

Wiener Model Identification using Genetic Programming

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Abstract—A Wiener model consists of a dynamic linear transfer function in series with a static nonlinear function. We can through the essences of GP, like robustness, domain independence and ability to search for satisfying solutions in solving complicated nonlinear problems, this study hoped that the evolved GP models could have a better applicability and accuracy of evaluations, and easily obtain the correct structure and parameters of the nonlinear function, and number of zeros and poles of the linear transfer function. GP is applied to the determine nonlinearity and unknown parameters in the nonlinear function and linear dynamic system model are estimated by a least square algorithm. The results of numerical studies indicate the usefulness of proposed approach to Wiener model identification.

Index Terms—Wiener model, system identification, Genetic Programming (GP), Akaike information criterion (AIC).

I. INTRODUCTION

The black box modeling based on the observed system input and output data for unknown dynamic system is called system identification. One of the difficulties in nonlinear system identification is that both the structure and the unknown parameters of model should be determined [1].

In [2], a fast method to get a first estimate of Wiener or Hammerstein systems has been introduced. However, methods must assume that the order of linear part is known and the search space is differentiable, and linear in the search parameters. In [3], Wiener model identification by evolutionary computation approach with piecewise linearization has been introduced. However, methods must assume that the nonlinear static part is invertible. Regarding these problems, we can use GP to improve.

Because GP exploit strategies of genetic information and survival of the fittest to guide their search, they need not calculate the gradient or assume that the search space is differentiable or continuous. GP simultaneously evaluate many points in the parameter space, so they are more likely to converge toward a global solution. The aim of system identification is to give the optimal mathematical model both

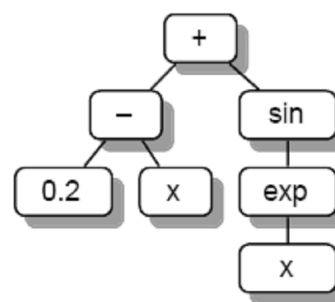
nonlinearity and linear dynamic system in an appropriate sense. We can use GP to determine the structure of nonlinear static block. The unknown parameters including linear dynamic system model's ones are estimated by the least square method.

Remainder of this paper is organized as follows. In section II, the expression tree is introduced. In section III, the Wiener model is introduced. Section IV describes our approach to the Wiener model identification and results of some numerical examples are shown in Section V. Finally, a conclusion is presented in Section VI.

II. EXPRESSION TREE

Each program or individual on the population is generally represented as a tree composed of functions and terminals appropriate to the problem domain. The function set may contain standard arithmetic operators, mathematical functions, logical operators, and domain-specific functions. The terminal set usually consists of variables and constants. The simple expression is represented as shown in Figure 1.

where the function set is $\{+, -, \sin, \exp\}$, and the terminal set is $\{x, 0.2\}$ [4].



$$(+ (- x 0.2) (\sin (\exp x)))$$

Figure 1: An example of GP Tree

III. WIENER MODEL IDENTIFICATION

A Wiener model consists of a linear dynamic system G followed by a static nonlinear part F as in Fig 2.



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Figure 2: A Wiener model.

Let consider the Wiener model

$$x(t) = G(q) \cdot u(t) = \frac{b_1 q^{-1} + \dots + b_{nb} q^{-nb}}{1 + a_1 q^{-1} + \dots + a_{na} q^{-na}} \cdot u(t) \quad (1)$$

$$y(t) = F(x(t)) + e(t) \quad (2)$$

Here q^{-1} is delay operator. $y(t), u(t)$ and $e(t)$ are the system output, input and measurement noise at time instant t respectively. The orders of linear dynamic block n_a and n_b are unknown and it should be determined from observed input and output data. $F(\cdot)$ is unknown nonlinear function.

We can rewrite (1) as,

$$x(t) = \sum_{j=1}^{nb} b_j u(t-j) - \sum_{i=1}^{na} a_i x(t-i) \quad (3)$$

For the sake of convenience, again let:

$$\theta = (a_1, \dots, a_{na}, b_1, \dots, b_{nb})^T$$

$$h(t) = (-x(t-1), \dots, -x(t-na), u(t-1), \dots, u(t-nb))$$

According to Eq. (3), we have:

$$x(t) = h(t) \cdot \theta \quad (4)$$

Suppose m represent sampling steps used to identify, let:

$$n = n_a + n_b,$$

$$X(t) = (x(1), x(2), \dots, x(m))^T,$$

$$H = \begin{bmatrix} h(1) \\ h(2) \\ \vdots \\ h(m) \end{bmatrix} = \begin{bmatrix} -x(0) & \dots & -x(1-n_a) & u(0) & \dots & u(1-n_b) \\ -x(1) & \dots & -x(2-n_a) & u(1) & \dots & u(2-n_b) \\ \vdots & \dots & \vdots & \vdots & \dots & \vdots \\ -x(m-1) & \dots & -x(m-n_a) & u(m-1) & \dots & u(m-n_b) \end{bmatrix}_{m \times n}$$

Hence:

$$X(t) = H\theta \quad (5)$$

Parameters are identified by the least square criterion as [5]:

$$\theta = (H^T H)^{-1} H^T X(t) \quad (6)$$

Then, we assume that nonlinear function is written by:

$$y(t) = F(x(t)) + e(t) = \sum_{k=1}^M \theta_k p_k(x(t)) + e(t) \quad (7)$$

Here, θ_k are model parameters and $p_k(\cdot)$ are the suitably selected nonlinear functions.

The function terms were determined by decomposing the tree starting from the root as far as reaching non-linear nodes (nodes which are not '+' or '-') [6] [7].

In Fig 3: The root node is a '+' operator, so it is possible to decompose the tree into two subtrees: 'p' and 'p₁' trees. The root node of the 'p' tree is again a linear operator, so it can be decomposed into 'p₂' and 'p₃' trees. The root node of the 'p₁' tree is a nonlinear node ('*') so it cannot be decomposed, so finally the decomposition procedure results in three subtrees: 'p₁', 'p₂' and 'p₃'.

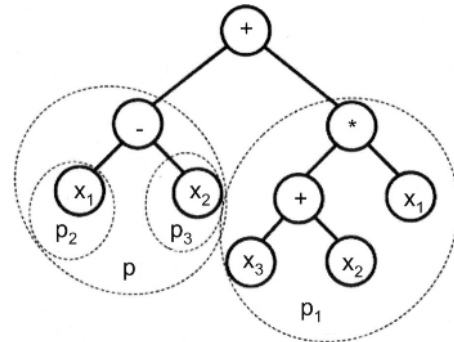


Figure 3: Decomposition of a tree to function terms

The parameters are assigned to the model after 'extracting' the p_i function terms from the tree. Least Squares Method can be used for the parameters identification.

The optimal $\Theta = [\theta_1, \dots, \theta_M]^T$ parameter vector can be analytically calculated:

$$\Theta = (P^T P)^{-1} P^T y \quad (8)$$

Where $y = [y(1), \dots, y(N)]^T$ is the measured output vector, and the P regression matrix is:

$$P = \begin{pmatrix} p_1(x(1)) & \dots & p_M(x(1)) \\ \vdots & \ddots & \vdots \\ p_1(x(N)) & \dots & p_M(x(N)) \end{pmatrix} \quad (9)$$

The compact matrix form corresponding to the linear-in-parameters model is

$$y = P\Theta + e \quad (10)$$

where P is the regression matrix, Θ is the parameter vector, e is the error vector [8].

Based on this representation, we can estimate unknown parameters, a_i, b_j and θ_k using the least square algorithm. The orders of this model, n_a, n_b and M should be determined by AIC, which is defined by [9],

$$AIC(na, nb, M) = N \log(E) + 2(na + nb + M) \quad (11)$$

$$E = \frac{1}{N} \sum_{t=1}^N (y(t) - \hat{y}(t))^2 \quad (12)$$

Where, E is the mean square error between actual output and estimated output, N is the number of sample points.

This kind of models is investigated in many actual processes such as Control Valve, where the measurement device with nonlinear characteristics is a component of the system.

The Control Valve is an opening with adjustable area. Normally it consists of an actuator, a valve body and a valve plug. The actuator is a device that transforms the control signal to movement of the stem and valve plug.

where $u(k)$ is the control pressure, $x(k)$ is the stem position, and $y(k)$ is the flow through the valve which is the controlled variable [10].

IV. THE FLOW CHART OF WIENER MODEL

The flow chart of the Wiener model identification with GP is as follows :

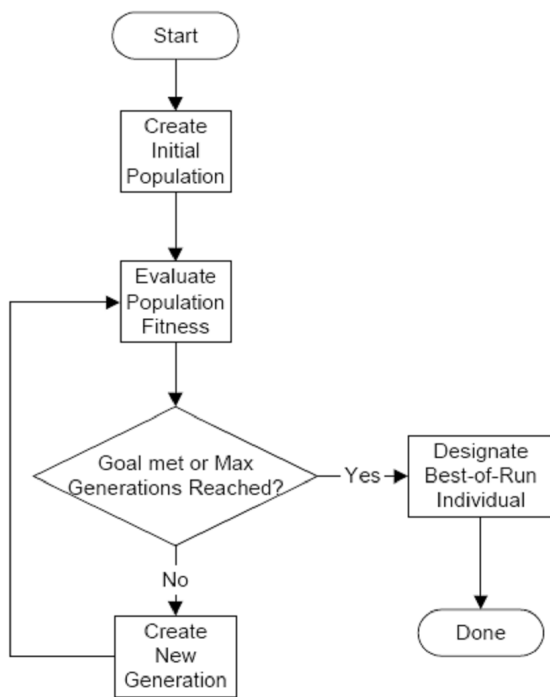


Figure 4: flowchart of GP

A. Creation of initial population

Our chromosome consists of two parts, parameters of the nonlinear function and order of linear discrete transfer function. We generate one hundred initial chromosomes of random compositions of the functions and terminals according to this method.

B. Evaluate Population Fitness

The fitness function reflects the goodness of a potential solution which is proportional to the probability of the selection of the individual. The fitness for each individual is evaluated by AIC to balance the accuracy and the complexity

of model. AIC is calculated with the number of nodes in the GP tree, the order of linear dynamic model and the accuracy of model. The fitness for each individual is evaluated with the minimum AIC.

C. Keep the better chromosome

We arrange the chromosome according to the fitness function, and keep the first groups of better chromosome as the parental generation of the next generation.

D. Use Genetic Operation :

The genetic operations include crossover and mutation. After the genetic operations are performed on the current population, the new generation replaces the now-old generation. This iterative process of measuring fitness and performing the genetic operations is repeated over many generations. The run of GP terminates when the termination criterion is satisfied. Sometimes we can't find the optimal solution quickly; therefore we can set up generation. We will stop the program when it reaches the generation, and select the best chromosome from inside.

V. EXPERIMENT RESULT

A. Experiment I

To show the validity of proposed approach to Wiener model, numerical simulation study is carried out.

Consider the following Wiener model [3]:

$$x(t) = 1.5x(t-1) - 0.7x(t-2) + u(t-1) + 0.5u(t-2) \quad (13)$$

$$y(t) = \frac{1}{1 + \exp\{x(t)\}} \quad (14)$$

Here, the nonlinear static part is invertible, measurement noise is a white noise normally distributed with mean zero, and the input signal $u(t)$ is assumed random input uniformly distributed over $[-1,1]$. GP is used to determine a mathematical function form for the static nonlinear block. The design parameters in the GP are as follows.

Table I: Parameters of GP

terminal sets	{ $x(t)$, \mathcal{R} }
function sets	{ +, -, *, exp, sinh, cosh, tanh }
Population size	100
generation	500
Max tree depth	6
Type of selection	Tournament selection
Crossover rate	0.7
mutation rate	0.3

Here \mathfrak{R} is constant. The identified linear dynamic system part is given by:

$$\hat{x}(t) = 1.532x(t-1) - 0.692x(t-2) + 1.044u(t-1) + 0.496u(t-2) \quad (15)$$

and the identified inverse function of nonlinear static part is shown in Fig. 5.

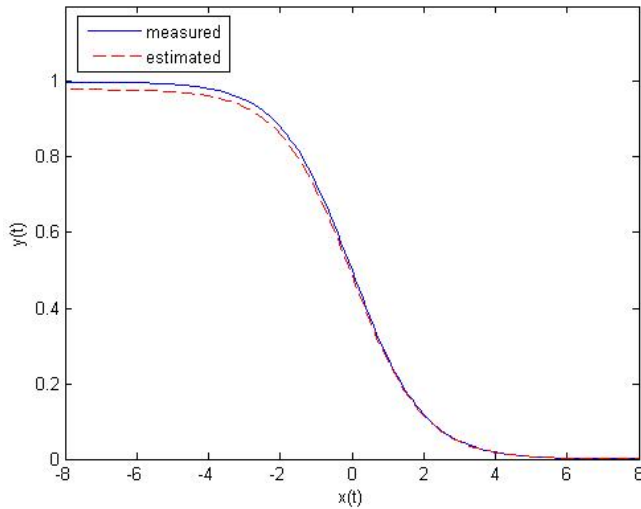


Figure 5: Estimated nonlinearity.

The output estimates $\hat{y}(t)$ are shown with the observations $y(t)$ in Fig 6.

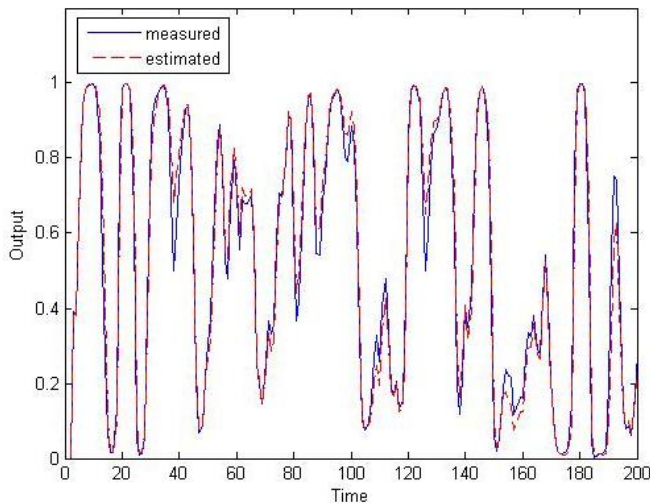


Figure 6: Output estimate and observation.

B. Experiment II

To show the validity of proposed approach to non-invertible static part, numerical simulation study is carried out.

Consider the following Wiener model:

$$x(t) = 1.5x(t-1) - 0.7x(t-2) + u(t-1) + 0.5u(t-2) \quad (16)$$

$$y(t) = \begin{cases} -1.5, & x(t) < -1.5 \\ x(t), & -1.5 \leq x(t) < 1.5 \\ 1.5, & 1.5 \leq x(t) \end{cases} \quad (17)$$

Here, the nonlinear static part is non-invertible, measurement noise is a white noise normally distributed with mean zero, and the input signal $u(t)$ is assumed random input uniformly distributed over $[-1,1]$. The design parameters in the GP are represented as shown in Table I.

The identified system part is given by

$$\hat{x}(t) = 1.532x(t-1) - 0.692x(t-2) + 1.044u(t-1) + 0.496u(t-2) \quad (18)$$

$$\hat{y}(t) = 1.48 \tanh(x(t)) \quad (19)$$

The estimated nonlinear function of nonlinear static part is given in Fig. 7.

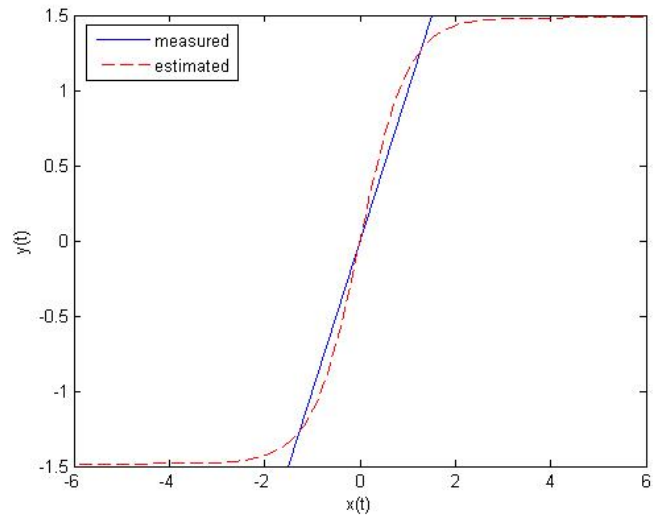


Figure 7: Estimated nonlinearity.

The output estimates $\hat{y}(t)$ are shown with the observations $y(t)$ in Fig 8.

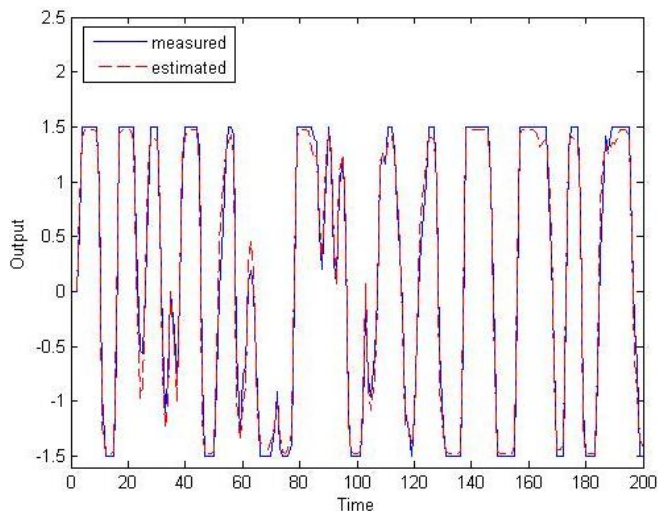


Figure 8: Output estimate and observation.

VI. SUMMARY

The GP is an effective algorithm to generate model structures from input-output data. Based on the GP method, methods needn't to assume that the order of linear part is known and the search space is differentiable, and linear in the search parameters. In addition, methods also needn't to assume that the nonlinear static part is invertible. Numerical simulation study result indicates the usefulness of the proposed approach to Wiener model identification. We can easily modify this approach for the other kind of nonlinear system models by selecting suitable nodes for GP.

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