

$$Z_{s1}^3\alpha_{s1}=-\frac{1}{2a_{s1}^2}Z_{s1}^6\hat{\theta}_s \qquad (71)$$

where a_{s1} is a design parameter. The inequality $0<S_{s1}^TS_{s1}\le N_{s1}$ is used and yields

$$
\begin{aligned}
LV_{s1}\ \le\ & -p_{s1}\|e_s\|^4-l_{s1}Z_{s1}^4+Z_{s1}^3(\hat{x}_{s2}-\widehat{\alpha}_{s1})\\
&-\frac{3}{4}Z_{s1}^4-\frac{1}{r_s}\tilde{\theta}_s\dot{\hat{\theta}}_s+\frac{1}{2}\theta_s^2+\frac{1}{2}\varepsilon_{s0}^4\\
&+\left(\frac{3n\sqrt{n}}{2\eta_{s0}}+\frac{3n\sqrt{n}\mu}{2\eta_{s1}^4}+\frac{3(1+\mu)}{4\eta_{s5}^2}\right)\|\psi(\hat{x}_{s1})\|^4\\
&+\frac{1}{2\eta_{s2}^4}h^4 \qquad (72)
\end{aligned}
$$

where $\Delta_{s1}=\left(\frac{3n\sqrt{n}}{2\eta_{s0}}+\frac{3n\sqrt{n}\mu}{2\eta_{s1}^4}+\frac{3(1+\mu)}{4\eta_{s5}^2}\right)\|\psi(\hat{x}_{s1})\|^4+\frac{1}{2}\theta_s^2+\frac{1}{2}\varepsilon_{s0}^4+\frac{1}{2}a_{s1}^2+\frac{1}{4}\varepsilon_{s1}^4+\frac{1}{2\eta_{s2}^4}h^4$.

By the definition of $\hat{x}_{s2}=Z_{s2}+\alpha_{s2f}$, then (72) can be rewritten as

$$
\begin{aligned}
LV_{s1}\ \le\ & -p_{s1}\|e_s\|^4-l_{s1}Z_{s1}^4+Z_{s1}^3\big(Z_{s2}+\alpha_{s2f}-\alpha_{s1}\big)\\
&+\frac{1}{r_s}\tilde{\theta}_s\left(\frac{r_s}{2a_{s1}^2}Z_{s1}^6-\dot{\hat{\theta}}_s\right)+\Delta_{s1}. \qquad (73)
\end{aligned}
$$

Similarly, a first-order filter with time constant κ_{s2} is introduced as:

$$\kappa_{s2}\dot{\alpha}_{s2f}+\alpha_{s2f}=\alpha_{s1},\quad \alpha_{s2f}(0)=\alpha_{s1}(0).$$

Let $\chi_{s2}=\alpha_{s2f}-\alpha_{s1}$, then $\dot{\alpha}_{s2f}=-(\chi_{s2}/\kappa_{s2})$, and

$$\dot{\chi}_{s2}=\dot{\alpha}_{s2f}-\dot{\alpha}_{s1}=-\frac{\chi_{s2}}{\kappa_{s2}}+B_{s2}(X_1)$$

where

$$B_{s2}(X_1)=\frac{3}{2a_{s1}^2}Z_{s1}^2\dot{Z}_{s1}\hat{\theta}_s+\frac{1}{2a_{s1}^2}Z_{s1}^2\dot{\hat{\theta}}_s.$$

Then, the following relation holds:

$$
\begin{aligned}
LV_{s1}\ \le\ & -p_{s1}\|e_s\|^4-l_{s1}Z_{s1}^4+Z_{s1}^3Z_{s2}+Z_{s1}^3\chi_{s2}\\
&+\frac{1}{r_s}\tilde{\theta}_s\left(\frac{r_s}{2a_{s1}^2}Z_{s1}^6-\dot{\hat{\theta}}_s\right)+\Delta_{s1}. \qquad (74)
\end{aligned}
$$

Step m: $(2\le m\le n-1)$. Define the Lyapunov function as

$$V_{sm}=V_{s(m-1)}+\frac{1}{4}Z_{sm}^4+\frac{1}{4}\chi_{sm}^4. \qquad (75)$$

Similarly, one has:

$$
\begin{aligned}
LV_{sm}\ \le\ & -p_{s1}\|e_s\|^4-\sum_{i=1}^{m-1}l_{si}Z_{si}^4+\sum_{i=1}^{m-1}Z_{si}^3Z_{s(i+1)}\\
&+\sum_{i=1}^{m-1}Z_{si}^3\chi_{s(i+1)}+\Delta_{s(m-1)}\\
&-\sum_{i=1}^{m-1}\left(\frac{\chi_{s(i+1)}^4}{\kappa_{s(i+1)}}-\chi_{s(i+1)}^3B_{s(i+1)}(X_i)\right)\\
&+\frac{1}{r_s}\tilde{\theta}_s\left(\sum_{i=1}^{m-1}\frac{r_s}{2a_{si}^2}Z_{si}^6-\dot{\hat{\theta}}_s\right)\\
&+Z_{sm}^3\big(\hat{x}_{s(m+1)}+\overline{f}_{sm}\big)-\frac{3}{4}Z_{sm}^4 \qquad (76)
\end{aligned}
$$

where

$$\overline{f}_{sm}(X_m)=k_{sm}e_{s1}'-\dot{\alpha}_{smf}+\frac{3}{4}Z_{sm}.$$

Take the intermediate control signal $\widehat{\alpha}_{sm}(X_{sm})$ as

$$\widehat{\alpha}_{sm} = -\big(l_{sm} Z_{sm} + \overline{f}_{sm}\big)$$

where $l_{sm} > 0$ is a design parameter. Then, adding and subtracting $\widehat{\alpha}_{sm}$ in (76) yields:

$$\begin{aligned} LV_{sm} \leq\ & -p_{s1}\|e_s\|^4 - \sum_{i=1}^{m} l_{si} Z_{si}^4 + \sum_{i=1}^{m-1} Z_{si}^3 Z_{s(i+1)} \\ & + \sum_{i=1}^{m-1} Z_{si}^3 \chi_{s(i+1)} + Z_{sm}^3\big(\hat{x}_{s(m+1)} - \widehat{\alpha}_{sm}\big) \\ & -\frac{3}{4} Z_{sm}^4 \\ & - \sum_{i=1}^{m-1}\left(\frac{\chi_{s(i+1)}^4}{\kappa_{s(i+1)}} - \chi_{s(i+1)}^3 B_{s(i+1)}(X_i)\right) \\ & + \frac{1}{r_s}\tilde{\theta}_s\left(\sum_{i=1}^{m-1}\frac{r_s}{2a_{si}^2} Z_{si}^6 - \dot{\hat{\theta}}_s\right) + \Delta_{s(m-1)} \end{aligned} \tag{77}$$

Similarly, $\widehat{\alpha}_{sm}(X_{sm})$ can be approximated by the RBF NN $W_{sm}^{*T} S_{sm}(X_{sm})$ as

$$\begin{aligned} \widehat{\alpha}_{sm}(X_{sm}) &= W_{sm}^{*T} S_{sm}(X_{sm}) + \delta_{sm}(X_{sm}), \\ |\delta_{sm}(X_{sm})| &\leq \varepsilon_{sm} \end{aligned}$$

where $\delta_{sm}(X_{sm})$ represents the approximation error and $X_{sm} \in \Omega_{X_{sm}} = \{X_{sm} \,|\, x \in \Omega\}$. Then one obtains

$$\begin{aligned} -Z_{sm}^3 \widehat{\alpha}_{sm} \leq\ & \frac{1}{2a_{sm}^2} Z_{sm}^6 \theta_s + \frac{1}{2} a_{sm}^2 \\ & + \frac{3}{4} Z_{sm}^4 + \frac{1}{4}\varepsilon_{sm}^4 \end{aligned} \tag{78}$$

$$Z_{sm}^3 \alpha_{sm} = -\frac{1}{2a_{sm}^2} Z_{sm}^6 \hat{\theta}_s. \tag{79}$$

Substituting (78) and (79) into (77) yields

$$\begin{aligned} LV_{sm} \leq\ & -p_{s1}\|e_s\|^4 - \sum_{i=1}^{m} l_{si} Z_{si}^4 + \sum_{i=1}^{m-1} Z_{si}^3 Z_{s(i+1)} \\ & + Z_{sm}^3(\hat{x}_{s(m+1)} - \alpha_{sm}) + \sum_{i=1}^{m-1} Z_{si}^3 \chi_{s(i+1)} \\ & + \frac{1}{r_s}\tilde{\theta}_s\left(\sum_{i=1}^{m}\frac{r_s}{2a_{si}^2} Z_{si}^6 - \dot{\hat{\theta}}_s\right) \\ & - \sum_{i=1}^{m-1}\left(\frac{\chi_{s(i+1)}^4}{\kappa_{s(i+1)}} - \chi_{s(i+1)}^3 B_{s(i+1)}(X_i)\right) \\ & + \Delta_{sm} \end{aligned} \tag{80}$$

where

$$\begin{aligned} \Delta_{sm} =\ & \left(\frac{3n\sqrt{n}}{2\eta_{s0}} + \frac{3n\sqrt{n}\mu}{2\eta_{s1}^4} + \frac{3(1+\mu)}{4\eta_{s5}^2}\right)\|\psi(\hat{x}_{s1})\|^4 \\ & + \frac{1}{2}\theta_s^2 + \frac{1}{2}\varepsilon_{s0}^4 + \frac{1}{2}\sum_{i=1}^{m}\left(a_{si}^2 + \frac{1}{2}\varepsilon_{si}^4\right) \\ & + \frac{1}{2\eta_{s2}^4} h^4. \end{aligned}$$

By the definition $\hat{x}_{sm} = Z_{sm} + \alpha_{smf}$, (80) can be rewritten as

$$\begin{aligned} LV_{sm} \leq\ & -p_{s1}\|e_s\|^4 - \sum_{i=1}^{m} l_{si} Z_{si}^4 + \sum_{i=1}^{m-1} Z_{si}^3 Z_{s(i+1)} \\ & + Z_{sm}^3(Z_{s(m+1)} + \alpha_{s(m+1)f} - \alpha_{sm}) \\ & + \frac{1}{r_s}\tilde{\theta}_s\left(\sum_{i=1}^{m}\frac{r_s}{2a_{si}^2} Z_{si}^6 - \dot{\hat{\theta}}_s\right) \\ & - \sum_{i=1}^{m-1}\left(\frac{\chi_{s(i+1)}^4}{\kappa_{s(i+1)}} - \chi_{s(i+1)}^3 B_{s(i+1)}(X_i)\right) \\ & + \Delta_{sm} + \sum_{i=1}^{m-1} Z_{si}^3 \chi_{s(i+1)}. \end{aligned} \tag{81}$$

Similarly, the filtering signal $\alpha_{s(m+1)f}$ is obtained by

$$\begin{aligned} \kappa_{s(m+1)}\dot{\alpha}_{s(m+1)f} + \alpha_{s(m+1)f} &= \alpha_{sm}, \\ \alpha_{s(m+1)f}(0) &= \alpha_{sm}(0). \end{aligned} \tag{82}$$

Define $\chi_{s(m+1)} = \alpha_{s(m+1)f} - \alpha_{sm}$, then

$$\dot{\alpha}_{s(m+1)f} = -\frac{\chi_{s(m+1)}}{\kappa_{s(m+1)}} \tag{83}$$

and

$$\begin{aligned} \dot{\chi}_{s(m+1)} &= \dot{\alpha}_{s(m+1)f} - \dot{\alpha}_{sm} \\ &= -\frac{\chi_{s(m+1)}}{\kappa_{s(m+1)}} + B_{s(m+1)}(X_m) \end{aligned} \tag{84}$$

where

$$B_{s(m+1)}(X_m) = \frac{3}{2a_{sm}^2} Z_{sm}^2 \dot{Z}_{sm}\hat{\theta}_s + \frac{1}{2a_{sm}^2} Z_{sm}^2 \dot{\hat{\theta}}_s.$$

Finally, it implies

$$\begin{aligned} LV_{sm} \leq\ & -p_{s1}\|e_s\|^4 - \sum_{i=1}^{m} l_{si} Z_{si}^4 + \sum_{i=1}^{m} Z_{si}^3 Z_{s(i+1)} \\ & + \sum_{i=1}^{m} Z_{si}^3 \chi_{s(i+1)} + \frac{1}{r_s}\tilde{\theta}_s\left(\sum_{i=1}^{m}\frac{r_s}{2a_{si}^2} Z_{si}^6 - \dot{\hat{\theta}}_s\right) \\ & - \sum_{i=1}^{m-1}\left(\frac{\chi_{s(i+1)}^4}{\kappa_{s(i+1)}} - \chi_{s(i+1)}^3 B_{s(i+1)}(X_i)\right) \\ & + \Delta_{sm}. \end{aligned} \tag{85}$$

Step n: Consider the following Lyapunov function:

$$V_{sn} = V_{s(n-1)} + \frac{1}{4} Z_{sn}^4 + \frac{1}{4}\chi_{sn}^4 \tag{86}$$

Similarly, one obtains

$$\begin{aligned} LV_{sn} \leq\ & -p_{s1}\|e_s\|^4 - \sum_{i=1}^{n-1} l_{si} Z_{si}^4 \\ & + \sum_{i=1}^{n-1} Z_{si}^3 Z_{s(i+1)} + \sum_{i=1}^{n-1} Z_{si}^3 \chi_{s(i+1)} \\ & + Z_{sn}^3(u_s + \bar{f}_{sn}) + \frac{1}{r_s}\tilde{\theta}_s\left(\sum_{i=1}^{n-1}\frac{r_s}{2a_{si}^2} Z_{si}^6 - \dot{\hat{\theta}}_s\right) \\ & - \sum_{i=1}^{n-1}\left(\frac{\chi_{s(i+1)}^4}{\kappa_{s(i+1)}} - \chi_{s(i+1)}^3 B_{s(i+1)}(X_i)\right) \\ & - \frac{3}{4} Z_{sn}^4 + \Delta_{s(n-1)} \end{aligned} \tag{87}$$

where

$$\bar{f}_{sn} = l_{sn}e'_{s1} - \dot{\alpha}_{snf} + \frac{1}{4}Z_{sn}^4 + \frac{3}{4}Z_{sn}.$$

Take the intermediate control signal $\widehat{\alpha}_{sn}(X_{sn})$ as $\widehat{\alpha}_{sn} = -(l_{sn}Z_{sn}+\bar{f}_{sn})$, where $l_{sn} > 0$. Then, subtracting $\widehat{\alpha}_{sn}(X_{sn})$ in (87) yields

$$\begin{aligned} LV_{sn} \leq\ & -p_{s1}\|e_s\|^4 - \sum_{i=1}^{n} l_{si}Z_{si}^4 + Z_{sn}^3(u_s - \widehat{\alpha}_{sn}) \\ & + \sum_{i=1}^{n-1} Z_{si}^3 Z_{s(i+1)} + \sum_{i=1}^{n-1} Z_{si}^3 \chi_{s(i+1)} \\ & + \frac{1}{r_s}\tilde{\theta}_s\left(\sum_{i=1}^{n-1}\frac{r_s}{2a_{si}^2}Z_{si}^6 - \dot{\hat{\theta}}_s\right) \\ & - \frac{3}{4}Z_{sn}^4 \\ & - \sum_{i=1}^{n-1}\left(\frac{\chi_{s(i+1)}^4}{\kappa_{s(i+1)}} - \chi_{s(i+1)}^3 B_{s(i+1)}(X_i)\right) \\ & + \Delta_{s(n-1)} \end{aligned} \tag{88}$$

Similarly, $\widehat{\alpha}_{sn}(X_{sn})$ can be approximated by the RBF NN $W_{sn}^{*T}S_{sn}(X_{sn})$ as

$$\begin{aligned} \widehat{\alpha}_{sn}(X_{sn}) &= W_{sn}^{*T}S_{sn}(X_{sn}) + \delta_{sn}(X_{sn}), \\ |\delta_{sn}(X_{sn})| &\leq \varepsilon_{sn} \end{aligned}$$

where $\delta_{sn}(X_{sn})$ represents the approximation error and $X_{sn} \in \Omega_{X_{sn}} = \{X_{sn}\,|\,x \in \Omega_x\}$. By the definition of u_s, we have

$$\begin{aligned} -Z_{sn}^3\hat{\alpha}_{sn} &\leq \frac{1}{2a_{sn}^2}Z_{sn}^6\theta_s + \frac{1}{2}a_{sn}^2 + \frac{3}{4}Z_{sn}^4 + \frac{1}{4}\varepsilon_{sn}^4, \\ Z_{sn}^3 u_s &= -\frac{1}{2a_{sn}^2}Z_{sn}^6\hat{\theta}_s. \end{aligned}$$

Then, substituting these into (88) yields

$$\begin{aligned} LV_{sn} \leq\ & -p_{s1}\|e_s\|^4 - \sum_{i=1}^{n} l_{si}Z_{si}^4 + \sum_{i=1}^{n-1} Z_{si}^3 Z_{s(i+1)} \\ & + \sum_{i=1}^{n-1} Z_{si}^3\chi_{s(i+1)} + \frac{1}{r_s}\tilde{\theta}_s\left(\sum_{i=1}^{n}\frac{r_s}{2a_{si}^2}Z_{si}^6 - \dot{\hat{\theta}}_s\right) \\ & - \sum_{i=1}^{n-1}\left(\frac{\chi_{s(i+1)}^4}{\kappa_{s(i+1)}} - \chi_{s(i+1)}^3 B_{s(i+1)}(X_i)\right) \\ & + \Delta_{sn} \end{aligned} \tag{89}$$

where

$$\begin{aligned} \Delta_{sn} =\ & \left(\frac{3n\sqrt{n}}{2\eta_{s0}} + \frac{3n\sqrt{n}\mu}{2\eta_{s1}^4} + \frac{3(1+\mu)}{4\eta_{s5}^2}\right)\|\psi(\hat{x}_{s1})\|^4 \\ & + \frac{1}{2}\theta_s^2 + \frac{1}{2}\varepsilon_{s0}^4 + \frac{1}{2}\sum_{i=1}^{n}\left(a_{si}^2 + \frac{1}{2}\varepsilon_{si}^4\right) \\ & + \frac{1}{2\eta_{s2}^4}h^4. \end{aligned}$$

By the definition of $\dot{\hat{\theta}}_s$, we obtain

$$\begin{aligned} LV_{sn} \leq\ & -p_{s1}\|e_s\|^4 - \sum_{i=1}^{n} l_{si}Z_{si}^4 + \sum_{i=1}^{n-1} Z_{si}^3 Z_{s(i+1)} \\ & + \sum_{i=1}^{n-1} Z_{si}^3\chi_{s(i+1)} + \frac{k_s}{r_s}\tilde{\theta}_s\hat{\theta}_s \\ & - \sum_{i=1}^{n-1}\left(\frac{\chi_{s(i+1)}^4}{\kappa_{s(i+1)}} - \chi_{s(i+1)}^3 B_{s(i+1)}(X_i)\right) \\ & + \Delta_{sn}. \end{aligned} \tag{90}$$

Using Young's inequality, one has

$$\begin{aligned} Z_{si}^3 Z_{s(i+1)} &\leq \frac{3}{4}Z_{si}^4 + \frac{1}{4}Z_{s(i+1)}^4 \\ Z_{si}^3\chi_{s(i+1)} &\leq \frac{3}{4}Z_{si}^4 + \frac{1}{4}\chi_{s(i+1)} \\ \tilde{\theta}_s\hat{\theta}_s &= \tilde{\theta}_s(\theta_s - \tilde{\theta}_s) \leq -\frac{1}{2}\tilde{\theta}_s^2 + \frac{1}{2}\theta_s^2 \\ \left|\chi_{s(i+1)}^3 B_{s(i+1)}\right| &\leq \frac{3}{4}\pi_s^{\frac{4}{3}}B_{s(i+1)}^{\frac{4}{3}}\chi_{s(i+1)}^4 + \frac{1}{4\pi_s^4} \end{aligned} \tag{91}$$

where $\pi_s > 0$. There exists $N_{s(i+1)} > 0$, such that $|B_{s(i+1)}| \leq N_{s(i+1)}$. Then, one has

$$\begin{aligned} LV_{sn} \leq\ & -p_{s1}\|e_s\|^4 - \sum_{i=1}^{n}(l_{si} - \frac{7}{4})Z_{si}^4 \\ & + \sum_{i=1}^{n-1}\left(\frac{1}{\kappa_{s(i+1)}} - \frac{1}{4} - \frac{3}{4}\pi_s^{\frac{4}{3}}B_{s(i+1)}^{\frac{4}{3}}\right)\chi_{s(i+1)}^4 \\ & - \frac{k_s}{2r_s}\tilde{\theta}_s^2 + \overline{\Delta}_{sn} \end{aligned} \tag{92}$$

where $\overline{\Delta}_{sn} = \Delta_{sn} + \frac{1}{4\pi_s^4} + \frac{1}{2}\theta_s^2$.

C. Stability analysis

Based on the preceding design and analysis, the stability of the closed-loop system described by equation (1) can be established.

Theorem 3: Consider the system (1). Under **Assumptions(1-2)**, if there exist positive definite matrix P , matrix K , and positive constants $(l_{cj} - \frac{7}{4})$, $(l_{sj} - \frac{7}{4})$ p_{c1}, p_{s1}, such that $c = \min\{\frac{2p_{c1}}{\lambda_{\min}^2(P)}, \frac{2p_{s1}}{\lambda_{\min}^2(P)}, 4(l_{ci} - \frac{7}{4}), 4(l_{si} - \frac{7}{4}), 4(\frac{1}{\kappa_{c(i+1)}} - \frac{1}{4} - \frac{3}{4}\pi_c^{\frac{4}{3}}B_{c(i+1)}^{\frac{4}{3}}), 4(\frac{1}{\kappa_{s(i+1)}} - \frac{1}{4} - \frac{3}{4}\pi_s^{\frac{4}{3}}B_{s(i+1)}^{\frac{4}{3}}), k_c, k_s\}$ is greater than 0, then the proposed state estimators and controllers u_s, u_s with the virtual control laws α_{ci}, α_{si} can guarantee that all the signals in the closed-loop system remain UUB in mean square.

Proof: Consider the expectation of the Lyapunov function $V = \theta_1 V_{cn} + \theta_2 V_{sn}$:

$$E[V] = \theta_1 E[V_{cn}] + \theta_2 E[V_{sn}],$$

where θ_1, θ_2 satisfy $\theta_1 + \theta_2 = 1$ and represent the probabilities of the normal case and the data-losing case, respectively. Its derivative satisfies

$$
\begin{aligned}
LV_{sn} \leq & -p_{s1}\|e_s\|^4 - \sum_{i=1}^{n} l_{si}Z_{si}^4 + \sum_{i=1}^{n-1} Z_{si}^3 Z_{s(i+1)} \\
& + \sum_{i=1}^{n-1} Z_{si}^3 \chi_{s(i+1)} + \frac{1}{r_s}\tilde{\theta}_s \left(\sum_{i=1}^{n-1} \frac{r_s}{2a_{si}^2} Z_{si}^6 - \dot{\hat{\theta}}_s \right) \\
& - \sum_{i=1}^{n-1} \left(\frac{\chi_{s(i+1)}^4}{\kappa_{s(i+1)}} - \chi_{s(i+1)}^3 B_{s(i+1)}(X_i) \right) \\
& + \Delta_{sn}, \qquad (93)
\end{aligned}
$$

where $b = \overline{\Delta}_{cn} + \overline{\Delta}_{sn}$.

Let $c > b/\Lambda$ for some $\Lambda \in \mathbb{R}$. Then $E[LV] < 0$ when $E[V] = \Lambda$. If $E[V(0)] < \Lambda$, it follows that $E[V(t)] < \Lambda$ for all $t > 0$. Therefore, this inequality holds for all $t > 0$, leading to

$$
0 < E[V] < V(0)e^{-ct} + \frac{b}{c}, \quad \forall t \geq 0, \qquad (94)
$$

which shows that $E[V(t)]$ is bounded by b/c. Consequently, all signals of system (1) are UUB in mean square.

IV. Simulation Example

To illustrate the effectiveness and applicability of the proposed adaptive NN control scheme, a representative simulation example is presented.

Consider a power network system [20], governed by the following stochastic differential equations:

$$
\begin{aligned}
d(\Delta\delta) &= \Delta\omega \, dt, \\
d(\Delta\omega) &= \frac{D}{2H}\Delta\omega \, dt + \frac{\omega_0}{2H}\Delta P \, dt, \\
d(\Delta P) &= \frac{1}{T}\left(-\Delta P - K\Delta\omega + u\right) dt + dw, \\
y(t) &= \Delta\delta. \qquad (95)
\end{aligned}
$$

Here, $\Delta\delta$, $\Delta\omega$, and ΔP represent deviations in rotor angle, rotor speed, and mechanical input power, respectively. The system parameters are fixed as $D = 3$, $H = 12$, $T = 1$, $K = 0.01$, and $\omega_0 = 314$. Defining state variables as $[x_1, x_2, x_3] = [\Delta\delta, \Delta\omega, \Delta P/2H]$ and scaled control input as $U = u\omega_0/2HT$, we establish initial conditions as $[x_1(0), x_2(0), x_3(0)]^T = [-0.011, 0.01, -0.1]^T$.

The simulation employs the following parameter values: $r_c = r_s = 21.5$, $a_{c1} = a_{s1} = 0.2$, $a_{c2} = a_{s2} = 0.21$, $a_{c3} = a_{s3} = 0.15$, $k_{c1} = k_{s1} = 110$, $k_{c2} = k_{s2} = 100$, $k_{c3} = k_{s3} = 90$, $k_c = k_s = 0.1$, $\kappa_{c2} = 0.5$, $\kappa_{c2} = 0.01$, $\kappa_{s3} = -2$, and initial adaptive parameters $\hat{\theta}_c(0) = \hat{\theta}_s(0) = 0$. The total simulation duration is 60 s, featuring data-loss intervals at $[5, 10] \cup [30, 34]$s.

For normal measurement conditions, the observer and controller are described by:

$$
\begin{aligned}
\alpha_{c1} &= -\frac{1}{2a_{c1}^2} Z_{c1}^3 \hat{\theta}_c, \\
\alpha_{c2} &= -\frac{1}{2a_{c2}^2} Z_{c2}^3 \hat{\theta}_c, \\
U_c &= -\frac{1}{2a_{c3}^2} Z_{c3}^3 \hat{\theta}_c, \\
\dot{\hat{x}}_{c1} &= \hat{x}_{c2} + k_{c1}(x_1 - \hat{x}_{c1}), \\
\dot{\hat{x}}_{c2} &= \hat{x}_{c3} + k_{c2}(x_1 - \hat{x}_{c1}), \\
\dot{\hat{x}}_{c3} &= U_c + k_{c3}(x_1 - \hat{x}_{c1}), \qquad (96)
\end{aligned}
$$

with the adaptive law defined as:

$$
\dot{\hat{\theta}}_c = \sum_{i=1}^{3} \frac{r_c}{2a_{ci}} Z_{ci}^6 - k_c \hat{\theta}_c. \qquad (97)
$$

During data-loss conditions, the modified observer equations become:

$$
\begin{aligned}
\alpha_{s1} &= -\frac{1}{2a_{s1}^2} Z_{s1}^3 \hat{\theta}_s, \\
\alpha_{s2} &= -\frac{1}{2a_{s2}^2} Z_{s2}^3 \hat{\theta}_s, \\
U_s &= -\frac{1}{2a_{s3}^2} Z_{s3}^3 \hat{\theta}_s, \\
\dot{\hat{x}}_{s1} &= \hat{x}_{s2} + k_{s1}(x_1' - \hat{x}_{s1}), \\
\dot{\hat{x}}_{s2} &= \hat{x}_{s3} + k_{s2}(x_1' - \hat{x}_{s1}), \\
\dot{\hat{x}}_{s3} &= U_s + k_{s3}(x_1' - \hat{x}_{s1}), \qquad (98)
\end{aligned}
$$

with the corresponding adaptive law:

$$
\dot{\hat{\theta}}_s = \sum_{i=1}^{3} \frac{r_s}{2a_{si}} Z_{si}^6 - k_s \hat{\theta}_s. \qquad (99)
$$

Simulation results, including state trajectories and adaptive parameters under both scenarios, are presented in Figs. 1–9.

A. Results

The proposed adaptive NN observer–controller was tested on the single-machine infinite-bus benchmark in two sensing regimes: continuous, error-free measurements and intermittent data loss. All gains were kept identical across regimes so that any change in behaviour could be attributed solely to the state-information channel.

Under uninterrupted sensing the three plant states converge quickly and smoothly. Figure 1 shows that the rotor-angle deviation x_1 settles into the $\pm 2\%$ band within roughly 1.2 s, while the NN estimate $\hat{x}_{c1}$ tracks the true state so closely that the peak estimation error never exceeds 4×10^{-3} rad. The speed deviation x_2 (Fig. 2) exhibits a modest overshoot of about 5% and reaches steady behaviour at $t \approx 2$ s. Mechanical power x_3 (Fig. 3) converges even faster, underscoring the effectiveness of the back-stepping structure and the single-parameter NN adaptation law. Together these curves confirm that, when data are available, the observer provides near-perfect state reconstruction and the controller maintains tight regulation of the nonlinear stochastic plant.

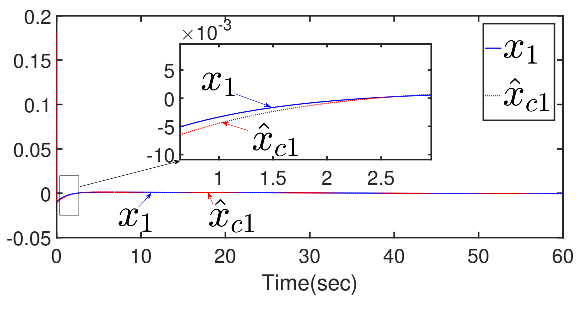

Fig. 1. State x_1 and estimate $\hat{x}_{c1}$ (normal case)

Robustness to missing information was examined by blocking the measurement stream during two 2 s intervals.

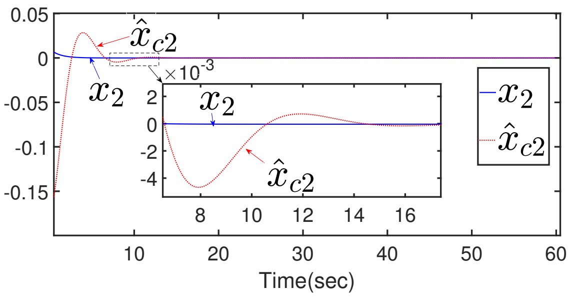

Fig. 2. State x_2 and estimate $\hat{x}_{c2}$ (normal case)

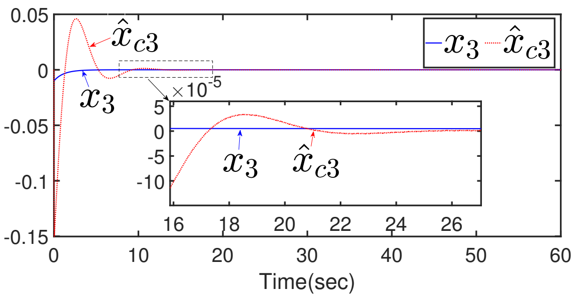

Fig. 3. State x_3 and estimate $\hat{x}_{c3}$ (normal case)

During each outage the observer substituted the most recent valid sample. Although this produces a temporary excursion—in x_1 the deviation rises to approximately 1.8×10^{-2} rad (Fig. 4)—all estimation errors remain bounded, and every state re-enters its $\pm 2\%$ band within 1.1 s after the channel is restored (Figs. 5–6). These observations validate the mean-square ultimate-bounded analysis and demonstrate that the scheme can tolerate realistic network interruptions without compromising overall stability.

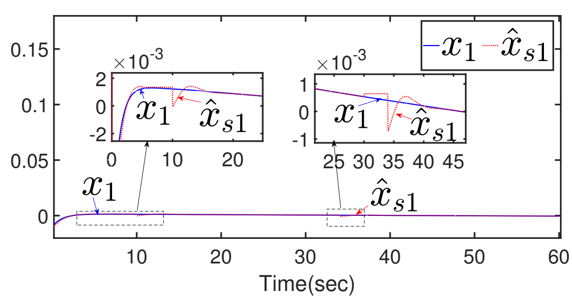

Fig. 4. State x_1 and estimate $\hat{x}_{s1}$ (data-loss case)

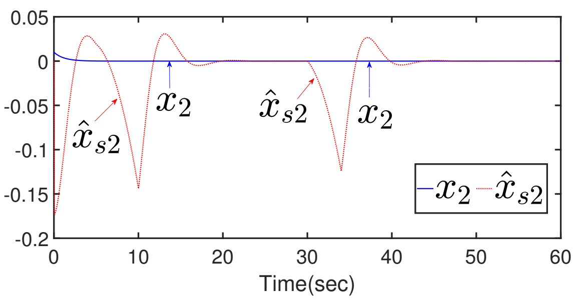

Fig. 5. State x_2 and estimate $\hat{x}_{s2}$ (data-loss case)

Control effort remains moderate in both regimes. Figure 7

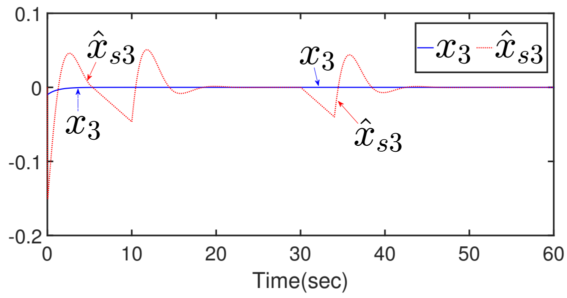

Fig. 6. State x_3 and estimate $\hat{x}_{s3}$ (data-loss case)

indicates that the input never exceeds ± 3 p.u. and displays no chattering, evidence that the DSC filters successfully suppress the high-frequency components introduced by adaptation. The filtered virtual-control signal $\alpha_{3\mathrm{f}}$ exhibits a peak-to-peak amplitude of 2.7 p.u. and attenuates the derivative's high-frequency content by more than 20 dB (Fig. 8), preserving robustness without sacrificing responsiveness.

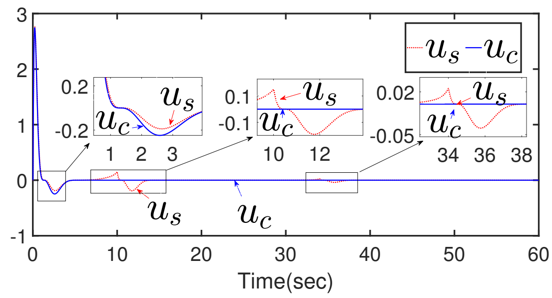

Fig. 7. Control inputs u_c (normal) and u_s (data-loss)

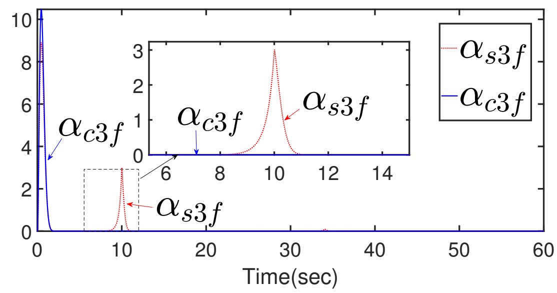

Fig. 8. Filtered states α_{c3f} (normal) and α_{s3f} (data-loss)

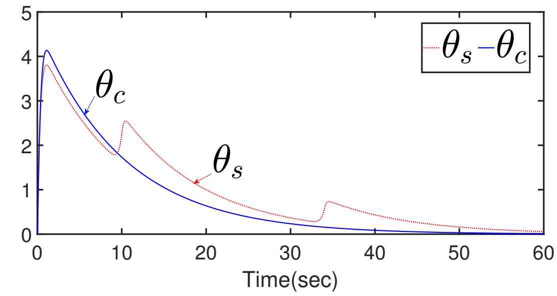

Fig. 9. Adaptive parameters θ_c (normal) and θ_s (data-loss)

In summary, the numerical evidence demonstrates that the proposed observer–controller delivers fast transient suppression (settling times below 2 s), low steady-state error (milliradian-level for angle and milliper-unit for speed and power), and bounded performance during sensor outages, all with modest control energy and minimal adaptive overhead. These properties collectively satisfy the stringent reliability and efficiency requirements of modern stochastic CPSs operating over unreliable communication networks.

B. Discussion

The simulation results confirm that the proposed adaptive NN control scheme performs well under both normal and data-loss measurement conditions. In particular, the observer can accurately estimate the system states even when full state information is unavailable, and the controller ensures that the system remains stable and tracks the desired trajectories. The adaptive laws also show good convergence properties, as the estimated parameters quickly approach steady values. During the periods of data loss, the system can still maintain boundedness and recover rapidly after the data resumes, which demonstrates the robustness of the control design. This feature is especially important for CPSs where communication interruptions are common.

V. Conclusion

This paper presents an adaptive NN control scheme for strict-feedback stochastic nonlinear CPSs subject to incomplete measurement scenarios. Combining output feedback control, observer-based estimation, and DSC effectively mitigates complexity challenges and ensures system stability. Simulations verify control efficacy, robustness, and adaptive performance under varied operational conditions, paving the way for future CPS enhancements.

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