Robust PID Control with Sliding Mode and Adaptive Rules for Uncertain Systems

T. C. Kuo, *Member, IAENG,* Y. J. Huang, *Member, IAENG,* Y. R. Wu, and C. Y Chen

Abstract—**This paper presents a neural network global PID-sliding mode control method for the tracking control of robot manipulators with bounded uncertainties. A certain sliding mode controller with PID sliding function is developed. Mathematical proof of the stability and convergence of the control system is given. Simulation results demonstrate that the chattering and the steady state errors are eliminated and satisfactory trajectory tracking is achieved.**

Index Terms—**PID controller, Robust adaptive control, Sliding mode control, Stability.**

I. Introduction

The proportional-integral-derivative (PID) controller is widely used in many control applications because of its simplicity and effectiveness. Though the use of PID control has a long history in control engineering, the three parameters of controller gain, i.e., proportional gain, K_P, integral gain, K_I, and derivative gain, K_D, was poorly tuned. In recent years, there has been extensive interest in self-tuning these three controller gains. The PID self-tuning methods based on the relay feedback technique were presented for a class of systems [1, 2]. An adaptive PID control PID control tuning was proposed to cope with the control problem for a class of uncertain chaotic systems with external disturbance [3]. A genetic algorithm was used to find the optimum tuning parameters of the PID controller by taking integral absolute error as fitting function [4].

Another disadvantage of PID controller is poor capability of dealing with system uncertainty, i.e., parameter variations and external disturbance. However, the uncertain systems are an important topic in the field of control. Sliding mode control (SMC) is one of the popular strategies to deal with uncertain control systems [5-7]. The main feature of SMC is the robustness against parameter variations and external disturbances. Various applications of SMC have been found, such as robotic manipulators, aircrafts, DC motors, chaotic systems, and so on [8-11].

In this paper, the adaptive PID with sliding mode controller is proposed for second-order uncertain systems. The goal is to achieve system robustness against parameter variations and external disturbances. In this study, the PID parameters can be systematically obtained according to the adaptive law. To reduce the high frequency chattering in the controller, the boundary layer technique is used [12]. The proposed method controller is applied to the brushless DC motor control system. The computer simulation results demonstrate that the chattering is eliminated and satisfactory trajectory tracking is achieved.

II. Definition of The Control System

Consider a second-order uncertain system,

$$\dot{x}_1(t) = \dot{x}_2(t), \tag{1}$$

$$\dot{x}_2(t) = f(x_1, x_2, t) + \Delta f(x_1, x_2, t) + d(t) + bu, \tag{2}$$

$$y(t) = x_1(t), \tag{3}$$

where $x_1(t)$ and $x_2(t)$ are measurable states, u is the input, y is the output, b is the input gain, $f(\cdot)$ is nominal parameter of plant, $\Delta f(\cdot)$ is the plant uncertainty applied to the system, and $d(t)$ denotes the external disturbance. It is assumed that there exist two positive upper bounds, g and α, satisfying $|\Delta f(\cdot)| \le g$ and $|d(t)| \le \alpha$.

Let e be the error between the desired trajectory y_d and the output y, i.e.,

$$e = y_d - y. \tag{4}$$

III. Controller Design

In order to have a second-order error dynamics, we define a signal x_r as

$$\dot{x}_r = \ddot{y}_d + K_1\dot{e} + K_0 e, \tag{5}$$

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where K_1 and K_0 are chosen by designers such that roots of $s^2 + K_1 s + K_0 = 0$ are in the open left-half complex plane. In general, we can choose $K_1 = 2\zeta\omega_n$ and $K_0 = \omega_n^2$, where ζ is the damping ratio and ω_n is the natural frequency.

In the following, the robust controller is developed. The design procedure of the proposed sliding mode control is divided into two steps. The first step is to define a sliding surface function such that in the sliding mode the system behaves equivalently as a linear system. The second step is to determine a control law such that the system will reach and stay on the sliding surface $s = 0$.

First, define the sliding function as

$$\sigma = x_2 - x_r \, . \tag{6}$$

If sliding mode occurs, i.e. $\sigma = 0$, then

$$x_r = x_2 \, . \tag{7}$$

Substituting (7) into (5), then

$$\ddot{e} + K_1\dot{e} + K_0 e = 0 \, . \tag{8}$$

This implies that error will tend to zero ($e \to 0$) as time goes to infinity ($t \to \infty$).

Next, let the control input u be

$$u = u_{PID} + u_s \, , \tag{9}$$

where

$$u_{PID} = \frac{1}{b}\left[K_P e + K_I \int e dt + K_D \dot{e} \right], \tag{10}$$

$$u_s = -\frac{1}{b}\left(|f| + g + \alpha + |\dot{x}_r| + K_2 \right)\mathrm{sgn}(\sigma) \, . \tag{11}$$

In (11), the gain K_2 is a positive scalar and $\mathrm{sgn}(\cdot)$ is the sign function, i.e.,

$$\mathrm{sgn}(\sigma) = \begin{cases} +1, & \sigma > 0, \\ -1, & \sigma < 0. \end{cases} \tag{12}$$

The three PID controller gains, K_P, K_I, and K_D, can be obtained by adaptive laws as following,

$$\dot{K}_P = -\eta_1 \sigma e \, , \tag{13}$$

$$\dot{K}_I = -\eta_2 \sigma \int e dt \, , \tag{14}$$

$$\dot{K}_D = -\eta_3 \sigma \dot{e} \, , \tag{15}$$

where $\eta_i > 0$ is the learning rate, $i = 1, 2, 3$.

In order to prove the stability, let the Lyapunov function be

$$V = \frac{1}{2}(\sigma^2 + \frac{1}{\eta_1}K_P^2 + \frac{1}{\eta_2}K_I^2 + \frac{1}{\eta_3}K_D^2) \, . \tag{16}$$

Taking derivative of (16) yields,

$$\dot{V} = \sigma\dot{\sigma} + \frac{1}{\eta_1}K_P\dot{K}_P + \frac{1}{\eta_2}K_I\dot{K}_I + \frac{1}{\eta_3}K_D\dot{K}_D \, , \tag{17}$$

Substituting adaptive laws (13)-(15) into (17), then

$$\dot{V} = |\sigma|\left[(|\Delta f| - g) + (|d| - \alpha) - K_2) \right]$$

$$< 0 \, . \tag{18}$$

Thus, the control law given by (9)-(12) and adaptive laws (13)-(15) guarantee the reaching and sustaining of the sliding mode.

In general, the inherent high-frequency chattering of the control input may limit the practical application of developed method. We further replace $\mathrm{sgn}(s)$ in (11) by the function $\mathrm{sat}(\frac{s}{\delta})$, i.e.,

$$\mathrm{sat}(\frac{\sigma}{\delta}) = \begin{cases} 1, & \frac{\sigma}{\delta} \geq 1, \\ \frac{\sigma}{\delta}, & -1 < \frac{\sigma}{\delta} < 1, \\ -1, & \frac{\sigma}{\delta} \leq -1, \end{cases} \tag{19}$$

where δ is the width of the boundary layer. With this replacement, the sliding surface function s with an arbitrary initial value will reach and stay within the boundary layer $|s| \leq \delta$.

IV. Simulation Results

In the computer simulation, we apply the proposed controller to the permanent magnet brushless DC motor with unknown but bounded parameter variations and external disturbance [11]. The dynamics of the brushless DC motor system is described as

$$\ddot{\theta} + a_1\dot{\theta} = b(u + d) \, , \tag{20}$$

where θ is the position angle, $\dot{\theta}$ is the angular velocity, $a_1 = \frac{B}{J}$ is composed of the viscous-friction coefficient B and an unknown but bounded J consisting of the rotor inertia and load, and $b = \frac{K_t K_c}{J}$ factors in the motor torque coefficient

K_t and the PWM inverter current coefficient K_c. The control input u is the voltage input.

Let $x_1 = \theta$ and $x_2 = \dot{\theta}$, therefore, the state-space equation of the brushless DC motor can be obtained as

$$\dot{x}_1 = x_2, \quad (21)$$

$$\dot{x}_2 = -a_1 x_2 + b(u + d), \quad (22)$$

$$y = x_1. \quad (23)$$

The bounds on the uncertain parameters and disturbances of the brushless DC motor were given in (Choi, *et al.*, 2001). The desired trajectory is

$$y_d = 2\pi - 2\pi \exp(-5t). \quad (24)$$

The control input can be implemented by using (9)-(11), adaptive laws (13)-(15), and (19). We choose damping ratio $\zeta = 1$ and natural frequency $\omega_n = 7$ so that roots of $s^2 + K_1 s + K_0 = 0$ are in the open left-half complex plane with $K_1 = 14$ and $K_0 = 49$. Three PID controller gains, K_P, K_I, and K_V, with initial values are $K_P(0) = 0$, $K_I(0) = 0$, and $K_D(0) = 0$. The learning rate η_i is 1, $i = 1, 2, 3$, boundary layer $\delta = 0.1$, and the gain K_2 of (11) is 50 in this example.

The simulation results are shown in Figs. 1 to 4. The sampling time is equal to 0.001 sec. The initial condition is $x_1(0) = x_2(0) = 0$. In Figs. 1 and 2, trajectory tracking results show that the output y converges to the desired trajectory y_d. From Fig. 3, it is obvious that chattering of the control input is eliminated by using boundary layer technology. Three PID controller gains are obtained by adaptive laws as shown in Fig. 4-6.

V. Conclusions

In this paper, a schematic robust controller design for second-order uncertain systems is proposed. The control law consists of a continuous adaptive PID control part and a discontinuous switching control input. The proposed method is simple and the three PID controller gains can be systematically obtained by adaptive laws. The high frequency chattering in the control input is eliminated by using boundary layer technology. The system stability is assured and the brushless DC motor is examined. The computer simulation results demonstrate that the chattering is eliminated and satisfactory trajectory tracking can be achieved.

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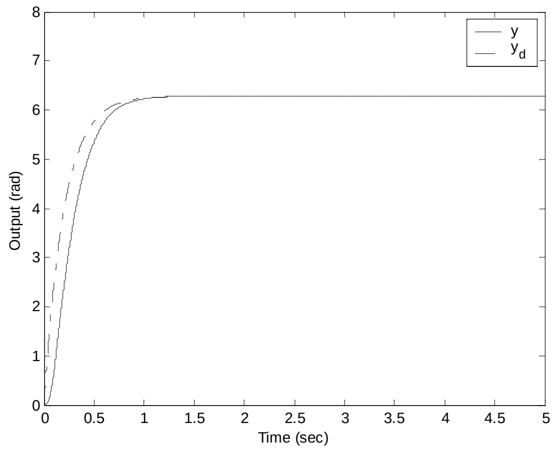

Fig. 1. Trajectory tracking.

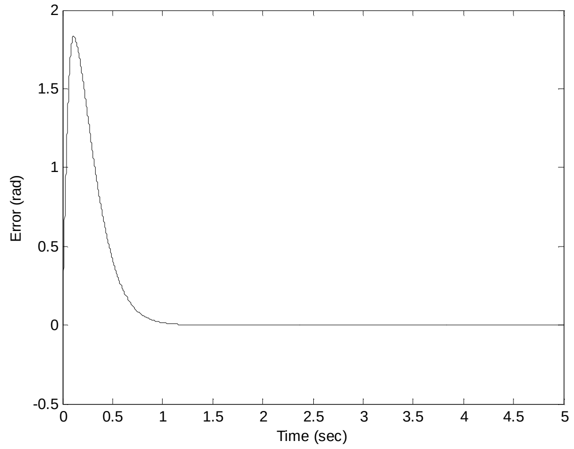

Fig. 2. Tracking error.

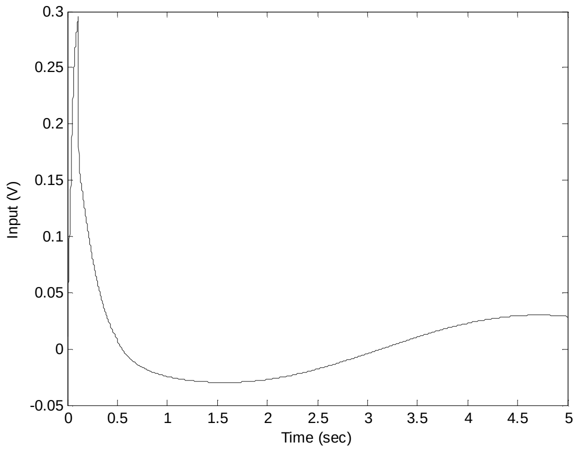

Fig. 3. Control input.

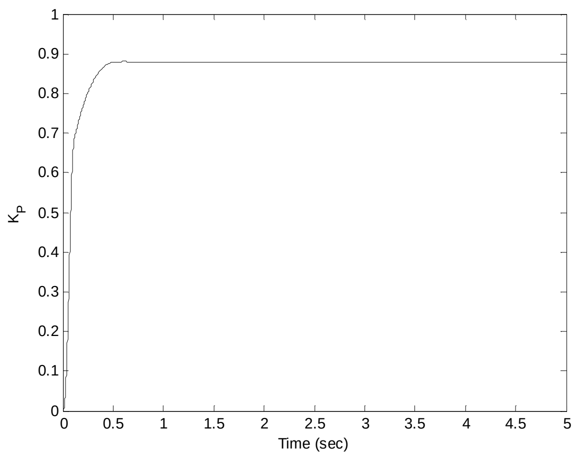

Fig. 4 Gain K_P.

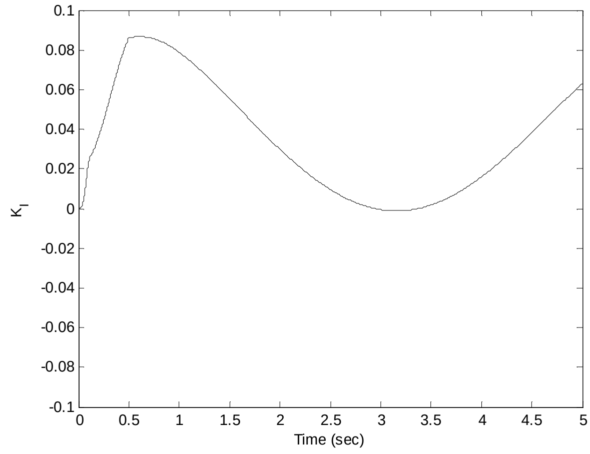

Fig. 5 Gain K_I.

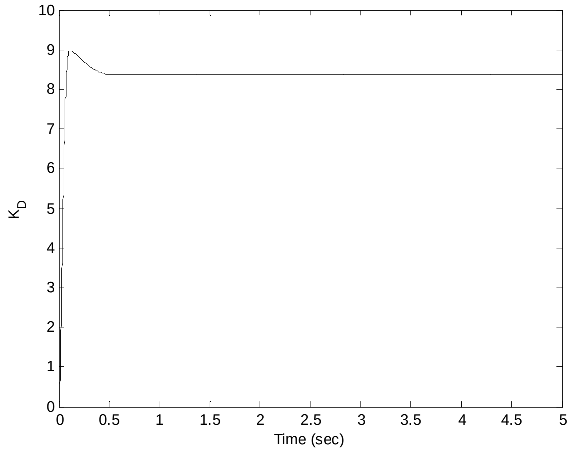

Fig. 6 Gain K_D.

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