Evaluation of Per Unit Length Parameters of Multiconductor lines of High Frequency Integrated Circuits by the Method of Rectangular Subareas

Saswati Ghosh, Ajay Chakrabarty

Abstract— **This paper presents the numerical technique for the evaluation of per unit length parameters of multiconductor lines used in high frequency integrated circuits. The conducting surfaces are modeled by planar rectangular subdomains. The Method of Moments is employed to calculate the charge distribution on the surface and hence the capacitance and inductance. The exact formulation for the matrix element is evaluated for rectangular subsection.**

Index Terms— **Multiconductor lines, method of moments, per-unit-length parameters, rectangular subareas.**

I. Introduction

The accurate evaluation of the per unit length capacitance and inductance of multiconducting lines and PCB lands is an important step in the design and packaging of high frequency integrated circuits. Considerable work has already been performed by other researchers on the development of different wideband microstrip interconnects and determination of capacitance of microstrip transmission lines [1-4]. For the capacitance calculation, the theory of dc field computation is used. The Method of Moments analysis with triangular and square subsections is available in other literatures [2-4]. In this paper, the authors have chosen the rectangular shape of the subsection because of its ability to conform easily to any geometrical surface or shape and at the same time to maintain the simplicity of approach compared to the triangular patch modeling. The Method of Moments with Pulse basis function and Point Matching has been used to evaluate the charge distribution and hence the capacitance and inductance of multiconducting bodies. The capacitances of different conducting structures such as square plates, circular disc are compared with other available data in literature [4-6]. Next the same method has been extended for multiconducting bodies e.g. parallel rectangular plates, parallel circular discs, circular coaxial conducting structures and the results for capacitance and inductance of these structures are presented.

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II. Theory

We consider a perfectly conducting surface is charged to a potential V. The unknown surface charge density distribution σ (r′) may then be determined by solving the following integral equation [4]

$$V = \iint_S \frac{\sigma(r')}{4\pi\varepsilon|r-r'|} ds' \qquad (1)$$

Here r and r' are the position vectors corresponding to observation and charge source points respectively, ds' is an element of surface S and ε is the permittivity of free space. The arbitrary-shaped bodies are approximated by planar rectangular subdomains (Figure 1). The Method of Moments with pulse basis function and point matching is then used to determine the approximate charge distribution [4]. On each subdomain, a pulse expansion function $P_n(r)$ is chosen such that $P_n(r)$ is equal to 1 when r is in the n-th rectangle and $P_n(r)$ is equal to 0 when r is not in the n-th rectangle. With the above definition of expansion function, the charge density, $\sigma(r')$ may be approximated as follows

$$\sigma(r') = \sum_{n=1}^{N} \sigma_n P_n(r') \quad where \quad P_n = \begin{matrix} 1 & for\ n-th\ subsection \\ 0 & elsewhere \end{matrix} \qquad (2)$$

Here N is the number of rectangles modeling the surface and σ_n's are the unknown weights (charge density).

Substitution of charge expansion (2) in (1) and point matching the resulting functional equation, yields an N x N system of linear equations which may be written in the following form

$$[V] = [K][Q] \qquad (3)$$

Here [K] is an **N** × N matrix and [Q] and [V] are column vectors of length N.

The elements of [K], [Q] and [V] are given as follows

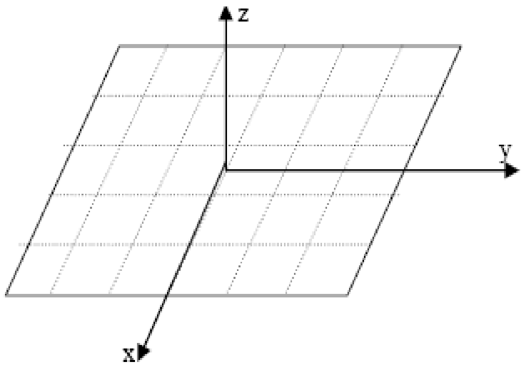

Fig. 1 Square plate divided into rectangular subsections.

$$K_{mn} = \iint_{rectangle} \frac{1}{4\pi\varepsilon |r_m - r'|} dA' \qquad (4)$$

$Q_n = \sigma_n$ =unknown charge density in subdomain n
$V_n = V$

r_m denotes the position vector of the center of the m[th] rectangle. *A*' is the area of the source rectangle.

$$|r_m - r'| = \sqrt{(x_m - x')^2 + (y_m - y')^2}$$

Here we have considered the conducting surface at z=0 plane.

Since the numerical formulation of (1) via the Method of Moments is well-known [4], we consider only the evaluation of the element of the moment matrix as given by equation (4). Each element corresponds to the potential at some point in space, r = (x, *y*, z), due to a rectangular patch of surface charge of unit charge density. In general, the patch is arbitrarily positioned and oriented in space.

The integration of equation (4) is quite tedious, but the final result is relatively simple [7].

For the diagonal elements of the matrix, the integration is evaluated as follows

$$K_{nn} = \frac{1}{\pi\varepsilon}\left(a \ln\left(\frac{b}{a} + \sqrt{\frac{b^2}{a^2} + 1}\right) + b \ln\left(\frac{a}{b} + \sqrt{\frac{a^2}{b^2} + 1}\right)\right) \qquad (5)$$

Here 2*a* and 2*b* are the sides of each rectangular subsection.

Using the standard integral formula the non-diagonal elements are evaluated as follows

$$K_{mn} = \frac{1}{4\pi\varepsilon}\left(-\left[|x_m - x'| \ln \frac{(|y_m - y_n + b| + \sqrt{(x_m - x')^2 + (y_m - y_n + b)^2})}{(|y_m - y_n - b| + \sqrt{(x_m - x')^2 + (y_m - y_n - b)^2})} \right]_{x_n - a}^{x_n + a} - \left[|y_m - y'| \ln \frac{(|x_m - x_n + a| + \sqrt{(y_m - y')^2 + (x_m - x_n + a)^2})}{(|x_m - x_n - a| + \sqrt{(y_m - y')^2 + (x_m - x_n - a)^2})} \right]_{y_n - b}^{y_n + b} \right) \qquad (6)$$

Here the source point is (x_n, y_n) and the field point is (x_m, y_m). The x′ and y′ of equation (6) are replaced by their respective limits. Solution of the matrix equation (3) yields values for the surface charge density at the centers of the subdomains. The capacitance, C, of the conducting surface is obtained from the following equation

$$C = Q/V = \frac{1}{V}\sum_{n=1}^{N} \sigma_n A_n \qquad (7)$$

where N is the total number of rectangular subsections.

The same method for a single conductor is extended for evaluating the capacitance of multiconducting bodies.

We consider two parallel rectangular conducting plates (2Lx2W) each divided into equal number of subsections (Figure 2).

The simplified formula achieved is as follows

$$V = \sum_{n=1}^{N} K_{mn}\sigma_n$$

$$\text{where } K_{mn} = \frac{1}{4\pi\varepsilon}\int_{x_n - a}^{x_n + a} dx' \int_{y_n - b}^{y_n + b} \frac{dy'}{\sqrt{(x_m - x')^2 + (y_m - y')^2 + (z_m - z')^2}} \qquad (8)$$

In matrix form, equation (8) can be written as follows

$$[K_{mn}][\sigma_n] = [V_n] \qquad (9)$$

where

$$[K_{mn}] = \begin{bmatrix} [K_{mn}^{11}] & [K_{mn}^{12}] \\ [K_{mn}^{21}] & [K_{mn}^{22}] \end{bmatrix}; \qquad [V_n] = \begin{bmatrix} [V_n^1] \\ [V_n^2] \end{bmatrix}$$

The diagonal sub matrices represent the effect of the plate itself and the non diagonal sub matrices represent the mutual interaction between the plates.

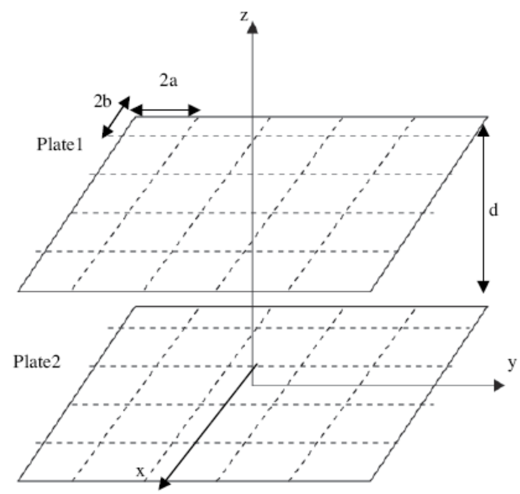

Fig. 2 Parallel plate divided into rectangular subsections.

The elements of the diagonal matrix remain same as the single element case. The elements of the non-diagonal matrix are evaluated following the same method. However, due to lack of space it is not possible to include the final expressions for the diagonal and non-diagonal elements of the non-diagonal matrices.

Similarly the exact expression for the elements of the non-diagonal sub matrices can be evaluated for two inclined plates (Figure 3). In this case, the expressions for the non diagonal elements remain almost same as for parallel plates, the only difference is that the value of d does not remain constant – it varies with the positions of the subsections.

For multiconductor lines surrounded by homogeneous medium the inductance of the line is evaluated from the following relation

$$C = \mu\varepsilon L^{-1} \qquad (10)$$

where the surrounding medium is characterized by μ and ε.

The characteristic impedance can be found out using the simple formula $Z=1/vC$ where $v=3x10^8$ *m/sec*.

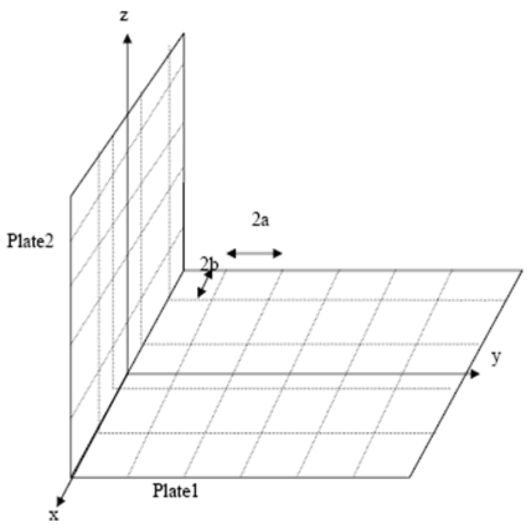

Fig. 3 Inclined plate divided into rectangular subsections.

III. Results and Discussions

A computer program based on the preceding formulation has been developed to determine the charge distribution and hence the capacitance of arbitrary shaped multiconducting bodies. The capacitance of different conducting surfaces e.g. rectangular plate, square plate, circular disc have been calculated (Figure 4 – 6). The capacitance data for a square and rectangular plate agrees with the available data in literature [4-6, 8-9]. Also the result for a circular disc (radius=1m, N=24, capacitance=68.36 pF) matches with the value available in literature [6]. Next the same method has been extended for multiconducting bodies e.g. parallel rectangular and circular plates, co-axial conductors with circular cross-section. The per-unit-length parameters for parallel square conductors, circular discs and co-axial conductors are presented and compared with other available data in Table 1 - 3.

For coaxial conductor, each circular cylinder is replaced by a cylinder with octagonal structure of surface area equal to that of the circular cylinder (Figure 7). Each side of the octagonal cylinder is divided into rectangular subsections. For co-axial conductors of finite length, there is appreciable fringing effect. The per unit length capacitance of the circular coaxial line is found by evaluating the capacitance of various lengths and then subtracting the part due to the fringing effect. Also the characteristic impedance of the coaxial conductor is evaluated and compared with the analytical value (Figure 8).

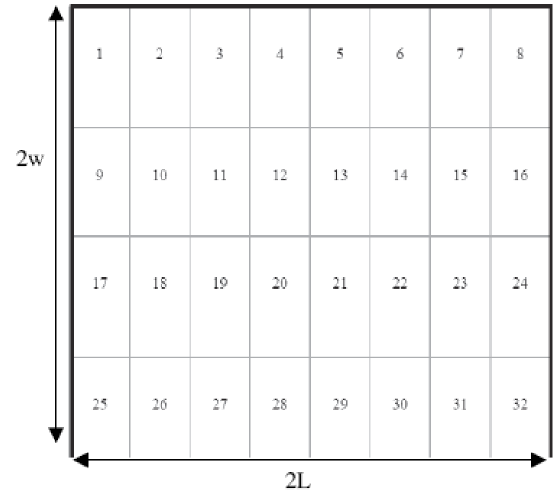

Fig. 4 Square plate (2L=1m; 2w=1m; V=1 volt) divided into N=32 subsections. Capacitance=38.69 pF.

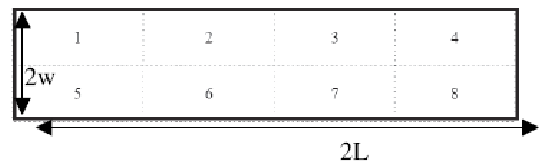

Fig. 5 Rectangular plate (2L=4m; 2w=1m; V=1 volt) divided into 4x2 subsections Capacitance=54.73 pF.

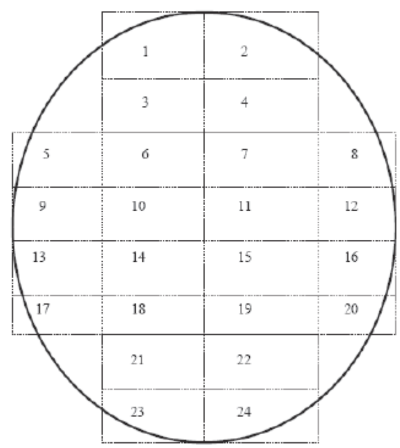

Fig. 6 Circular disc (radius=1m, N=24). Capacitance=68.36 pF agrees with the value in literature =70.73 pF [9].

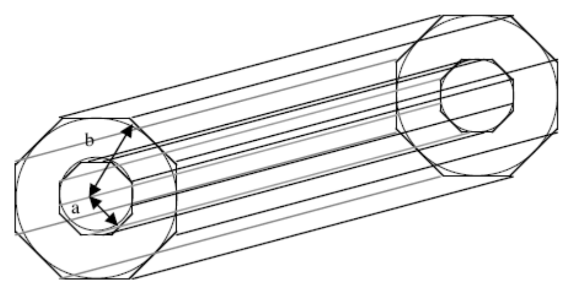

Fig. 7 Circular coaxial conductor approximated with octagonal cross-sectional coaxial structure.

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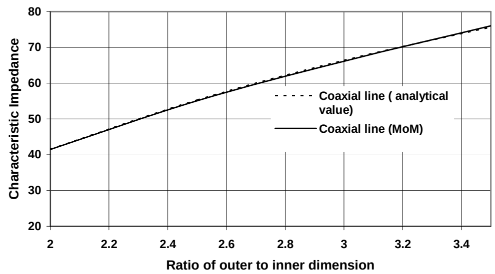

Fig. 8 Plot of the characteristic impedance versus ratio of outer to inner dimension of circular coaxial line.

TABLE I
CAPACITANCE OF PARALLEL SQUARE CONDUCTING PLATE
(LENGTH= WIDTH= 2L=1M)

d/2L	Capacitance in pF (calculated)	Capacitance in pF (C_0=εA/d)	C/C_0	C/C_0 [4]
0.01	904.48	884.14	1.023	1.024
0.025	378.43	353.67	1.07	1.05
0.05	203.35	176.83	1.15	1.15
0.10	105.92	88.41	1.198	1.2

TABLE II
CAPACITANCE OF PARALLEL CIRCULAR CONDUCTING PLATES
(RADIUS=1M)

d in meter	Capacitance in pF (MoM)	Capacitance in pF using analytical formula ($C_0=\varepsilon\pi r^2/d$)	C/C_0	C/C_0 [12]
0.02	1405.86	1388.8	1.015	
0.03	981.16	925.92	1.0597	1.062
0.05	617.64	555.56	1.11	
0.07	471.5	396.82	1.18	

TABLE III
CAPACITANCE AND INDUCTANCE OF CIRCULAR COAXIAL LINES (RADIUS=1M)

Ratio of outer to inner dimension	Capacitance in pF (including fringing effect)		Capacitance / unit length in pF/meter	Analytical value $C=2\pi\varepsilon/\ln(b/a)$ pF/meter	Inductance / unit length in μH / m
	Length =1m	Length =2 m			
2	109.44	189.75	80.21	80.15	0.1386

IV. CONCLUSION

A simple and efficient numerical procedure based on Method of Moments is presented for the evaluation of the per-unit length parameters of multiconducting bodies. The conducting structure is divided into rectangular subareas. The data for capacitance of different planar and non-planar conducting structures show well agreement with their analytical value. This method can be used for the determination of equivalent circuit models of multiconductor or multiwire arrangements used in electronic systems.

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