Proceedings of the International MultiConference of Engineers and Computer Scientists 2008 Vol II
IMECS 2008, 19-21 March, 2008, Hong Kong

Real time Study of a System with n Non-Identical Elements by Increasable Failure Rates

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Abstract- Reliability models based on Markov chain (Except in queuing systems) have extensive applications in electrical and electronically devices. In this paper we consider a system with n parallel and not necessarily identical elements with increasable failure rates (Failure rates are wiebull distributed) and the elements are non repairable. The system works until all elements fail. The system of equations are established and the exact equations are sought for the parameters like MTTF and the probability that system working at the time t. A numerical example has been solved to demonstrate the procedure which clarifies the theoretical development. It seems that the model can tackle more realistic situations.

***Index Terms*- Reliability, Markov chain, Parallel systems, Wiebull distribution.**

1. Nomenclature

The notations used in this paper are as follows:

n : Number of elements,

λ_i : Failure rate of the i st element,

$P_i(t)$: Probability that the system is in state i at the time t,

$R_P(t)$: Probability that system works at time t,

$MTTF$: Mean time to failure of the system,

X_j : Interval between $(j-1)$st and j st failure.

2. Introduction

$k-out-of-n$ Models, are one of the most useful models to calculate the reliability of electrical and electronically devices and systems. Lots of studies have been done so far in this area. We try to categorically classify them. At the first glance, they may be classifying in to two namely steady state and real time distinct classes. The element in both classes may be repairable or non repairable. The failure rates of the elements can be considered constant, increasing or decreasing whereas the repair rate is constant. Most of studies in this field are in steady state and a few studies are in real time conditions.

Gera [1] use a matrix formulation and solution for these systems, Lam, Keong and Tony [2] use a general model for consecutive k-out-of-n: F repairable system with exponential distribution and (k-1) - step Markov dependence, Sarhan, Abouammoh [3] work on nonrepairable system with no independent components subject to common shocks, Arumozhi [4] calculate the Exact equation and an algorithm for reliability evaluation of k-out-of-n systems, Koucky [5] calculate Exact reliability formula and bounds for general systems, Flynn and Chung [6] use a heuristic algorithm for determining replacement policies in consecutive k-out-of-n systems, Guan and Wu [7] work on Repairable consecutive system with fuzzy states, Li, Zho and Yam [8] works on a system with some components being suspended when the system is down, Sharifi and Moosakhani [9] works on a system with two element with constant and increasable fuzzy failure rates in real time situations.

In this paper we work on a system with n parallel and not necessarily identical element with increasable failure rates for real time conditions. The system works until more than all elements failed. The paper is divided into five parts. First part is nomenclature and second part is introduction. The third part explores the models. Numerical examples are presented in the fourth part and the final section deals with the conclusion.

3. Modeling

Assume a system with n parallel and non identical elements. The system works until all elements fail.

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Therefore each element has two states and consequently the system will have 2^n states. Let $A_1A_2...A_k$ be the state where the elements $A_1, A_2,\ldots$and A_k are working and other $(n-k)$ elements are failed. Also $A_1A_2...A_k\eta_j$ is the state that the element η_j works in addition to other k elements. The state O indicates that all the elements are failed. The state structure of the system is shown in figure 1.

In this figure, the category i, $i=1,2,...,n$, indicates that the states in this category are with i elements working and $(n-i)$ elements failed. Each state is closely related to the states in the antecedent and precedent categories. I.e., if an element failed in any state, then the state is transferred to the next category. In other words, if the system is in any state in category k, with the failure of one element, the state will be in category $(k-1)$. The system works if at least k elements work. Therefore we have:

The system failed in state O and now we have:

$$R_P(t)=\sum_{\substack{k=1\\1\le A_1<A_2<...<A_k\le n}}^{n}P_{A_1A_2...A_k}(t) \qquad (01)$$

And we know that:

$$\sum_{\substack{k=1\\1\le A_1<A_2<...<A_k\le n}}^{n}P_{A_1A_2...A_k}(t)+P_O(t)=1 \qquad (02)$$

In this system, the purpose is to find $P_i(t)$ for all states. From state $A_1A_2...A_n$ through O in figure 1 we have:

$$\begin{aligned}P_{A_1A_2...A_k}(t+\Delta t)&=P_{A_1A_2...A_k}(t)-\\&\sum_{i=1}^{k}\lambda_{A_i}\times t\times\Delta t\times P_{A_1A_2...A_k}(t)+\\&\sum_{\substack{j=1\\\eta_j\ne A_i\quad i=1,2,...,k}}^{n}\lambda_{\eta_j}\times t\times\Delta t\times P_{A_1A_2...A_k\eta_j}(t)\end{aligned} \qquad (03)$$

$P_{A_1A_2...A_k}(t)$ is the probability that the system be in the state $A_1A_2...A_k$ at time t. $\sum_{\substack{j=1\\\eta_j\ne A_i\quad i=1,2,...,k}}^{n}\lambda_{\eta_j}\times t\times\Delta t\times P_{A_1A_2...A_k\eta_j}(t)$ is the rate of transfer from category $(k+1)$ to this state and $\sum_{i=1}^{k}\lambda_{A_i}\times t\times\Delta t\times P_{A_1A_2...A_k}(t)$ is the rate of transfer from this state to states of category $(k-1)$ at time Δt. By solving equation (03) we can calculate the values of $P_i(t)$ as follows:

$$\begin{aligned}R_P(t)=\sum_{i=1}^{n}&e^{-\frac{1}{2}\lambda_i\times t^2}-\sum_{1\le i_1<i_2\le n}e^{-\frac{1}{2}(\lambda_{i_1}+\lambda_{i_2})\times t^2}\\&+...+(-1)^{(k-1)}\sum_{\substack{1\le i_1<i_2<...<i_k\le n\\k=1,2,...,n}}e^{-\frac{1}{2}\sum_{j=1}^{k}\lambda_{i_j}\times t^2}\\&+...\\&+(-1)^{(n-1)}\sum_{i=1}^{n}e^{-\frac{1}{2}\lambda_i\times t^2}\end{aligned} \qquad (05)$$

And the system $MTTF$ is also calculated as followed:

$$\begin{aligned}MTTF&=\int_0^{+\infty}R_P(t)dt=\\&\sum_{i=1}^{n}\sqrt{\frac{2\pi}{\lambda_i}}-\sum_{1\le i_i<i_2\le n}\sqrt{\frac{2\pi}{\lambda_{i_1}+\lambda_{i_2}}}\\&+...+\\&(-1)^{(k-1)}\sum_{\substack{1\le i_1<i_2<...<i_k\le n\\k=1,2,...,n}}\sqrt{\frac{2\pi}{\sum_{j=1}^{k}\lambda_{i_j}}}\\&+...+(-1)^{(n-1)}\sqrt{\frac{2\pi}{\sum_{i=1}^{n}\lambda_i}}\end{aligned} \qquad (06)$$

4. Numeric Samples

In this Example, the system has three non identical elements. Assume that λ_i, $i=1,2,3$, is the failure rate of the i st element. The states of the system are shown in figure 2:

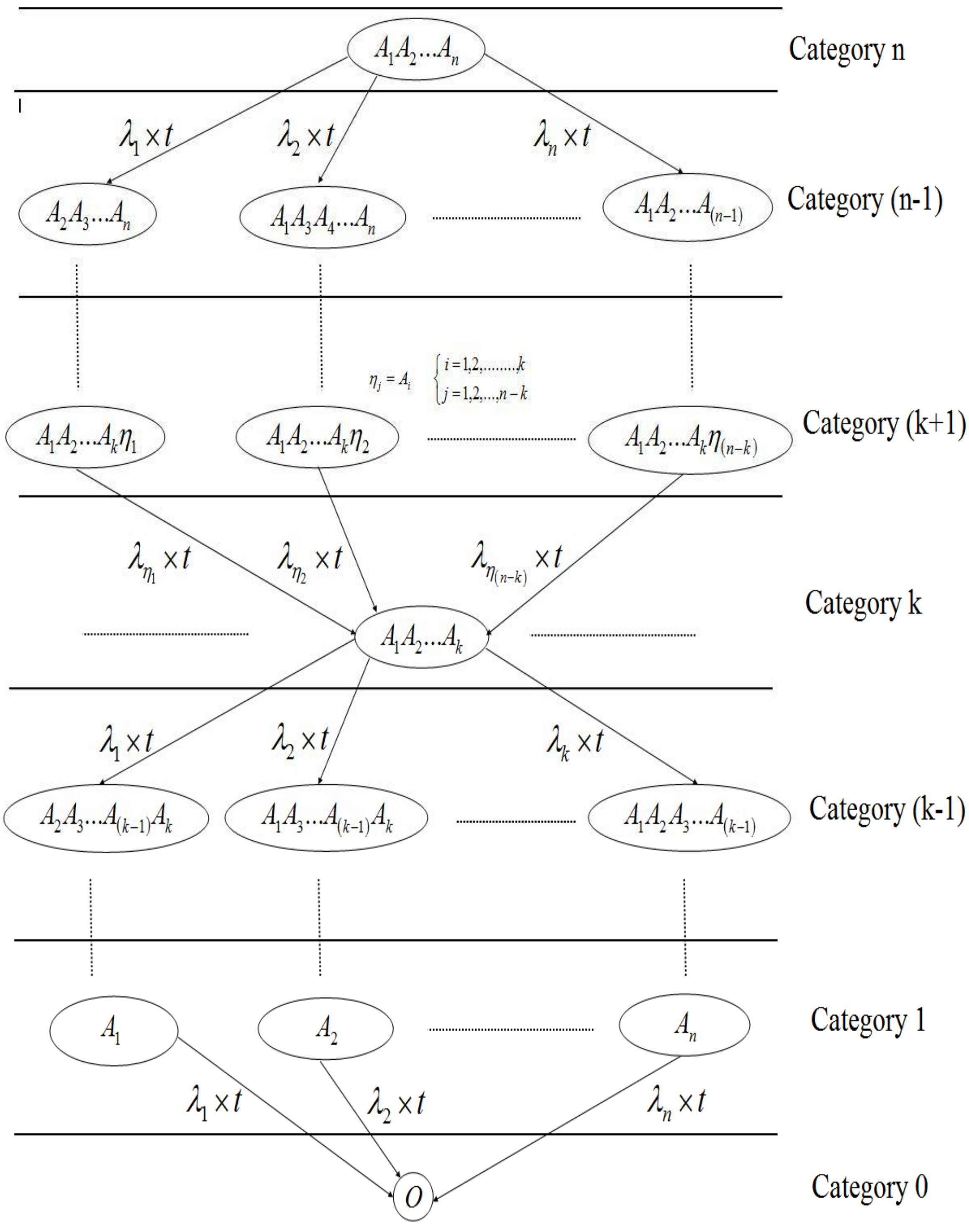

Fig 1: the state of the system

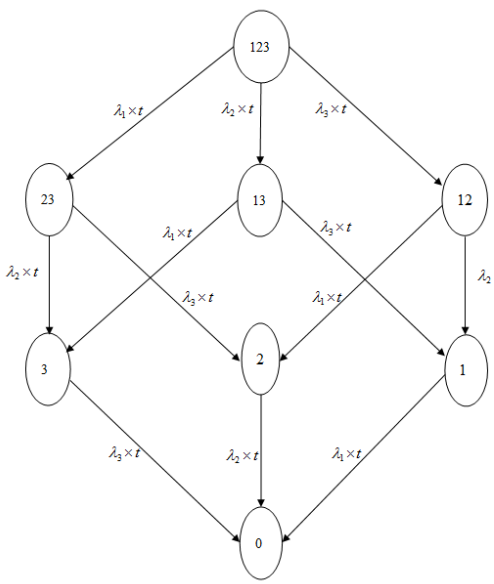

Figure 2: The states of example 1

By the equation (03) we can calculate the $P_i(t)$ as follow:

$$P_{123}(t+\Delta t)=P_{123}(t)-(\lambda_1+\lambda_2+\lambda_3)\times t\times\Delta t\times P_{123}(t) \quad (07)$$

$$P_{12}(t+\Delta t)=P_{12}(t)-(\lambda_1+\lambda_2)\times t\times\Delta t\times P_{12}(t)+\lambda_3\times t\times\Delta t\times P_{123}(t) \quad (08)$$

$$P_{13}(t+\Delta t)=P_{13}(t)-(\lambda_1+\lambda_3)\times t\times\Delta t\times P_{13}(t)+\lambda_2\times t\times\Delta t\times P_{123}(t) \quad (09)$$

$$P_{23}(t+\Delta t)=P_{23}(t)-(\lambda_2+\lambda_3)\times t\times\Delta t\times P_{23}(t)+\lambda_1\times t\times\Delta t\times P_{123}(t) \quad (10)$$

$$P_1(t+\Delta t)=P_1(t)+\lambda_3\times t\times\Delta t\times P_{13}(t)+\lambda_2\times t\times\Delta t\times P_{12}(t)-\lambda_1\times t\times\Delta t\times P_1(t) \quad (11)$$

$$P_2(t+\Delta t)=P_2(t)+\lambda_3\times t\times\Delta t\times P_{23}(t)+\lambda_1\times t\times\Delta t\times P_{12}(t)-\lambda_2\times t\times\Delta t\times P_2(t) \quad (12)$$

$$P_3(t+\Delta t)=P_3(t)+\lambda_1\times t\times\Delta t\times P_{13}(t)+\lambda_2\times t\times\Delta t\times P_{12}(t)-\lambda_3\times t\times\Delta t\times P_3(t) \quad (13)$$

And then we have:

$$R_P(t)=\left(e^{-\frac{1}{2}\lambda_1 t^2}+e^{-\frac{1}{2}\lambda_2 t^2}+e^{-\frac{1}{2}\lambda_3 t^2}\right)-\left[e^{-\frac{1}{2}(\lambda_1+\lambda_2)t^2}+e^{-\frac{1}{2}(\lambda_1+\lambda_3)t^2}+e^{-\frac{1}{2}(\lambda_2+\lambda_3)t^2}\right]+e^{-\frac{1}{2}(\lambda_1+\lambda_2+\lambda_3)t^2} \quad (14)$$

And at last the $MTTF$ is:

$$MTTF=\int_0^{+\infty}R_P(t)dt=\left(\sqrt{\frac{2\pi}{\lambda_1}}+\sqrt{\frac{2\pi}{\lambda_2}}+\sqrt{\frac{2\pi}{\lambda_3}}\right)-\left(\sqrt{\frac{2\pi}{\lambda_1+\lambda_2}}+\sqrt{\frac{2\pi}{\lambda_1+\lambda_3}}+\sqrt{\frac{2\pi}{\lambda_2+\lambda_3}}\right)+\left(\sqrt{\frac{2\pi}{\lambda_1+\lambda_2+\lambda_3}}\right) \quad (15)$$

Assume with a independent sample with size n, the parameters of failure rates are $\lambda_1=0.01$, $\lambda_2=0.02$ and $\lambda_3=0.03$, now the $MTTF$ is:

$$MTTF=\left(\sqrt{\frac{2\pi}{0.01}}+\sqrt{\frac{2\pi}{0.02}}+\sqrt{\frac{2\pi}{0.03}}\right)-\left(\sqrt{\frac{2\pi}{0.01+0.02}}+\sqrt{\frac{2\pi}{0.01+0.03}}+\sqrt{\frac{2\pi}{0.02+0.03}}\right)+\left(\sqrt{\frac{2\pi}{0.01+0.02+0.03}}\right)=16.52\times\sqrt{\pi}=29.28\ h \quad (16)$$

And the probability that system works after 150 hour is:

$$R_P(150)=\left(e^{-\frac{1}{2}\times 0.01\times 150^2}+e^{-\frac{1}{2}\times 0.02\times 150^2}+e^{-\frac{1}{2}\times 0.03\times 150^2}\right)-\left(e^{-\frac{1}{2}\times(0.01+0.02)\times 150^2}+e^{-\frac{1}{2}\times(0.01+0.03)\times 150^2}+e^{-\frac{1}{2}\times(0.02+0.03)\times 150^2}\right)+e^{-\frac{1}{2}\times(0.01+0.02+0.03)\times 150^2}\approx 0 \quad (16)$$

5. Conclusion

In this paper, we present a real time model to calculate the reliability of systems. This model can be expanded and a $k-out-of-n$ system with n identical and increasable failure rates can be considered. In this system, we must determine a policy to decide when the whole system fails.

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