A Design for the Pitch Curve of Noncircular Gears with Function Generation

Jen-Yu Liu and Yen-Chuan Chen

***Abstract*— In this paper, we employ the Fourier series to design the pitch curve of noncircular gears with function generation. With the speed ratio is represented as Fourier series, the nonlinear function of angular displacements can be acquired through integration. And then, based on known finite separated angular displacements of two gears, the order of Fourier series is decided and a set of simultaneous equation is obtained. By solving the simultaneous equations, the Fourier series function regarding angular displacement and speed ratio can be obtained. Finally, synthesizing these two formulas creates the pitch curves for the noncircular pinion and gear, respectively. In this research, we demonstrate two designs examples to interpret the design process of the pitch curves of noncircular gears. In addition, the results derived from this paper can be taken as a reference to design the pitch curves of noncircular gears with function generation.**

***Index Terms*—pitch curve, noncircular gears, function generation.**

I. Introduction

The applications of noncircular gears are often applied in the transmission systems which may need variable speed ratio, nonlinear motion function, or variable input and output loadings, respectively. Basically, most of those applications are using elliptic gears therefore the elliptical pitch curve is more arresting [1-2]. In the field of research, most studies focus on the elliptic and the modified elliptic gears [3-11]. For instance, Emura and Arakawa [12-15] employed the elliptic and the modified elliptic gears at a design of the steering mechanism for automobiles and ships. Taking the linkage mechanism with a sliding slot as an example, Danieli [16] concluded a relationship of the speed ratio between drive and driven links, and further constructed the pitch curves of the equivalent noncircular gears according to this relationship. Dooner [17] used the noncircular gears to eliminate unnecessary loading in transmission and analyzed the relationship of the loading and the speed ratio of gears with input and output on the basis of the law of conservation of energy. With this relationship, he designed the outline of two pitch curves of gears and calculated the reduced torque on noncircular gears.

Usually, the pitch curves of the noncircular gears have to match a specific input/output relationship. The common procedure is to employ a known functional relationship, tooth number, and modules to synthesize the pitch curves of the noncircular gears which fit this relationship. Sometimes, as regards a design for the pitch curves of noncircular gears, the requirements are to satisfy finite separated angular positions, i.e., input and output angular displacement regarding the finite separated positions. The relevant literatures for designing noncircular gears with such function generation are rare.

Owing to the periodicity in the Fourier series, its first order derivative, its second order derivative or above [18-19], the demand on periodical features for angular displacement, angular velocity, and angular acceleration of the noncircular gear, the Fourier series is an adequate tool to generate the finite separated angular displacement function of the noncircular gears. In this paper, based on the known finite separated angular displacement position, we construct angular displacement function using Fourier series and deduce the pitch curve equations of noncircular gears meeting this finite separated position according to this function. In addition, some examples are created to interpret the method for designing the pitch curves of noncircular gears with function generation.

II. Synthesizing Principles of Noncircular Pitch Curves

In Figure 1, the noncircular gears 1 and 2 with angular velocity ω_1 and ω_2 and angular displacement ϕ_1 and ϕ_2, respectively, reversely rotate around the fixed axes O_1 and O_2. Coordinate systems x_1y_1 and x_2y_2 are the moving coordinate attached to gears 1 and 2, respectively.

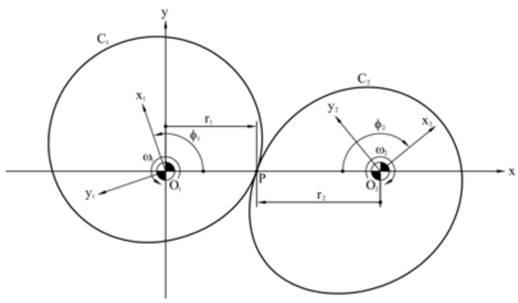

Figure 1 Pitch curves of noncircular gears

The rotating-speed ratio $m_{21}(\phi_1)$ is:

$$m_{21}(\phi_1) = \frac{\omega_2}{\omega_1} = \frac{d\phi_2/dt}{d\phi_1/dt} = \frac{d\phi_2}{d\phi_1} \quad (1)$$

when ϕ_1=0 then ϕ_2 also be zero.

Integrating Equation (1) with respect to ϕ_1 the relationship of ϕ_1 and ϕ_2 is obtained:

$$\phi_2 = \int_0^{\phi_1} m_{21}(\phi)d\phi \quad (2)$$

The distance from the contacting point P of gear 1 or gear 2 to the rotating center of each gear, and the center distances for two gears are $r_1(\phi_1)$, $r_2(\phi_2)$ and D, respectively. Based on the

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synthesizing principle [20], $r_1(\phi_1)$ and $r_2(\phi_2)$ are acquired as follows:

$$r_1(\phi_1) = \frac{Dm_{21}(\phi_1)}{1+m_{21}(\phi_1)} \quad (3)$$

$$r_2(\phi_1) = \frac{D}{1+m_{21}(\phi_1)} \quad (4)$$

Then the Equations of pitch curves of gears 1 and 2, $R_1(\phi_1)$ and $R_2(\phi_2)$ respectively, are:

$$\begin{aligned} R_1(\phi_1) &= r_1(\phi_1)\cos\phi_1\,\mathbf{i_1} + r_1(\phi_1)\sin\phi_1\,\mathbf{j_1} \\ &= \frac{Dm_{21}(\phi_1)}{1+m_{21}(\phi_1)}\cos\phi_1\,\mathbf{i_1} + \frac{Dm_{21}(\phi_1)}{1+m_{21}(\phi_1)}\sin\phi_1\,\mathbf{j_1} \quad (5) \end{aligned}$$

$$\begin{aligned} R_2(\phi_2) &= r_2(\phi_1)\cos\phi_2\,\mathbf{i_2} + r_2(\phi_1)\sin\phi_2\,\mathbf{j_2} \\ &= \frac{D\cos\phi_2}{1+m_{21}(\phi_1)}\mathbf{i_2} - \frac{D\sin\phi_2}{1+m_{21}(\phi_1)}\mathbf{j_2} \quad (6) \end{aligned}$$

III. Fourier Series Function for Finite Separated Angular Displacements

Table 1 shows N finite separated angular displacement positions of ϕ_1 and ϕ_2. As gear 1 rotates with a constant speed, the angular displacement of gear 2 displays a nonlinear variation. Thus, the rotating speed ratio is not a constant. Science gear 1 and gear 2 are periodical rotating, the rotating speed ratio displays a periodical variation. For an arbitrary integral n, Equation 1 should satisfy the following relationship:

$$m_{21}(\phi_1) = m_{21}(2n\pi + \phi_1) \quad (7)$$

Let the period time of the noncircular gear is T, Equation (7) can be represented with M order Fourier series [18-19]:

$$m_{21}(\phi_1) = \frac{d\phi_2}{d\phi_1} = A_0 + 2\sum_{n=1}^{M} A_n \cos\left(\frac{2n\pi}{T}\phi_1\right) + 2\sum_{n=1}^{M} B_n \sin\left(\frac{2n\pi}{T}\phi_1\right) \quad (8)$$

Table 1 N finite separated angular displacement

	ϕ_1	ϕ_2
1	$\phi_{1,1}$	$\phi_{2,1}$
2	$\phi_{1,2}$	$\phi_{2,2}$
3	$\phi_{1,3}$	$\phi_{2,3}$
:	:	:
N-1	$\phi_{1,N-1}$	$\phi_{2,N-1}$
N	$\phi_{1,N}$	$\phi_{2,N}$

Substituting Equation (8) into Equation (2), the relationship between ϕ_1 and ϕ_2 is:

$$\phi_2 = \int_0^{\phi_1} m_{21}(\phi)d\phi = A_0\phi_1 + 2\sum_{n=1}^{M}\frac{TA_n}{2n\pi}\sin\left(\frac{2n\pi}{T}\phi_1\right) + 2\sum_{n=1}^{M}\frac{TB_n}{2n\pi}\left[1-\cos\left(\frac{2n\pi}{T}\phi_1\right)\right] \quad (9)$$

From Equation (7), the period of angular displacement of noncircular gears are 2π, Equations (8) and (9) can be modified as:

$$m_{21}(\phi_1) = A_0 + 2\sum_{n=1}^{M} A_n \cos(n\phi_1) + 2\sum_{n=1}^{M} B_n \sin(n\phi_1) \quad (10)$$

$$\phi_2 = A_0\phi_1 + 2\sum_{n=1}^{M}\frac{A_n}{n}\sin(n\phi_1) + 2\sum_{n=1}^{M}\frac{B_n}{n}\left[1-\cos(n\phi_1)\right] \quad (11)$$

Equation (11) must satisfy conditions such as ϕ_1=0 together with ϕ_2=0 and $\phi_1=2\pi$ together with $\phi_2=2\pi$. Substituting these conditions into Equation (11), we can obtain A_0=1. In Equation (11), let $A_n/n=a_n$, $B_n/n=b_n$, $A_n=na_n$, and $B_n=nb_n$. Equations (10) and (11) can be written as:

$$m_{21}(\phi_1) = 1 + 2\sum_{n=1}^{M} na_n \cos(n\phi_1) + 2\sum_{n=1}^{M} nb_n \sin(n\phi_1) \quad (12)$$

$$\phi_2 = \phi_1 + 2\sum_{n=1}^{M} a_n \sin(n\phi_1) + 2\sum_{n=1}^{M} b_n\left[1-\cos(n\phi_1)\right] \quad (13)$$

In order to find the values of various coefficients of Fourier series, such as a_1, b_1, a_2, b_2, …, a_M, and b_M, respectively, Equation (13) can be modified as :

$$\sum_{n=1}^{M} a_n \sin(n\phi_1) + \sum_{n=1}^{M} b_n\left[1-\cos(n\phi_1)\right] = \phi_2 - \phi_1 \quad (14)$$

Substituting various separated positions listed in Table 1 into Equation (14), we can obtain the simultaneous formulas to solve various coefficients of Fourier series:

$$\begin{cases} a_1\alpha_{1,1} + b_1\beta_{1,1} + a_2\alpha_{2,1} + b_2\beta_{2,1} \\ +\cdots+a_M\alpha_{M,1} + b_M\beta_{M,1} = \phi_{2,1} - \phi_{1,1} \\ a_1\alpha_{1,2} + b_1\beta_{1,2} + a_2\alpha_{2,2} + b_2\beta_{2,2} \\ +\cdots+a_M\alpha_{M,2} + b_M\beta_{M,2} = \phi_{2,2} - \phi_{1,2} \\ a_1\alpha_{1,3} + b_1\beta_{1,3} + a_2\alpha_{2,3} + b_2\beta_{2,3} \\ +\cdots+a_M\alpha_{M,3} + b_M\beta_{M,3} = \phi_{2,3} - \phi_{1,3} \\ \vdots \\ a_1\alpha_{1,N-1} + b_1\beta_{1,3} + a_2\alpha_{2,N-1} + b_2\beta_{2,3} \\ +\cdots+a_M\alpha_{M,N-1} + b_M\beta_{M,N-1} = \phi_{2,N-1} - \phi_{1,N-1} \\ a_1\alpha_{1,N} + b_1\beta_{1,N} + a_2\alpha_{2,N} + b_2\beta_{2,N} \\ +\cdots+a_M\alpha_{M,N} + b_M\beta_{M,N} = \phi_{2,N} - \phi_{1,N} \end{cases} \quad (15)$$

where

$$\alpha_{i,j} = \sin(i\phi_{1,j}) \quad (16)$$

$$\beta_{i,j} = 1 - \cos(i\phi_{1,j}) \quad (17)$$

In Equations (16) and (17), $1 \le i \le M$ and $1 \le j \le N$. Further, only with the total number for unknown coefficients of a_1, b_1, a_2, b_2, …, a_M, and b_M been equal to N will there be only one set of solutions for the simultaneous formulas of Equation (15). Obviously, if N is an even number, M is N/2; however, for an odd N, the situation that one of those unknown coefficients of a_1, b_1, a_2, b_2, …, a_M, and b_M must be 0 in order to match the above condition. Through a numerical computation and analysis, we find when $b_M = 0$ for the case of an odd N, the solutions of Equation (15) have a better effect for transmission and facilitates processing of the shape of gears. Thus,

$$M = \begin{cases} N/2 & ,N \text{ is even.} \\ (N+1)/2, & N \text{ is odd, } b_N = 0. \end{cases} \tag{18}$$

Equation (15) can be expressed as the following matrix form:

$$\mathbf{A}_{N\times N}\,\mathbf{a}_N = \mathbf{V}_N \tag{19}$$

where

$$\mathbf{V}_N = \begin{bmatrix} \phi_{2,1}-\phi_{1,1} \\ \phi_{2,2}-\phi_{1,2} \\ \phi_{2,3}-\phi_{1,3} \\ \vdots \\ \phi_{2,N-1}-\phi_{1,N-1} \\ \phi_{2,N}-\phi_{1,N} \end{bmatrix} \tag{20}$$

For an even N:

$$\mathbf{A}_{N\times N} = \begin{bmatrix} \alpha_{1,1} & \beta_{1,1} & \cdots & \alpha_{N/2,1} & \beta_{N/2,1} \\ \alpha_{1,2} & \beta_{1,2} & \cdots & \alpha_{N/2,2} & \beta_{N/2,2} \\ \alpha_{1,3} & \beta_{1,3} & \cdots & \alpha_{N/2,3} & \beta_{N/2,3} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \alpha_{1,N-1} & \beta_{1,N-1} & \cdots & \alpha_{N/2,N-1} & \beta_{N/2,N-1} \\ \alpha\phi_{1,N} & \beta_{1,N} & \cdots & \alpha_{N/2,N} & \beta_{N/2,N} \end{bmatrix} \tag{21}$$

$$\mathbf{a}_N = \begin{bmatrix} a_1 \\ b_1 \\ \vdots \\ \vdots \\ a_{N/2} \\ b_{N/2} \end{bmatrix} \tag{22}$$

For an odd N:

$$\mathbf{A}_{N\times N} = \begin{bmatrix} \alpha_{1,1} & \beta_{1,1} & \cdots & \alpha_{(N-1)/2,1} & \beta_{(N-1)/2,1} & \alpha_{(N+1)/2,1} \\ \alpha_{1,2} & \beta_{1,2} & \cdots & \alpha_{(N-1)/2,2} & \beta_{(N-1)/2,2} & \alpha_{(N+1)/2,2} \\ \alpha_{1,3} & \beta_{1,3} & \cdots & \alpha_{(N-1)/2,3} & \beta_{(N-1)/2,3} & \alpha_{(N+1)/2,3} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \alpha_{1,N-1} & \beta_{1,N-1} & \cdots & \alpha_{(N-1)/2,N-1} & \beta_{(N-1)/2,N-1} & \alpha_{(N+1)/2,N-1} \\ \alpha_{1,N} & \beta_{1,N} & \cdots & \alpha_{(N-1)/2,N} & \beta_{(N-1)/2,N} & \alpha_{(N+1)/2,N} \end{bmatrix} \tag{23}$$

$$\mathbf{a}_N = \begin{bmatrix} a_1 \\ b_1 \\ \vdots \\ a_{(N-1)/2} \\ b_{(N-1)/2} \\ a_{(N+1)/2} \end{bmatrix} \tag{24}$$

Therefore,

$$\mathbf{a}_N = \mathbf{A}^{-1}_{N\times N}\,\mathbf{V}_N \tag{25}$$

The solutions for coefficients of Fourier series under even and odd known conditions are displayed from Equation (19) to Equation (25).

Design example 1 (N is even)

For a pair of noncircular gears, the finite separated angular displacements for gears 1 and 2 are shown in Table 2. The pitch curve of noncircular gears satisfy these specified angular displacements are designed as what follows.

From Table 2, owing to 6 finite separated angular displacements, N equals to 6. By Equation (18), M = 3. Substituting data shown in Table 2 into Equations (20) and (22), we can obtain:

Table 2 Even finite separated angular displacement

	ϕ_1	ϕ_2
1	π/9	π/9
2	4π/9	5π/9
3	5π/6	π
4	11π/9	25π/18
5	25π/18	3π/2
6	5π/3	31π/18

$$\mathbf{V}_6 = \begin{bmatrix} 0.000000 \\ 0.349066 \\ 0.523599 \\ 0.523599 \\ 0.349066 \\ 0.174533 \end{bmatrix} \tag{26}$$

$$\mathbf{A}_{6\times 6} = \begin{bmatrix} 0.342 & 0.060 & 0.643 & 0.234 & 0.866 & 0.500 \\ 0.985 & 0.826 & 0.342 & 1.940 & -0.866 & 1.500 \\ 0.500 & 1.866 & -0.866 & 0.500 & 1.000 & 1.000 \\ -0.643 & 1.766 & 0.985 & 0.826 & -0.866 & 0.500 \\ -0.940 & 1.342 & 0.643 & 1.766 & 0.500 & 0.134 \\ -0.866 & 0.500 & -0.866 & 1.500 & 0.000 & 2.000 \end{bmatrix} \tag{27}$$

Employ Guass Method [21] to get the inverse matrix of Equation (25):

$$\mathbf{A}^{-1}_{6\times 6} = \begin{bmatrix} 0.022 & 0.387 & 0.187 & -0.208 & -0.103 & -0.330 \\ -0.157 & -0.607 & 0.338 & 0.296 & -0.010 & -0.153 \\ 0.645 & -0.022 & -0.256 & 0.334 & -0.063 & -0.096 \\ -0.269 & 0.322 & -0.102 & -0.411 & 0.516 & -0.055 \\ 0.445 & -0.181 & 0.142 & -0.242 & 0.213 & 0.000 \\ 0.530 & -0.066 & -0.038 & 0.288 & -0.457 & 0.394 \end{bmatrix} \tag{28}$$

Substitute Equations (26) and (28) into Equation (25) to acquire:

$$\mathbf{a}_6 = \begin{bmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \\ a_3 \\ b_3 \end{bmatrix} = \begin{bmatrix} 0.030686 \\ 0.278672 \\ -0.005610 \\ 0.014658 \\ -0.041404 \\ 0.017464 \end{bmatrix} \tag{29}$$

By Equation (13), it can be obtained:

$$\begin{aligned}\phi_2 &= \phi_1 + \sum_{n=1}^{3} a_n \sin(n\phi_1) + \sum_{n=1}^{3} b_n \left[1-\cos(n\phi_1)\right] \\ &= \phi_1 + 0.030686 \sin\phi_1 + 0.278672(1-\cos\phi_1) \\ &\quad -0.005610 \sin 2\phi_1 + 0.014658(1-\cos 2\phi_1) \\ &\quad -0.041404 \sin 3\phi_1 + 0.017464(1-\cos 3\phi_1) \quad (30)\end{aligned}$$

Differentiate Equation (30) with respect to ϕ_1, we obtain rotating speed ratio $m_{21}(\phi_1)$ as:

$$\begin{aligned}m_{21}(\phi_1) &= \sum_{n=1}^{3} na_n \cos(n\phi_1) + \sum_{n=1}^{3} nb_n \sin(n\phi_1) \\ &= 1 + 0.030686 \cos\phi_1 + 0.278672 \sin\phi_1 \\ &\quad -0.011220 \cos 2\phi_1 + 0.029316 \sin 2\phi_1 \\ &\quad -0.124212 \cos 3\phi_1 + 0.052392 \sin 3\phi_1 \quad (31)\end{aligned}$$

With Equations (30) and (31) substituted into Equations (5) and (6), the equations of pitch curve of gear 1 and 2 are:

$$R_1(\phi_1) = \frac{Dm_{21}(\phi_1)}{1+m_{21}(\phi_1)} \cos\phi_1 \, \mathbf{i_1} + \frac{Dm_{21}(\phi_1)}{1+m_{21}(\phi_1)} \sin\phi_1 \, \mathbf{j_1} \quad (32)$$

$$R_2(\phi_2) = \frac{D\cos\phi_2}{1+m_{21}(\phi_1)} \mathbf{i_2} - \frac{D\sin\phi_2}{1+m_{21}(\phi_1)} \mathbf{j_2} \quad (33)$$

Figure 2 shows Fourier series satisfying the finite separated angular displacements by solving Equation (30) wherein the black bold curve is representing Equation (30), and the dotted line is the corresponding values of finite separated angular displacements. Figure 3 indicates the drawn pitch curves of gears 1 and 2 by Equations (32) and (33).

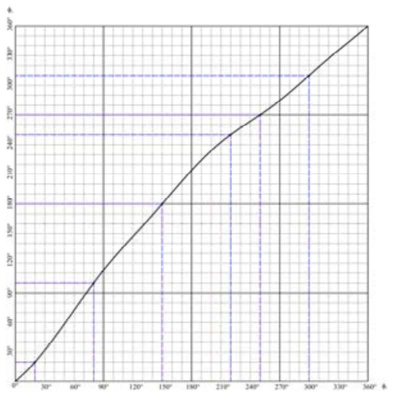

Figure 2 Fourier series function satisfy Table 2

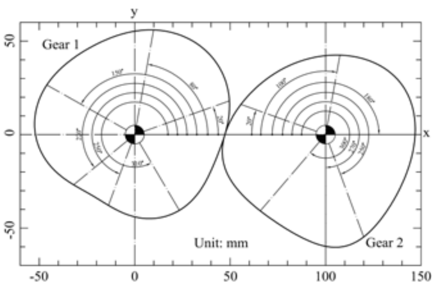

Figure 3 Pitch curve of gears 1 and 2 by Equations (32) and (33)

Design example 2 (N is odd)

The finite separated anular displacements for gears 1 and 2 are shown in Table 3. The design of pitch curves of noncircular gears must satisfy these odd finite separated angular displacements. In Table 3, 7 known finite separated angular displacements are listed, N equals to 7. By Equation (18), M = 4. Substituting data in Table 3 into Equations (20), (23), and (24), respectively, we have:

Table 3 Odd finite separated angular displacement

	ϕ_1	ϕ_2
1	$\pi/9$	$\pi/9$
2	$5\pi/8$	$\pi/3$
3	$\pi/9$	$5\pi/9$
4	$5\pi/6$	π
5	$11\pi/9$	$25\pi/18$
6	$25\pi/18$	$3\pi/2$
7	$5\pi/3$	$31\pi/18$

$$\mathbf{V}_7 = \begin{bmatrix} 0.000000 \\ 0.174533 \\ 0.349066 \\ 0.698132 \\ 0.523599 \\ 0.349066 \\ 0.174533 \end{bmatrix} \quad (34)$$

$$\mathbf{A}_{7\times7} = \begin{bmatrix} 0.342 & 0.060 & 0.643 & 0.234 & 0.866 & 0.500 & 0.985 \\ 0.766 & 0.357 & 0.985 & 1.174 & 0.500 & 1.866 & -0.342 \\ 0.985 & 0.826 & 0.342 & 1.940 & -0.866 & 1.500 & -0.643 \\ 0.500 & 1.866 & -0.866 & 0.500 & 1.000 & 1.000 & -0.866 \\ -0.643 & 1.766 & 0.985 & 0.826 & -0.866 & 0.500 & 0.342 \\ -0.940 & 1.342 & 0.643 & 1.766 & 0.500 & 0.134 & -0.985 \\ -0.866 & 0.500 & -0.866 & 1.500 & 0.000 & 2.000 & 0.866 \end{bmatrix} \quad (35)$$

The reverse matrix of Equation (35) is:

$$\mathbf{A}_{7\times7}^{-1} = \begin{bmatrix} 0.456 & -0.290 & 0.508 & 0.156 & -0.147 & -0.186 & 0.253 \\ 0.129 & -0.192 & 0.012 & 0.318 & 0.336 & -0.064 & -0.101 \\ -0.002 & 0.433 & -0.202 & -0.209 & 0.243 & 0.060 & -0.213 \\ 0.456 & 0.486 & 0.523 & -0.154 & -0.308 & 0.377 & 0.076 \\ 0.351 & 0.063 & -0.207 & 0.148 & -0.255 & 0.231 & -0.017 \\ -0.468 & 0.669 & -0.344 & 0.034 & 0.147 & -0.266 & 0.215 \\ 0.617 & 0.450 & 0.187 & -0.048 & 0.095 & -0.129 & 0.121 \end{bmatrix} \quad (36)$$

Substitute Equations (34) and (36) into Equation (25), we obtain:

$$\mathbf{a}_7 = \begin{bmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \\ a_3 \\ b_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 0.049878 \\ 0.328853 \\ -0.030062 \\ -0.025730 \\ -0.013766 \\ 0.042001 \\ -0.020946 \end{bmatrix} \quad (37)$$

Thus, from Equation (13), we have:

$$\begin{aligned}\phi_2 &= \phi_1 + \sum_{n=1}^{4} a_n \sin(n\phi_1) + \sum_{n=1}^{4} b_n \left[1-\cos(n\phi_1)\right] \\ &= \phi_1 + 0.049878 \sin\phi_1 + 0.328853(1-\cos\phi_1) \\ &\quad -0.030062 \sin 2\phi_1 - 0.025730(1-\cos 2\phi_1) \\ &\quad -0.013766 \sin 3\phi_1 + 0.042001(1-\cos 3\phi_1)\end{aligned}$$

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$$-0.020946\sin 4\phi_1 \quad (38)$$

Differentiate Equation (38) with respect to ϕ_1, the speed ratio $m_{21}(\phi_1)$ is:

$$\begin{aligned} m_{21}(\phi_1) &= \sum_{n=1}^{4} na_n \cos(n\phi_1) + \sum_{n=1}^{4} nb_n \sin(n\phi_1) \\ &= 1 + 0.049878\cos\phi_1 + 0.328853\sin\phi_1 \\ &\quad -0.060124\cos 2\phi_1 - 0.051460\sin 2\phi_1 \\ &\quad -0.041298\cos 3\phi_1 + 0.126003\sin 3\phi_1 \\ &\quad -0.083784\cos 4\phi_1 \end{aligned} \quad (39)$$

Substituting Equations (38) and (39) into Equations (5) and (6), the equations of pitch curves of gears 1 and 2 are as follows:

$$\begin{aligned} R_1(\phi_1) &= r_1(\phi_1)\cos\phi_1\,\mathbf{i_1} + r_1(\phi_1)\sin\phi_1\,\mathbf{j_1} \\ &= \frac{Dm_{21}(\phi_1)}{1+m_{21}(\phi_1)}\cos\phi_1\,\mathbf{i_1} + \frac{Dm_{21}(\phi_1)}{1+m_{21}(\phi_1)}\sin\phi_1\,\mathbf{j_1} \end{aligned} \quad (40)$$

$$\begin{aligned} R_2(\phi_2) &= r_2(\phi_1)\cos\phi_2\,\mathbf{i_2} + r_2(\phi_1)\sin\phi_2\,\mathbf{j_2} \\ &= \frac{D\cos\phi_2}{1+m_{21}(\phi_1)}\,\mathbf{i_2} - \frac{D\sin\phi_2}{1+m_{21}(\phi_1)}\,\mathbf{j_2} \end{aligned} \quad (41)$$

Figure 4 shows Fourier series meeting the finite separated angular displacements by solving Equation (38). In the figure, the black bold curve is related to Equation (38) and the dotted line is corresponding values of finite separated angular displacements. Figure 5 shows the pitch curves of gears 1 and 2 according to Equations (40) and (41) when the center distance D is 100mm.

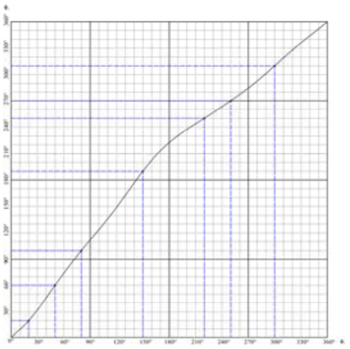

Figure 4 Fourier series function satisfy Table 3

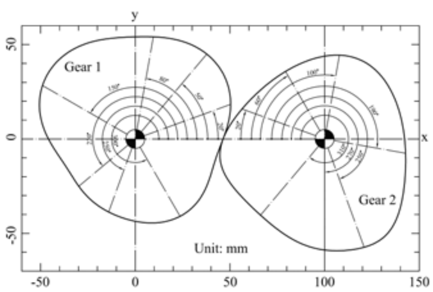

Figure 5 Pitch curve of gears 1 and 2 by Equations (40) and (41)

IV. CONCLUSION

In this paper, we provide the angular displacement curves of the noncircular gears by employing Fourier series to synthesize finite separated angular displacements and further synthesize the pitch curves of the noncircular gears. Based on the known finite separated angular displacements, simultaneous equations for the coefficients of M order Fourier series which satisfy these odd as well as even finite separated angular displacements respectively are derived. By solving the simultaneous equations, we obtain M order Fourier series. And the pitch curves of noncircular gears by means of the synthesizing principle for the pitch curves of noncircular gears. Two design examples in this study interpret the method to synthesize the functions and the pitch curves of the noncircular gears with finite separated angular displacements through Fourier series. As a reference for a research of designing the pitch curve of the noncircular gear, this study result can contribute to design a noncircular gear, which meets the function of finite separated angular displacement.

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