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Remarks on the Optimal Probing Lot Size for Probing the Semiconductor Wafers

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Abstract—**In this study, we reformulated the problem of wafer probe operation in semiconductor manufacturing to consider a probe machine (PM) which has a discrete shift distribution with a nondecreasing failure rate. To maintain the imperfect PM during the probing of a lot of wafers, a minimal repair policy is introduced with type II inspection error. This paper aims to find an optimal probing lot size that minimizes the expected average processing time per wafer. The adequacy of using a geometric shift distribution is examined when the actual shift distribution has an increasing failure rate.**

Keywords: Lot size; Wafer probing operation

1. Introduction

When on-line process control is impossible, Porteus [3] studied the optimal production lot size when the imperfect process has a geometric shift distribution. Djamaludin et al. [1] extended Porteus's [3] model to consider the situation where outputs are sold under a free warranty repair policy. Later, the work of Djamaludin et al. [1] was further studied by Wang and Sheu [4] under the assumption that the process has a discrete general shift distribution.

Unlike the works described above, where the on-line control is assumed to be infeasible, Lee [2] studied a particular wafer probe problem where the detection and correction of the out of control problems for the probe machine (PM) are feasible during the probing of a batch of wafers. The PM is assumed to have a geometric shift distribution. Once the PM shifts into an out-of control state, wafers would be misprobed. However, the misprobed wafers can be reworked by cleaning the ink and reprobing. Obviously, the number of misprobed wafers will be affected by the lot size. As a result, it is important to determine the optimal probing lot size that minimizes the expected average processing time per wafer. An upper bound and two heuristic solutions for the optimal probing lot size are provided by Lee [2].

This study is attempts to extend Lee's [2] wafer probe problem by further considering the following two features in the developed wafer probe model: (i) It is assumed that the PM possesses a discrete Weibull shift distribution with nondecreasing failure rate. (ii) Minimal repair is used to maintain the PM during probing wafers in a lot.

The rest of this paper is organized as follows. In Section 2, the mathematical model is established. Properties for the optimal probing lot size are explored. In Section 3, numerical examples are used to investigate the adequacy of using the geometric distribution when the PM has an increasing failure rate. Finally, concluding remarks are made.

2. Mathematical model

Consider a lot of size Q to be probed, $1 \leq Q \leq \bar{Q}$, where $\bar{Q}$ is the load for the maximum lot size. Before probing a batch of wafers, a setup time K_b is required for related software installation and inking machine calibration etc., which is independent of Q. The PM is in an in-control state every time after it is setup. Wafers are probed sequentially and the time to probe each wafer is μ_b. Denote $p_j > 0$ as the probability that the PM shifts into an out-of-control state while probing the jth wafer since the last setup given that the PM is in an in-control state immediately before probing the jth wafer. Furthermore, we denote $\bar{P}_j = \sum_{i=j+1}^{\infty} p_i$ as the reliability of a PM while probing the jth wafer. In addition, according to the real operation situation where only a few wafers would be misprobed in a lot of wafers, it is reasonable to assume that $\bar{P}_j$ is large, for $j \leq \bar{Q}$. Hence, it is reasonable to assume that the probability would be small for a wafer in a lot to be misprobed repeatedly. Therefore, it is assumed that there are no misprobed wafers or corrections for the PM while probing the reprobed lots.

Once the PM is out of control, wafers will be misprobed until the problems are detected and corrected. Let $0 \leq \alpha < 1$ be the probability that the out of control problems of the PM are detected given that the PM is in an out-of-control state. Once an out-of-control state is detected, the PM is corrected as an in-control state by minimal repair before probing the next wafer. The ex-

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pected number of corrections for the PM after Q wafers are probed is denoted as $C(Q)$, which excludes the possible correction immediately after probing the Qth wafer. Hence, the expected total time required for corrections for probing a lot of Q wafers is given by $\mu_c C(Q)$, where μ_c is the expected time for correcting the out of control problems each time.

After a lot of Q wafers are finished probing, the whole lot of wafers will move into the final inspection station to identify the misprobed wafers. The expected number of the misprobed wafers is denoted as $R(Q)$. Reworking the misprobed wafers is feasible by cleaning the ink on the misprobed wafers, followed by reprobing. As a result, the expected total processing time for the reprobed lot is given by $K_r(1-\bar{P}_Q)+\mu_r R(Q)$, where K_r is the setup time to prepare for probing a reprobed lot and μ_r is the time required to reprobe a wafer.

Based on the above description, the expected average processing time of a lot with Q wafers is given by

$$\begin{aligned} A(Q) &= \frac{1}{Q}\Big\{K_b + K_r\left(1-\bar{P}_Q\right) \\ &+ \mu_b Q + \mu_r R(Q) + \mu_c C(Q)\Big\}, \end{aligned} \qquad (1)$$

where $R(Q)$ and $C(Q)$ can be easily obtained as follows:

$$\begin{aligned} R(Q) &= Q - \bar{P}_1 - \alpha\sum_{n=2}^{Q}\bar{P}_n/\bar{P}_{n-1} \\ &- (1-\alpha)\sum_{n=2}^{Q}\bar{P}_n\{\sum_{i=1}^{n-2}[\alpha(1-\alpha)^{n-i-2}/\bar{P}_i] \\ &+ (1-\alpha)^{n-2}\}, \quad Q\geq 1, \end{aligned}$$

$$\begin{aligned} C(Q) &= (Q-1)\alpha \\ &- \alpha\sum_{n=2}^{Q}\bar{P}_{n-1}\{\sum_{i=1}^{n-2}[\alpha(1-\alpha)^{n-i-2}/\bar{P}_i] \\ &+ (1-\alpha)^{n-2}\} \quad Q\geq 1. \end{aligned}$$

Our objective here is to find an optimal probing lot size Q^* that minimizes $A(Q)$ given in Eq.(1).

The necessary condition for a local minimum solution to $A(Q)$ is to satisfy $A(Q) < A(Q-1)$ and $A(Q) \leq A(Q+1)$, which are equivalent to

$$G(Q-1) < K_b \quad \text{and} \quad G(Q) \geq K_b,$$

where

$$G(Q) = \begin{cases} K_r\left(\bar{P}_Q + Qp_{Q+1} - 1\right) \\ \quad +\mu_r\left\{Q\left[R(Q+1)-R(Q)\right]-R(Q)\right\} \\ \quad\quad +\mu_c\left\{Q\left[\alpha\left(1-P(Q+1)\right)\right]-C(Q)\right\}, & Q\geq 1, \\ 0, \quad Q=0, \end{cases}$$

where $P(Q) = \bar{P}_{Q-1}\left\{\sum_{i=1}^{Q-2}\left[\alpha(1-\alpha)^{Q-i-2}\Big/\bar{P}_i\right] + (1-\alpha)^{Q-2}\right\}$, $Q\geq 2$ and $P(1)=1$.

For $Q\leq\bar{Q}$, we have

$$\begin{aligned} &G(Q+1)-G(Q) \\ &= (Q+1)\Big\{K_r\left(p_{Q+2}-p_{Q+1}\right) \\ &+\mu_r\left[P(Q+2)-P(Q+3)\right]+\mu_c\alpha\left[P(Q+1)-P(Q+2)\right]\Big\}. \end{aligned}$$

From the last equation, a sufficient condition for $G(Q)$ is increasing in $Q\leq\bar{Q}$ which is given by

$$\mu_r P(Q+1) + \alpha\mu_c P(Q) - K_r p_Q \qquad (2)$$

is decreasing in $Q\leq\bar{Q}$.

3. Numerical examples

In this section, six different types of wafer products, designated A to F as shown in Table 1 as described in Lee [2], are used to evaluate the effects of minimal repair for the imperfect PM on both the optimal probing lot size (Q^*) and its corresponding expected average processing time ($A(Q^*)$). The reliability of the PM that the first j wafers are probed in an in-control state is given by $\bar{P}_j = (1-p)^{j^\delta}$, where p is dependent on different types of wafer products produced (e.g., p=0.8 for product B), and δ is increased from 1 to 1.9 to evaluate the change of Q^* and $A(Q^*)$. The %Penalty $= 100\%[A(Q^*(\delta=1)) - A(Q^*(\delta>1))]/A(Q^*(\delta>1))$ is used to evaluate the adequacy of the assumption that the PM follows a geometric shift distribution when the actual shift distribution is a discrete Weibull with an increasing failure rate. For the same product, we observe that Lee's [2] optimal lot size (processing time) often overestimates (underestimates) the real optimum. The geometric solution may not be a good approximative solution when a PM actually has a Weibull shift distribution with increasing failure rate, especially when δ is large. For example, we observe that product E has %Penalty=145.02% at $\delta=1.7$.

4. Concluding remarks

In this paper, a lot-size model has been developed for the wafer probe operation in the manufacturing of semiconductors, where the PM is assumed to have a discrete shift distribution with non-decreasing failure rate. Also, a minimal repair policy is introduced to maintain the imperfect PM; however, a type II inspection error exists for this model. A condition is explored for the uniqueness of the optimal probing lot size. The adequacy of using the geometric distribution when the actual shift distribution is discrete Weibull with an increasing failure rate is also examined. The geometric solution may not be a good approximative solution when the PM has worse reliability.

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