Stability and Stabilizability of Linear Parameter Dependent System with Time Delay

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Abstract—**This paper presents sufficient condition for exponential stability with a given convergence rate and asymptotically stability of linear parameter dependent (LPD) delay system and gives sufficient condition for stabilizability of LPD delay control system. We use appropriate Lyapunov functions and derive stability condition in term of linear matrix inequality (LMI).**

Keywords: exponential stability, asymptotically stability, linear parameter dependent (LPD) delay system, stabilizability

1 Introduction

The stability problem of linear parameter dependent (LPD) system has been investigated in many works [1]-[2] and [4]-[8]. There are different conditions given to deal with the stability problem for LPD system [1]-[2] and [6]-[7]. Now, this problem is an increasing interest because it can apply to many engineering systems. The LPD system is defined from uncertain linear time varying system[3]. When the system matrices of uncertain system are formulated by a polytope of matrices. The Lyapunov function method is a important tool for studying LPD system stability.

In this paper, we will employ Lyaponov function for establishing exponential stability condition with a given convergence rate and asymptotically stable of linear parameter dependent (LPD) delay system. Our condition will been expressed in terms of linear matrix inequality (LMI). Then we will extend LPD delay system to LPD delay control system for to find stabilizability condition also numerical example. We let some important notations

R^+- the set of all non-negative real number;
R^n- the n-dimensional space;
$\langle x, y\rangle$ or $x^\top y-$ the scalar product of two vector x, and y ;
$\|x\|-$ the Euclidean vector norm of x;
$M^{n\times m}-$ the space of all $(n\times m)$ matrices;
$A^\top-$ the transpose of the matrix A;
A is symmetric if $A = A^\top$;
$C([-h,0],R^n)-$ the Banach space of all piecewise continuous vector function mapping [-h,0] into R^n;
$\lambda(A)-$ the set of all eigienvalues of A;
$\lambda_{max}(A)-$ max $\{Re\lambda : \lambda\in\lambda(A)\}$,
$\lambda_{min}(A)-$ min $\{Re\lambda : \lambda\in\lambda(A)\}$;
$I-$ the identity matrix.

We consider the linear parameter dependent delay systems

$$\begin{cases} \dot{x}(t) = A(\alpha)x(t) + B(\alpha)x(t-h), & \forall t \geq 0; \\ x(t) = \phi(t), & \forall t \in [-h, 0] \end{cases} \quad (1)$$

where $x(t) \in R^n$ is the state, , $h \in R^+$ is the delay, and $\phi(t)$ is a continuous vector-valued initial function. $A(\alpha)$ and $B(\alpha)$ are matrices belonging to the polytope Ω_1 using for theorem 2.1

$$\Omega_1 := [A(\alpha), B(\alpha)] = \{[\sum_{i=1}^N \alpha_i A_i, \sum_{i=1}^N \alpha_i B_i],$$

$$\sum_{i=1}^N \alpha_i = 1, \alpha_i \geq 0, i = 1, ..., N\}.$$

$A(\alpha)$ and $B(\alpha)$ are uncertain time varying matrices belonging to the polytope Ω_2 using for theorem 2.2

$$\Omega_2 := [A(\alpha), B(\alpha)] = \{[\sum_{i=1}^N \alpha_i(t) A_i,$$

$$\sum_{i=1}^N \alpha_i(t) B_i], \sum_{i=1}^N \alpha_i(t) = 1, \alpha_i(t) \geq 0, i = 1, ..., N\}.$$

We also assume the following bounds of the parameter values:

$$\exists \beta_i > 0 : \quad \|\dot{\alpha}_i(t)\| \leq \beta_i, \quad \forall t > 0.$$

Definition 1.1 The system (1) is said to be $\beta-$ stable, if there is a function $\xi(.) : R^+ \to R^+$ such that for each $\phi(t) \in C([-h, 0], R^n)$, the solution $x(t, \phi)$ of the system satisfies

$$\|x(t, \phi)\| \leq \xi(\|\phi\|)e^{-\beta t}, \quad \forall t \in R^+.$$

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Lemma 1.2 *Assume that $S \in R^{n\times n}$ is a symmetric positive definite matrix. Then for every $Q \in R^{n\times n}$:*

$$2\langle Qy, x\rangle - \langle Sy, y\rangle \leq \langle QS^{-1}Q^\top x, x\rangle, \quad \forall x, y \in R^n.$$

Definition 1.3 The equilibrium point $x_{eq} \in R^n$ of (1) is asymptotically stable if it is Lyapunov stable and for every solution that exists on $[0,\infty)$ such that $x(t) \to x_{eq}$ as $t \to \infty$.

2 STABILITY CONDITION

From the system (1), we can change the form of the state variable

$$y(t) = e^{\beta t}x(t), \quad t \in R^+,$$

then the system (1) is transformed to the following delay system

$$\dot{y}(t) = A_\beta(\alpha)y(t) + B_\beta(\alpha)y(t-h), \quad t \in R^+, \tag{2}$$

where

$$A_\beta(\alpha) = A(\alpha) + \beta I, B_\beta(\alpha) = e^{\beta h}B(\alpha), P_\epsilon = P + \epsilon I.$$

Theorem 2.1 *The system (1) is $\beta-$ stable if there exists P and Q be positive definite matrices and $h,\beta,\epsilon > 0$ such that the following condition holds.*

1. $A_i^\top P_\epsilon + 2\beta P_\epsilon + P_\epsilon A_i + Q + e^{2\beta h}P_\epsilon B_i Q^{-1}B_i^\top P_\epsilon \leq -I, \quad i = 1, ..., N.$
2. $A_i^\top P_\epsilon + 4\beta P_\epsilon + P_\epsilon A_i + 2Q + A_j^\top P_\epsilon + P_\epsilon A_j + e^{2\beta h}P_\epsilon B_i Q^{-1}B_j^\top P_\epsilon + e^{2\beta h}P_\epsilon B_j Q^{-1}B_i^\top P_\epsilon \leq \frac{2I}{N-1}, \quad i = 1, ..., N-1, j = i+1, ..., N.$

Proof. We define the following Lyapunov function for system (2) :

$$V(t, y(t)) = y^\top(t)Py(t) + \epsilon\|y(t)\|^2 + \int_{t-h}^{t} y^\top(s)Qy(s)ds.$$

The derivative of V along the trajectories of system (2) is given by

$$\begin{aligned}\dot{V} &= 2y^\top(t)A_\beta^\top(\alpha)P_\epsilon y(t) + 2y^\top(t-h)B_\beta^\top(\alpha)P_\epsilon y(t) \\ &\quad + y^\top(t)Qy(t) - y^\top(t-h)Qy(t-h).\end{aligned}$$

Using lemma 1.2, we have

$$\begin{aligned}&2y^\top(t-h)B_\beta^\top(\alpha)P_\epsilon y(t) - y^\top(t-h)Qy(t-h) \\ &\leq y^\top(t)P_\epsilon B_\beta(\alpha)Q^{-1}B_\beta^\top(\alpha)P_\epsilon y(t).\end{aligned}$$

Thus, we obtain that

$$\begin{aligned}\dot{V} &\leq y^\top(t)[A_\beta^\top(\alpha)P_\epsilon + P_\epsilon A_\beta(\alpha) \\ &\quad + P_\epsilon B_\beta(\alpha)Q^{-1}B_\beta^\top(\alpha)P_\epsilon + Q]y(t) \\ &= y^\top(t)[\{A^\top(\alpha) + \beta I\}P_\epsilon + P_\epsilon\{A(\alpha) + \beta I\} \\ &\quad + P_\epsilon\{e^{\beta h}B(\alpha)\}Q^{-1}\{e^{\beta h}B^\top(\alpha)\}P_\epsilon + Q]y(t) \\ &= y^\top(t)[\{\sum_{i=1}^{N}\alpha_i A_i^\top + \beta I\}P_\epsilon + P_\epsilon\{\sum_{i=1}^{N}\alpha_i A_i + \beta I\} \\ &\quad + P_\epsilon\{e^{\beta h}\sum_{i=1}^{N}\alpha_i B_i\}Q^{-1}\{e^{\beta h}\sum_{i=1}^{N}\alpha_i B_i^\top\}P_\epsilon + Q]y(t).\end{aligned}$$

Then, we get that

$$\begin{aligned}\dot{V} &\leq y^\top(t)[\sum_{i=1}^{N}\alpha_i[\sum_{i=1}^{N}\alpha_i[A_i^\top P_\epsilon + 2\beta P_\epsilon + P_\epsilon A_i + Q]] \\ &\quad + \{\sum_{i=1}^{N}\alpha_i e^{\beta h}P_\epsilon B_i Q^{-1}\}\{\sum_{i=1}^{N}\alpha_i e^{\beta h}B_i^\top P_\epsilon\}]y(t) \\ &= y^\top(t)[\sum_{i=1}^{N}\alpha_i^2[A_i^\top P_\epsilon + 2\beta P_\epsilon + P_\epsilon A_i + Q \\ &\quad + e^{2\beta h}P_\epsilon B_i Q^{-1}B_i^\top P_\epsilon] + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_i\alpha_j[A_i^\top P_\epsilon + 4\beta P_\epsilon \\ &\quad + P_\epsilon A_i + 2Q + A_j^\top P_\epsilon + P_\epsilon A_j] + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_i\alpha_j[e^{2\beta h}P_\epsilon \\ &\quad \times B_i Q^{-1}B_j^\top P_\epsilon + e^{2\beta h}P_\epsilon B_j Q^{-1}B_i^\top P_\epsilon]]y(t).\end{aligned}$$

Since $\sum_{i=1}^{N}\alpha_i = 1$ and

$$\sum_{i=1}^{N}\alpha_i A_i\sum_{i=1}^{N}\alpha_i B_i = \sum_{i=1}^{N}\alpha^2 A_i B_i + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_i\alpha_i[A_i B_j + A_j B_i].$$

By the condition 1., 2. and since

$$(N-1)\sum_{i=1}^{N}\alpha_i^2 - 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_i\alpha_j = \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}[\alpha_i - \alpha_j]^2 \geq 0.$$

Therefore, we have

$$\dot{V}(t, y(t)) \leq 0, \quad \forall t \in R^+.$$

Integrating both sides of (4) from 0 to t, we find

$$V(t, y(t)) - V(0, y(0)) \leq 0, \quad \forall t \in R^+,$$

and hence

$$\begin{aligned}&y^\top(t)Py(t) + \epsilon\|y(t)\|^2 + \int_{t-h}^{t} y^\top(s)Qy(s)ds \\ &\leq y^\top(0)Py(0) + \epsilon\|y(0)\|^2 + \int_{0-h}^{0} y^\top(s)Qy(s)ds\end{aligned}$$

where $P \geq 0$. Since

$$y^\top P y \geq 0, \quad \int_{t-h}^{t} y^\top(s) Q y(t) ds \geq 0$$

and since

$$\int_{-h}^{0} y^\top(s) Q y(s) ds$$
$$\leq \lambda_{max}(Q)\|\phi\| \int_{-h}^{0} e^{\beta s} ds = \frac{\lambda_{max}(Q)}{\beta}(1 - e^{-\beta h})\|\phi\|,$$

we have

$$\begin{aligned} \epsilon\|y(t)\|^2 &\leq \lambda_{max}(P)\|y(0)\|^2 + \epsilon\|y(0)\|^2 \\ &+ \frac{\lambda_{max}(Q)}{\beta}(1 - e^{-\beta h})\|\phi\|. \end{aligned}$$

Therefore, the solution $y(t, \phi)$ of the system (2) is bounded. Returning to the solution $x(t, \phi)$ of system (1) and noting that

$$\|y(0)\| = \|x(0)\| = \phi(0) \leq \|\phi\|,$$

we have

$$\|x(t, \phi)\| \leq \xi(\|\phi\|) e^{-\beta t}, \quad \forall t \in R^+,$$

where

$$\begin{aligned} \xi(\|\phi\|) &:= \{\epsilon^{-1}\lambda_{max}(P)\|\phi\|^2 + \|\phi\|^2 \\ &+ \frac{\lambda_{max}(Q)}{\beta\epsilon}(1 - e^{-\beta h})\|\phi\|\}^{\frac{1}{2}}. \end{aligned}$$

This means that the system (1) is $\beta-$ stable. The proof of the theorem is complete. □

Consider the system (1). Let $P_j, Q_j, j = 1, 2, ..., N$ be symmetric matrices and S be positive definite

$$M_i(P_j, Q_j) = \begin{bmatrix} \sum_{k=1}^{N} \beta_k P_k + A_i^\top P_j + P_j A_i + Q_j & P_j B_i \\ B_i^\top P_j & -Q_j \end{bmatrix},$$

$$N_{i,j}(R, h) = \begin{bmatrix} hA_i^\top R A_j - \frac{R}{h} & hA_i^\top R B_j + \frac{R}{h} \\ hB_i^\top R A_j + \frac{R}{h} & hB_i^\top R B_j - \frac{R}{h} \end{bmatrix}, S \in R^{2n \times 2n}.$$

Theorem 2.2 The system (1) is asymptotically stable if there exist $P_j, Q_j, j = 1, 2, ..., N$, let R be symmetric positive definite matrices and S be symmetric semi-positive definite matrix and $h \in R^+$ which satisfy the following matrix inequality holds.

1. $M_i(P_i, Q_i) + N_{i,i}(R, h) < -S, \quad i = 1, ..., N.$
2. $M_j(P_i, Q_i) + M_i(P_i, Q_i) + N_{j,i}(R, h) + N_{i,j}(R, h) < \frac{2S}{N-1}, i = 1, ..., N-1, j = i+1, ..., N.$

Proof. We define the following Lyapunov-Krasovskii function for system (1) :

$$V(x(t)) = V_1 + V_2 + V_3$$

where $V_1 := x^\top(t) P(\alpha) x(t)$, $V_2 := \int_{t-h}^{t} x^\top(\theta) Q(\alpha) x(\theta) d\theta$ and $V_3 := \int_{t-h}^{t} \int_{s}^{t} \dot{x}^\top(\theta) R \dot{x}(\theta) d\theta ds$ with $P(\alpha) = \sum_{i=1}^{N} \alpha_i(t) P_i, Q(\alpha) = \sum_{i=1}^{N} \alpha_i(t) Q_i$. The derivative of V along the trajectories of system (1) is given by $\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3$. Therefore,

$$\begin{aligned} \dot{V}_1 &= x^\top(t)\dot{P}(\alpha)x(t) + 2\dot{x}^\top(t)P(\alpha)x(t) \\ &= x^\top(t)\dot{P}(\alpha)x(t) + 2x^\top(t)A^\top(\alpha)P(\alpha)x(t) \\ &\quad +2x^\top(t-h)B^\top(\alpha)P(\alpha)x(t) \end{aligned}$$

$\dot{V}_2 = x^\top(t)Q(\alpha)x(t) - x^\top(t-h)Q(\alpha)x(t-h)$ and $\dot{V}_3 = h\dot{x}^\top(t)R\dot{x}(t) - \int_{t-h}^{t} \dot{x}^\top(\theta)R\dot{x}(\theta)d\theta$ Thus, using the jensen's inequality the last term can be bounded as follows:

$$-\int_{t-h}^{t} \dot{x}^\top(\theta)R\dot{x}(\theta)d\theta \quad < \quad -[x(t) - x(t-h)]^\top \frac{R}{h}[x(t) - x(t-h)]$$

We obtain that

$$\begin{aligned} \dot{V} \quad < \quad & x^\top(t)\dot{P}(\alpha)x(t) + 2x^\top(t)A^\top(\alpha)P(\alpha)x(t) \\ & +2x^\top(t-h)B^\top(\alpha)P(\alpha)x(t) + x^\top(t)Q(\alpha)x(t) \\ & -x^\top(t-h)Q(\alpha)x(t-h) + x^\top(t)A^\top(\alpha)hRA(\alpha)x(t) \\ & +2x^\top(t-h)B^\top(\alpha)hRA(\alpha)x(t) + x^\top(t-h)B^\top(\alpha)hR \\ & \times B(\alpha)x(t-h) - x^\top(t)\frac{R}{h}x(t) + 2x^\top(t)\frac{R}{h}x^\top(t-h) \\ & -x^\top(t-h)\frac{R}{h}x(t-h). \end{aligned}$$

We obtain that

$$\begin{aligned} \dot{V} \quad < \quad & \sum_{i=1}^{N} \alpha_i(t)[x^\top(t)\sum_{k=1}^{N}\beta_k P_k x(t) + 2x^\top(t)\sum_{j=1}^{N}\alpha_j(t)A_j^\top P_i x(t) \\ & +2x^\top(t-h)\sum_{j=1}^{N}\alpha_j(t)B_j^\top P_i x(t) + x^\top(t)Q_i x(t) \\ & -x^\top(t-h)Q_i x(t-h) + x^\top(t)\sum_{j=1}^{N}\alpha_j(t)A_j^\top hRA_i x(t) \\ & +2x^\top(t-h)\sum_{j=1}^{N}\alpha_j(t)B_j^\top hRA_i x(t) \\ & +x^\top(t-h)\sum_{j=1}^{N}\alpha_j(t)B_j^\top hRB_i x(t-h) - x^\top(t)\frac{R}{h}x(t) \\ & +2x^\top(t-h)\frac{R}{h}x(t) - x^\top(t-h)\frac{R}{h}x(t-h)] \end{aligned}$$

since $\sum_{i=1}^{N} \dot{\alpha}_i(t)P_i \leq \sum_{i=1}^{N} \beta_i P_i$. We can rewrite as

$$\begin{aligned} \dot{V} \quad < \quad & \sum_{i=1}^{N}\alpha_i(t)[\sum_{j=1}^{N}\alpha_j(t)\{x^\top(t)\sum_{k=1}^{N}\beta_k P_k x(t) + 2x^\top(t)A_j^\top P_i x(t) \\ & +x^\top(t)A_j^\top hRA_i x(t) + x^\top(t)Q_i x(t) \\ & +2x^\top(t-h)B_j^\top hRA_i x(t) + 2x^\top(t-h)B_j^\top P_i x(t) \\ & -x^\top(t-h)Q_i x(t-h) + x^\top(t-h)B_j^\top hRB_i x(t-h) \\ & -x^\top(t)\frac{R}{h}x(t) + 2x^\top(t-h)\frac{R}{h}x(t) - x^\top(t-h)\frac{R}{h}x(t-h)\}]. \end{aligned}$$

Since $\sum_{i=1}^{N}\alpha_i(t) = 1$ and

$$1 = (\sum_{i=1}^{N}\alpha_i(t))^2 = \sum_{i=1}^{N}\alpha_i^2(t) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_i(t)\alpha_j(t).$$

Let us set

$$\begin{aligned}\dot V &< \sum_{i=1}^{N}\alpha_i^2(t)\Big[x^\top(t)\sum_{k=1}^{N}\beta_k P_k x(t) + 2x^\top(t)A_i^\top P_i x(t)\\
&+x^\top(t)Q_i x(t) - x^\top(t-h)Q_i x(t-h)\\
&+2x^\top(t-h)B_i^\top P_i x(t) + x^\top(t)A_i^\top hRA_i x(t)\\
&+2x^\top(t-h)B_i^\top hRA_i x(t) - x^\top(t)\frac{R}{h}x(t)\\
&+2x^\top(t-h)\frac{R}{h}x(t) + x^\top(t-h)B_i^\top hRB_i x(t-h)\\
&-x^\top(t-h)\frac{R}{h}x(t-h)\Big]\\
&+\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_i(t)\alpha_j(t)\Big[x^\top(t)\sum_{k=1}^{N}\beta_k P_k x(t)\\
&+2x^\top(t)A_j^\top P_i x(t) + 2x^\top(t-h)B_j^\top P_i x(t)\\
&+x^\top(t)Q_i x(t) - x^\top(t-h)Q_i x(t-h)\\
&+x^\top(t)A_j^\top hRA_i x(t) + 2x^\top(t-h)B_j^\top hRA_i x(t)\\
&+x^\top(t-h)B_j^\top hRB_i x(t-h) - x^\top(t)\frac{R}{h}x(t)\\
&+2x^\top(t-h)\frac{R}{h}x(t) - x^\top(t-h)\frac{R}{h}x(t-h)\\
&+x^\top(t)\sum_{k=1}^{N}\beta_k P_k x(t) + 2x^\top(t)A_i^\top P_j x(t)\\
&+2x^\top(t-h)B_i^\top P_j x(t) + x^\top(t)Q_j x(t)\\
&+x^\top(t)A_i^\top hRA_j x(t) + 2x^\top(t-h)B_i^\top hRA_j x(t)\\
&+x^\top(t-h)B_i^\top hRB_j x(t-h) - x^\top(t)\frac{R}{h}x(t)\\
&+2x^\top(t-h)\frac{R}{h}x(t) - x^\top(t-h)Q_j x(t-h)\\
&-x^\top(t-h)\frac{R}{h}x(t-h)\Big].\end{aligned}$$

Therefore, we have

$$\begin{aligned}\dot V &< \begin{bmatrix}x(t)\\x(t-h)\end{bmatrix}^\top \{\sum_{i=1}^{N}\alpha_i^2(t)[M_i(P_i,Q_i) + N_{i,i}(R)]\\
&+\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_i(t)\alpha_j(t)[M_j(P_i,Q_i)\\
&+M_i(P_i,Q_i) + N_{j,i}(R,h) + N_{i,j}(R,h)]\}\begin{bmatrix}x(t)\\x(t-h)\end{bmatrix}.\end{aligned}$$

We use the following condition as

$$\begin{aligned}\dot V &< \begin{bmatrix}x(t)\\x(t-h)\end{bmatrix}^\top [(-\sum_{i=1}^{N}\alpha_i^2(t)S)\\
&+\frac{2}{N-1}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_i(t)\alpha_j(t)S]\begin{bmatrix}x(t)\\x(t-h)\end{bmatrix},\end{aligned}$$

we have

$$\begin{aligned}&(N-1)\sum_{i=1}^{N}\alpha_i^2(t) - 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_i(t)\alpha_j(t)\\
&= \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}[\alpha_i(t) - \alpha_j(t)]^2 \geq 0.\end{aligned}$$

Thus, $\dot V < 0$. Therefore, this means that the system (1) is asymptotically stable. The proof of the theorem is complete. □

Example 2.1 Consider the following linear parameter dependent delay system :

$$\dot x(t) = A(\alpha)x(t) + B(\alpha)x(t - \frac{1}{2}), \quad t \in R^+, \tag{3}$$

with any initial function $\phi(t) \in C([-\frac{1}{2}, 0], R^+)$ where

$$A(\alpha) = \alpha_1\begin{bmatrix}-5 & 1\\1 & -3\end{bmatrix} + \alpha_2\begin{bmatrix}-4 & 1\\-2 & -3\end{bmatrix},$$

$$B(\alpha) = \alpha_1\begin{bmatrix}-0.5 & 0\\1 & 0\end{bmatrix} + \alpha_2\begin{bmatrix}-0.05 & 0\\1 & 0\end{bmatrix}.$$

We have $h = \frac{1}{2}, N = 2$. Taking $\epsilon = \beta = 1$, we can find $Q = \begin{bmatrix}16 & -4\\-4 & 9\end{bmatrix}$ and $P = \begin{bmatrix}4 & -1\\-1 & 3\end{bmatrix}$ satisfy all conditions of Theorem 2.1. Therefore, the system (3) is 1- stable. □

Example 2.2 Consider the following linear parameter dependent delay system :

$$\dot x(t) = A(\alpha)x(t) + B(\alpha)x(t-1), \quad t \in R^+, \tag{4}$$

where

$$A(\alpha) = \alpha_1(t)\begin{bmatrix}-9 & 1\\1 & -6\end{bmatrix} + \alpha_2(t)\begin{bmatrix}-9 & 1\\1 & -5\end{bmatrix},$$

$$B(\alpha) = \alpha_1(t)\begin{bmatrix}-2 & 1\\1 & -1\end{bmatrix} + \alpha_2(t)\begin{bmatrix}-2 & 1\\1 & -2\end{bmatrix}.$$

We have $h = 1, N = 2$. Taking $|\dot\alpha_i(t)| \leq \frac{1}{2}, i = 1, 2$ (such as $\alpha_1 = e^{-\frac{t}{2}}$ and $\alpha_2 = 1 - e^{-\frac{t}{2}}$), $Q_1 = \begin{bmatrix}30 & -1\\-1 & 25\end{bmatrix}$, $Q_2 = \begin{bmatrix}30 & -1\\-1 & 26\end{bmatrix}$ and $R = \begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$ then

$$P_1 = P_2 = \begin{bmatrix}100 & -1\\-1 & 80\end{bmatrix}, \quad S = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

satisfy all conditions of Theorem 2.2. Therefore, the system (4) is asymptotically stable. □

3 STABILIZABILITY CONDITION

Consider the following LPD delay control system

$$\dot x(t) = A(\alpha)x(t) + B(\alpha)x(t-h) + C(\alpha)u(t), \quad \forall t \geq 0 \tag{5}$$

where $A(\alpha), B(\alpha)$ and $C(\alpha)$ are uncertain time varying matrices belonging to the polytope Ω

$$\begin{aligned}\Omega \;&:=\; \big[A(\alpha), B(\alpha), C(\alpha)\big] \\ &= \{\big[\sum_{i=1}^{N}\alpha_i(t)A_i, \sum_{i=1}^{N}\alpha_i(t)B_i, \sum_{i=1}^{N}\alpha_i(t)C_i\big], \\ &\sum_{i=1}^{N}\alpha_i(t) = 1, \quad \alpha_i(t) \geq 0, \quad i = 1, ..., N\}.\end{aligned}$$

The $u(t) \in R^m$ is the control of the system.

Definition 3.1 The LPD delay control system (5) is said to be $\beta-$ stabilizable if there exists control $u(t) = Kx(t)$, where $K \in R$ such that the closed loop system

$$\dot{x}(t) = [A(\alpha) + KC(\alpha)]x(t) + B(\alpha)x(t-h)$$

is $\beta-$ stable. The control $u(t) = Kx(t)$ is the feedback stabilizing control of the system.

Definition 3.2 The LPD delay control system (5) is said to be stabilizable if there exists control $u(t) = Kx(t)$, where K is nonnegative real number such that the closed loop system

$$\dot{x}(t) = [A(\alpha) + KC(\alpha)]x(t) + B(\alpha)x(t-h)$$

is asymptotically stable. The control $u(t) = Kx(t)$ is the feedback stabilizing control of the system.

Theorem 3.1 *The system (5) is $\beta-$ stabilizable if there exists P and Q be positive definite matrices and $\beta, \epsilon, K > 0$ such that the following condition holds.*

1. $A_i^\top P_\epsilon + 2\beta P_\epsilon + P_\epsilon A_i + Q + KC_i^\top P_\epsilon + KP_\epsilon C_i + e^{2\beta h}P_\epsilon B_i Q^{-1}B_i^\top P_\epsilon \leq -I, \quad i = 1, ..., N.$
2. $A_i^\top P_\epsilon + 4\beta P_\epsilon + P_\epsilon A_i + 2Q + A_j^\top P_\epsilon + P_\epsilon A_j + KC_i^\top P_\epsilon + KP_\epsilon C_j + KC_i^\top P_\epsilon + KP_\epsilon C_j + e^{2\beta h}P_\epsilon B_i Q^{-1}B_j^\top P_\epsilon + e^{2\beta h}P_\epsilon B_j Q^{-1}B_i^\top P_\epsilon \leq \dfrac{2I}{N-1}, \quad i = 1, ..., N-1, j = i+1, ..., N.$

The feedback control is given by

$$u(t) = Kx(t).$$

Consider the system (5). Let $P_j, Q_j, j = 1, 2, ..., N$ be symmetric matrices and S be positive definite $S \in R^{2n\times 2n}$

$$\overline{M}_i(P_j, Q_j) = \begin{bmatrix} m_{ij} & P_jB_i \\ B_i^\top P_j & -Q_j \end{bmatrix},$$

$m_{ij} := \sum_{k=1}^{N}\beta_k P_k + A_i^\top P_j + P_jA_i + Q_j + KC_i^\top P_j + KP_jC_i,$

$$\overline{N}_{i,j}(R,h) = \begin{bmatrix} 1_{ij} & hA_i^\top RB_j + hKC_i^\top RB_j + \frac{R}{h} \\ 2_{ij} & hB_i^\top RB_j - \frac{R}{h} \end{bmatrix},$$

$1_{ij} := hA_i^\top RA_j + hKC_i^\top RA_j + hKA_i^\top RC_j + hKC_i^\top RC_j - \dfrac{R}{h}$

$2_{ij} := hB_i^\top RA_j + hKB_i^\top RC_j + \frac{R}{h}$

Theorem 3.2 *The system (5) is stabilizable if there exist $P_j, Q_j, j = 1, 2, ..., N$, let R be symmetric positive definite matrices and S be symmetric semi-positive definite matrix and $h, K \in R^+$ which satisfy the following matrix inequality holds.*

1. $\overline{M}_i(P_i, Q_i) + \overline{N}_{i,i}(R,h) < -S, \quad i = 1, ..., N.$
2. $\overline{M}_j(P_i, Q_i) + \overline{M}_i(P_i, Q_i) + \overline{N}_{j,i}(R,h), + \overline{N}_{i,j}(R,h) < \dfrac{2S}{N-1}, \quad i = 1, ..., N-1, j = i+1, ..., N.$

The feedback control is given by

$$u(t) = Kx(t).$$

4 CONCLUSIONS

In this paper, we study linear parameter dependent (LPD) delay system and LPD control delay system. We gave sufficient condition for exponential stability with a given convergence rate and asymptotically stability of linear parameter dependent (LPD) delay system and also sufficient condition for stabilizability of LPD delay control system. We use appropriate Lyapunov functions and derive stability condition in term of linear matrix inequality (LMI).

References

[1] D. henrion, D. Arzelier and J.B. Lasserre, “ On parameter dependent Lyapunov functions for robust stability of linear systems,” *IEEE Conference* ,V1, pp. 887 - 892, 12/04

[2] E. Fridman and U. Shaked,“ Parameter dependent stability and stabilization of uncertain time delay systems,” *IEEE Trans.Automat.Contr.*, , V48, N5, pp. 861-866, 6/03

[3] F. Gouaisbaut and D. Peaucelle,“ Delay dependent robust stability of time delay systems,” *Phd.Thesis*, Toulouse, France

[4] G. Zhai, H. Lin and P.J. Antsaklis, Quadratic stabilizability of switched linear systems with polytopic uncertainties,” *Phd.Thesis*, Wakayama university, Japan, 2002

[5] J.C. Geromel and P. Colaneri,“ Robust stability of time varying polytopic systems,” *Systems Control Lett.*, V55, p. 81-85, 2006

[6] P.T. Nam and V.N. Phat, “Exponential stability and stabilization of linear uncertain polytopic time delay systems,”*Phd.Thesis*, Qui Nhon University, vietnam

[7] V. Jeyakumar, P.T. Nam and V.N. Phat, “New conditions for stability and stabilizability of linear parameter depentdent systems,” *Phd. Thesis*, Qui Nhon University, vietnam

[8] V.F. Montagner and P.L.D. Peres , “Stability of linear time varying through quadratically parameter dependent Lyapunov function,” *XXVIII CNMAC - Congresso Nacional de Matematica Aplicada e Computacional* , Vol. 1, pp.1-3, 01/05