A New Frequency-domain Regularization for the GMDF Algorithm

Junghsi Lee and Hsu Chang Huang

***Abstract*—Frequency domain adaptive filters are attractive in applications requiring a large number of coefficients such as acoustic echo cancellation (AEC). Our recent paper derived a not-so-restrictive fixed common step-size bound for the generalized multidelay adaptive filter (GMDF). Based on this bound, this paper introduces a new frequency-domain regularization for the GMDF algorithm. Extensive simulation results demonstrate the usefulness of our algorithm in the scenario of speech input signals.**

***Index Terms*—Acoustic echo cancellation, frequency domain adaptive filters, GMDF, regularization parameter.**

I. Introduction

The normalized least-mean-square (NLMS) scheme has been the most popular adaptive filtering algorithm in many applications. It is well known that the NLMS and its variation, normalized block LMS (NBLMS) with block length N, converge at the same rate and achieve the same misadjustment if the fixed step-size parameter of NBLMS is N times as large as that of the NLMS [4]. However, both algorithms have the same convergence bounds for the step-size parameter. Therefore, even for a moderate block length N, the NBLMS has to employ a fairly small step-size parameter to meet the very restrictive convergence bound.

Frequency-domain block LMS (FBLMS) adaptive filters are attractive alternatives for acoustic echo cancellation (AEC) partly because of the very low computational complexity that is due to the usage of fast Fourier transform (FFT). Soo proposed a variation of FBLMS, which was referred to as the multidelay frequency domain adaptive filter (MDF) to alleviate the delay problem associated with the large filter size [10]. MDF segments the filter into several partitions and employ as many sub-filters as well. The MDF belongs to the class of partitioned FBLMS (PFBLMS) algorithms. Moulines [9] proposed the generalized MDF (GMDF) that allows one to select FFT size and the block delay separately. This advantage is owing to the controlling of the overlap between the successive input blocks. The PFBLMS and GMDF can be implemented with normalization in each of the frequency bins. Some researchers thought that the frequency-bin normalization procedure resolves the problem of slow modes of the NLMS algorithm and the resulting algorithm converges faster than the NLMS [2,6]. However, there were researchers reasoned that because of the restriction on the step-size bounds, the frequency domain algorithms actually do not perform better than the NLMS in convergence and tracking properties [1].

Researchers have had different views on the convergence performance of PFBLMS, and the derivations of step-size bounds in the literature are not consistent [2,5,9]. Our recent paper [8] showed that bound of the fixed common step-size of the GMDF is N times larger than that of the NBLMS. Because of this new analysis, we can choose proper step-size so that the well-designed GMDF can maintain good tracking and convergence performance and have great saving in computations as well.

However, input signal is very often in low level in AEC application. Just like that for NLMS algorithms, a regularization parameter vector $\boldsymbol{\delta}(n)$ has to be added in the normalization process to avoid divergence. There have been some regularized schemes presented to tackle this problem in time-domain NLMS [3,7] and frequency-domain adaptive filters as well. Based on our recent discovery of the step-size bound of the GMDF, we propose a new regularization that works well in the scenario of speech input signals.

The rest of the paper is organized as follows. Section 2 introduces the frequency-domain regularized GMDF algorithm. Extensive simulation results that demonstrate the usefulness of our filter are presented in Section 3. The conclusions are made in the last section of the paper.

II. A New Regularized GMDF

Consider the GMDF with L sub-filters, each is of order N and FFT size is $2N$. Without loss of generality, we assume that $M = NL$ where M is the filter order. The GMDF uses a positive integer α to control the overlap between the successive input blocks. Consequently, it updates the coefficients every $R = N/\alpha$ samples. In the k^{th} iteration, define reference input vector $\mathbf{x}_k$ and desired response vector $\mathbf{d}_k$, respectively, as

$$\mathbf{x}_k = \left[u(kR), u(kR+1), \cdots, u(kR+N-1)\right]^T \tag{1}$$

$$\mathbf{d}_k = \left[d(kR), d(kR+1), \cdots, d(kR+N-1)\right]^T \tag{2}$$

Frequency-domain input vector for l^{th} sub-filter, denoted as $\mathbf{X}_{l,k}$, $l = 1, 2, \cdots, L$ is computed as

$$\mathbf{X}_{l,k} = FFT\left[\mathbf{x}_{k-l\alpha}^T, \mathbf{x}_{k-(l-1)\alpha}^T\right]^T \tag{3}$$

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H. C. Huang is with Department of Electrical Engineering, Yuan-Ze University, Chung-Li, Taoyuan, 32026, TAIWAN, (Fax: +886-3-4639355; e-mail: s958501@mail.yzu.edu.tw.)

J. Lee is with the Department of Electrical Engineering, Yuan-Ze University, Chung-Li, Taoyuan, 32026, TAIWAN (Phone: +886-3-4638800 Ext. 2419; fax: +886-3-4639355; e-mail: eejlee@saturn.yzu.edu.tw.)

The corresponding frequency-domain coefficient vector $\mathbf{H}_{l,k}$ is defined accordingly as

$$\mathbf{H}_{l,k} = FFT\left[\mathbf{h}_{l,k}^T, \mathbf{0}_{N\times 1}^T\right]^T \tag{4}$$

where $\mathbf{h}_{l,k}$ is the l^{th} sub-filter's time-domain coefficient vector. Filter output vector $\hat{\mathbf{d}}_k$ is calculated as

$$\hat{\mathbf{d}}_k = \text{ last N points of } FFT^{-1}\left[\sum_{l=1}^{L}\mathbf{H}_{l,k}\otimes\mathbf{X}_{l,k}\right] \tag{5}$$

where $\otimes$ denotes element-wise multiplication. Frequency-domain error vector $\mathbf{E}_k$ is obtained as follows.

$$\mathbf{e}_k = \mathbf{d}_k - \hat{\mathbf{d}}_k \tag{6}$$

$$\mathbf{E}_k = FFT\left[\mathbf{0}_{(2N-R)\times 1}^T, \mathbf{e}_k^T\right]^T \tag{7}$$

The frequency power of the l^{th} subfilter at k^{th} iteration is calculated as

$$\mathbf{Z}_{l,k} = \beta\mathbf{Z}_{l,k-1} + (1-\beta)\bar{\mathbf{X}}_{l,k}\otimes\mathbf{X}_{l,k} \tag{8}$$

where $\bar{\mathbf{X}}_{l,k}$ denotes the complex conjugate of $\mathbf{X}_{l,k}$, and β is a forgetting factor. The coefficient vector $\mathbf{H}_{l,k}$ is updated as

$$\mathbf{H}_{l,k+1} = \mathbf{H}_{l,k} + \frac{2\mu_{GMDF}}{M}\mathbf{\Phi}_{l,k}, \tag{9}$$

where μ_{GMDF} is a fixed common un-normalized step-size parameter of the GMDF filter. In (9), $\mathbf{\Phi}_{l,k}$, the new information for updating, is obtained as

$$\mathbf{\Phi}_{l,k} = FFT\left[\phi_{l,k}^T, \mathbf{0}^T\right]^T \tag{10}$$

where

$$\phi_{l,k} = \textbf{ first part of } FFT^{-1}\left[\left(\mathbf{E}_k\otimes\bar{\mathbf{X}}_{l,k}\right) \odot\left(\mathbf{Z}_{l,k} + \delta\cdot\mathbf{1}_{2N\times 1}\right)\right] \tag{11}$$

where $\odot$ denotes element-wise division and δ denotes the regularization parameter.

We have recently derived a bound for the fixed common step-size parameter as [8]

$$0 < \mu_{GMDF} < 2N. \tag{12}$$

This bound is obtained under the assumption that input signal is a white Gaussian process with zero mean and the variance is σ_u^2. Therefore, the expected value of averaged frequency domain power $\mathbf{Z}_{l,k}$ can be approximated as $2N\sigma_u^2$. Now consider situations that input signals are at very low level for some time, i.e., $x(n)\approx 0$. So, $\mathbf{Z}_{l,k}$ would be very small too. We choose a regularization parameter δ to restrict the change of coefficient adjustment in (9) as

$$0 < \frac{2\mu_{GMDF}}{M(0+\delta)} < \frac{2\cdot 2N}{M(2N\sigma_u^2)}. \tag{13}$$

This implies that the regularization parameter δ has to be greater than $\sigma_u^2\mu_{GMDF}$. A workable and more conservative choice is

$$3\mu_{GMDF}\sigma_u^2 < \delta. \tag{14}$$

III. Simulation Results

In this section, we present the results of several experiments that demonstrate the usefulness of the proposed regularization scheme of the GMDF algorithm. The adaptive filter was used to identify a 512-tap acoustic echo impulse response. We have compared the performance of our proposed to two other alternatives, i.e., NLMS and a GMDF. We have used the normalized squared coefficient error (NSCE) to evaluate the performance of the algorithms. The NSCE is defined as

$$NSCE(n) = 10\log_{10}\frac{\left\|\mathbf{h}_o(n)-\hat{\mathbf{h}}(n)\right\|^2}{\left\|\mathbf{h}_o(n)\right\|^2} \tag{15}$$

where $\hat{\mathbf{h}}(n)$ is the filter coefficient vector. In order to make the comparison fair, the step-sizes μ_{NLMS} and μ_{GMDF} are chosen by the following relationship

$$\mu_{GMDF} = R\cdot\mu_{NLMS} = \left(\frac{N}{\alpha}\right)\cdot\mu_{NLMS}. \tag{16}$$

A. AR and MA processes

We have used AR processes and MA processes as the reference input signals. The acoustic echo impulse response was set to be time-varying from seconds 2.4 to 6.4. The evolution of coefficients is described by

$$\mathbf{h}_o(n) = \mathbf{h}_o + g(n), \tag{17}$$

where $g(n)$ is a white Gaussian noise with variance 10^{-3}. In these examples, the adaptive filter was run with the same structure and the same number of coefficients as the acoustic echo system. The additive noise is a white Gaussian process with variance 10^{-2}. The NSCE curves shown here are results of ensemble averages over 20 independent runs. The simulation results of the cases that GMDF employs $L=8$ and $\alpha=4$ are illustrated in Figures 1-4. For AR inputs, our filter exhibits better performance as shown in Figures 1 and 2 for $\mu_{GMDF}=8$ and $\mu_{GMDF}=16$, respectively. For the relatively ill-conditioned MA inputs, our filter clearly outperforms the other two filters with fast convergence behavior and great tracking properties as well. The results are illustrated in Figures 3 and 4 for two different step sizes.

B. Speech Signals

In this experiment, the excitations are 8-second-long Chinese speech signals. A 512-tap time-invariant acoustic echo system is used in the experiments. We compared our filter with the ε-NLMS and the conventional regularized GMDF filters with $\delta=0.01$ and $\delta=1$. The results for Speech-I are given in Figures 5 and 6. GMDF with $\delta=1$ performs slightly better than our filter after 5.5 second for the case $\mu_{GMDF}=16.0$ as demonstrated in Figure 6. However, GMDF with $\delta=1$ performs poorly for the case $\mu_{GMDF}=8.0$ as shown in Figure 5. Results of Speech-II are shown in Figures 7 and 8. ε-NLMS does not perform well in the scenario of speech inputs. GMDF with $\delta=0.01$ does not always exhibit better behavior than that of GMDF with $\delta=1$. Our filter does exhibits great performance for Speeches I and II, and for all other experiments we have conducted as well.

IV. CONCLUSIONS

This paper introduced a new frequency-domain regularized GMDF filter. This regularization is based on our recent discovery of the step-size bound of the GMDF. Extensive simulation results of different types of inputs including AR processes, MA processes and speech signals showed that the proposed filter works well and outperforms the other competing techniques.

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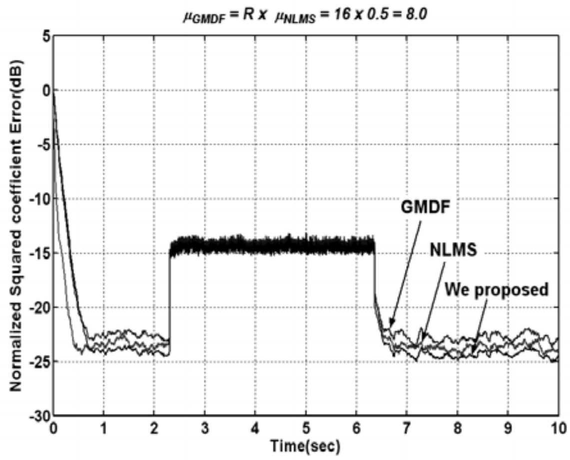

Fig. 1, NSCE curves of NLMS ($\mu_{NLMS} = 0.5$), GMDF ($\mu_{GMDF} = 8.0$), and our filter. AR input processes.

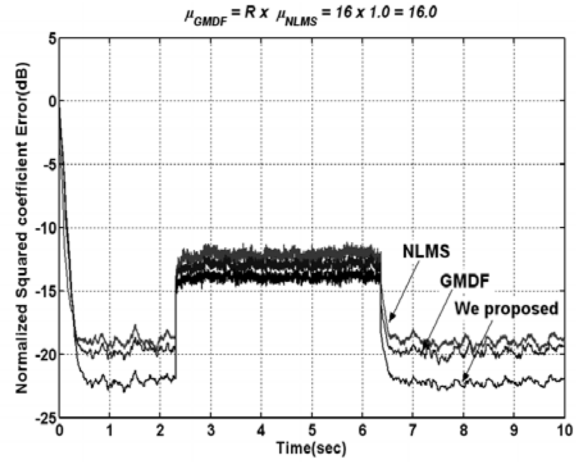

Fig. 2, NSCE curves of NLMS ($\mu_{NLMS} = 1.0$), GMDF ($\mu_{GMDF} = 16.0$), and our filter. AR input processes.

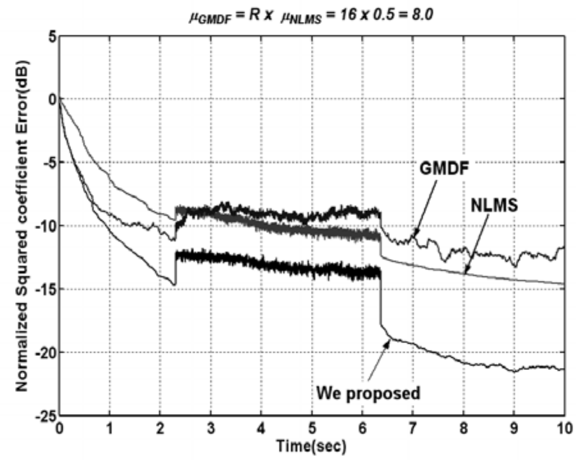

Fig. 3, NSCE curves of NLMS ($\mu_{NLMS} = 0.5$), GMDF ($\mu_{GMDF} = 8.0$), and our filter. MA input processes.

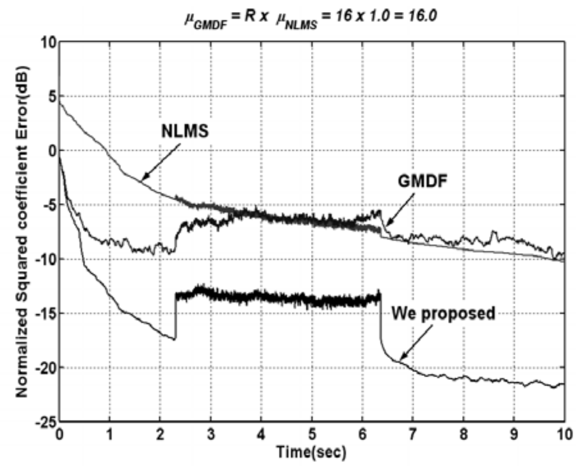

Fig. 4, NSCE curves of NLMS ($\mu_{NLMS} = 1.0$), GMDF ($\mu_{GMDF} = 16.0$), and our filter. MA input processes.

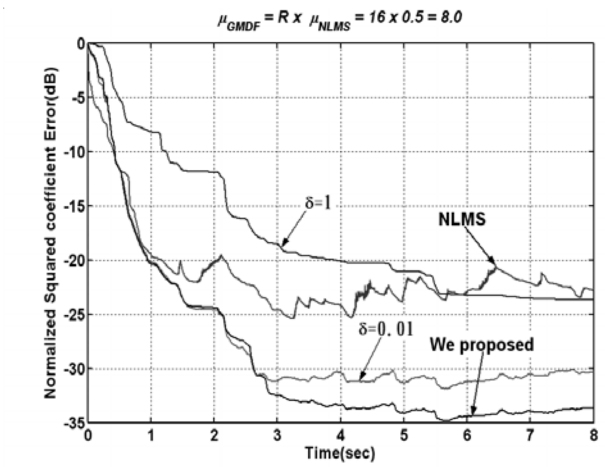

Fig. 5, NSCE curves of NLMS ($\mu_{NLMS} = 0.5$), GMDF filters with $\delta = 0.01$ and $\delta = 1$, respectively, ($\mu_{GMDF} = 8.0$), and our filter. Speech input I.

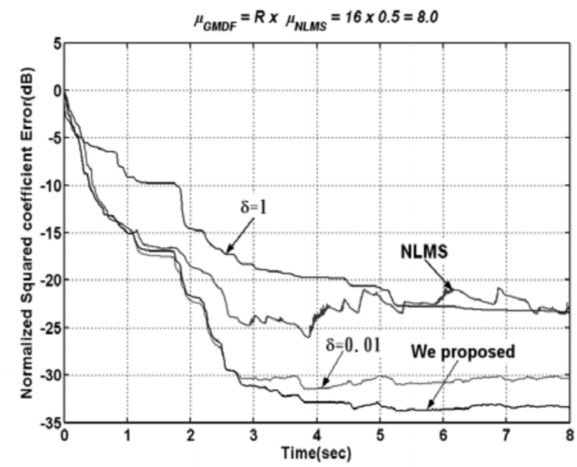

Fig. 7, NSCE curves of NLMS ($\mu_{NLMS} = 0.5$), GMDF filters with $\delta = 0.01$ and $\delta = 1$, respectively, ($\mu_{GMDF} = 8.0$), and our filter. Speech input II.

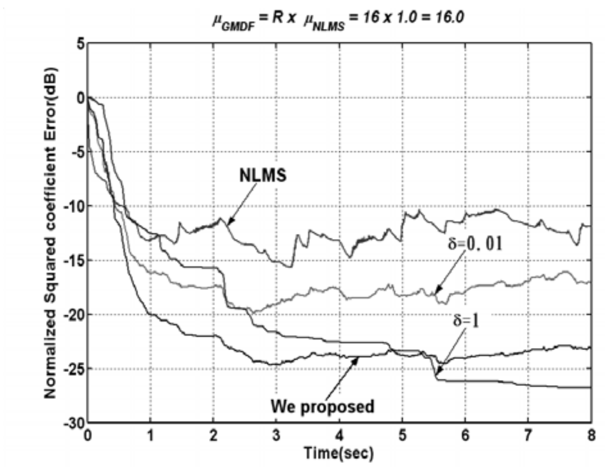

Fig. 6, NSCE curves of NLMS ($\mu_{NLMS} = 1.0$), GMDF filters with $\delta = 0.01$ and $\delta = 1$, respectively, ($\mu_{GMDF} = 16.0$), and our filter. Speech input I.

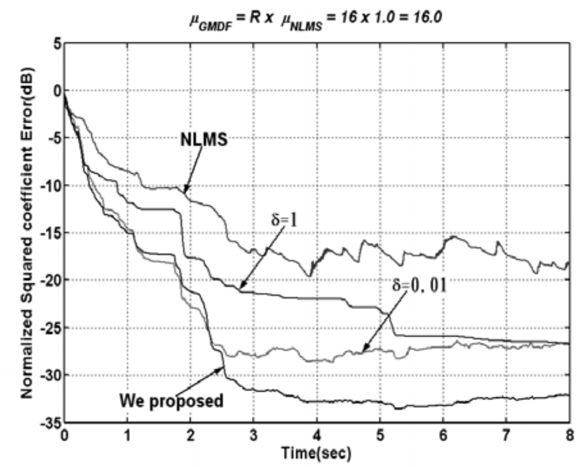

Fig. 8, NSCE curves of NLMS ($\mu_{NLMS} = 1.0$), GMDF filters with $\delta = 0.01$ and $\delta = 1$, respectively, ($\mu_{GMDF} = 16.0$), and our filter. Speech input II.